DISCUSSION

Critical Evaluation of Equivalent Moment Factor Procedures for Laterally Unsupported Beams

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First Quarter, 2010

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The authors of the paper are to be commended for studying various issues related to the use of moment gradient coefficients for estimating lateral-torsional buckling strength. By comparing approaches used in different specifications (historically and in other countries' codes) and by quantifying deficiencies in empirical formulations, they have provided an opportunity for practicing engineers to understand the limitations of the 2005 AISC *Specification* (AISC, 2005). An important conclusion of the paper is that empirical relationships for estimating moment gradient coefficients can be unconservative in some cases and may be too aggressive for design purposes.

Clark and Hill (1960) elegantly presented the use of coefficients to extend the uniform moment case. This extension had a theoretical basis corresponding to the minimum potential energy solution for the buckling moment. It also included tabular data for common cases, which have been widely published. The formulation was completely general in that it provided a means of calculating coefficients without placing any assumptions on the beam twist and lateral displacement. However, direct evaluation of the required definite integrals is practically limited to cases that are closely approximated by a single half-sine or half-cosine curve (corresponding to either fixed or free torsional-rotation boundary conditions). This approach is adequate for the majority of cases in common use, but it is not necessarily accurate in cases that are largely anti-symmetric. Historically, the specifications have relied on empirical relationships for estimating the moment gradient coefficient C_b in order to bypass the need for evaluation of definite integrals.

Wilkerson (2005) presented a family of approximate relationships with varying accuracy that have estimable error and are therefore suitable for general use. The basis for the method was a form of quadrature, or numerical integration, which was tailored for the application by choosing weighting constants so that the error was exactly zero if the integrand was piecewise-equal to the function in the assumed quadrature rule (either the trapezoidal rule or Simpson's rule). It was shown in that paper that two approximate relationships would be sufficient for the estimation of C_b for cases of (1) linearly (or nearly linearly) varying loads and (2) concentrated loads. The latter expression recognized nonsmoothness in the moment diagram and the potential for large errors when using only moment data, and thus included the use of shear data to increase the accuracy. Abrupt discontinuities in the moment diagram were not addressed (no current method other than finite element modeling is able to give accurate results in these cases).

Both expressions were derived as "quarter-point" methods to simplify the procedure, but the derivations were general enough to be used for the development of more refined models. It was determined by studying the error in estimation of the definite integrals that the use of higher order derivative information (i.e., shears) had a greater effect on reducing the error than could be realized by adding additional moment data. Using the terminology of the original paper and the 2005 AISC *Specification*, the recommended expressions were as shown here:

For cases with linearly varying loads only:

$$C_b = \frac{M_{max}}{\sqrt{\dfrac{M_A^2}{4} + \dfrac{M_B^2}{2} + \dfrac{M_C^2}{4}}} \le 2.6$$

For cases with concentrated loads:

$$C_b = \frac{M_{max}}{\sqrt{\left(\dfrac{1}{4} + \dfrac{\pi}{48}\right)M_A^2 + \dfrac{M_B^2}{2} + \left(\dfrac{1}{4} - \dfrac{\pi}{48}\right)M_C^2 + \dfrac{L_b}{12}\left(\dfrac{M_A V_A}{4} + \dfrac{M_B V_B}{2} + \dfrac{M_C V_C}{4}\right)}} \le 2.6$$

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where V_A, V_B, and V_C were shears corresponding to the quarter-point moments, and the sign convention for shears and moments followed a "strength of materials" approach consistent with the relationship $M(z) = \int_0^z V(x)dx$. It was demonstrated by comparison with 10 cases from Clark and Hill that the use of an upper bound on C_b of 2.6 was sufficient to limit the error to no more than 10% for various linear and nonlinear moment diagrams.

Additionally, Wilkerson (2005) made recommendations on the use of a second coefficient to include the effects of load position when not located at the shear center of the beam. It was demonstrated through the use of finite element modeling that the recommendations in the 2005 AISC *Specification* for the consideration of an unbraced beam loaded at the top flange were sometimes unconservative. Moment strength was overestimated by as much as 43% in cases reviewed. The paper gave approximate expressions for evaluation of the load position coefficient, C_a, in cases with linearly varying and concentrated loads. The expressions for this coefficient were derived similarly to those for C_b by approximating the definite integrals developed by Clark and Hill. The incorporation of load position into the evaluation of moment strength requires an extension to the usual equation for critical moment. A preliminary section in Wilkerson (2005) included a derivation of this expression based on the differential equations for lateral-torsional buckling of a transversely loaded beam.

It is the writer's hope that this discussion will further encourage adoption of a more rigorous approach to the estimation of coefficients that extend the uniform moment strength solution to the more general cases needed for everyday design. Because approximate relationships are clearly necessary to allow for a practical implementation of this extension, the writer advocates the use of methods that have a basis in theory with reliable error estimates, such as those derived in Wilkerson (2005).

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