CLOSURE

Critical Evaluation of Equivalent Moment Factor Procedures for Laterally Unsupported Beams

Paper by EDGAR WONG and ROBERT G. DRIVER
First Quarter, 2010

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The authors thank the discusser for his insightful comments and the favorable review of their critical approach that exposed the shortcomings of several methods for determining equivalent moment factors for laterally unsupported beams. We also agree that although accurate methods can be developed based on fundamental theory, simple conservative methods are more appropriate for design applications. This is true not only for the savings in computational effort, but also as recognition that not all basic assumptions inherent in an "exact" method will be met in the context of a real structure.

We regret that we were unaware of the discusser's paper published in the proceedings of the Structural Dynamics and Materials Conference (Wilkerson, 2005), as it is indeed a relevant contribution to the literature on this subject. It provides a lucid derivation of the generalized critical moment equation presented by Clark and Hill (1960), as well as approximate methods for determining equivalent moment factors based on theoretical formulations. The discusser also shows in his paper how results for beams with concentrated loads can be improved by the inclusion of quarter-point shear values in the equation. An important contribution of the discusser's paper is the clear demonstration of the reason that implementing the square-root format in the quarter-point moment method is advantageous.

The equivalent moment factor equation presented by Wong and Driver (2010) for use with any moment distribution and loading case:

$$C_b = \frac{4M_{\max}}{\sqrt{M_{\max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}} \leq 2.5 \quad (9)$$

is similar in format to the one presented by Wilkerson for use with distributed loads, including the recommended maximum value. His equation for use with beams with concentrated loads is slightly more complex, because it requires the evaluation of shears at the beam quarter-points as well. Due to the singularity in the shear function at concentrated loads, when a load falls at one of the quarter-points, the value of C_b is undefined. This results in a discrete jump in the value of C_b when comparing a case where the point load is immediately to the left of the quarter-point with one where the load is immediately to the right, which is contrary to what the pure theory would suggest. We realize that this shortcoming can be mitigated by choosing additional moment and shear sampling points to reduce the magnitudes of these jumps (although they could not be eliminated), but it is at the cost of increased complexity of the resulting equation—and this may render it unsuitable for design use.

A caution is offered about the contention that the error in C_b can be calculated using the method described in the discussion, in that this assumes that without discretization of the sampling points along the length, the method is exact. The assumption that the angle of twist can be represented as a half-sine-wave, although reasonable, tends toward nonconservative estimates of C_b. The inevitable geometric imperfections in real structures cannot be included in an exact solution because they can only be quantified on a statistical basis. Also, it must be kept in mind that it is not only the variations of the internal force effects (moment and shear) along the length of the unbraced segment of the beam that affect the "true" value of C_b, but also the torsional properties of the cross-section, despite the fact that these effects are most often neglected.

Because the comparisons of C_b values presented by Wilkerson (2005) using the loading cases that were discussed by Clark and Hill (1960) cover a very limited scope of possibilities, it is instructive to insert the curves derived from the equations given in the discussion into the comparison graphs presented in the original paper (Wong and Driver, 2010). Three such graphs are presented in Figures 17 through 19.

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Figure 17 shows that for moment type 1 (Figure 2 in the original paper), the Wilkerson equation tends toward an upper bound to the numerical data presented, so the equation—although likely accurate for certain torsional stiffness properties—does not lend itself to general design use without modifications to account for empirical observations. The same is observed for moment types 2 and 3 (Figures 3 and 4), although the upper bound of 2.6 on the value of C_b diminishes this concern considerably. Figure 18 shows that for moment type 4 (Figure 5 in the original paper), the outcome of the C_b equation recommended for beams with concentrated loads is highly dependent upon whether the midspan shear is taken to the left or to the right of the load point (because the discusser did not indicate which shear value should be applied, both scenarios are presented and compared). The C_b equation recommended for use with distributed loads, which is independent of shear, falls between these two extremes. It should be noted that two of the three cases effectively address the nonconservative outcome arising from the 2005 AISC *Specification* for fixed-ended beams. Figure 19 shows that for moment type 6 (Figure 7 in the original paper), which is a case that accounts for the application of a concentrated load at any point along the beam, the prediction of C_b using the method described in the discussion includes discrete jumps at the quarter-points and an asymmetry about the beam centerline that do not reflect expectations derived from theory. These irregularities are not seen in the other methods presented by Wong and Driver (2010) or with the Wilkerson equation that was recommended for use with distributed loads.

Since the time the paper by Wong and Driver (2010) was accepted for publication in the AISC *Engineering Journal*, Equation 9 was adopted by Standard S16 of the Canadian Standards Association (CSA, 2009) for general use in the design of laterally unsupported steel beams.

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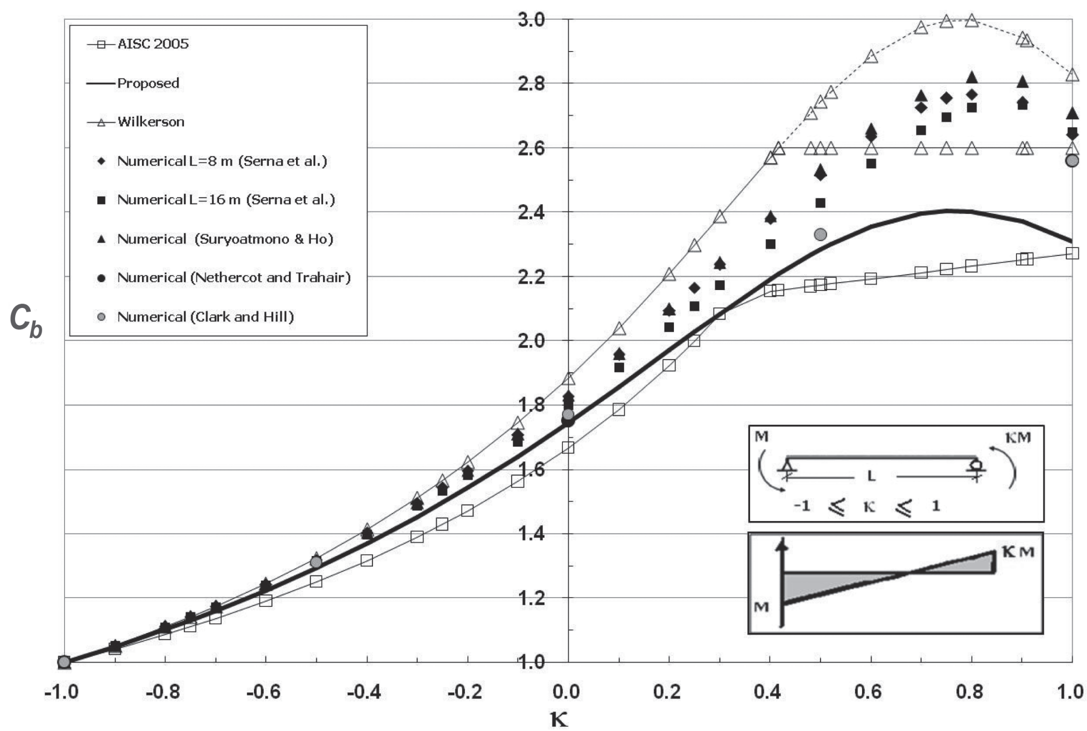

Fig. 17. C_b *results for moment type 1.*

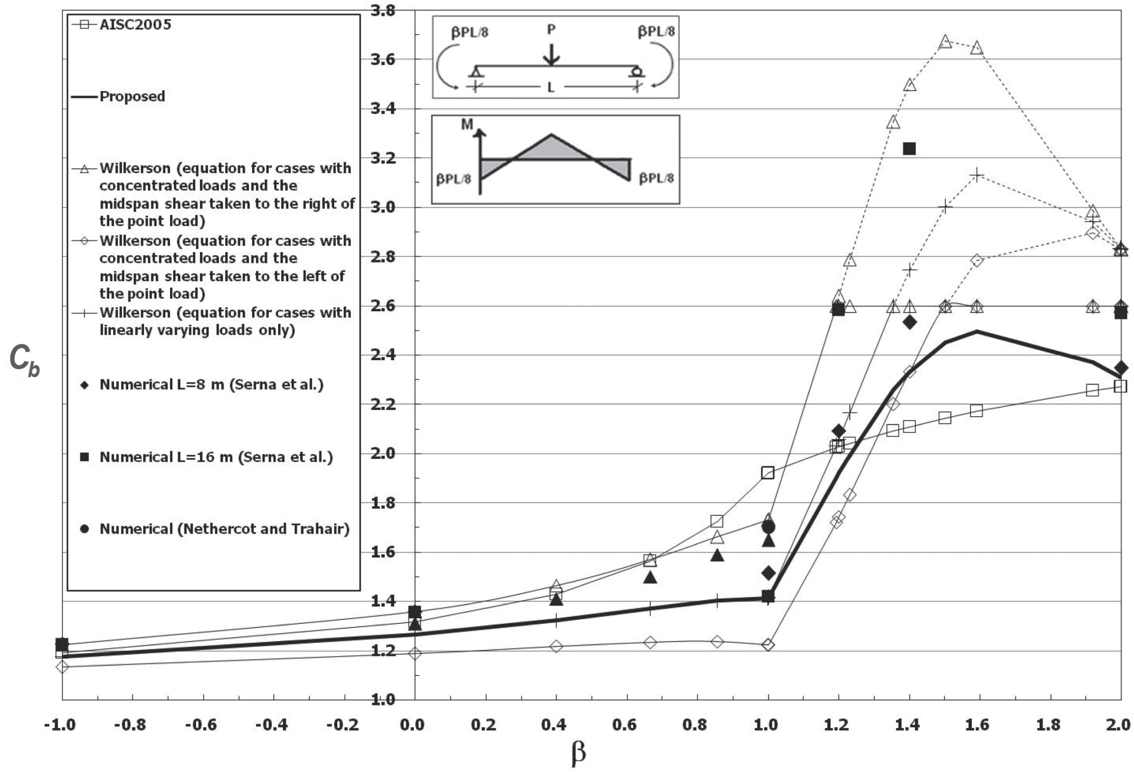

Fig. 18. C_b *results for moment type 4.*

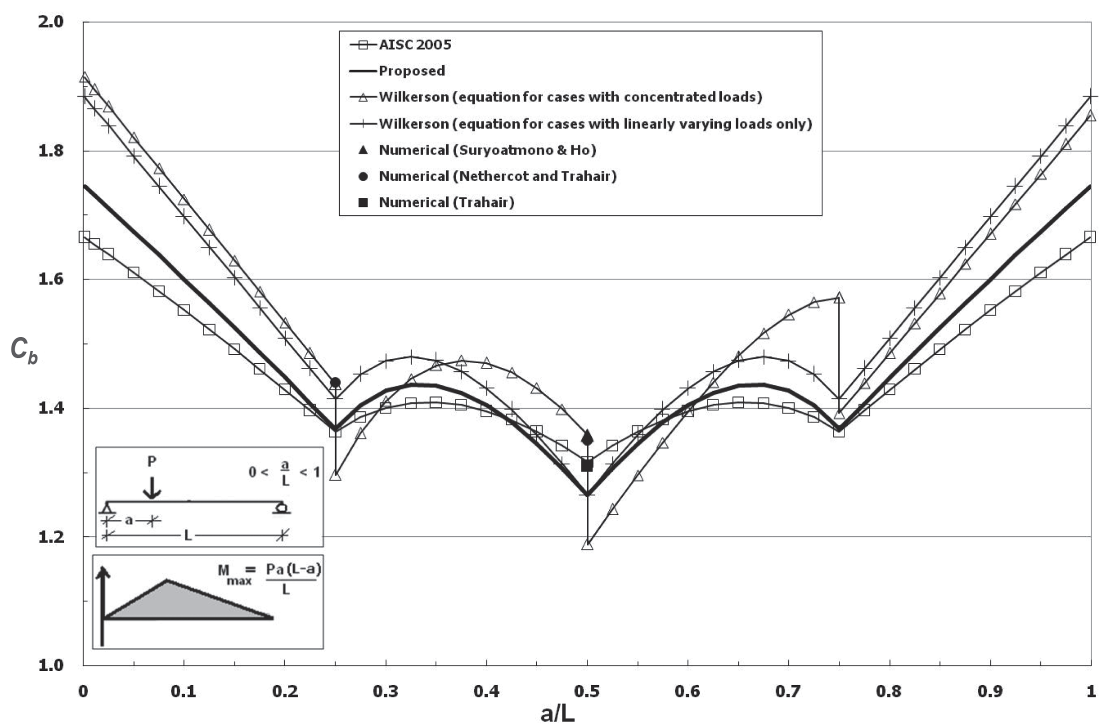

Fig. 19. C_b *results for moment type 6.*