SPECIAL FOCUS: **NON-BUILDING STRUCTURES**

Bending of Top Plates in Base Chair Connections

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ABSTRACT

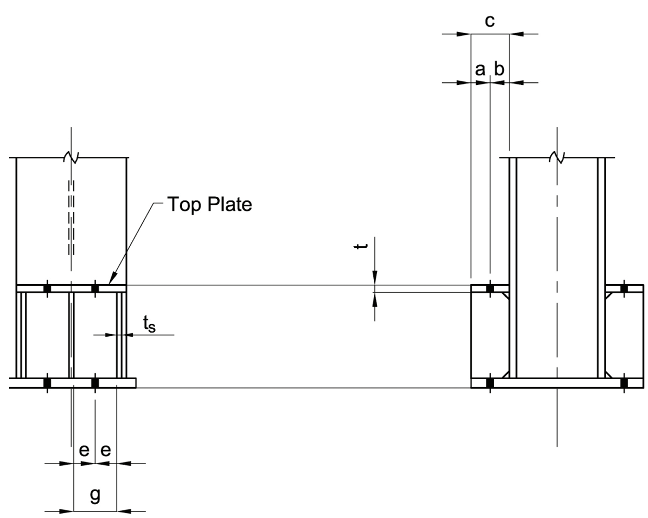
In many heavy industrial facilities, column bases must transfer large uplift loads to the foundation, and if the tension load is too large to carry with a standard base plate, chairs are used to transfer the load. The top plate thickness has traditionally been determined using either the elastic one-way bending capacity of a beam spanning the distance between the vertical stiffener plates or the two-way bending capacity based on elasticity theory. These design methods can be used where the nominal stress must be limited to the elastic range, such as structures subject to fatigue. For static loads, a more realistic design model can be achieved by considering the plastic capacity of the plate in two-way bending. In this paper, the author develops a method to calculate the ultimate bending capacity of top plates in base chair connections based on yield line theory. The limit-analysis solution does not rely on the conservative assumptions inherent in calculation methods commonly used in design practice. The proposed equation can be used to calculate the capacity of many different base chair configurations, including those commonly used on plate and shell structures.

Keywords: base chairs, base plates, uplift, yield line theory

In many heavy industrial facilities, column bases must transfer large uplift loads to the foundation. If the tension load is too large to carry with a standard base plate, chairs can be used to transfer the load. A base plate connection with a chair welded to each flange is shown in Figure 1.

Frequently, a vertical bracing gusset plate is welded to the column flange at the location of a base chair. At these locations, the top plate can be split as shown in Figure 2a or the chair height can be adjusted to accommodate the gusset plate height as shown in Figure 2b. For some projects, the second option is not viable, because the chair height must be

(a) Base chair for heavy industrial facility



(b) Base chair arrangement and dimensions

Fig. 1. Column base plate connection with chair.

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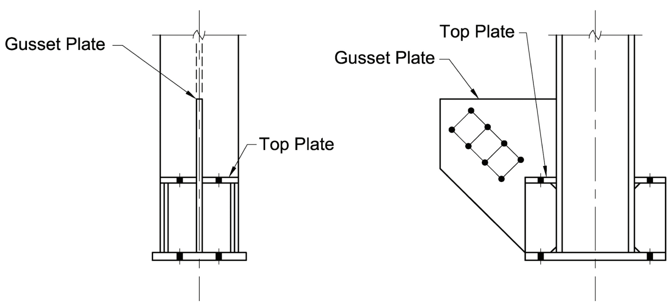
determined early in the design process to allow fabrication of the anchor rods, and the height of the gusset plate is usually not determined until much later in the design process.

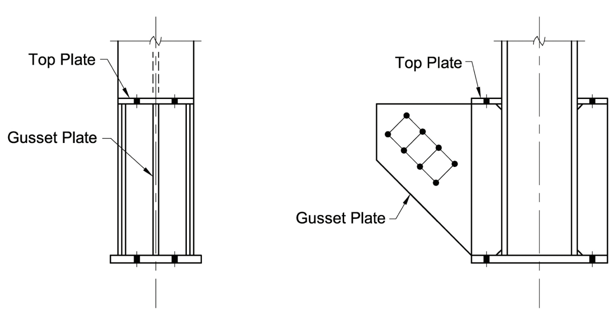
In some cases, the gusset plate is notched to clear the base chair, but this practice should be avoided if possible, because it causes a discontinuity in the gusset plate, disrupting the flow of forces from the gusset plate into the anchor rods. This condition is usually caused by a standard base chair detail on the contract drawings that does not account for vertical brace locations.

Base chairs are also used at the bottom of plate and shell structures such as tanks, silos and stacks, as shown in Figure 3. For these structures, the top plate can be a continuous ring or separate plates at each anchor rod.

The top plate thickness has traditionally been determined using the elastic one-way bending capacity of a beam spanning the distance between the vertical stiffener plates (AISI, 1992; Bednar, 1986; Mahajan, 1975) or the two-way bending capacity based on elasticity theory (Troitsky, 1982). These design methods can be used where the nominal stress must be limited to the elastic range, such as structures subject to fatigue. However, for static loads, a more realistic design model can be achieved by considering the plastic capacity of the plate in two-way bending. In this paper, the yield line method will be used to derive an equation to determine the ultimate bending capacity of continuous and noncontinuous top plates.



(a) Split top plate



(b) Increased chair height

Fig. 2. Base chair configurations with a gusset plate at the column flange.

THE YIELD LINE METHOD

The yield line method was developed by Hognestad (1953) and Johansen (1962) to determine the ultimate capacity of concrete slabs. The shape of the failure pattern must be known in order to calculate the collapse load. Many patterns may be valid for a particular joint configuration. Because the solution is upper-bound, the pattern that gives the lowest load will provide results closest to the true failure load. Therefore, selection of the proper yield line pattern is important because an incorrect failure pattern will produce unsafe results.

The collapse load is calculated using the principle of virtual work, assuming a plastic mechanism forms along each line of the assumed failure pattern. To maintain equilibrium, the external work done by the load moving through the virtual displacement, δ, must equal the strain energy due to the plastic moment rotating through virtual rotations, θ_i. The virtual rotations are assumed small, so $\theta_i = \tan(\theta_i) = \sin(\theta_i)$. The influence of strain-hardening and membrane effects are not accounted for in yield line analysis; therefore, there is potentially a large reserve capacity beyond the calculated collapse load.

The general procedure for deriving a yield line solution is

1. Select a valid yield line pattern.
2. Determine the equation that describes the external work done by the load moving through the virtual displacement,

$$W_E = P\delta \tag{1}$$

where
P = load, kips (N)
δ = virtual displacement, in. (mm)

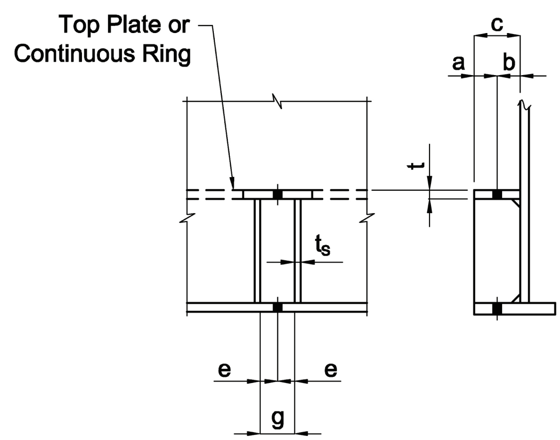


Fig. 3. Base chair configuration for plate and shell structures.

3. Determine the equation that describes the internal work done by the rotations along the yield lines,

$$W_I = \sum M_{pi}\theta_i \tag{2}$$

where
M_{pi} = plastic moment capacity of yield line i, kip-in. (N-mm)
= $m_p L_i$
θ_i = virtual rotation of yield line i
mp = plastic moment capacity per unit length of the fitting, kip-in./in. (N-mm/mm)
L_i = length of yield line i, in. (mm)

4. Set the external work equal to the internal work and solve for the load.

Kennedy and Goodchild (2003) present practical information for design of concrete slabs using the yield line method. The yield line method has also been used to determine the local capacity of steel connections. The yield line pattern derived by Kapp (1974) can be used to calculate the local capacity of wide flange webs subjected to transverse loads. Dranger (1977) developed yield line equations for bolted flange connections with tension loads. Wardenier (1982) used the yield line method to derive equations for the local wall bending capacity of HSS truss joints. Zoetemeijer (1974), Packer and Morris (1977), Mann and Morris (1979) and Zoetemeijer (1981) used the yield line method to derive equations to calculate the local bending capacity of end plates and column flanges in end plate moment connections.

YIELD LINE SOLUTION FOR TOP PLATE

To determine the capacity of a top plate in bending, the yield line pattern in Figure 4 will be used. All points on the edge of the hole and along yield line 2 have been displaced vertically by an amount δ.

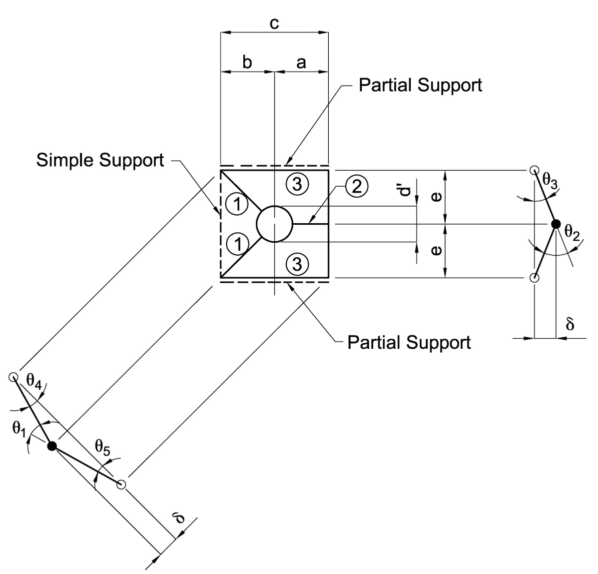


Fig. 4. Yield line model for top plate bending.

Determine the length of the yield lines:

$$L_1 = \sqrt{b^2 + e^2} - d'/2 \tag{3}$$

$$L_2 = a - d'/2 \tag{4}$$

$$L_3 = c \tag{5}$$

where
a = distance from the center of the hole to the edge of the top plate, in. (mm)
b = distance from the center of the hole to the face of the support, in. (mm)
$c = a + b$, in. (mm)
d' = hole diameter, in. (mm)
e = distance from the center of the hole to the inside edge of the vertical stiffener plate, in. (mm)

Determine the rotation angles of the yield lines:

$$\theta_3 = \frac{\delta}{e} \tag{6}$$

$$\begin{aligned}\theta_2 &= 2\theta_3 \\ &= 2\,\delta/e\end{aligned} \tag{7}$$

$$\theta_4 = \frac{\delta}{(b/e)L_1} \tag{8}$$

$$\theta_5 = \frac{\delta}{(e/b)L_1} \tag{9}$$

$$\begin{aligned}\theta_1 &= \theta_4 + \theta_5 \\ &= \left(\frac{e}{b} + \frac{b}{e}\right)\frac{\delta}{L_1}\end{aligned} \tag{10}$$

The plastic moment capacity per unit length along yield lines 1 and 2 (see Figure 4) is

$$m_p = \frac{F_y t^2}{4} \tag{11}$$

where
F_y = specified minimum yield strength of the top plate, ksi (MPa)
t = thickness of the top plate, in. (mm)

Because the fixity at yield lines 3 can vary between pinned and rigid, the plastic moment capacity per unit length can be expressed as

$$m'_p = \alpha \frac{F_y t^2}{4} \tag{12}$$

where

α = reduction factor to account for the effect of partial fixity of the outer yield lines (discussed in detail in the next section)

The internal work is

$$\begin{aligned} W_I &= \sum L_i \theta_i m_{pi} \\ &= 2L_1\theta_1 m_p + L_2\theta_2 m_p + 2L_3\theta_3 m'_p \\ &= 2L_1\left(\frac{e}{b}+\frac{b}{e}\right)\frac{\delta}{L_1} m_p + \left(a - d'/2\right)\left(2\delta/e\right) m_p + \\ &\quad 2c\left(\delta/e\right) m'_p \end{aligned} \tag{13}$$

The external work is

$$W_E = T\delta \tag{14}$$

where

T = tension in the anchor rod, kips (N)

Substitute Equations 11 and 12 into Equation 13, set internal work equal to external work, and solve for T to get Equation 15:

$$T = \frac{F_y t^2}{2}\left[\frac{e}{b} + \left(1+\alpha\right)\frac{c}{e} - \frac{d'}{2e}\right] \tag{15}$$

PARTIAL FIXITY AT OUTER YIELD LINES

Equation 12 contains a factor, α, to account for the effect of partial fixity of the outer yield lines (yield lines 3), where $0 \le \alpha \le 1$. For top plates that are continuous over the stiffeners, the outer yield lines can be assumed fully fixed. For discontinuous top plates, the design can be based on the conservative assumption that the outer yield lines are simply supported. For these cases, the values for α are

α = 1 for continuous top plates fixed against rotation at both outer yield lines

= 0 for top plates that are free to rotate at both outer yield lines

If flexural continuity is provided between the top plate and the vertical side plates, the bending capacity of the vertical plates can be used to provide partial fixity to the outer yield lines on the top plate. In the presence of axial loading, Neal (1961) showed that the plastic capacity of a member with rectangular cross section is reduced according to Equation 16, which gives the reduced moment capacity per inch of the vertical side plate:

$$m'_{ps} = m_{ps}\left[1 - \left(\frac{P}{P_y}\right)^2\right] \tag{16}$$

where

m_{ps} = full plastic moment capacity per inch of the vertical side plate, kip-in./in. (N-mm/mm)

= $F_{ys}t_s^2/4$

P = compression load in the vertical side plate, kips (N)

P_y = yield load of the vertical side plate, kips (N)

= $F_{ys}b_s t_s$s

F_{ys} = specified minimum yield strength of the vertical side plates, ksi (MPa)

b_s = width of the vertical side plates, in. (mm)

t_s = thickness of the vertical side plates, in. (mm)

If the vertical side plate has the same width as the top plate, the fixity factor for the outer yield lines can be calculated using Equation 17:

$$\alpha = \frac{F_{ys}}{F_y}\left(\frac{t_s}{t}\right)^2\left[1 - \left(\frac{P}{P_y}\right)^2\right] \tag{17}$$

VERIFICATION OF PROPER YIELD LINE PATTERN

Because the yield line solution is upper-bound, the pattern that gives the lowest load will provide results closest to the true failure load. To verify that the pattern in Figure 4 is correct, the collapse load will be compared to that from other admissible patterns. To simplify the comparisons, the diameter of the hole will be neglected.

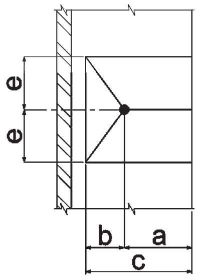
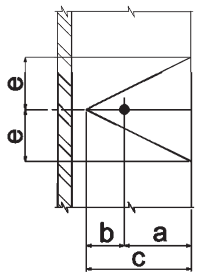
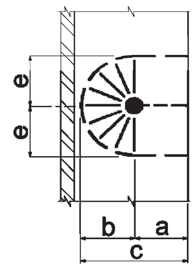
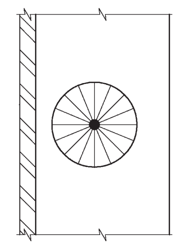
The number of possible yield line patterns for any joint is infinite, but there are at least four admissible patterns that lead to practical design equations for top plate connections. These are labeled 1 through 4 in Table 1, where pattern 1 is the basis for the derivation of Equation 15. The normalized load is calculated for each pattern as

$$\frac{2P}{F_y t^2} \tag{18}$$

The normalized load for each pattern is compared to that of pattern 1 to determine the conditions that will cause one of the other three patterns to yield a lower capacity than that predicted with Equation 15 (i.e., $P_{pn} < P_{p1}$). The results are shown for $\alpha = 0$ in the top part of the table and $\alpha = 1$ in the bottom part.

A comparison of the normalized load equations of pattern 2 to those of pattern 1 proves that pattern 2 can never control the design. For the case of $\alpha = 0$, the conditions for $P_{pn} < P_{p1}$

Table 1. Comparison of Admissible Yield Line Patterns

Pattern Geometry				
Pattern No.	**1**	**2**	**3**	**4**
$\alpha = 0$				
$\left(\frac{2P}{F_y t^2}\right)_{\alpha=0}$	$\frac{e}{b}+\frac{c}{e}$	$\frac{e}{b}+\frac{4c^2}{eb}$	$\pi-1+\frac{c}{e}$	2π
Condition for $P_{pn} < P_{p1}$	—	NC	$\frac{e}{b} > \pi-1$	$\frac{e}{b}+\frac{c}{e} > 2\pi$
$\alpha = 1$				
$\left(\frac{2P}{F_y t^2}\right)_{\alpha=1}$	$\frac{e}{b}+\frac{2c}{e}$	$\frac{e}{b}+\frac{4c^2}{eb}$	$\pi-2+\frac{2c}{e}$	2π
Condition for $P_{pn} < P_{p1}$	—	NC	$\frac{e}{b} > \pi-2$	$\frac{e}{b}+\frac{2c}{e} > 2\pi$

NC = Never controls

for patterns 3 and 4 are beyond the range of geometry typical of base chair connections; therefore, they can be neglected. For the case of $\alpha = 1$, the conditions for $P_{pn} < P_{p1}$ for patterns 3 and 4 are more common, and these limits should be recognized by the designer.

PROPOSED DESIGN METHOD

It is proposed that Equation 15 be used for design of top plates in base chair connections. To account for the upper-bound nature of the solution and the corner effect, Wood (1961) recommended an additional design margin of 15% for yield line solutions. Kennedy and Goodchild (2003) recommended an additional margin of 10%. Based on these values, it is recommended that $\phi = 0.80$ be used as the reduction factor for load and resistance factor design (LRFD) in lieu of the traditional value of 0.90 for members subject to bending. For allowable stress design (ASD), $\Omega = 1.88$ is recommended.

If the top plate is split around a gusset plate as shown in Figure 2a, free rotation should be assumed at the boundary welded to the gusset plate, unless the weld is strong enough to develop the plastic bending capacity of the top plate. For top plates with different boundary conditions at each of the outer yield lines, the average of the two independent values of α can be used.

If $\alpha = 1$ is used, the following geometric constraints should be met in order to ensure that Equation 15 is valid:

$$\frac{e}{b} \le \pi - 2 \tag{19}$$

$$\frac{e}{b} + \frac{2c}{e} \le 2\pi \tag{20}$$

EXAMPLE

For the column base chair connection in Figure 5, determine the top plate thickness required to carry the factored uplift force of 250 kips. The plate material is ASTM A36 and the anchor rod holes have a diameter of 2c in.

Solution A: Assume simple supports at the outer yield lines

The plate is continuous over the middle stiffener; therefore, rotation is restrained at that location. For simplicity, the yield line pattern will be assumed free to rotate at the outer stiffeners. Therefore, yield lines 3 will be assumed fixed at one side of the pattern and free at the other side, and $\alpha = (1 + 0)/2 = 0.5$.

$a = 3$ in.
$b = 4$ in.
$c = 7$ in.
$d' = 2.31$ in.
$e = 2.75$ in.
$F_y = 36$ ksi

Try a 1.25-in.-thick plate.

$t = 1.25$ in.

$$T_u = \frac{250 \text{ kips}}{4 \text{ anchor rods}}$$
$$= 62.5 \text{ kips per anchor rod}$$

$$T_n = \frac{(36 \text{ ksi})(1.25 \text{ in.})^2}{2}\left[\frac{2.75 \text{ in.}}{4.00 \text{ in.}} + 0.5)\left(\frac{7.00 \text{ in.}}{2.75 \text{ in.}}\right) - \frac{(2.31 \text{ in.})}{(2)(2.75 \text{ in.})}\right]$$
$$= 115 \text{ kips}$$

$$\phi T_n = (0.80)(115 \text{ kips})$$
$$= 92.0 \text{ kips} > 62.5 \quad \textbf{o.k.}$$

Solution B: Assume partial fixity at the outer yield lines

Try a 1-in.-thick plate.

$t = 1$ in.
$F_{ys} = 36$ ksi
$b_s = 7$ in.
$t_s = w$ in.
$P = 250$ kips/8 = 31.3 kips
$P_y = (36 \text{ ksi})(7.00 \text{ in.})(0.750 \text{ in.}) = 189$ kips

The fixity factor for the outer yield line, α_o, is

$$\alpha_o = \frac{36.0 \text{ ksi}}{36.0 \text{ ksi}}\left(\frac{0.750 \text{ in.}}{1.00 \text{ in.}}\right)^2\left[1 - \left(\frac{31.3 \text{ kips}}{189 \text{ kips}}\right)^2\right]$$
$$= 0.547$$

The plate is continuous over the middle stiffener; therefore, rotation is restrained at that location, and $\alpha = (1 + 0.547)/2 = 0.774$.

$$T_n = \frac{(36 \text{ ksi})(1.00 \text{ in.})^2}{2}\left[\frac{2.75 \text{ in.}}{4.00 \text{ in.}} + (1 + 0.774)\left(\frac{7.00 \text{ in.}}{2.75 \text{ in.}}\right) - \frac{(2.31 \text{ in.})}{(2)(2.75 \text{ in.})}\right]$$
$$= 86.1 \text{ kips}$$

Using the AISI (1992) method, a required plate thickness of 1.35 in. is required. If the plate is modeled as a simple beam with a net width of 7 in. − 2.31 in. = 4.69 in., the required

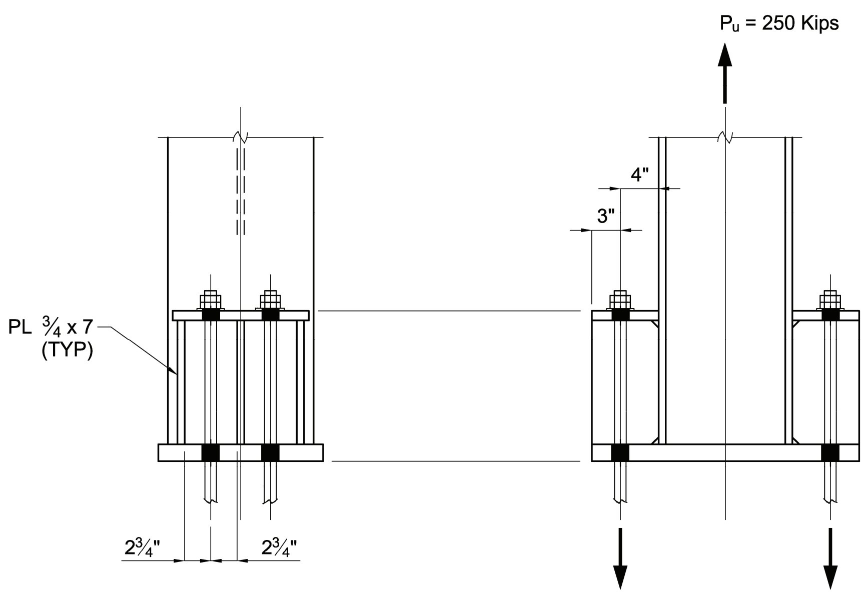


Fig. 5. Example: column with large uplift.

thickness is 1.50 in. The use of Equation 15 clearly provides a more economical solution for top plates compared to the traditional methods of analysis.

CONCLUSIONS

A method has been developed to calculate the ultimate bending capacity of top plates in base chair connections. The proposed design procedure is based on yield line theory and provides a limit-analysis solution that does not rely on the conservative assumptions inherent in the calculation methods commonly used in design practice. The proposed equation can be used to calculate the capacity of many different base chair configurations, including those commonly used on plate and shell structures.

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