A Strength Design Approach to Ponding

EDWARD SILVER

ABSTRACT

This article proposes a strength design approach to ponding resistance. It derives methods to include consideration of the impact of camber, nonuniform loading, and end fixity. The intent is to expand on the existing American Institute of Steel Construction (AISC) provisions so they may be adapted to unique framing and loading conditions.

Keywords: ponding, roof loads.

This article proposes a strength design approach to ponding resistance. The proposed procedures are constructed from the fundamentals of ponding analysis. From these fundamentals, we derive equations that help predict the behavior of beams subjected to ponding loads, then develop simplified procedure where possible.

Because the ponding demand is proportional to the ponding load—which is proportional to the total deflection—the following issues are discussed in this article:

- Beam deflection with ponding.
- Effect of camber and end fixity.
- Ponding load demand.
- Ponding moment demand.
- Beam and girder systems.

BEAM DEFLECTION WITH PONDING

The vertical load on a beam causes the beam to deflect. Water accumulates in the beam's deflected shape, inducing an additional ponding load, which induces additional deflection, which induces more water accumulation. This relationship between loading and deflection is the basis for calculating the ponded beam's deflection.

We begin by analyzing a uniformly loaded simple-span beam to determine the total deflection, including ponding effects.

Uniformly Loaded Beam

The total load along the span equals the initial, uniform load plus the ponded fluid load filling the final deflected shape of the beam:

Edward Silver, S.E., Principal, Silver and Associates, Inc., 7543 Woodley Ave., Suite 201, Van Nuys, CA, 91406. E-mail: edward.silver@esala.com

Loading = Initial load + Ponded water

$$w(x) = w_i + \gamma S[y(x)] \tag{1}$$

where

w_i	=	initial uniform loading
γ	=	density of ponded liquid (62.4 lb/ft^3 for water)
S	=	beam spacing
$y(x)$	=	total deflection along span, including ponding

From beam theory, the loading is proportional to the fourth derivative of the deflection:

$$w(x) = EI\,\frac{d^4 y}{dx^4} \equiv EI\, y^{iv}(x) \tag{2}$$

Combining Equation 2 into Equation 1 gives a fourth-order linear differential equation:

$$EI\, y^{iv}(x) - \gamma S\, y(x) = w_i \tag{3}$$

The general solution to this differential equation is

$$\begin{aligned} y(x) &= c_1 \cos\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right) + c_2 \sin\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right) \\ &\quad + c_3 \cosh\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right) + c_4 \sinh\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right) - \frac{w_i}{\gamma S} \end{aligned} \tag{4}$$

This provides the deflected shape of any prismatic, uniformly loaded beam, regardless of its end conditions.

To determine the undefined constants, c_1, c_2, c_3 and c_4, we apply the end conditions—boundary conditions—for a simply supported beam:

$$y(0) = y(L) = M(0) = M(L) = 0$$

From beam theory, $M = EI y''$. So, the boundary conditions are:

$$y(0) = y(L) = y''(0) = y''(L) = 0$$

Applying these values gives:

$$y''(0) = \sqrt{\frac{\gamma S}{EI}}\left[c_1 \cancelto{1}{\cos(0)} + c_2 \cancelto{0}{\sin(0)} - c_3 \cancelto{1}{\cos(0)} - c_4 \cancelto{0}{\sinh(0)}\right]$$

$$= \sqrt{\frac{\gamma S}{EI}}\left[c_1 - c_3\right] = 0 \qquad \text{so,} \qquad c_3 = c_1$$

$$y(0) = c_1 \cancelto{1}{\cos(0)} + c_2 \cancelto{0}{\sin(0)} + c_1 \cancelto{1}{\cosh(0)} + c_4 \cancelto{0}{\sinh(0)} - \frac{w_i}{\gamma S}$$

$$= 2c_1 - \frac{w_i}{\gamma S} = 0 \qquad \text{so,} \qquad c_1 = \frac{w_i}{2\gamma S}$$

$$y(L) = \frac{w_i}{2\gamma S}\left[\cos\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot L\right) + \cosh\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot L\right) - 2\right]$$

$$+ c_2 \sin\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot L\right) + c_4 \sinh\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot L\right) = 0$$

$$y''(L) = \sqrt{\frac{\gamma S}{EI}}\left\{\begin{array}{l} \dfrac{w_i}{2\gamma S}\left[\cos\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right) + \cosh\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right)\right] \\ + c_2 \sin\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right) + c_4 \sinh\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right) \end{array}\right\} = 0$$

Solving for the remaining unknowns:

$$c_2 = \frac{w_i}{2\gamma S}\,\frac{1 - \cos\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right)}{\sin\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right)}$$

$$c_4 = \frac{w_i}{2\gamma S}\,\frac{1 - \cosh\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right)}{\sinh\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot L\right)}$$

Applying the following identities:

$$1 - \cos(A) = 2\sin^2\left(\frac{A}{2}\right)$$

$$\sin(A) = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$

and

$$1 - \cosh(A) = -2\sinh^2\left(\frac{A}{2}\right)$$

$$\sinh(A) = 2\sinh\left(\frac{A}{2}\right)\cosh\left(\frac{A}{2}\right)$$

gives

$$c_2 = \frac{w_i}{2\gamma S}\,\frac{\sin\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot \dfrac{L}{2}\right)}{\cos\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot \dfrac{L}{2}\right)}$$

and

$$c_4 = -\frac{w_i}{2\gamma S}\,\frac{\sinh\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot \dfrac{L}{2}\right)}{\cosh\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot \dfrac{L}{2}\right)}$$

Substituting these last two constants and applying the following trigonometric identities:

$$\cos\left(\frac{A}{2} - x\right) = \cos\left(\frac{A}{2}\right) \cdot \cos(x) + \sin\left(\frac{A}{2}\right) \cdot \sin(x)$$

$$\cosh\left(\frac{A}{2} - x\right) = \cosh\left(\frac{A}{2}\right) \cdot \cosh(x) - \sinh\left(\frac{A}{2}\right) \cdot \sinh(x)$$

gives the deflected shape of a uniformly loaded, simple-span beam under ponding loads:

$$y(x) = \frac{w_i}{2\gamma S}\left\{\begin{array}{l} \left[\dfrac{\cos\left(\sqrt[4]{\dfrac{\gamma S}{EI}}\left(\dfrac{L}{2} - x\right)\right)}{\cos\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot \dfrac{L}{2}\right)} - 1\right] \\ - \left[1 - \dfrac{\cosh\left(\sqrt[4]{\dfrac{\gamma S}{EI}}\left(\dfrac{L}{2} - x\right)\right)}{\cosh\left(\sqrt[4]{\dfrac{\gamma S}{EI}} \cdot \dfrac{L}{2}\right)}\right] \end{array}\right\} \tag{5}$$

which can be written as:

$$y(x) = \frac{\Delta_i}{\dfrac{5\pi^4}{192}C}\left\{\begin{array}{l} \left[\dfrac{\cos\left(\pi\sqrt[4]{C}\left(\dfrac{1}{2} - \dfrac{x}{L}\right)\right)}{\cos\left(\dfrac{\pi}{2}\sqrt[4]{C}\right)} - 1\right] \\ - \left[1 - \dfrac{\cosh\left(\pi\sqrt[4]{C}\left(\dfrac{1}{2} - \dfrac{x}{L}\right)\right)}{\cosh\left(\dfrac{\pi}{2}\sqrt[4]{C}\right)}\right] \end{array}\right\} \tag{6}$$

where

Δ_i = initial midspan deflection $= \dfrac{5w_iL^4}{384EI}$

C = ponding factor $= \dfrac{\gamma SL^4}{\pi^4 EI}$

The ponding factor, C, is identical to the flexibility coefficient used in the 2005 AISC *Specification* Appendix 2. The AISC factor includes the density of water and adjustments for mixing units of measurements:

$$C_{\text{AISC}} = \frac{32\, SL^4}{10^7 I} \approx \frac{62.4 \text{ pcf}\left(144 \,\frac{\text{in.}^2}{\text{ft}^2}\right) SL^4}{\pi^4 \left(29 \cdot 10^6 \text{ksi}\right) I}$$

The maximum deflection, Δ, occurs at midspan:

$$\Delta = \frac{\Delta_i}{\dfrac{\dfrac{5\pi^4}{192} C}{\left[\dfrac{1}{\cos\left(\dfrac{\pi}{2}\sqrt[4]{C}\right)} + \dfrac{1}{\cosh\left(\dfrac{\pi}{2}\sqrt[4]{C}\right)} - 2\right]}} \tag{7}$$

The odd formatting of this equation is intentional, so it can be more easily applied later in the article.

Nonuniform Loading

Roof loading is not always uniform. Off-center concentrated loads occur due to framing configurations and rooftop equipment. Snow drifts and rain loads on sloped roofs cause trapezoidal loading. A simple, but extreme, form of nonuniform loading is a triangular load, shown in Figure 1.

We can analyze triangular loading as we did for the uniform loads:

$$\text{Loading} = \text{Initial load} + \text{Ponded water}$$

$$w(x) = w_i \frac{x}{L} + \gamma S\, y(x) \tag{8}$$

where

w_i = the peak load at the end of the beam

Combining Equation 8 into Equation 2 gives the differential equation:

$$EI\, y^{iv}(x) - \gamma S\, y(x) = w_i \frac{x}{L} \tag{9}$$

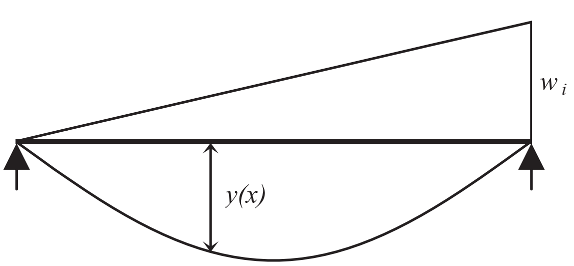

Fig. 1. Simple span with triangular load.

The general solution is:

$$y(x) = c_1 \cos\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right) + c_2 \sin\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right)$$

$$+ c_3 \cosh\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right) + c_4 \sin\left(\sqrt[4]{\frac{\gamma S}{EI}} \cdot x\right) - \frac{w_i}{\gamma S}\frac{x}{L}$$

Applying the same boundary conditions as provided earlier gives:

$$y(x) = \frac{w_i}{2\gamma S} \left\{ \begin{array}{l} \left[\dfrac{\sin\left(\sqrt[4]{\dfrac{\gamma S L^4}{EI}}\, \dfrac{x}{L}\right)}{\sin\left(\sqrt[4]{\dfrac{\gamma S L^4}{EI}}\right)} - \dfrac{x}{L} \right] \\ - \left[\dfrac{x}{L} - \dfrac{\sinh\left(\sqrt[4]{\dfrac{\gamma S L^4}{EI}}\, \dfrac{x}{L}\right)}{\sinh\left(\sqrt[4]{\dfrac{\gamma S L^4}{EI}}\right)} \right] \end{array} \right\} \tag{10}$$

which we can write as

$$y(x) = \Delta_i \frac{76.66}{\pi^4 C} \left\{ \begin{array}{l} \left[\dfrac{\sin\left(\sqrt[4]{C}\, \dfrac{\pi x}{L}\right)}{\sin\left(\sqrt[4]{C} \cdot \pi\right)} - \dfrac{x}{L} \right] \\ - \left[\dfrac{x}{L} - \dfrac{\sinh\left(\sqrt[4]{C}\, \dfrac{\pi x}{L}\right)}{\sinh\left(\sqrt[4]{C} \cdot \pi\right)} \right] \end{array} \right\} \tag{11}$$

where

Δ_i = initial midspan deflection $= \dfrac{w_i L^4}{2(76.66)EI}$

C = ponding factor $= \dfrac{\gamma\, S\, L^4}{\pi^4 EI}$

For a triangular loaded beam, the maximum deflection occurs at $x = \sqrt{1 - \sqrt{\dfrac{8}{15}}}\,(L) = 0.5193L$. This location shifts toward 0.50L with increasing ponding loads. To get the maximum moment location, we differentiate Equation 11 to get the slope and set it equal to zero, giving:

$$\frac{\cos\left(\pi\sqrt[4]{C} \cdot \dfrac{x}{L}\right)}{\sin\left(\pi\sqrt[4]{C}\right)} + \frac{\cosh\left(\pi\sqrt[4]{C} \cdot \dfrac{x}{L}\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} - \frac{2}{\pi\sqrt[4]{C}} = 0 \tag{12}$$

Solving for x/L, numerically, over the range of possible values of C, gives the results shown in Figure 2.

Because the slope is zero at the maximum deflection, the deflection nearby the maximum location is fairly constant. The maximum deflection can be calculated, reasonably accurately, anywhere between $0.5L$ and $0.5193L$. As Figure 2 shows, we can more accurately use:

$$\frac{x}{L} = 0.5193 - 0.193C\left(0.96 + 0.04C\right) \tag{13}$$

Plugging Equation 13 into Equation 11 gives the maximum deflection:

$$\Delta = \frac{\Delta_i}{\dfrac{\pi^4 C}{76.66}\left[\begin{array}{c} \dfrac{\sin\left(\pi\sqrt[4]{C}\left[0.5193 - 0.193C\left(0.96 + 0.04C\right)\right]\right)}{\sin\left(\pi\sqrt[4]{C}\right)} \\ +\dfrac{\sinh\left(\pi\sqrt[4]{C}\left[0.5193 - 0.193C\right]\left(0.96 + 0.04C\right)\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} \\ -1.02 \end{array}\right]} \tag{14}$$

Simplified Ponding Deflection

The deflection Equations 7 and 14 are cumbersome, so a simplified solution would be useful. The current 2005 AISC ponding analysis uses the simplifying assumption that the final deflected shape of the beam can be approximated as a sine curve:

$$y(x) = \Delta \sin\left(\frac{\pi x}{L}\right) \tag{15}$$

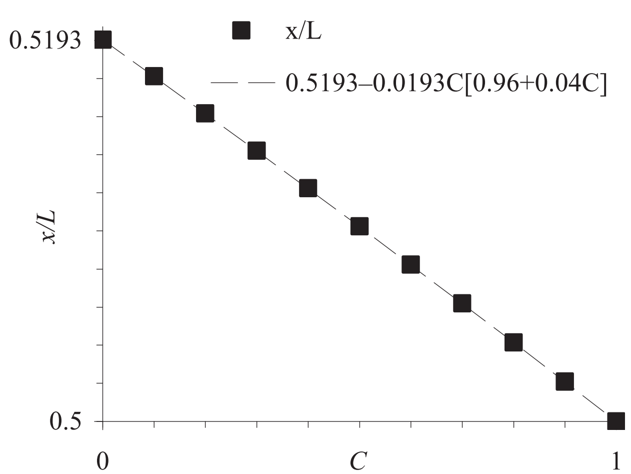

Fig. 2. Maximum deflection location.

where
Δ = total midspan deflection

Liquid ponded into this deflected shape produces a ponding load

$$w_p(x) = \gamma S \Delta \sin\left(\frac{\pi x}{L}\right) \tag{16}$$

From beam theory, the deflection is proportional to the fourth integral of the loading:

$$\begin{aligned} y_p(x) &= \int\int\int\int \frac{w_p(x)}{EI} \\ &= \frac{\gamma S}{EI}\int\int\int\int \Delta \sin\frac{\pi x}{L} \\ &= \Delta_p \sin\frac{\pi x}{L} \end{aligned}$$

where

$$\Delta_p = C\Delta \tag{17}$$

and

$$C = \text{ponding factor} = \frac{\gamma S L^4}{\pi^4 EI}$$

The total deflection is the sum of the initial deflection plus the ponding deflection:

$$\Delta = \Delta_i + \Delta_p \tag{18}$$

Substituting Equation 17 into Equation 18:

$$\Delta - C\Delta = \Delta_i$$

results in the basic ponding deflection equation:

$$\Delta = \frac{\Delta_i}{1 - C} \tag{19}$$

If we could use Equation 19 to estimate the deflection for beams with general loading, there will be some error, but the procedure would be greatly simplified.

Comparing the actual deflection of uniform and triangular loaded beams—Equations 7 and 14—to the approximate Equation 19 yields the data shown in Figure 3.

The graph in Figure 3 shows that Equation 19 accurately provides the total beam deflection, including ponding effects. The error between Equation 19 and the exact deflection equations is:

$$\text{Error} = \frac{\left[\dfrac{\Delta_i}{1 - C}\right]}{\Delta_{actual}} - 1$$

Plotting this error over the range of possible values of C yields the graph in Figure 4.

The error is less than half of 1 percent. So, this is nearly exact, for practical purposes.

Because the ponded fluid depth equals the total deflection, this gives the basic ponding depth as:

$$h_p = \Delta = \frac{\Delta_i}{1-C} \tag{20}$$

where

h_p = the maximum ponding depth

The maximum ponding depth term, h_p, is redundant here, but becomes useful when working with cambered beams.

End Fixity—Rotational Restraint

Though it is common to analyze simple span beams with perfectly pinned end connections, this is rarely a realistic model. For instance, conventional bolted connections on wide-flange beams provide rotational restraint, especially if the bolts are fully tensioned. In such situations, it is common for the actual beam deflection to be 20 to 30% smaller than predicted by a perfectly pinned model.

Because ponding loads are directly proportional to deflections, it is more accurate to include the effects of partial end fixity in the calculations. One way to accomplish this is to adjust the pin-ended deflection. For infinitely rigid end restraints and uniform loading:

$$\Delta_{i-FIXED} = \frac{w_i L^4}{384EI} = 0.2\frac{5w_i L^4}{384EI} = 0.2\Delta_{i-PINNED}$$

Design engineers do the same adjustment when determining camber on composite beams, where they adjust the pin-ended deflection to get the expected deflection. The adjustment is based on personal experience with some examples being shown in the table below:

Condition	$C_R = \dfrac{\Delta_{ACTUAL}}{\Delta_{PINNED}}$
Fixed ends	0.2
WF beam with shear connection	0.8
Open web steel joists	1.15

The values in the table are approximate numbers, and slight variations occur depending on the rotational stiffness of the supporting member. The open web joist factor is an increase due to web deformations, but it applies in the same way as the end fixity adjustment. Because the joist adjustment increases the deflection load, it should always be included in the ponding calculation.

Applying these factors to the initial deflection gives:

$$\Delta_{ACTUAL} = C_R \Delta_{PINNED}$$

With this factor, the ponding calculations are the same as for a pin-ended beam, except the deflection is reduced due to partial end fixity. Analyzing Equations 15 through 19 using this factor gives:

$$h_p = \Delta = \frac{\Delta_i}{1-C} \tag{21}$$

This is the same as Equation 20, except the ponding factor becomes $C = \dfrac{C_R \gamma S L^4}{\pi^4 EI}$.

The initial deflection, Δ_i, also should be adjusted for end fixity. The rest of the ponding calculations stay the same.

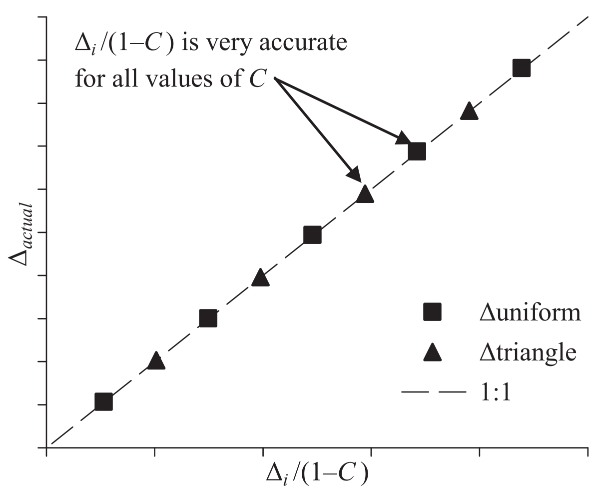

Fig. 3. Accuracy of approximate deflection equation.

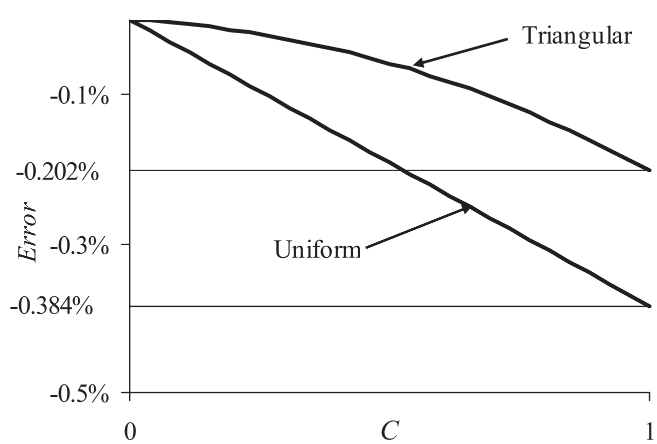

Fig. 4. Error in approximate equation $\Delta = \dfrac{\Delta_i}{1-C}$.

CAMBERED BEAMS

Uniform camber reduces the deflection and thus reduces the ponding load. On a cambered beam, the ponding depth is

$$h_p = \Delta - \Delta_c$$

where
- Δ = the total beam deflection, including ponding
- Δ_c = the upward midspan camber

If we assume the cambered shape is also a sign curve, the ponding load is:

$$w_p(x) = \gamma S h_p \sin\frac{\pi x}{L}$$

where
- h_p = midspan fluid depth = $\Delta - \Delta_c$

Integrating the loading to get the ponding deflection:

$$y_p(x) = \int\int\int\int w_p(x) = \frac{\gamma S L^4}{\pi^4 EI} h_p \sin\frac{\pi x}{L}$$

giving

$$\Delta_p = \frac{\gamma S L^4}{\pi^4 EI} h_p = C h_p \tag{22}$$

Because the total deflection is the sum of the initial deflection plus the ponding deflection,

$$\Delta = \Delta_i + \Delta_p$$

and the ponding depth is the total deflection minus the camber:

$$h_p = \Delta - \Delta_c = \Delta_i + \Delta_p - \Delta_c$$

and the ponding deflection is:

$$\Delta_p = h_p - \left(\Delta_i - \Delta_c\right) = h_p - h_i$$

Substituting Equation 22:

$$C h_p = h_p - h_i$$

Solving for the midspan ponding depth:

$$h_p = \frac{h_i}{1 - C} \tag{23}$$

where
- h_p = total midspan pond depth = $\Delta - \Delta_c$
- h_i = initial midspan pond depth = $\Delta_i - \Delta_c$

Except for the midspan camber term, this is identical to Equations 20 and 21.

PONDING LOAD DEMAND

Summarizing the previous results provides the ponding load of a single beam:

$$w_p(x) = \gamma S h_p \sin\frac{\pi x}{L} \tag{24}$$

where

$$h_p = \frac{h_i}{1 - C} \tag{25}$$

$$C = \frac{C_R \gamma S L^4}{\pi^4 EI} \tag{26}$$

- h_i = Initial midspan pond depth = $\Delta_i - \Delta_c$

PONDING MOMENT DEMAND

The water accumulated in the beam's deflected shape will add loading to the beam, thereby increasing the end shears and moments. These can be calculated from the deflected shapes.

Uniform Loading

The moment in a uniformly loaded beam with ponding can be derived from the deflection:

$$M(x) = -EI\, y''(x) \tag{27}$$

Substituting the deflection Equation 5 into Equation 27:

$$M(x) = -\frac{w_i EI}{2\gamma S}\frac{d^2}{dx^2}\left\{\begin{array}{l}\left[\dfrac{\cos\left(\sqrt[4]{\dfrac{\gamma S}{EI}}\left(\dfrac{L}{2} - x\right)\right)}{\cos\left(\sqrt[4]{\dfrac{\gamma S}{EI}}\cdot\dfrac{L}{2}\right)} - 1\right] \\ -\left[1 - \dfrac{\cosh\left(\sqrt[4]{\dfrac{\gamma S}{EI}}\left(\dfrac{L}{2} - x\right)\right)}{\cosh\left(\sqrt[4]{\dfrac{\gamma S}{EI}}\cdot\dfrac{L}{2}\right)}\right]\end{array}\right\}$$

Differentiating and simplifying gives:

$$M(x) = \frac{w_i L^2}{2\pi^2\sqrt{C}}\left\{\begin{array}{l}\dfrac{\cos\left[\pi\sqrt[4]{C}\left(\dfrac{1}{2} - \dfrac{x}{L}\right)\right]}{\cos\left(\dfrac{\pi}{2}\sqrt[4]{C}\right)} \\ -\dfrac{\cosh\left[\pi\sqrt[4]{C}\left(\dfrac{1}{2} - \dfrac{x}{L}\right)\right]}{\cosh\left(\dfrac{\pi}{2}\sqrt[4]{C}\right)}\end{array}\right\}$$

The maximum moment occurs at midspan:

$$M = \frac{w_i L^2}{2\pi^2 \sqrt{C}} \left[\frac{1}{\cos\left(\frac{\pi}{2}\sqrt[4]{C}\right)} - \frac{1}{\cosh\left(\frac{\pi}{2}\sqrt[4]{C}\right)} \right]$$

This is the total moment, including ponding:

$$M = \frac{w_i L^2}{8} + M_p$$

Solving for the ponding moment:

$$M_p = \frac{w_i L^2}{2\pi^2 \sqrt{C}} \left[\frac{1}{\cos\left(\frac{\pi}{2}\sqrt[4]{C}\right)} - \frac{1}{\cosh\left(\frac{\pi}{2}\sqrt[4]{C}\right)} \right] - \frac{w_i L^2}{8}$$

which can be written as:

$$M_p = \frac{\dfrac{\Delta_i \gamma S L^4}{\pi^2}}{\dfrac{\dfrac{5\pi^4}{192} C^{3/2}}{\dfrac{1}{\cos\left(\frac{\pi}{2}\sqrt[4]{C}\right)} - \dfrac{1}{\cosh\left(\frac{\pi}{2}\sqrt[4]{C}\right)} - \dfrac{\pi^2 \sqrt{C}}{4}}} \tag{28}$$

where

Δ_i = initial midspan deflection $= \dfrac{5 w_i L^4}{384 EI}$

Triangular Loading

From the deflection Equations 10 and 27:

$$M(x) = -\frac{w_i EI}{2\gamma S} \frac{d^2}{dx^2} \left\{ \begin{array}{l} \left[\dfrac{\sin\left(\pi\sqrt[4]{C}\,\frac{x}{L}\right)}{\sin\left(\pi\sqrt[4]{C}\right)} - \dfrac{x}{L} \right] \\ - \left[\dfrac{x}{L} - \dfrac{\sinh\left(\pi\sqrt[4]{C}\,\frac{x}{L}\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} \right] \end{array} \right\} \tag{29}$$

Differentiating and simplifying gives:

$$M(x) = \frac{w_i L^2}{2\pi^2 \sqrt{C}} \left\{ \frac{\sin\left(\pi\sqrt[4]{C}\,\frac{x}{L}\right)}{\sin\left(\pi\sqrt[4]{C}\right)} - \frac{\sinh\left(\pi\sqrt[4]{C}\,\frac{x}{L}\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} \right\} \tag{30}$$

For the triangular loaded beam, the maximum moment occurs at $x = \frac{1}{\sqrt{3}} L = 0.5774L$. This location shifts toward $0.50L$ with increasing ponding loads. To get the maximum moment location, we differentiate Equation 30 to get the shear and set it equal to zero, giving:

$$\frac{\cos\left(\pi\sqrt[4]{C} \cdot \frac{x}{L}\right)}{\sin\left(\pi\sqrt[4]{C}\right)} - \frac{\cosh\left(\pi\sqrt[4]{C} \cdot \frac{x}{L}\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} = 0 \tag{31}$$

Solving this numerically for *x/L*, we get the data plotted in Figure 5.

As Figure 5 shows, the maximum moment occurs very near:

$$\frac{x}{L} = 0.5774 - 0.0774\,C\,(1.07 - 0.07C) \tag{32}$$

Plugging Equation 32 into Equation 30 gives the maximum total moment:

$$M = \frac{w_i L^2}{2\pi^2 \sqrt{C}} \tag{33}$$

$$\times \left\{ \begin{array}{l} \dfrac{\sin\left(\pi\sqrt[4]{C}\,0.5774 - 0.0774\,C\,(1.07 - 0.07C)\right)}{\sin\left(\pi\sqrt[4]{C}\right)} \\ - \dfrac{\sinh\left(\pi\sqrt[4]{C}\,0.5774 - 0.0774\,C\,(1.07 - 0.07C)\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} \end{array} \right\}$$

This is the total moment, which is the sum of the initial moment and the ponding moment:

$$M = M_i + M_p = \frac{w_i L^2}{9\sqrt{3}} + M_p \tag{34}$$

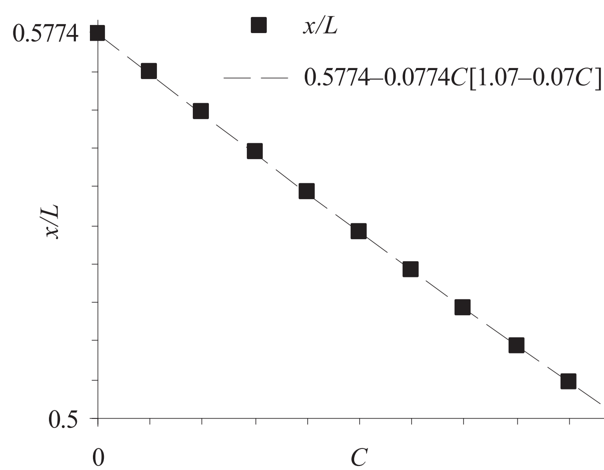

Fig. 5. Maximum deflection location.

Solving for the ponding moment:

$$M_p = \frac{w_i L^2}{2\pi^2\sqrt{C}} \tag{35}$$

$$\times \left\{ \begin{array}{l} \dfrac{\sin\left(\pi\sqrt[4]{C}\left[0.5774 - 0.0774\,C\,(1.07 - 0.07C)\right]\right)}{\sin\left(\pi\sqrt[4]{C}\right)} \\ -\dfrac{\sinh\left(\pi\sqrt[4]{C}\left[0.5774 - 0.0774\,C\,(1.07 - 0.07C)\right]\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} \end{array} \right\}$$

$$-\frac{w_i L^2}{9\sqrt{3}}$$

which can be written as:

$$M_p = \frac{\dfrac{\Delta_i \gamma S L^4}{\pi^2}}{\dfrac{1.271\,C^{3/2}}{\left\{ \begin{array}{l} \dfrac{\sin\left(\pi\sqrt[4]{C}\left[0.5774 - 0.0774\,C\,(1.07 - 0.07C)\right]\right)}{\sin\left(\pi\sqrt[4]{C}\right)} \\ -\dfrac{\sinh\left(\pi\sqrt[4]{C}\left[0.5774 - 0.0774\,C\,(1.07 - 0.07C)\right]\right)}{\sinh\left(\pi\sqrt[4]{C}\right)} \\ -1.266\sqrt{C} \end{array} \right\}}} \tag{36}$$

where

Δ_i = initial midspan deflection $= \dfrac{w_i L^4}{153.32EI}$

Simplified Ponding Moment Demand

With a sinusoidal deflected shape, the ponding load is:

$$w_p(x) = \gamma S h_p \sin\frac{\pi x}{L}$$

The ponding moment is:

$$M_p(x) = -\int\int w_p(x) = \frac{\gamma S L^2}{\pi^2} h_p \sin\frac{\pi x}{L}$$

and the ponding moment demand is:

$$M_p = \frac{\gamma S h_p L^2}{\pi^2} \tag{37}$$

where

$$h_p = \frac{h_i}{1 - C}$$

If we use Equation 37 to estimate the moment for beams with general loading, there will be some error, but the procedure is be greatly simplified.

Comparing the actual moment for a uniform and triangular loaded beam—Equations 28 and 36—to the approximate Equation 37, we get the data plotted in Figure 6.

Equation 19 accurately provides the total beam deflection, including ponding effects. The error between Equation 19 and the exact deflection equations is:

$$\text{Error} = \frac{\left[\dfrac{\Delta_i}{1 - C}\dfrac{\gamma S L^2}{\pi^2}\right]}{M_{P-actual}} - 1$$

Plotting this error over the range of possible values of C yields Figure 7.

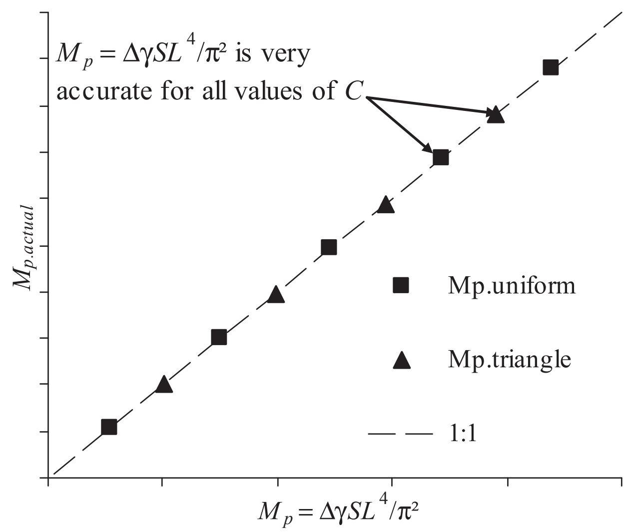

Fig. 6. Accuracy of approximate moment equation.

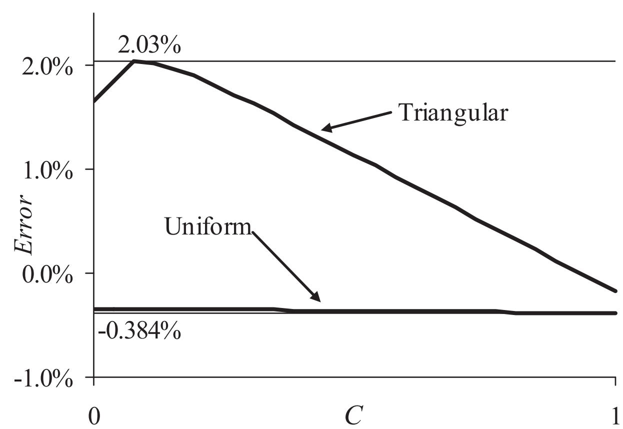

Fig. 7. Error in approximate equation $\dfrac{\Delta_i}{1-C}\dfrac{\gamma S L^2}{\pi^2}$.

The error is very small, except for certain values of C under triangular loading, which has up to about 2% error. This is an extreme loading condition, so under common loading, the approximation is very accurate for practical purposes.

Summarizing the ponding moment demand procedure:

$$\text{Ponding moment} = M_p = \frac{h_p \gamma S L^4}{\pi^4} \tag{38}$$

where

h_p = ponding depth = $\dfrac{h_i}{1-C}$

C = ponding factor = $\dfrac{C_R \gamma S L^4}{\pi^4 EI}$

h_i = initial midspan pond depth = $\Delta_i - \Delta_c$

γ = ponding liquid density (62.4 pcf for water)

S = beam spacing

L = beam span

Example 1—Beam with Ponding

W12×14 beam; $I_x = 88.6$ in.^{4}, span = 30 ft; spacing = 6 ft; end restraint factor = $C_R = 0.8$; camber = 1 in.

Initial deflection due to $D + R$ or S:

$$\Delta_i = C_R \frac{5 w_i L^4}{384 EI} = 1.2 \text{ in.}$$

Initial depth:

$$h_i = \Delta_i - \Delta_c = 1.2 \text{ in.} - 1 \text{ in.} = 0.2 \text{ in.}$$

Ponding factor:

$$C = C_R \frac{\gamma S L^4}{\pi^4 EI} = 0.8 \frac{62.4(6)30^4(144)}{\pi^4(29\times10^6)88.6} = 0.14$$

Midspan ponding depth:

$$h_p = \frac{h_i}{1-C} = \frac{0.2 \text{ in.}}{1-0.14} = 0.23 \text{ in.}$$

Added ponding moment:

$$\begin{aligned} M_P &= \frac{S\gamma h_p L^2}{\pi^2} \\ &= \frac{6(62.4 \text{ pcf} \times 0.23 \text{ in.})30^2}{\pi^2 \times 12 \text{ in./ft}} = 0.7 \text{ k-ft} \end{aligned}$$

Added ponding reaction:

$$\begin{aligned} R_P &= \frac{S\gamma h_p L}{\pi} \\ &= \frac{6(62.4 \text{ pcf} \times 0.23 \text{ in.})30}{\pi \times 12 \text{ in./ft}} = 0.1 \text{ k} \end{aligned}$$

BEAM AND GIRDER SYSTEMS

We can expand the preceding procedures to beam and girder systems as follow:

Beam Supported by Girders

This beam has added trapezoidal ponding load due to the girder ponding depths, h_{Gp}, at each end of the beam.

Ponding load:

$$w_{Bp}(x_B) = \gamma S \begin{bmatrix} h_{Bp} \sin \dfrac{\pi x_B}{L_B} \\ + h_{GpR} \sin \dfrac{\pi x_{GR}}{L_{GR}} \left(\dfrac{x_B}{L_B}\right) \\ + h_{GpL} \sin \dfrac{\pi x_{GL}}{L_{GL}} \left(1 - \dfrac{x_B}{L_B}\right) \end{bmatrix} \tag{39}$$

where

h_{GpL} = left girder ponding depth

h_{GpR} = right girder ponding depth

Note that the girder ponding depth varies along the length of the girder. The adjacent girders may have staggered columns, so their maximum deflections may occur at different locations. Equation 39 represents the general condition for a beam framing into any point along the girder.

The resulting beam's ponding moment demand would then be:

$$M_{Bp}(x_G) = \gamma S L_B^{\,2} \begin{bmatrix} \dfrac{h_{Bp}}{\pi^2} \\ + \dfrac{h_{GpR}}{16} \sin \dfrac{\pi x_{GR}}{L_{GR}} \\ + \dfrac{h_{GpL}}{16} \sin \dfrac{\pi x_{GL}}{L_{GL}} \end{bmatrix} \tag{40}$$

The deflection caused by the ponding load in Equation 39 is:

$$\Delta_{Bp}(x_G) = C_B \left[h_{Bp} + \frac{5\pi^4}{768} \begin{pmatrix} h_{GpR} \sin \dfrac{\pi x_{GR}}{L_{GR}} \\ + h_{GpL} \sin \dfrac{\pi x_{GL}}{L_{GL}} \end{pmatrix} \right]$$

where

$$C_B = \frac{C_R \gamma S L_B^{\,4}}{\pi^4 EI_B}$$

Substituting:

$$h_{Bp} = \Delta_{Bp} + h_{Bi}$$

and solving for the beam ponding depth,

$$h_{Bp}(x_G) = \frac{h_{Bi}}{1-C_B} + \frac{5\pi^4}{768}\frac{C_B}{1-C_B}\begin{pmatrix} h_{GpR}\sin\dfrac{\pi x_{GR}}{L_{GR}} \\ +\, h_{GpL}\sin\dfrac{\pi x_{GL}}{L_{GL}} \end{pmatrix} \tag{41}$$

If the beam frames into both girder midspans, then the maximum beam ponding depth is:

$$h_{Bp} = \frac{h_{Bi}}{1-C_B} + \frac{5\pi^4}{384}\frac{C_B}{1-C_B}\left(\frac{h_{GpR}+h_{GpL}}{2}\right) \tag{42}$$

and the corresponding maximum beam ponding moment demand is:

$$M_{Bp} = \gamma S L_B{}^2\left[\frac{h_{Bp}}{\pi^2} + \frac{h_{GpL}+h_{GpR}}{16}\right] \tag{43}$$

Solving Equation 39 for the end reaction:

$$R_{BpL}(x_G) = \gamma S L_B\begin{bmatrix} \dfrac{h_{Bp}}{\pi} \\ +\dfrac{h_{GpL}}{3}\sin\dfrac{\pi x_{GL}}{L_{GL}} \\ +\dfrac{h_{GpR}}{6}\sin\dfrac{\pi x_{GR}}{L_{GR}} \end{bmatrix} \tag{44}$$

Substituting Equation 41:

$$R_{BpL}(x_G) = \frac{\gamma S L_B}{\pi}\begin{bmatrix} \dfrac{h_{Bp}}{1-C_B} \\ +\left(\dfrac{\pi}{3}+\dfrac{C_B}{1-C_B}\dfrac{5\pi^4}{768}\right)h_{GpL}\sin\dfrac{\pi x_{GL}}{L_{GL}} \\ +\left(\dfrac{\pi}{6}+\dfrac{C_B}{1-C_B}\dfrac{5\pi^4}{768}\right)h_{GpR}\sin\dfrac{\pi x_{GR}}{L_{GR}} \end{bmatrix} \tag{45}$$

Girder Ponding

The girder supports the end reactions from the beams on each side of the girder.

$$w_G(x_G) = \frac{R_{BpL}+R_{BpR}}{S}$$

Substituting from Equation 45, we get the ponding load on the girder:

$$w_{Gp}(x_G) = \frac{\gamma}{\pi}\left\{\begin{array}{l} \dfrac{h_{Bp1}L_{B1}}{1-C_{B1}} + \dfrac{h_{Bp2}L_{B2}}{1-C_{B2}} \\ +\begin{bmatrix} \left(\dfrac{\pi}{3}+\dfrac{C_{B1}}{1-C_{B1}}\dfrac{5\pi^4}{768}\right)L_{B1} \\ +\left(\dfrac{\pi}{3}\quad\dfrac{C_{B2}}{1-C_{B2}}\dfrac{5\pi^4}{768}\right)L_{B2} \end{bmatrix} h_{Gp}\sin\dfrac{\pi x_G}{L_G} \\ +\left(\dfrac{\pi}{6}+\dfrac{C_{B1}}{1-C_{B1}}\dfrac{5\pi^4}{768}\right)L_{B1}h_{Gp1}\sin\dfrac{\pi x_{G1}}{L_{G1}} \\ +\left(\dfrac{\pi}{6}+\dfrac{C_{B2}}{1-C_{B2}}\dfrac{5\pi^4}{768}\right)L_{B2}h_{Gp2}\sin\dfrac{\pi x_{G2}}{L_{G2}} \end{array}\right\} \tag{46}$$

The adjacent girders may have staggered columns, so their maximum deflections may occur at different locations. Equation 46 represents the general loading condition. It can be used to solve complex framing patterns by modeling all the girders and beams, then iterating to find the ponding depths of all the framing members.

We will next look at some conditions that allow for closed-form solutions.

Identical Beams and Girders

If the nearby beams and girders are all identical, with the same loading, then the solution can be simplified. The beam ponding depth becomes:

$$h_{Bp}(x_G) = \frac{h_{Bi}}{1-C_B} + \frac{5\pi^4}{384}\frac{C_B}{1-C_B}h_{Gp}\sin\frac{\pi x_G}{L_G} \tag{47}$$

The girder loading is

$$w_{Gp}(x_G) = \gamma L_B\begin{bmatrix} \dfrac{2}{\pi}\dfrac{h_{Bi}}{1-C_B} \\ +\left(1+\dfrac{5\pi^3}{192}\dfrac{C_B}{1-C_B}\right)h_{Gp}\sin\dfrac{\pi x_G}{L_G} \end{bmatrix} \tag{48}$$

The girder deflection due to ponding is

$$\Delta_{Gp} = C_G\left[\frac{5\pi^3}{192}\frac{h_{Bi}}{1-C_B} + \left(1+\frac{5\pi^3}{192}\frac{C_B}{1-C_B}\right)h_{Gp}\right]$$

where

$$C_G = \frac{C_R\gamma L_B L_G{}^4}{\pi^4 E I_G}$$

Substituting $h_{Gp} = h_{Gi} + \Delta_{Gp}$:

$$h_{Gp} = \frac{\dfrac{h_{Gi}}{C_G} + \dfrac{5\pi^3}{192}\dfrac{h_{Bi}}{1-C_B}}{\dfrac{1-C_G}{C_G} - \dfrac{5\pi^3}{192}\dfrac{C_B}{1-C_B}} \tag{49}$$

The beam framing into the girder midspan has a ponding depth of:

$$h_{Bp} = \frac{h_{Bi}}{1-C_B} + \frac{5\pi^4}{384}\frac{C_B}{1-C_B}h_{Gp} \tag{50}$$

Using the ponding depths from Equations 49 and 50, the ponding moment demands can be computed as follows.

Beam at girder midspan from Equation 40:

$$M_{Bp} = \gamma S L_B^2\left(\frac{h_{Bp}}{\pi^2} + \frac{h_{Gp}}{8}\right) \tag{51}$$

Using Equation 40 to calculate the moment demand for beams framing into different locations on the girder yields the following.

Beam at girder third-points:

$$M_{Bp} = \gamma S L_B^2\left(\frac{h_{Bp}}{\pi^2} + \frac{\sqrt{3}}{16}h_{Gp}\right) \tag{52}$$

Beam and girder quarter-points:

$$M_{Bp} = \gamma S L_B^2\left(\frac{h_{Bp}}{\pi^2} + \frac{h_{Gp}}{8\sqrt{2}}\right) \tag{53}$$

Girder moment:

$$\begin{aligned} M_{Gp} &= \gamma L_B L_G^2\left\{\frac{h_{Bp}}{4\pi} + \left[1 - \left(\frac{\pi^2}{8} - 1\right)\frac{5\pi^3}{192}\cdot\frac{C_B}{1-C_B}\right]\frac{h_{Gp}}{\pi^2}\right\} \\ &= \frac{\gamma L_B L_G^2}{4\pi}\left[h_{Bp} + \left(5.3 - \frac{C_B}{1-C_B}\right)\frac{h_{Gp}}{4.2}\right] \end{aligned} \tag{54}$$

The total moment demand, including rain and/or snow, can then be compared against the beam's flexural capacity.

Example 2—Uniform Beam and Girder System

End restraint factor = $C_R = 0.8$

W12×14 beam; I_x = 88.6 in.^{4}; span = 30 ft; spacing = 6 ft; camber = 1 in.; deflection due to $D + R$ or S: $\Delta_i = 1.2$ in.
$h_{Bi} = 1.2 - 1 = 0.2$ in.

W18×35 girder; I_x = 510 in.^{4}; span = 31 ft; camber = 1 in.; deflection due to $D + R$ or S: $\Delta_i = 1.3$ in.
$h_{Gi} = 1.3 - 1 = 0.3$ in.
$C_G = 0.14$

The girder ponding depth is:

$$\begin{aligned} h_{Gp} &= \frac{\dfrac{h_{Gi}}{C_G} + \dfrac{5\pi^3}{192}\dfrac{h_{Bi}}{1-C_B}}{\dfrac{1-C_G}{C_G} - \dfrac{5\pi^3}{192}\dfrac{C_B}{1-C_B}} \\ &= \frac{\dfrac{0.3}{0.14} + \dfrac{5\pi^3}{192}\dfrac{0.2}{1-0.14}}{\dfrac{1-0.14}{0.14} - \dfrac{5\pi^3}{192}\dfrac{0.14}{1-0.14}} = 0.39 \text{ in.} \end{aligned} \tag{55}$$

The beam framing into the girder midspan has a ponding depth of:

$$\begin{aligned} h_{Bp} &= \frac{h_{Bi}}{1-C_B} + \frac{5\pi^4}{384}\frac{C_B}{1-C_B}h_{Gp} \\ &= \frac{0.2}{1-0.14} + \frac{5\pi^4}{384}\frac{0.14}{1-0.14}0.39 = 0.31 \text{ in.} \end{aligned} \tag{56}$$

The moment in the beam at the girder midspan is:

$$\begin{aligned} M_{Bp} &= \gamma S L_B^2\left(\frac{h_{Bp}}{\pi^2} + \frac{h_{Gp}}{8}\right) \\ &= \frac{62.4(6)30^2}{12{,}000}\left(\frac{0.39}{\pi^2} + \frac{0.31}{8}\right) \\ &= 2.2 \text{ k-ft} \end{aligned} \tag{57}$$

The girder moment is:

$$\begin{aligned} M_{Gp} &= \frac{\gamma L_B L_G^2}{4\pi}\left[h_{Bp} + \left(5.3 - \frac{C_B}{1-C_B}\right)\frac{h_{Gp}}{4.2}\right] \\ &= \frac{62.4(30)31^2}{4\pi(12{,}000)}\left[0.31 + \left(5.3 - \frac{C_B}{1-C_B}\right)\frac{0.39}{4.2}\right] \\ &= 9.4 \text{ k-ft} \end{aligned} \tag{58}$$

Beam Framing into a Wall—Identical Adjacent Framing

The beam ponding depth is

$$h_{Bp}(x_G) = \frac{h_{Bi}}{1-C_B} + \frac{5\pi^4}{768}\frac{C_B}{1-C_B}h_{Gp}\sin\frac{\pi x_G}{L_G} \tag{59}$$

If the adjacent beams and girders are identical and similarly loaded, then the girder loading is:

$$w_{Gp}(x_G) = \gamma L_B\left[\begin{aligned} &\frac{2}{\pi}\frac{h_{Bi}}{1-C_B} \\ &+\left(\frac{5}{6} + \frac{5\pi^3}{256}\frac{C_B}{1-C_B}\right)h_{Gp}\sin\frac{\pi x_G}{L_G}\end{aligned}\right] \tag{60}$$

The girder deflection due to ponding is:

$$\Delta_{Gp} = C_G \left[\frac{5\pi^3}{192} \frac{h_{Bi}}{1-C_B} + \left(\frac{5}{6} + \frac{5\pi^3}{256} \frac{C_B}{1-C_B} \right) h_{Gp} \right]$$

The girder ponding depth is:

$$h_{Gp} = \frac{\dfrac{h_{Gi}}{C_G} + \dfrac{5\pi^3}{192} \dfrac{h_{Bi}}{1-C_B}}{\dfrac{1-\frac{5}{6}C_G}{C_G} - \dfrac{5\pi^3}{256} \dfrac{C_B}{1-C_B}} \tag{61}$$

The beam framing into the wall and the girder midspan has a ponding depth of:

$$h_{Bp} = \frac{h_{Bi}}{1-C_B} + \frac{5\pi^4}{768} \frac{C_B}{1-C_B} h_{Gp} \tag{62}$$

The ponding moment demands are as follows.

Beam at girder midspan from Equation 40:

$$M_{Bp} = \gamma S L_B^2 \left(\frac{h_{Bp}}{\pi^2} + \frac{h_{Gp}}{16} \right) \tag{63}$$

Beam at girder third-points:

$$M_{Bp} = \gamma S L_B^2 \left(\frac{h_{Bp}}{\pi^2} + \frac{\sqrt{3}}{32} h_{Gp} \right) \tag{64}$$

Beam and girder quarter-points:

$$M_{Bp} = \gamma S L_B^2 \left(\frac{h_{Bp}}{\pi^2} + \frac{h_{Gp}}{16\sqrt{2}} \right) \tag{65}$$

Girder moment:

$$\begin{aligned} M_{Gp} &= \gamma L_B L_G^2 \left\{ \frac{h_{Bp}}{4\pi} + \left[\frac{5}{6} + \left(1 - \frac{\pi^2}{12} \right) \frac{5\pi^3}{256} \cdot \frac{C_B}{1-C_B} \right] \frac{h_{Gp}}{\pi^2} \right\} \\ &= \frac{\gamma L_B L_G^2}{4\pi} \left[h_{Bp} + \left(7.75 - \frac{C_B}{1-C_B} \right) \frac{h_{Gp}}{7.3} \right] \end{aligned} \tag{66}$$

STRENGTH CAPACITY PONDING CHECK

To design for ponding, a designer can include the ponding moment demands—from this article—in the appropriate ASCE 7-05 load combination and then compare it to the capacity from Chapter F of the 2005 AISC *Specification*:

$$\begin{aligned} M_u &\le \phi M_n \\ 1.2M_D + 1.6\left(M_{R\,or\,S} + M_P \right) &\le 0.9 M_n \end{aligned} \tag{67}$$

or

$$\begin{aligned} M_u &\le \frac{M_n}{\Omega_b} \\ M_D + M_{R\,or\,S} + M_P &\le \frac{M_n}{1.67} \end{aligned} \tag{68}$$

This is consistent with the general provisions of ASCE 7-05 and AISC; however, it conflicts with the provisions of Appendix 2 of the 2005 AISC *Specification*.

Present Code Provisions

In Appendix 2 of the 2005 AISC *Specification*, the ponding check is a service load stress check:

$$f_b = f_o + f_w = \frac{M_o + M_w}{S_x} \le 0.8F_y$$

At the time this ponding stress check was adopted, the AISC flexural capacity was also a service load stress check, but with a different safety factor:

$$M_o \le 0.66F_y$$

Thus, the present AISC ponding design provision allows a stress increase for the ponding moment of 0.8/0.66 = 1.21. Conversely, the ponding demand was reduced by 0.66/0.8 = 0.825.

The reason behind this stress increase appears to be an assumption that the current snow and rain load demands already account for some amount of ponding load. Thus, combining ponding with the other loading is slightly redundant. Incorporating this into the demand capacity check gives:

$$0.83\left[1.2M_D + 1.6\left(M_{R\,or\,S} + M_P \right) \right] \le 0.9M_n \tag{69}$$

and

$$0.83\left(M_D + M_{R\,or\,S} + M_P \right) \le \frac{M_n}{1.67} \tag{70}$$

Another inherent assumption in the current AISC ponding provision is that the beam's compression flanges can be assumed fully braced regardless of the actual conditions. By allowing $0.8F_y$ flexural stress, without checking for flange bracing, the current code implies that lateral-torsional buckling can be ignored.

Rain Loading on Roof Areas away from Drains

For rain loading, ASCE 7-05 specifies the depth of water due to clogged primary roof drains. If the roof is sloped, then this loading only applies near the roof drains. No loading is specified for the roof area away from the water pooled around the drain.

It would be reasonable to include a rain load for ponding checks away from this area. This can be calculated using the rainfall rate, the tributary area of roof uphill from the beam, and an open channel flow depth calculation. For most roofs, this would be a very shallow flow depth and loading. This author recommends a minimum rain load of 5 psf.

CONCLUSION

The current ponding provisions of the 2005 AISC *Specification* can be expanded to include more unique conditions. There also may be a benefit to formatting the analysis to more closely mesh with the other code provisions. This would allow the ponding analysis to be interpreted, adapted to unique conditions, improved, and calibrated to other provisions of the code.

The current AISC procedure frames a specific condition simply and directly, but obscures its inner workings, making it difficult to adapt to other conditions.

There are some actions not included in this model. Inelastic action would induce larger deflections for moments above the beam's yield limit, though this is usually not the case. Residual stresses in rolled beams can affect the deflection in ways similar to inelastic action. Camber variability is very common and would also induce errors in this calculation.

There have been few, if any, documented failures due to ponding—though it is possible that ponding played a role in past snow load– or clogged-roof-drain–induced failures. It is likely that the current procedure is too conservative. This is a paradox, because the current procedure uses a lower factor of safety than the ASCE and AISC codes. With a more accurate design calculation, and refinement in the procedures, it is possible that less structural steel could be used on future roofs.

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