CLOSURE

Limit State Response of Composite Columns and Beam-Columns Part II: Application of Design Provisions for the 2005 AISC Specification

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The authors thank Dr. Geschwindner for his comments on the derivation of the plastic section modulus, Z_s, of circular HSS as shown in Table 5 of the original paper. (Equation numbers referenced in this Closure are the same as those used in the Discussion, for clarity.)

The derivation of Equation 16 of the Discussion was omitted from the original paper for brevity. It is shown in the attached Appendix. Equation 17 was intended as a straightforward lower-bound curve fit to Equation 16; many similar expressions are possible. The primary assumption that was made in Equation 16 is that the wall of the circular HSS is assumed to be thin (i.e., that $\theta \approx \theta_s$ using the nomenclature of the Discussion). As noted by Dr. Geschwindner, this assumption results in the area of the steel being underestimated and that for the concrete being overestimated.

The authors appreciate Dr. Geschwindner's efforts in developing new exact and approximate equations for Z_s, represented by Equations 15 and 8, respectively, in the Discussion. The authors agree that his equations are applicable to thick-walled circular CFTs ($\theta \neq \theta_s$) and provide results with better accuracy than those stated in the original paper.

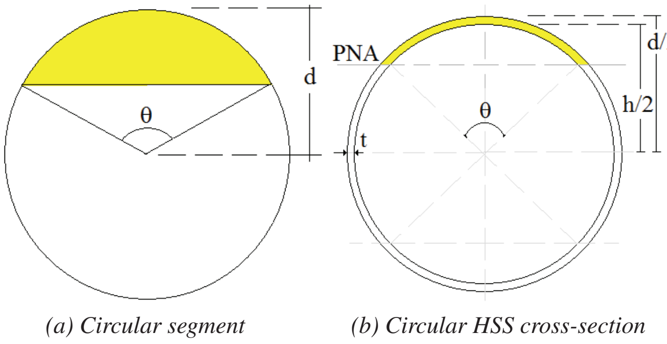
The authors agree that Equation 8 in the Discussion is a reasonable replacement for both the original equation for Z_{sB} in Table I-1d on the CD companion to the 13th edition AISC *Manual* (Equation 4 in the Discussion) and the proposed equation for Z_{sB} in our original paper (Equation 17 in the Discussion). Finally, the authors will like to note that Equation 8 is the same as those given by the Architectural Institute of Japan (AIJ) in their provisions for composite members once a number of geometric transformations are made.

APPENDIX

Derivation of the Plastic Section Modulus of a Circular HSS Thin Tube (Equation 16 in the Discussion)

This appendix derives the equation to get the plastic section modulus (Z_s) of a circular HSS stated in Table 5 in the original paper (Equation 15 in this appendix). The derivation assumes a thin-walled HSS cross section as shown in Figure A.1b.

The area and the centroidal distance of a circular segment (Figure A.1a) including both concrete and steel sections are given by:



(a) Circular segment *(b) Circular HSS cross-section*

Fig. A.1. Variables used in the derivation.

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$$A_d = \frac{d^2}{8}\left(\theta - \sin\theta\right) \tag{A.1}$$

$$Y_d = \frac{2d\sin^3\left(\theta/2\right)}{3\left(\theta - \sin\theta\right)} \tag{A.2}$$

Note in Figure A.1 that the angle θ in (a) is not the same as the angle θ in (b). This difference is small for thin-walled sections. Assuming the steel wall is thin enough that the difference can be neglected, the area and centroidal distance of the circular segment in the concrete only can be approximated as:

$$A_h = \frac{h^2}{8}\left(\theta - \sin\theta\right) \tag{A.3}$$

$$Y_h = \frac{2h\sin^3\left(\theta/2\right)}{3\left(\theta - \sin\theta\right)} \tag{A.4}$$

Thus, the area and the first moment of the area of the shaded ring segment in Figure A.1b are as follows:

$$A_r = A_d - A_h \tag{A.5}$$

$$Q_r = A_d Y_d - A_h Y_h \tag{A.6}$$

Then, the centroidal distance of a ring segment (i.e., only the steel) is given by:

$$Y_r = \frac{Q_r}{A_r} = \frac{2\left(d^3 - h^3\right)\sin^3\left(\theta/2\right)}{3\left(d^2 - h^2\right)\left(\theta - \sin\theta\right)} \tag{A.7}$$

The last equation can be adjusted for the complement ring segment when θ is changed by 2π – θ. Thus:

$$Y_r = \frac{Q_r}{A_r} = \frac{2\left(d^3 - h^3\right)\sin^3\left(\theta/2\right)}{3\left(d^2 - h^2\right)\left(2\pi - \theta + \sin\theta\right)} \tag{A.8}$$

The compression and tension forces on the steel ring segments, their respective centroidal distances, and the total bending moment are given by the following equations.

For the compression zone defined by the angle θ, where $r_m = (d - t)/2$ and $t = (d - h)/2$:

$$\begin{aligned} C_s &= 2\pi r_m t\left(\frac{\theta}{2\pi}\right)F_y = \left(\frac{d-t}{2}\right)\left(t\right)\left(\theta\right)F_y \\ &= \frac{\left(d^2 - h^2\right)\left(\theta\right)F_y}{8} \end{aligned} \tag{A.9}$$

$$Y_{cs} = \frac{2\left(d^3 - h^3\right)\sin^3\left(\theta/2\right)}{3\left(d^2 - h^2\right)\left(\theta - \sin\theta\right)} \tag{A.10}$$

For the tension zone defined by the complement of the angle θ, where $r_m = (d - t)/2$ and $t = (d - h)/2$:

$$T_s = \left(\frac{d-t}{2}\right)\left(t\right)\left(2\pi - \theta\right)F_y = \frac{\left(d^2 - h^2\right)\left(2\pi - \theta\right)F_y}{8} \tag{A.11}$$

$$Y_{ts} = \frac{2\left(d^3 - h^3\right)\sin^3\left(\theta/2\right)}{3\left(d^2 - h^2\right)\left(2\pi - \theta + \sin\theta\right)} \tag{A.12}$$

Then, the nominal moments in the steel cross section can be summed as:

$$C_s Y_{cs} + T_s Y_{ts} = F_y Z_{s\theta} \tag{A.13}$$

From Equation A.13, the plastic modulus of the steel cross-section for any angle theta is given by:

$$\begin{aligned} Z_{s\theta} &= \frac{\theta\left(d^3 - h^3\right)\sin^3\left(\theta/2\right)}{12\left(\theta - \sin\theta\right)} \\ &\quad + \frac{\left(2\pi - \theta\right)\left(d^3 - h^3\right)\sin^3\left(\theta/2\right)}{12\left(2\pi - \theta + \sin\theta\right)} \end{aligned} \tag{A.14}$$

In the denominator of the second term of Equation A.14, the sin(θ) term may be taken as its trigonometric equivalent, –sin(2π – θ). With all like terms based upon the same angle, 2π – θ, Equation A.14 can be restated as:

$$\begin{aligned} Z_{s\theta} &= \frac{\left(d^3 - h^3\right)\sin^3\left(\theta/2\right)}{12} \\ &\quad \times \left[\frac{\theta}{\left(\theta - \sin\theta\right)} + \frac{\left(2\pi - \theta\right)}{\left(2\pi - \theta\right) - \sin\left(2\pi - \theta\right)}\right] \end{aligned} \tag{A.15}$$

Equation A.15 is the one shown in Table 5 (Point B) in the original paper to get the plastic section modulus (Z_s) of a circular HSS.