Tables for Eccentrically Loaded WT Shapes in Compression

MARK E. GORDON

ABSTRACT

WT shapes are often used for bracing, and they are typically connected to their supports with gusset plates on the flange. This attachment creates an eccentric axial load on the WT, which is not considered by design tables in the 13th edition AISC *Manual of Steel Construction.* This paper demonstrates one method for generating design tables that account for this eccentric loading.

Keywords: WT shapes, compression, eccentric loading.

Horizontal WT braces commonly connect to their supports with gusset plates to the WT flange. This connection creates an eccentric axial loading. The axial compression tables in Section 4 of the 13th edition of the AISC *Steel Construction Manual* do not consider connection eccentricity. An Excel spreadsheet was developed to generate allowable stress design (ASD) and load and resistance factor design (LRFD) tables to assist engineers in considering these eccentric connections using the 13th edition *Manual*. Tables are located at the end of this paper. The available strengths in Table 1 (ASD) and Table 2 (LRFD) for WTs were determined by inputting different lengths and loads until a maximum load was found for which the WT still passed. The reduction factors in Table 3 (ASD) and Table 4 (LRFD) were developed by taking the maximum allowable P load with the eccentric connection and dividing by the maximum allowable P load without the eccentric connection. These factors are useful in reducing the allowable stresses in analysis and design programs, instead of having to check the WTs with eccentricities by hand. The tables were developed with the following assumptions:

1. ASD and LRFD, 2005 AISC *Specification for Structural Steel Buildings*.
2. WT member yield strength, F_y, of 50 ksi.
3. The WT members are horizontal, connected to a gusset plate at the flange, with the gusset plate on top.
4. Gusset plates are ½ in. thick.
5. The ends of the WT are pinned ($K = 1$).
6. Eccentricity taken from centroid of WT to the centroid of the gusset plate.
7. Design moment includes self-weight of WT.
8. For the LRFD method, a dead load factor of 1.2 is applied to the self-weight of the member.

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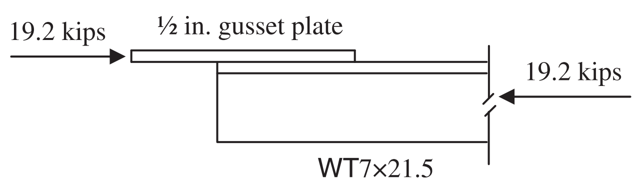
The following examples demonstrate the procedure that is incorporated in the spreadsheet that was used to make the tables. Equation numbers refer to the 2005 AISC *Specification*.

EXAMPLE 1

Slender WT in Compression Using ASD



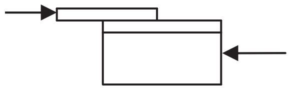
Given:

A 25 ft. horizontal WT7×21.5 brace with an axial compression load of 19.2 kips that is connected on top of the flange with a ½-in. gusset plate.

WT7×21.5 properties from Table 1-8 of the AISC *Manual* and from the AISC Shapes Database:

$A_g = 6.31$ in.^{2}
$d = 6.83$ in.
$t_w = 0.305$ in.
$b_f = 8.00$ in.
$t_f = 0.530$ in.
$I_x = 21.9$ in.^{4}
$S_x = 3.98$ in.^{3}
$r_x = 1.86$ in.
$\bar{y} = 1.31$ in.
$Z_x = 7.05$ in.^{3}
$I_y = 22.6$ in.^{4}

Table 1 (ASD)



Horizontal WT Shapes

Available Strength (P_n/Ω_c)
for Compression Loads* with Connection Eccentricity
(kips)

F_y = 50 ksi

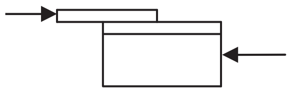
Shape	Span Length (ft)															
	2.5	**5.0**	**7.5**	**10.0**	**12.5**	**15.0**	**17.5**	**20.0**	**22.5**	**25.0**	**27.5**	**30.0**	**32.5**	**35.0**	**37.5**	**40.0**
WT4×9	27.1	24.0	19.9	15.4	11.4	8.64	6.62									
WT5×11	31.7	30.0	26.8	22.6	18.3	14.3	11.3	9.00								
WT5×13	38.4	36.1	31.8	26.7	21.5	16.9	13.3	10.6	8.49							
WT5×15	47.1	43.2	37.6	31.4	25.2	19.7	15.6	12.4	10.0							
WT6×11	30.0	27.3	22.2	16.2	11.6											
WT6×13	31.6	30.7	29.0	26.4	23.2	19.8	16.5	13.5	11.0	9.04						
WT6×15	40.9	39.5	36.8	32.8	28.3	23.7	19.3	15.6	12.8	10.5						
WT6×17.5	52.6	50.5	46.4	40.7	34.6	28.5	23.0	18.6	15.2	12.5						
WT6×20	58.1	55.7	51.1	44.1	37.0	30.3	24.3	19.5	15.8	12.9						
WT6×22.5	68.7	65.6	59.4	51.1	42.6	34.7	27.7	22.4	18.2	14.8						
WT6×25	78.0	73.3	65.5	56.4	47.2	38.5	30.9	25.0	20.3	16.6						
WT7×11	23.1	22.3	20.6	18.0	14.9	11.7										
WT7×13	31.3	30.1	27.4	23.5	19.2	14.8	11.6									
WT7×15	37.8	36.9	35.1	32.2	28.4	24.4	20.3	16.5	13.6							
WT7×17	45.7	44.5	42.0	38.2	33.5	28.6	23.7	19.3	15.9	13.1						
WT7×19	54.6	53.0	49.6	44.6	38.8	32.8	26.8	21.9	18.0	14.9						
WT7×21.5	61.7	59.9	56.6	51.6	45.6	39.5	33.7	28.1	23.2	19.3	16.0	13.4				
WT7×24	73.0	70.7	66.3	59.9	52.7	45.4	38.3	31.7	26.2	21.8	18.2	15.2				
WT7×26.5	82.9	80.0	74.6	67.1	58.8	50.3	42.2	34.8	28.8	24.0	20.0	16.8				
WT7×30.5	93.0	89.9	84.4	74.8	64.5	54.5	45.0	36.8	30.3	25.1	20.8					
WT7×34	108	103	94.1	83.3	71.9	60.8	50.2	41.2	33.9	28.1	23.3	19.4				
WT7×45	138	130	117	102	85.9	70.9	57.3	46.4	37.8	30.8	25.3					
WT8×33.5	102	99.3	95.6	90.1	82.5	73.4	64.3	55.6	47.5	40.1	34.0	28.9	24.6	21.0		
WT8×38.5	123	121	116	108	97.6	86.4	75.4	64.8	54.9	46.5	39.5	33.6	28.7	24.5		
WT8×44.5	148	143	135	125	113	100	87.6	75.6	64.2	54.5	46.4	39.6	33.8	29.0	24.8	
WT8×50	164	159	150	139	126	112	97.9	84.6	72.1	61.3	52.2	44.6	38.2	32.7	28.1	
WT9×38	114	112	109	104	98.0	90.2	81.6	73.0	64.5	56.2	48.5	41.7	36.0	31.1	26.9	23.3
WT9×43	137	135	130	124	116	106	95.0	84.3	73.8	64.0	55.0	47.4	40.9	35.4	30.6	26.5
WT9×48.5	164	159	152	143	132	120	108	95.5	83.5	72.1	62.1	53.5	46.3	40.0	34.7	30.1
WT9×53	179	174	166	156	144	131	118	105	91.8	79.6	68.6	59.2	51.3	44.4	38.6	33.5
WT9×59.5	199	194	185	174	161	147	132	117	103	89.4	77.1	66.7	57.8	50.2	43.6	37.9
WT9×65	214	208	199	187	173	157	141	125	109	94.9	81.8	70.7	61.2	53.1	46.0	39.9
WT9×71.5	232	226	216	203	188	171	154	137	120	105	90.4	78.2	67.8	58.8	51.1	44.4
WT10.5×83	278	272	262	250	235	219	201	183	165	148	131	115	101	88.9	78.2	68.9

*Based on the following:

- Horizontal WT member attached to a ½-in. gusset plate
- K = 1 (pinned ends)
- Ω_c = 1.67

Note: Strength values only shown for KL/r_{min} < 200.

Table 2 (LRFD)



Horizontal WT Shapes

Available Strength ($\phi_c P_n$)
for Compression Loads* with Connection Eccentricity
(kips)

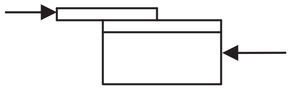
F_y = 50 ksi

Shape	Span Length (ft)															
	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	40.0
WT4×9	40.8	36.3	30.3	23.6	17.7	13.5	10.4									
WT5×11	47.7	45.3	40.6	34.5	28.1	22.1	17.5	14.1								
WT5×13	57.8	54.5	48.3	40.7	33.1	26.0	20.7	16.6	13.4							
WT5×15	70.9	65.2	57.1	47.9	38.8	30.5	24.2	19.5	15.8							
WT6×11	45.1	41.1	33.5	24.4	17.5											
WT6×13	47.5	46.3	43.9	40.0	35.3	30.4	25.5	20.9	17.2	14.2						
WT6×15	61.6	59.6	55.7	49.8	43.2	36.3	29.8	24.3	19.9	16.5						
WT6×17.5	79.2	76.2	70.2	61.9	52.9	43.8	35.5	28.9	23.8	19.7						
WT6×20	87.4	84.1	77.5	67.3	56.9	46.9	37.8	30.7	25.1	20.6						
WT6×22.5	103	99.0	90.2	78.0	65.5	53.6	43.2	35.1	28.7	23.7						
WT6×25	117	111	99.3	86.1	72.5	59.6	48.1	39.1	32.1	26.5						
WT7×11	34.8	33.6	31.0	27.1	22.5	17.7										
WT7×13	47.1	45.3	41.3	35.6	29.1	22.5	17.6									
WT7×15	56.8	55.5	52.9	48.7	43.2	37.1	31.0	25.4	21.0							
WT7×17	68.8	67.0	63.5	57.9	51.0	43.6	36.3	29.7	24.5	20.4						
WT7×19	82.1	79.8	74.9	67.6	59.0	50.0	41.1	33.6	27.8	23.1						
WT7×21.5	92.8	90.3	85.6	78.3	69.7	60.8	52.1	43.8	36.4	30.4	25.6	21.6				
WT7×24	110	107	100	91.1	80.5	69.8	59.3	49.4	41.2	34.5	29.0	24.5				
WT7×26.5	125	121	113	102	89.8	77.3	65.2	54.2	45.2	37.9	31.9	27.0				
WT7×30.5	140	136	128	114	98.8	83.9	69.8	57.5	47.7	39.8	33.3					
WT7×34	163	155	142	127	110	93.6	77.9	64.3	53.4	44.6	37.4	31.5				
WT7×45	207	195	177	155	132	110	89.2	72.8	59.8	49.4	41.0					
WT8×33.5	153	150	144	137	126	112	99.1	86.2	74.0	63.0	53.8	46.1	39.6	34.1		
WT8×38.5	186	182	175	164	149	132	116	100	85.5	72.9	62.3	53.5	46.0	39.7		
WT8×44.5	222	215	204	190	172	153	135	117	100	85.4	73.2	62.9	54.3	46.9	40.6	
WT8×50	247	239	227	210	191	171	151	131	112	95.9	82.3	70.9	61.2	52.9	45.9	
WT9×38	171	169	164	158	149	138	125	112	100	87.7	76.1	66.0	57.3	49.9	43.6	38.1
WT9×43	206	203	197	188	176	161	146	130	115	99.8	86.3	74.9	65.2	56.8	49.6	43.4
WT9×48.5	247	240	230	216	200	183	166	147	129	112	97.3	84.5	73.6	64.2	56.1	49.2
WT9×53	269	262	250	236	219	200	181	162	142	124	108	93.5	81.5	71.2	62.3	54.6
WT9×59.5	299	291	279	263	244	224	203	181	160	139	121	105	91.8	80.3	70.3	61.7
WT9×65	321	313	300	282	262	239	216	192	170	148	128	112	97.4	85.1	74.5	65.3
WT9×71.5	349	340	326	308	286	261	236	211	186	163	142	123	108	94.3	82.7	72.5
WT10.5×83	418	409	396	378	356	333	307	281	255	229	204	180	159	141	125	111

*Based on the following:
- Horizontal WT member attached to a ½-in. gusset plate
- K = 1 (pinned ends)
- ϕ_c = 0.90

Note: Strength values only shown for KL/r_{min} < 200.

Table 3 (ASD)



Horizontal WT Shapes

Reduction Factor for Compression Loads*
with Connection Eccentricity
$P_r/(P_n/\Omega_c)$

F_y = 50 ksi

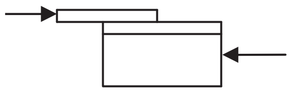
Shape	Span Length (ft)															
	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	40.0
WT4×9	0.391	0.389	0.398	0.439	0.501	0.545	0.568									
WT5×11	0.479	0.468	0.466	0.481	0.512	0.558	0.593	0.613								
WT5×13	0.435	0.425	0.428	0.445	0.479	0.527	0.562	0.582	0.590							
WT5×15	0.390	0.394	0.406	0.428	0.468	0.520	0.556	0.578	0.587							
WT6×11	0.539	0.563	0.624	0.709	0.776											
WT6×13	0.581	0.573	0.563	0.559	0.562	0.572	0.589	0.613	0.628	0.634						
WT6×15	0.519	0.511	0.504	0.508	0.521	0.543	0.575	0.604	0.620	0.626						
WT6×17.5	0.463	0.455	0.454	0.466	0.488	0.521	0.563	0.593	0.611	0.618						
WT6×20	0.436	0.421	0.408	0.416	0.432	0.458	0.495	0.520	0.533	0.536						
WT6×22.5	0.403	0.389	0.383	0.395	0.417	0.451	0.491	0.517	0.531	0.535						
WT6×25	0.376	0.372	0.378	0.390	0.411	0.445	0.485	0.512	0.527	0.532						
WT7×11	0.663	0.665	0.676	0.700	0.733	0.772										
WT7×13	0.606	0.609	0.626	0.658	0.701	0.749	0.783									
WT7×15	0.588	0.584	0.580	0.584	0.598	0.619	0.649	0.679	0.699							
WT7×17	0.550	0.545	0.543	0.549	0.565	0.589	0.622	0.654	0.675	0.687						
WT7×19	0.510	0.505	0.505	0.516	0.537	0.568	0.608	0.642	0.664	0.676						
WT7×21.5	0.487	0.477	0.465	0.459	0.458	0.463	0.473	0.493	0.515	0.528	0.532	0.529				
WT7×24	0.447	0.438	0.428	0.426	0.430	0.440	0.459	0.486	0.509	0.523	0.528	0.526				
WT7×26.5	0.419	0.410	0.403	0.404	0.411	0.427	0.452	0.484	0.507	0.520	0.526	0.524				
WT7×30.5	0.415	0.403	0.387	0.393	0.405	0.425	0.455	0.486	0.506	0.517	0.519					
WT7×34	0.378	0.373	0.376	0.384	0.397	0.418	0.450	0.482	0.502	0.513	0.516	0.511				
WT7×45	0.361	0.361	0.366	0.376	0.395	0.424	0.462	0.489	0.503	0.508	0.503					
WT8×33.5	0.472	0.464	0.452	0.439	0.433	0.437	0.445	0.457	0.475	0.495	0.508	0.514	0.514	0.508		
WT8×38.5	0.429	0.421	0.410	0.401	0.403	0.412	0.425	0.444	0.470	0.491	0.504	0.511	0.512	0.507		
WT8×44.5	0.397	0.396	0.393	0.391	0.396	0.404	0.418	0.436	0.462	0.483	0.498	0.505	0.507	0.503	0.495	
WT8×50	0.389	0.388	0.386	0.386	0.391	0.400	0.413	0.432	0.458	0.480	0.495	0.503	0.505	0.502	0.495	
WT9×38	0.499	0.493	0.483	0.472	0.462	0.457	0.455	0.456	0.461	0.471	0.486	0.498	0.504	0.505	0.501	0.494
WT9×43	0.456	0.450	0.441	0.431	0.426	0.424	0.427	0.434	0.447	0.464	0.483	0.495	0.502	0.503	0.500	0.493
WT9×48.5	0.412	0.411	0.409	0.408	0.408	0.410	0.414	0.424	0.440	0.461	0.480	0.493	0.500	0.501	0.499	0.492
WT9×53	0.407	0.406	0.405	0.405	0.405	0.407	0.411	0.420	0.435	0.455	0.475	0.488	0.496	0.498	0.496	0.490
WT9×59.5	0.399	0.398	0.397	0.397	0.398	0.400	0.406	0.417	0.432	0.452	0.472	0.486	0.494	0.498	0.496	0.490
WT9×65	0.388	0.387	0.386	0.386	0.386	0.391	0.399	0.411	0.426	0.447	0.466	0.480	0.487	0.490	0.488	0.482
WT9×71.5	0.381	0.381	0.380	0.380	0.382	0.387	0.395	0.406	0.421	0.441	0.461	0.475	0.483	0.487	0.485	0.479
WT10.5×83	0.395	0.395	0.395	0.396	0.397	0.399	0.403	0.409	0.417	0.429	0.443	0.460	0.473	0.482	0.486	0.487

*Based on the following:

- Horizontal WT member attached to a ½-in. gusset plate
- K = 1 (pinned ends)
- Ω_c = 1.67

Note: Strength values only shown for KL/r_{min} < 200.

Table 4 (LRFD)



Horizontal WT Shapes

Reduction Factor for Compression Loads* with Connection Eccentricity

$P_u/(\phi_c P_n)$

F_y = 50 ksi

Shape	Span Length (ft)															
	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	40.0
WT4×9	0.391	0.392	0.404	0.448	0.515	0.564	0.594									
WT5×11	0.479	0.470	0.470	0.488	0.523	0.573	0.612	0.637								
WT5×13	0.435	0.427	0.432	0.452	0.490	0.542	0.581	0.607	0.621							
WT5×15	0.390	0.396	0.410	0.435	0.478	0.534	0.575	0.603	0.618							
WT6×11	0.539	0.564	0.626	0.713	0.781											
WT6×13	0.582	0.574	0.566	0.564	0.570	0.584	0.605	0.632	0.652	0.663						
WT6×15	0.520	0.512	0.507	0.513	0.529	0.554	0.590	0.623	0.644	0.656						
WT6×17.5	0.463	0.456	0.457	0.471	0.496	0.532	0.578	0.612	0.634	0.647						
WT6×20	0.437	0.423	0.412	0.422	0.441	0.472	0.513	0.543	0.562	0.571						
WT6×22.5	0.403	0.390	0.387	0.401	0.426	0.464	0.508	0.539	0.559	0.569						
WT6×25	0.376	0.373	0.381	0.395	0.420	0.457	0.502	0.534	0.555	0.565						
WT7×11	0.663	0.666	0.678	0.703	0.738	0.779										
WT7×13	0.606	0.610	0.628	0.661	0.706	0.756	0.792									
WT7×15	0.589	0.585	0.582	0.588	0.604	0.628	0.660	0.694	0.717							
WT7×17	0.551	0.546	0.545	0.553	0.571	0.598	0.634	0.670	0.694	0.710						
WT7×19	0.510	0.506	0.507	0.520	0.544	0.577	0.620	0.657	0.683	0.699						
WT7×21.5	0.487	0.478	0.468	0.463	0.466	0.473	0.487	0.511	0.538	0.556	0.565	0.568				
WT7×24	0.448	0.439	0.431	0.431	0.437	0.450	0.472	0.504	0.531	0.550	0.560	0.563				
WT7×26.5	0.420	0.412	0.406	0.409	0.419	0.437	0.465	0.501	0.529	0.547	0.558	0.562				
WT7×30.5	0.415	0.404	0.390	0.398	0.413	0.436	0.470	0.505	0.530	0.546	0.553					
WT7×34	0.378	0.374	0.379	0.389	0.404	0.429	0.464	0.500	0.526	0.542	0.550	0.551				
WT7×45	0.361	0.362	0.369	0.382	0.403	0.436	0.478	0.510	0.531	0.541	0.543					
WT8×33.5	0.473	0.465	0.454	0.443	0.439	0.445	0.456	0.472	0.493	0.517	0.534	0.545	0.550	0.549		
WT8×38.5	0.429	0.422	0.412	0.404	0.409	0.420	0.436	0.458	0.487	0.512	0.530	0.541	0.547	0.547		
WT8×44.5	0.398	0.397	0.395	0.395	0.401	0.412	0.428	0.449	0.478	0.504	0.523	0.535	0.541	0.543	0.539	
WT8×50	0.389	0.389	0.388	0.390	0.397	0.408	0.423	0.445	0.474	0.500	0.519	0.532	0.539	0.541	0.538	
WT9×38	0.499	0.493	0.485	0.475	0.467	0.464	0.464	0.468	0.475	0.489	0.508	0.524	0.534	0.540	0.541	0.538
WT9×43	0.456	0.450	0.442	0.434	0.430	0.431	0.435	0.445	0.461	0.481	0.504	0.520	0.531	0.537	0.539	0.536
WT9×48.5	0.412	0.411	0.411	0.411	0.413	0.416	0.422	0.435	0.454	0.478	0.501	0.517	0.529	0.535	0.537	0.535
WT9×53	0.407	0.407	0.407	0.408	0.409	0.413	0.420	0.431	0.449	0.472	0.495	0.513	0.524	0.531	0.534	0.532
WT9×59.5	0.399	0.399	0.399	0.400	0.402	0.406	0.414	0.428	0.446	0.469	0.492	0.510	0.523	0.530	0.533	0.532
WT9×65	0.388	0.388	0.388	0.389	0.391	0.398	0.408	0.421	0.440	0.463	0.487	0.504	0.516	0.523	0.525	0.524
WT9×71.5	0.381	0.381	0.382	0.383	0.386	0.393	0.403	0.416	0.434	0.457	0.481	0.499	0.511	0.519	0.522	0.521
WT10.5×83	0.395	0.396	0.397	0.398	0.400	0.404	0.410	0.418	0.428	0.442	0.459	0.479	0.496	0.508	0.517	0.521

*Based on the following:

- Horizontal WT member attached to a ½-in. gusset plate
- K = 1 (pinned ends)
- ϕ_c = 0.90

Note: Strength values only shown for KL/r_{min} < 200.

$r_y\ = 1.89$ in.
$Q_s = 0.776$
$J\ = 0.522$ in.^{4}
$\bar{r}_o\ = 2.86$ in. (Equation E4-7, $\bar{r}_o^2$)
$H\ = 0.865$ (Equation E4-8)

$S_{xc} = I_x/y_c = 21.9/1.31 = 16.72$ in.^{3}

Check for slender elements:

From Table B4.1 Case 8,

$$\frac{d}{t_w} = \frac{6.83}{0.305} = 22.4 > \lambda_r = 0.75\sqrt{\frac{E}{F_y}} = 0.75\sqrt{\frac{29{,}000}{50}} = 18.1$$

Therefore, the web is slender.

From Table B4.1 Case 3,

$$\frac{b_f}{2t_f} = \frac{8.00}{2(0.530)} = 7.5 < \lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.5$$

Therefore, the flange is noncompact.

There are slender elements. Specification Section E7 is applicable.

The cross section is composed of only unstiffened compression elements. Therefore, $Q_a = 1.0$.

$Q = Q_sQ_a = (0.776)(1.0) = 0.776$

Flexural buckling about the x-x axis:

$$\frac{KL}{r_x} = \frac{1.0(25\text{ft})(12\text{ in./ft})}{1.86\text{ in.}} = 161.3$$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{(0.776)50}} = 128.8 < 161.3$$

Therefore, Equation E7-3 applies.

From Equation E3-4,

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r_x}\right)^2} = \frac{\pi^2(29{,}000)}{(161.3)^2} = 11.0\text{ ksi}$$

From Equation E7-3,

$F_{cr} = 0.877F_e = 0.877(11.0) = 9.6$ ksi; **controls**

Flexural buckling about the y-y axis:

$$\frac{KL}{r_y} = \frac{1.0(25\text{ ft})(12\text{ in./ft})}{1.89\text{ in.}} = 158.7$$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{(0.776)50}} = 128.8 < 158.7$$

Therefore, Equation E7-3 applies.

From Equation E3-4,

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r_y}\right)^2} = \frac{\pi^2(29{,}000)}{(158.7)^2} = 11.4\text{ ksi}$$

From Equation E7-3,

$F_{cr} = 0.877F_e = 0.877(11.4) = 10.0$ ksi; **does not control**

Torsional and flexural-torsional buckling of members with slender elements:

From Equation E4-11,

$$F_{ez} = \left[\frac{\pi^2 EC_w}{(K_zL)^2} + GJ\right]\frac{1}{A_g\bar{r}_o^2}$$

Omit term with C_w per User Note at end of Section E4.

$$F_{ez} = \frac{GJ}{A_g\bar{r}_o^2} = \frac{11{,}200(0.522)}{6.31(2.86)^2} = 113.3\text{ ksi}$$

Calculate F_e using Equation E4-5,

$$F_e = \left(\frac{F_{ey}+F_{ez}}{2H}\right)\left[1-\sqrt{1-\frac{4F_{ey}F_{ez}H}{(F_{ey}+F_{ez})^2}}\right]$$

$$= \left(\frac{11.4+113.3}{2(0.865)}\right)\left[1-\sqrt{1-\frac{4(11.4)(113.3)(0.865)}{(11.4+113.3)^2}}\right]$$

$= 11.2$ ksi

$0.44QF_y = 0.44(0.776)(50) = 17.1 > 11.2$ ksi

Therefore, use equation E7-3.

$F_{cry} = 0.877F_e = 9.8$ ksi; **does not control**

Nominal compressive strength:

$P_n = F_{cr}A_g = (9.6)(6.31) = 60.9$ kips

Calculate the required flexural strength:

Moment due to axial load, $M_{ecc} = P(y + y')$ where $y' = ¼$ in., half of ½-in.-thick gusset plate.

$M_{ecc} = (19.2)(1.31 + 0.25)$
$\quad = 30.0$ kip-in.

Moment due to weight of WT, $M_0 = wL^2/8$

$M_0 = 20.2$ kip-in.

$M_{nt} = M_{ecc} + M_0 = 30.0 + 20.2 = 50.1$ kip-in. (ASD)

Second-order effects with C_m based on Section C2.1b,

$\alpha = 1.6$ (ASD)

$C_m = 1$

From Equation C2-5,

$$P_{e1} = \frac{\pi^2 EI}{\left(K_1 L\right)^2} = \frac{\pi^2 (29{,}000)(21.9)}{\left[(25)(12)\right]^2} = 69.6 \text{ kips}$$

From Equation C2-2,

$$B_1 = \frac{1}{1 - \alpha P_r / P_{e1}} \geq 1.0$$

$$B_1 = \frac{1}{1 - 1.6(19.2)/69.6} = 1.79 \text{ (ASD)}$$

$M_1 = B_1 M_{nt} = (1.79)(50.1) = 89.7$ kip-in (ASD)

Calculate the nominal flexural strength:

Flexural yielding limit state is $M_p = F_y Z_c < 1.6M_y$

Using Equation F9-2,

$M_p = F_y Z_x < 1.6M_y$, for stems in tension

$1.6M_y = 1.6F_y S_x = 1.6(50)(3.98) = 318.4$ kip-in.

$M_p = F_y Z_x = (50)(7.05) = 352.5$ kip-in.

From Equation F9-1,

$M_n = M_p = 318.4$ kip-in.; **controls**

Flange local buckling limit state:

Check flange compactness using Table B4.1 Case 7,

$$\lambda = \frac{b_f}{2t_f} = \frac{8.0}{2(0.530)} = 7.5$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.2 > 7.5$$

Therefore, the flange is compact.

Check flange slenderness using Table B4.1 Case 7,

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29{,}000}{50}} = 24.08 > 7.5$$

Therefore, the flange is not slender.

Calculate critical flange local buckling stress (only applicable if noncompact or slender),

For noncompact sections (Equation F9-7),

$$F_{cr} = F_y\left(1.19 - 0.50\left(\frac{b_f}{2t_f}\right)\sqrt{\frac{F_y}{E}}\right)$$

For slender sections (Equation F9-8),

$$F_{cr} = 0.69\frac{E}{\left(\frac{b_f}{2t_f}\right)^2}$$

Calculate the nominal flexural strength (Equation F9-6),

$M_n = F_{cr} S_x$ not applicable

Lateral-torsional buckling:

From Equation F9-4,

$$M_n = M_{cr} = \pi\frac{\sqrt{EI_y GJ}}{L_b}\left[B + \sqrt{\left(1 + B^2\right)}\right]$$

From Equation F9-5,

$$B = \pm 2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}}$$

$$= +2.3\left(\frac{6.83}{25(12)}\right)\sqrt{\left(\frac{22.6}{0.522}\right)}$$

$$= +0.345$$

$$B + \sqrt{\left(1 + B^2\right)} = \left[+0.345 + \sqrt{\left(1 + \left(+0.345\right)^2\right)}\right]$$
$$= 1.403$$

$$M_n = M_{cr} = \pi\frac{\sqrt{\left(29{,}000\right)\left(22.6\right)\left(11{,}200\right)\left(0.522\right)}}{25(12)}\left(1.403\right)$$

$M_n = 909.0$ k-in.; **does not control**

Design of WT member for combined forces:

Since $I_{yc}/I_y \approx 1.0 > 0.9$, use H2-1.

From Equation H2-1,

$$\frac{f_a}{F_a} + \frac{f_{bw}}{F_{bw}} \leq 1.0$$

Which can be rewritten for ASD as,

$$\frac{P_r}{\left(P_n/\Omega_c\right)}+\frac{M_r}{\left(M_n/\Omega_b\right)}\le 1.0$$

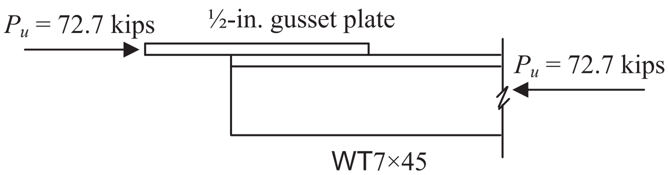
$$\frac{19.2}{\left(60.9/1.67\right)}+\frac{89.7}{\left(318.4/1.67\right)}=1.0\le 1.0$$

Calculate the reduction factor for the compression load with connection eccentricity:

$$\frac{P_r}{\left(P_n/\Omega_c\right)}=\frac{19.2}{\left(60.9/1.67\right)}=0.528$$

EXAMPLE 2

Nonslender WT in Compression and Noncompact in Bending Using LRFD



Given:
A 20-ft. horizontal WT7×45 brace with an ultimate axial compression load of 72.7 kips that is connected on top of the flange with a ½-in. gusset plate.

WT7×45 Properties from Table 1-8 of the AISC *Manual* and the AISC Shapes Database:

A_g = 13.20 in.^{2}
d = 7.01 in.
t_w = 0.440 in.
b_f = 14.50 in.
t_f = 0.710 in.
I_x = 36.5 in.^{4}
S_x = 6.16 in.^{3}
r_x = 1.66 in.
$\bar{y}$ = 1.09 in.
Z_x = 11.50 in.^{3}
I_y = 181 in.^{4}
r_y = 3.70 in.
Q_s = 1.0
J = 2.030 in.^{4}
$\bar{r}_o$ = 4.12 in. (Equation E4-7, $\bar{r}_o^2$)
H = 0.968 (Equation E4-8)

$S_{xc} = I_x/y_c$ = 36.5/1.09 = 33.49 in.^{3}

Check for slender elements:

From Table B4.1 Case 8,

$$\frac{d}{t_w}=\frac{7.01}{0.440}=15.9<\lambda_r=0.75\sqrt{\frac{E}{F_y}}=0.75\sqrt{\frac{29{,}000}{50}}=18.1$$

Therefore, the web is noncompact.

From Table B4.1 Case 3,

$$\frac{b_f}{2t_f}=\frac{14.50}{2(0.710)}=10.2<\lambda_r=0.56\sqrt{\frac{E}{F_y}}=0.56\sqrt{\frac{29{,}000}{50}}=13.5$$

Therefore, the flange is noncompact.

There are no slender elements. AISC *Specification* Sections E3 and E4 apply.

Flexural buckling about the x-x axis:

$$\frac{KL}{r_x}=\frac{1.0(20\text{ ft})(12\text{ in./ft})}{1.66\text{ in.}}=144.6$$

$$4.71\sqrt{\frac{E}{F_y}}=4.71\sqrt{\frac{29{,}000}{50}}=113.4<144.6$$

Therefore, Equation E3-3 applies.

From Equation E3-4,

$$F_e=\frac{\pi^2E}{\left(\dfrac{KL}{r_x}\right)^2}=\frac{\pi^2(29{,}000)}{\left(144.6\right)^2}=13.7\text{ ksi}$$

From Equation E3-3,

$F_{cr} = 0.877F_e = 0.877(13.7) = 12.0$ ksi; **controls**

Flexural buckling about the y-y axis:

$$\frac{KL}{r_y}=\frac{1.0(20\text{ ft})(12\text{ in./ft})}{3.70\text{ in.}}=64.9$$

$$4.71\sqrt{\frac{E}{F_y}}=4.71\sqrt{\frac{29{,}000}{50}}=113.4>64.9$$

Therefore, Equation E3-2 applies.

From Equation E3-4,

$$F_e=\frac{\pi^2E}{\left(\dfrac{KL}{r_y}\right)^2}=\frac{\pi^2(29{,}000)}{\left(64.9\right)^2}=68.0\text{ ksi}$$

From Equation E3-2,

$$F_{cry} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{50}{68.0}}\right] 50$$

$F_{cry} = 36.8$ ksi; **does not control**

Torsional and flexural-torsional buckling of members without slender elements:

From Equation E4-3,

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2} = \frac{11,200(2.030)}{13.20\left(4.12\right)^2} = 101.5 \text{ ksi}$$

From Equation E4-2,

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{\left(F_{cry} + F_{crz}\right)^2}}\right]$$

$$= \left(\frac{36.8 + 101.5}{2(0.968)}\right)\left[1 - \sqrt{1 - \frac{4(36.8)(101.5)(0.968)}{\left(36.8 + 101.5\right)^2}}\right]$$

$F_{cr} = 36.1$ ksi; **does not control**

Nominal compressive strength:

$P_n = F_{cr}A_g = (12.0)(13.20) = 158.5$ kips

Calculate the required flexural strength:

Moment due to axial load, $M_{ecc} = P(y + y')$ where $y' = ¼$ in., half of ½-in.-thick gusset plate.

$M_{ecc} = (72.7)(1.09 + 0.25)$

$= 97.4$ kip-in.

Moment due to weight of WT, $M_0 = wL^2/8$

$M_0 = 27.0$ kip-in.

$M_{nt} = 1.2M_0 + M_{ecc}$
$= 1.2(27.0) + 97.4 = 129.8$ kip-in. (LRFD)

Second-order effects with C_m based on section C2.1b,

$\alpha = 1.0$ (LRFD)

$C_m = 1$

From Equation C2-5,

$$P_{e1} = \frac{\pi^2 EI_x}{\left(K_1 L\right)^2} = \frac{\pi^2 (29,000)(36.5)}{\left[(20)(12)\right]^2} = 181.4 \text{ kips}$$

From Equation C2-2,

$$B_1 = \frac{1}{1 - \alpha P_r / P_{e1}} \geq 1.0$$

$$B_1 = \frac{1}{1 - 1.0(72.7)/181.4} = 1.67 \quad \text{(LRFD)}$$

$M_1 = B_1 M_{nt} = (1.67)(129.8) = 216.7$ kip-in (LRFD)

Calculate the nominal flexural strength:

Flexural yielding limit state is $M_p = F_y Z_x < 1.6M_y$

Using Equation F9-2,

$M_p = F_y Z_x < 1.6\ M_y$ for stems in tension

$1.6M_y = 1.6F_y S_x = 1.6(50)(6.16) = 492.8$ kip-in.

$M_p = F_y Z_x = (50)(11.50) = 575$ kip-in.

From Equation F9-1,

$M_n = M_p = 492.8$ kip-in.; **controls**

Flange local buckling limit state:

Check flange compactness using Table B4.1 Case 7,

$$\lambda = \frac{b_f}{2t_f} = \frac{14.5}{2(0.710)} = 10.2$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.2 < 10.2$$

Therefore, the flange is noncompact.

Check flange slenderness using Table B4.1 Case 7,

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29,000}{50}} = 24.1 > 10.2$$

Therefore, the flange is noncompact.

Calculate critical flange local buckling stress:

For noncompact sections (Equation F9-7),

$$F_{cr} = F_y\left(1.19 - 0.50\left(\frac{b_f}{2t_f}\right)\sqrt{\frac{F_y}{E}}\right)$$

$$= 50\left(1.19 - 0.50\left(\frac{14.5}{2(0.71)}\right)\sqrt{\frac{50}{29000}}\right)$$

$= 48.9$ ksi

Calculate the nominal flexural strength (Equation F9-6),
$M_n = F_{cr}S_{xc} = (48.9)(33.49)$

$M_n = 1637.5$ kip-in.; **does not control**

Lateral-torsional buckling:

From Equation F9-4,

$$M_n = M_{cr} = \pi \frac{\sqrt{EI_y GJ}}{L_b}\left[B + \sqrt{\left(1 + B^2\right)}\right]$$

From Equation F9-5,

$$B = \pm 2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}}$$

$$= +2.3\left(\frac{7.01}{20(12)}\right)\sqrt{\left(\frac{181}{2.03}\right)}$$

$$= +0.634$$

$$B + \sqrt{\left(1 + B^2\right)} = \left[+0.634 + \sqrt{\left(1 + \left(+0.634\right)^2\right)}\right]$$

$$= 1.818$$

$$M_n = M_{cr} = \pi \frac{\sqrt{\left(29{,}000\right)\left(181\right)\left(11{,}200\right)\left(2.03\right)}}{20(12)}\left(1.818\right)$$

$M_n = 8223.7$ k-in.; **does not control**

Design of WT member for combined forces:

Since $I_{yc}/I_y \approx 1.0 > 0.9$ use H2-1.

From Equation H2-1,

$$\frac{f_a}{F_a} + \frac{f_{bw}}{F_{bw}} \leq 1.0$$

which can be rewritten for LRFD as,

$$\frac{P_u}{(\phi_c P_n)} + \frac{M_u}{(\phi_b M_n)} \leq 1.0$$

$$\frac{72.7}{(0.9)(158.5)} + \frac{216.7}{(0.9)(492.8)} = 1.0 \leq 1.0$$

Calculate the reduction factor for the compression load with connection eccentricity:

$$\frac{P_u}{(\phi_c P_n)} = \frac{72.7}{(0.9)(158.5)} = 0.510$$

REFERENCES

AISC (2005), *Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL.