Stiffener Requirements to Prevent Edge Buckling

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ABSTRACT

Steel connection elements such as gusset plates and coped beam webs feature unsupported edges that are sometimes stiffened to prevent buckling. The stiffener requirements for structural elements with large aspect ratios are well established. However, for typical connections with an aspect ratio of ½ to 2, the existing requirements may not provide accurate results. The results from 123 elastic finite element models were analyzed to determine the stiffness requirements for edge stiffeners with smaller aspect ratios. A design procedure based on a simplified model is proposed as a starting point for stiffener design.

Keywords: edge stiffeners, coped beams, gusset plates.

In steel connection elements such as gusset plates and coped beam webs, unsupported edges are sometimes stiffened to prevent buckling. The top edges of coped beam webs are subjected to compressive flexural stresses as shown in Figure 1, which can cause a local instability. The AISC *Steel Construction Manual* (AISC, 2005) provides equations to determine the buckling capacity. To increase the critical stress of the coped section, the edge can be stiffened as shown in Figure 2.

In seismic applications, Nast et al. (1999), and Rabinovitch and Cheng (1993) showed that the addition of gusset plate edge stiffeners results in significantly improved energy absorption capability and a more stable post-buckling response. The AISC *Seismic Design Manual* (AISC, 2006) recommends stiffening the edge when the unsupported length exceeds

$$L_{fg} = 0.75t\sqrt{\frac{E}{F_y}} \tag{1}$$

where

L_{fg} = free length of the gusset plate at the edge, in., as shown in Figure 3
t = gusset plate thickness, in.
E = modulus of elasticity, ksi
F_y = yield stress of the material, ksi

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Design limits for the maximum unsupported length of the free edge of gusset plates are also provided by AASHTO (2004), Astaneh (1998), Reno and Duan (1997), and Caltrans (2001). When these limits are not met, the edge can be stiffened as shown in Figure 4.

For the stiffener to be effective, it must be stiff enough to alter the buckled shape of the plate. The stiffener requirements for structural elements with large aspect ratios are well established (AISI, 1997). In this paper, the aspect ratio is defined as the length-to-width ratio of the plate, *a*/*b*, as shown in Figure 5. For typical connections, the aspect ratio varies from ½ to 2, and the existing requirements may not provide accurate results. This paper will examine the effect of the aspect ratio on the stiffener requirements to prevent edge buckling.

The results from 123 elastic finite element models were used to determine the critical flexural stiffness of edge stiffeners. The effects of width-to-thickness ratio and aspect ratio of the braced element were studied, resulting in a simple equation that can be used to predict the minimum moment of inertia required to force the plate to buckle in a stiffened mode.

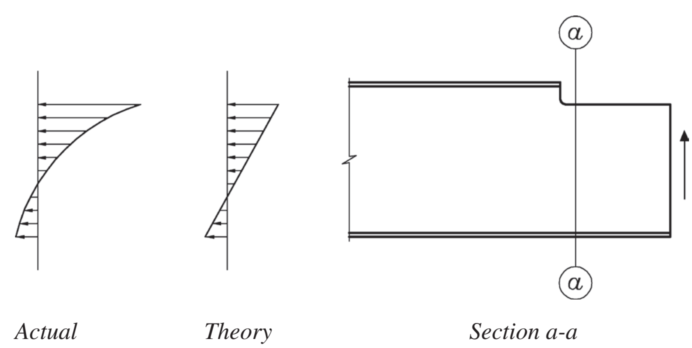


Fig. 1. Flexural stress at a beam cope.

BACKGROUND

Plate Buckling

For infinitely long plates, the well known plate buckling equation is (Galambos, 1998)

$$\sigma_c = k \frac{\pi^2 E}{12\left(1-\nu^2\right)\left(b/t\right)^2} \tag{2}$$

where

σ_c = critical stress, ksi
k = buckling coefficient
E = modulus of elasticity, ksi
ν = Poisson's ratio
b = plate width, in.
t = plate thickness, in.

For plates in pure compression with both non-loaded edges simply supported, $k = 4.00$. For plates in pure compression with one non-loaded edge simply supported and one free, $k = 0.425$.

Gerard and Becker (1957) presented equations for plates of finite length with simple supports at the loaded edges. For plates with one non-loaded edge simply supported and one free,

$$k = \frac{6}{\pi^2}\left[1 - \nu + \frac{\left(\pi b/\lambda\right)^2}{6}\right] \tag{3}$$

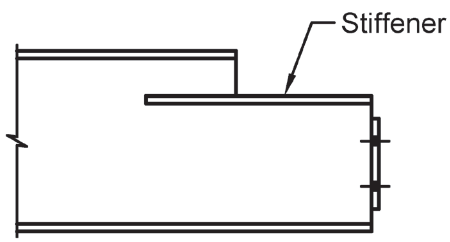


Fig. 2. Coped beam with a stiffened edge.

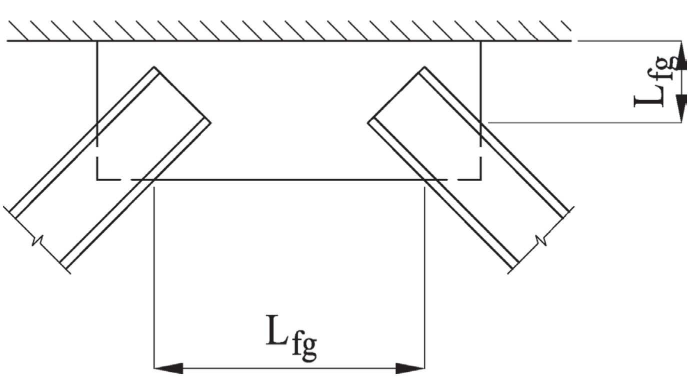


Fig. 3. Free length of gusset plate edges.

where

λ = a/m
a = plate length, in.
m = integer that gives the lowest k

For plates with both non-loaded edges simply supported

$$k = \left(\frac{\lambda}{b} + \frac{b}{\lambda}\right)^2 \tag{4}$$

Stiffener Requirements

Timoskenko and Gere (1961) solved the differential equation for a plate supported on both non-loaded edges by an elastic beam and provided a graphical solution for various values of beam stiffness. The CRC *Handbook of Structural Stability* (CRC, 1971) provides graphical solutions for plates simply supported on three edges and stiffened on one non-loaded edge. The curves indicate that the stiffener requirements increase as the aspect ratio of the plate increases.

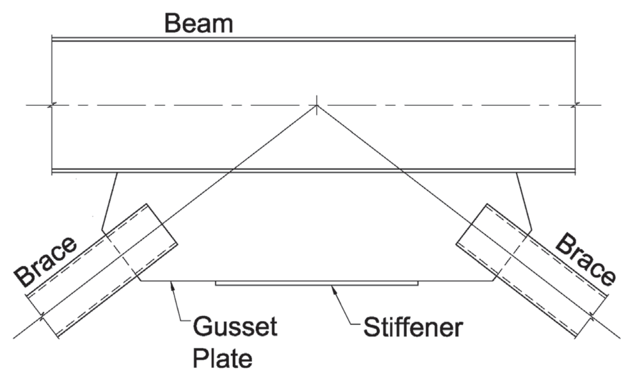


Fig. 4. Gusset plate with a stiffened edge.

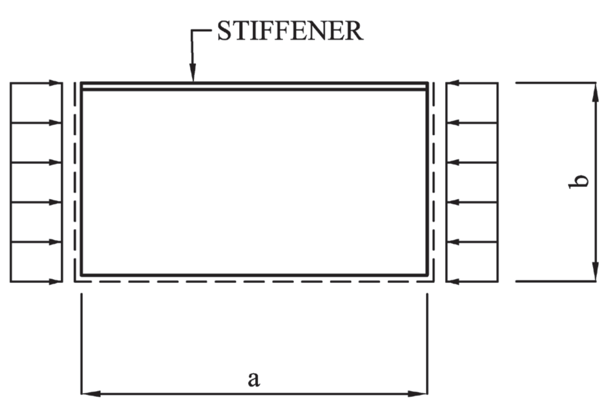


Fig. 5. Model used for finite element analysis.

Table 1. Critical Stress for Models with b = 25 in., ksi												
Model	***a*, in.**	**Stiffener Moment of Inertia, *I*, in.^{4}**										
		0	**1.254**	**2.508**	**5.017**	**10.03**	**15.05**	**20.07**	**30.10**	**40.14**	**60.20**	**Infinite**
25-1	12.50	187.6	253.6	268.1	274.6	278.1	279.5	280.2	281.1	281.6	282.2	286.0
25-2	18.75	91.56	140.4	162.3	178.1	187.4	190.9	192.9	–	–	–	185.4
25-3	25.00	59.04	91.68	115.8	141.6	161.2	169.0	173.3	178.0	180.6	183.5	195.0
25-4	50.00	27.96	35.46	45.36	63.28	93.44	117.9	138.4	170.0	184.0	187.7	212.1

Table 2. Critical Stress for Models with b = 35 in., ksi												
Model	***a*, in.**	**Stiffener Moment of Inertia, *I*, in.^{4}**										
		0	**1.880**	**3.760**	**7.521**	**15.04**	**22.56**	**30.08**	**45.13**	**60.17**	**90.25**	**Infinite**
35-1	17.50	94.49	129.5	134.5	137.3	139.0	139.6	140.0	140.5	140.7	141.0	142.8
35-2	26.25	46.57	73.49	82.37	90.69	94.89	96.51	97.40	–	–	–	93.83
35-3	35.00	30.06	49.11	61.34	73.60	82.40	85.94	87.89	90.03	91.26	92.69	102.3
35-4	70.00	14.28	19.27	25.01	35.37	52.23	65.66	76.40	91.14	92.57	94.29	105.7

Table 3. Critical Stress for Models with b = 45 in., ksi												
Model	***a*, in.**	**Stiffener Moment of Inertia, *I*, in.^{4}**										
		0	**2.480**	**4.961**	**9.921**	**19.84**	**29.76**	**39.68**	**59.53**	**79.37**	**119.1**	**Infinite**
45-1	22.50	56.78	77.93	80.76	82.38	83.33	83.71	83.93	84.18	84.36	84.53	85.56
45-2	33.75	28.09	44.87	50.64	54.64	57.04	57.98	58.49	–	–	–	56.47
45-3	45.00	18.16	30.36	37.69	44.76	49.78	51.78	52.87	54.11	54.82	55.64	61.09
45-4	90.00	8.644	12.04	15.74	22.40	33.16	41.49	47.93	54.62	55.42	56.40	62.84

According to Caltrans (2001), the moment of inertia of a gusset plate stiffener should be the largest of:

$$I_s = 1.83t^4\sqrt{\left(b/t\right)^2 - 144} \tag{5}$$

$$I_s = 9.2t^4 \tag{6}$$

These equations were originally specified in the 1962 edition of the *Specification for the Design of Light Gage Cold-Formed Steel Structural Members* (AISI, 1962) to provide a minimum stiffness for bracing the edge of an infinitely-long plate element within a member. The 1996 AISI *Specification* (AISI, 1997) has a more refined approach which is based on the research of Desmond et al. (1981), and Pekoz (1986). The newer provisions account for the post-buckling strength as well as the critical buckling capacity, and provide a method to calculate the effect of partially effective stiffeners.

FINITE ELEMENT MODELS

The study consisted of 123 models with width-to-thickness ratios, b/t, of 25, 35 and 45. The aspect ratios, a/b, were 0.50, 0.75, 1.00 and 2.00, and each of the 12 base model geometries were modeled with various stiffener sizes as shown in Tables 1, 2 and 3.

Figure 5 shows the loading and edge conditions for the finite element models. The simply supported edges, designated with dashed lines in Figure 5, were modeled with out-of-plane translation fixed and all three rotational degrees of freedom free. The stiffener was centered on the plate and a uniform axial stress was applied parallel to the stiffened edge. The modulus of elasticity was 29,000 ksi, and Poisson's ratio was 0.3.

The finite element program used for the buckling analysis is BASP, which was developed at the University of Texas at

Austin. The program uses a two-dimensional idealization. The analysis is performed in two steps. First, the in-plane analysis is performed to calculate the stresses arising from the applied loading. Using these stresses, an out-of-plane analysis is performed to solve for the critical buckling load. The program provides an elastic solution and does not account for pre-buckling deformations or initial out-of-flatness of the plate. The program is described in more detail in Akay et al. (1977).

The accuracy of the models were verified by comparing the critical loads from the program to the theoretical critical loads for the case where the plate has one non-loaded edge free and the remaining edges simply supported. The theoretical critical loads were calculated using Equation 2 with the buckling coefficient, k, from Equation 3. The critical loads from the finite element models were obtained using a 2.5-in. square mesh. Calculations were carried out for each of the 12 base model geometries used in this study and the ratio of BASP load to theoretical load varied from 0.98 to 1.01. Therefore, the 2.5-in. mesh size is adequate to capture the critical loads in sufficient accuracy and the remaining models used a maximum mesh size of 2.5-in.

RESULTS

The effect of stiffener moment of inertia on the critical load of the plates is shown in Tables 1, 2 and 3. The results are plotted in Figures 6, 7 and 8 for the models with plate width-to-thickness ratios, b/t, of 25, 35 and 45, respectively. The critical stress versus stiffener moment of inertia curves are nonlinear, and in each case, the plots show that the stiffener moment of inertia reaches a critical value where a further increase in stiffness provides only marginal gains in the critical buckling stress of the plate.

The sharp knee on the curves in Figures 6, 7 and 8 indicate the point where the plate buckling shape changed from the classical unstiffened mode to the stiffened mode. This can be seen by observing the buckled shapes of model 35-4 in Figures 9, 10 and 11, which show the effect of the stiffener moment of inertia on the buckled shape. Figure 9 shows that a stiffener with a moment of inertia of 1.88 in.^{4} does not restrain the lateral translation at the stiffened edge adequately. In Figure 10, the model with $I = 60.17$ in.^{4}, shows some lateral translation at the stiffened edge, but the stiffener's moment of inertia was large enough to alter the buckled shape. When the moment of inertia of the stiffener is increased to 90.25 in.^{4}, Figure 11 shows the lateral translation at the stiffened edge is very small; however, the critical stress was only about 2% higher than for the specimen with $I = 60.17$ in.^{4}

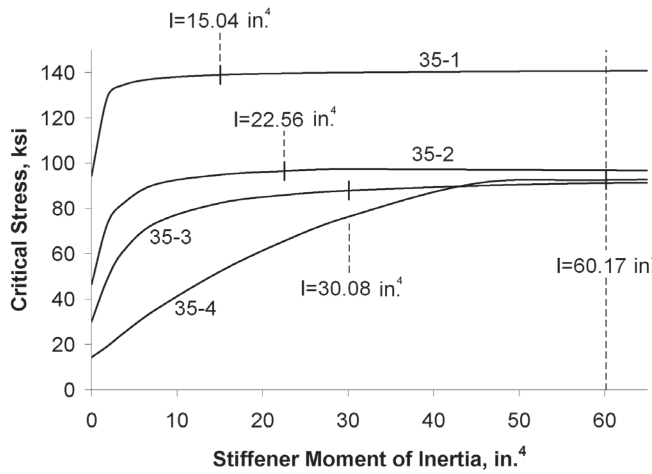


Fig. 7. Critical stress versus stiffener moment of inertia for models with b/t *= 35.*

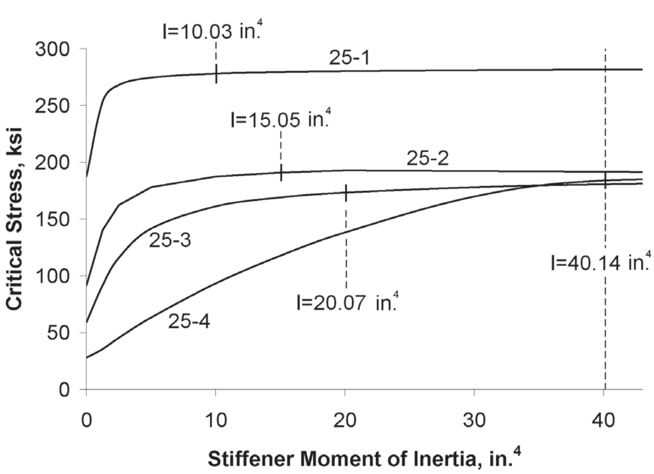


Fig. 6. Critical stress versus stiffener moment of inertia for models with b/t *= 25.*

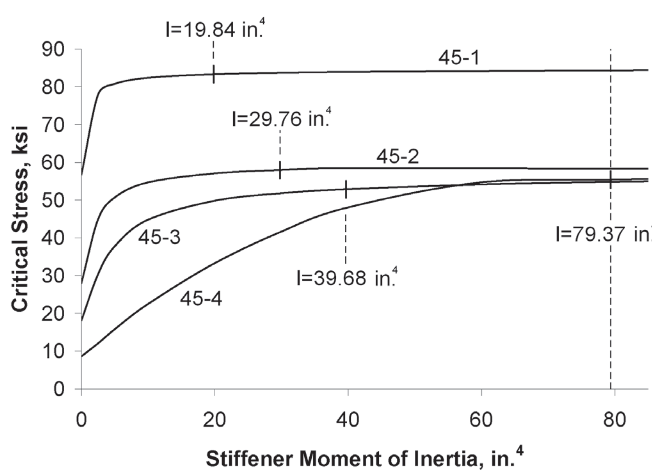


Fig. 8. Critical stress versus stiffener moment of inertia for models with b/t *= 45.*

PROPOSED DESIGN METHOD

For use in design, a critical stiffener moment of inertia must be determined that, if exceeded, will be adequate to brace the plate against buckling in an unstiffened mode. Proposed stiffness requirements should produce critical loads in the plate approximately equal to the case of infinite stiffness. This occurs at the transition point where the plate buckling shape changes from the classical unstiffened mode to the stiffened mode. To meet this objective, the Caltrans (2001) requirements can be modified to account for the plate aspect ratio. A simple design equation can be obtained by multiplying I_s from Equation 5 by the aspect ratio, and dividing by 2, which gives

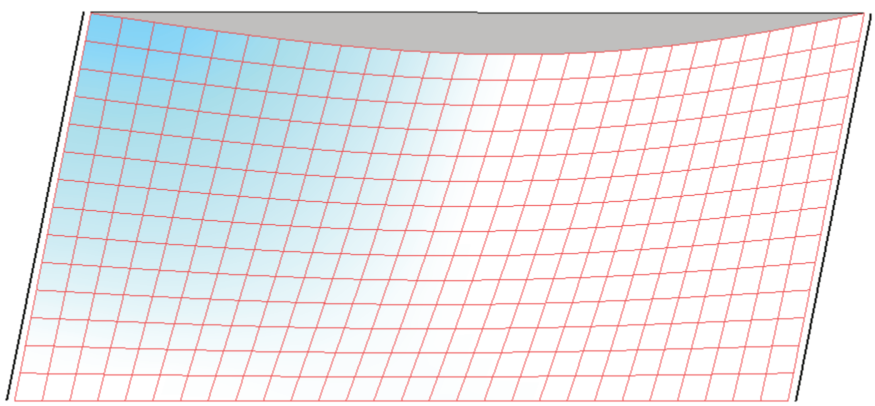
$$I_c = \left(\frac{a}{b}\right)\left(\frac{I_s}{2}\right) \tag{7}$$

Equation 6 controls the design only for connections with b/t < 13, which is much stockier than most connection elements; therefore, it will not be considered further in this paper. Substituting Equation 5 into Equation 7, the final equation is

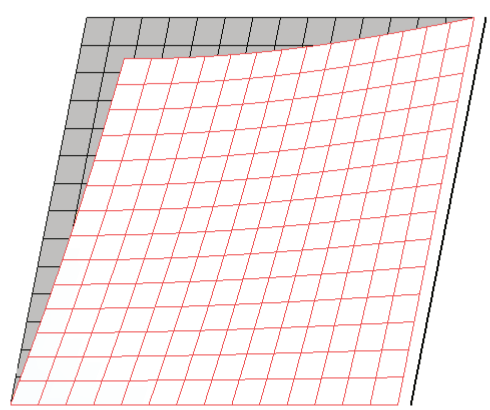
$$I_c = 0.92\left(\frac{a}{b}\right)t^4\sqrt{\left(\frac{b}{t}\right)^2 - 144} \tag{8}$$

This equation should only be used for aspect ratios between ½ and 2. For an aspect ratio of 2, it gives the same results as Equation 5.

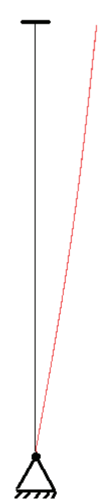
The critical moment of inertia calculated with Equation 8 is indicated in Figures 6, 7 and 8 with a vertical line for each of the 12 base model geometries. In each case, the calculated critical moment of inertia is at a location on the curve beyond the sharp knee, where the rate of change is low, indicating



a. Isometric view



b. Sectioned isometric view



c. Cross-sectional view

Fig. 9. Buckled shape for Model 35-4 with stiffener I = *1.880 in.*^{4}.

that the buckled shape has transitioned from an unstiffened mode to a stiffened mode. Once the stiffener critical moment of inertia is reached, additional stiffness provides only very small gains in the critical stress of the plate. In Tables 1, 2 and 3, the values to the right of the heavy line are for models with stiffeners that meet or exceed the stiffness requirements of Equation 8.

CONCLUSION

For an edge stiffener to be effective, it must have adequate flexural stiffness to alter the buckled shape of the plate. The results from 123 elastic finite element models were analyzed to determine the stiffness requirements for edge stiffeners with aspect ratios between ½ and 2. The parameters studied were the width-to-thickness ratio and the aspect ratio of the braced element.

Equation 8 was proposed as a simple method to calculate the minimum stiffener moment of inertia required to brace connection elements against buckling in an unstiffened mode. Use of the proposed equation should be limited to the range of aspect ratios studied. At aspect ratios larger than two, Equation 5 should be used.

The design procedure outlined in this paper was based on a simplified model that does not include many factors that are present in real structures. Some of these factors are initial out-of-flatness of the element/stiffener assembly, residual stresses and inelastic material behavior, non-idealized boundary conditions, and non-uniform stress distribution in the braced element. Further research is needed to quantify the effects of these items; therefore, sound judgment is required when applying the proposed design procedure.

ACKNOWLEDGMENTS

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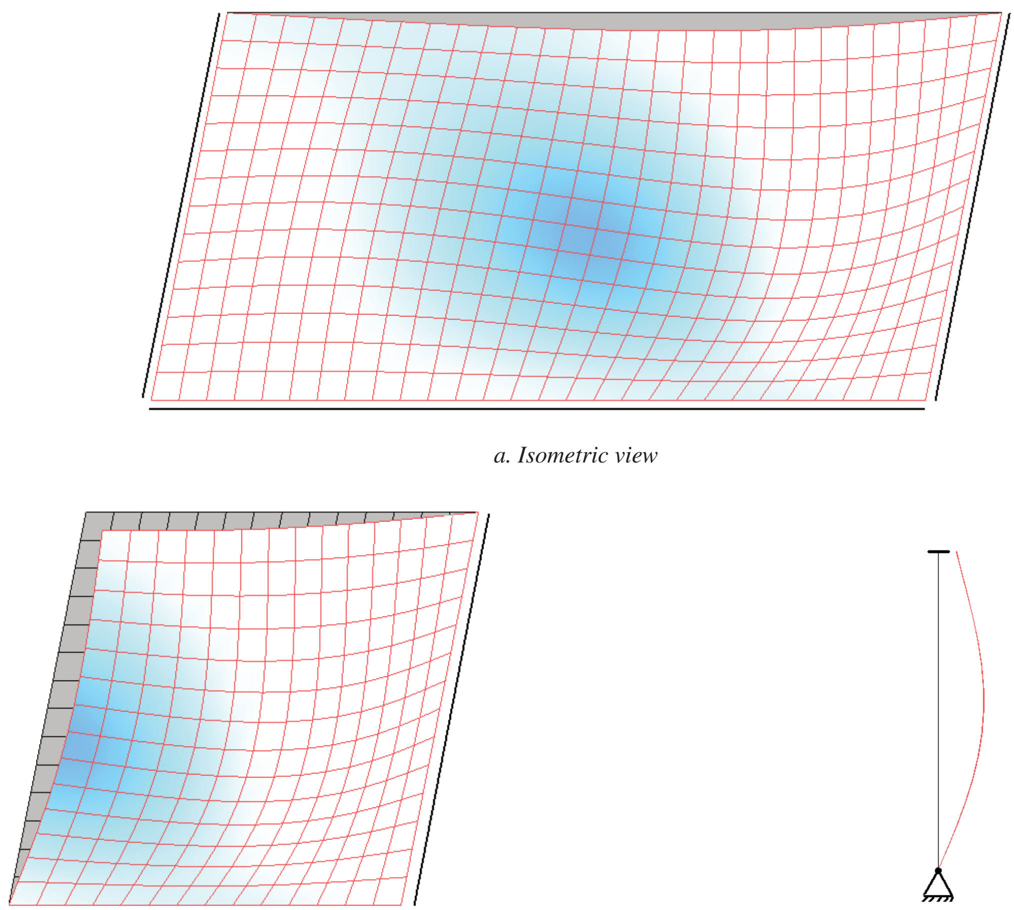

a. Isometric view

b. Sectioned isometric view

c. Cross-sectional view

Fig. 10. Buckled shape for Model 35-4 with stiffener $I = 60.17$ *in.*^{4}.

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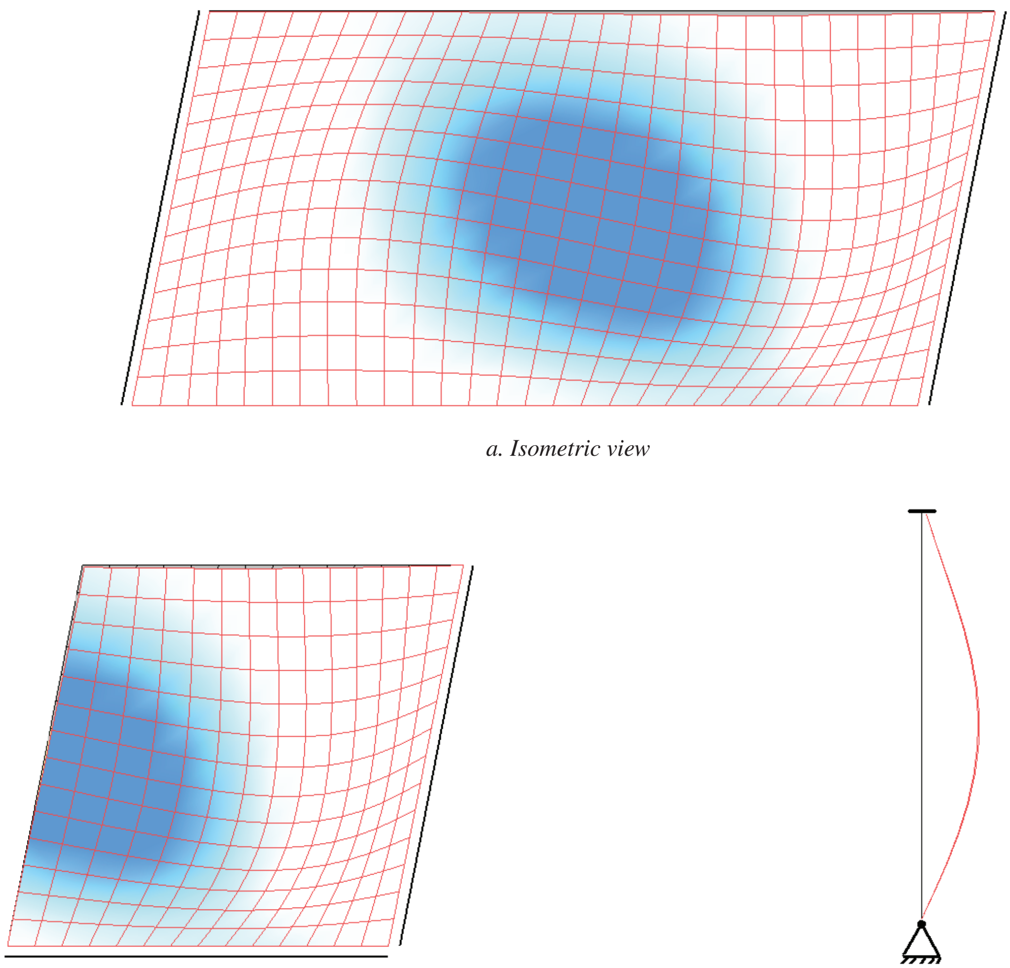

a. Isometric view

b. Sectioned isometric view

c. Cross-sectional view

Fig. 11. Buckled shape for Model 35-4 with stiffener I = *90.25 in.*^{4}.

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