Notes on the Impact of Hole Reduction on the Flexural Strength of Rolled Beams

LOUIS F. GESCHWINDNER

ABSTRACT

The use of $\phi = 0.9$ and $\Omega = 1.67$ with the provisions in Section F13.1 of AISC 360-05 (AISC, 2005) to account for the reduction in flexural strength for a beam with holes in the tension flange has been questioned several times since the publication of the *Specification for Structural Steel Buildings* in 2005. The intent of this paper is to review and provide justification for the use of the resistance/safety factors within the 2005 *Specification* provisions for the impact on flexural strength of holes in the tension flange.

Keywords: bolt holes, tension flange, resistance factors, safety factors.

INTRODUCTION

The use of $\phi = 0.9$ and $\Omega = 1.67$ with the provisions in Section F13.1 of AISC 360-05 (AISC, 2005) to account for the reduction in flexural strength for a beam with holes in the tension flange has been questioned several times since the publication of the *Specification for Structural Steel Buildings* in 2005. The research basis for the 2005 provisions originated from a report by Dexter et al. (2002). Their report includes a proposed formulation for the limits on when the impact of the holes must be considered and how the strength should be determined in those cases; however, the report does not address the appropriate resistance or safety factors to be used. The intent of this paper is to review and provide justification for the use of the resistance/safety factors within the 2005 *Specification* provisions for the impact on flexural strength of holes in the tension flange.

THEORY

AISC 360-05 Equation F13-1 is intended to present a simple yet reasonable approach to account for holes in the tension flange of beams. Although not presented this way in the *Specification*, Equation F13-1 can be rewritten in terms of critical stress, always less than F_y, times the full elastic section modulus as illustrated by the following two equations:

$$M_n = F_{cr} S_x \tag{1}$$

Louis F. Geschwindner, Ph.D., P.E., Vice President, American Institute of Steel Construction, 1 E. Wacker Drive, Suite 700, Chicago, IL, 60601. E-mail: geschwindner@aisc.org

$$F_{cr} = \left[\frac{F_u A_{fn}}{Y_t F_y A_{fg}}\right] F_y \tag{2}$$

Because ASTM A36 and A992 steels meet the limit given in Section F13 for $Y_t = 1.0$, and the term

$$\left[\frac{F_u A_{fn}}{F_y A_{fg}}\right]$$

in Equation 2 is always less than 1.0 if this check is applicable, the critical stress is always less than the yield stress. Therefore, use of the resistance/safety factors associated with yielding (i.e., $\phi = 0.9$, $\Omega = 1.67$) appears to be warranted.

To examine this interpretation more closely, three models are developed for determining flexural strength when holes are present in the tension flange of W-shapes.

Model 1

For ease of calculation, the W-shape is modeled with holes in both the tension and compression flanges. The flange forces are taken as the rupture force and the web is assumed to be yielding throughout. Thus,

$$M_n = F_u A_{fn}\left(d - t_f\right) + F_y\left[Z_x - A_{fg}\left(d - t_f\right)\right] \tag{3}$$

If the ratio of flange-rupture strength to flange-yield strength, which is always less than 1.0, is taken as

$$\Psi = \frac{F_u A_{fn}}{F_y A_{fg}} \tag{4}$$

Equation 3 can be stated as

$$M_n = F_y Z_x - \left(1 - \Psi\right)\left(F_y A_{fg}\right)\left(d - t_f\right) \tag{5}$$

Since this equation represents a rupture failure mode, the resistance factor, $\phi = 0.75$, and safety factor, $\Omega = 2.00$, are used to determine the available strength.

When there is no reduction for the presence of holes, $\Psi = 1.0$, Equation 5 reduces to $M_n = F_y Z_x$. Thus, the design strength is $\phi M_n = 0.75 F_y Z_x$ and the allowable strength is $M_n/\Omega = 0.5 F_y Z_x$. Similarly, the provisions of Section F13, as represented by Equations 1 and 2, reduce to $M_n = F_y S_x$. Thus, the design strength, using $\phi = 0.9$, becomes $\phi M_n = 0.9 F_y S_x$ and the allowable strength, using $\Omega = 1.67$, becomes $M_n/\Omega = 0.6 F_y S_x$. In all cases where the shape factor, Z_x/S_x, is greater than or equal to 1.2 (0.9/0.75 = 1.2 or 2.00/1.67 = 1.2), the *Specification* approach gives a lower or equal available strength when compared to this model. But, if the shape factor is less than 1.2, this model, which was initially thought to be conservative, gives a lower value than the *Specification* approach.

Model 2

A second model is investigated to see if this underprediction can be reversed by eliminating the holes at the compression flange which were included for convenience only.

For this model, only the holes in the tension flange are accounted for and the compression flange is not reduced. It takes a bit more calculation effort to determine the nominal strength with this approach, but it is expected to yield a more accurate representation of the true behavior. In this case, provided the plastic neutral axis remains in the web,

$$M_n = F_y Z_x - \left(1-\Psi\right)\left(F_y A_{fg}\right)\left(\frac{d-t_f}{2}\right) - \frac{\left[\left(1-\Psi\right)F_y A_{fg}\right]^2}{4F_y t_w} \tag{6}$$

and the resistance/safety factor is again taken as $\phi = 0.75$ or $\Omega = 2.00$ since the strength calculation considered rupture of the tension flange. This model yields higher available flexural strength for most of the range of the ratio of flange-rupture strength to flange-yield strength. However, as the reduction for holes gets smaller—that is, as Ψ approaches 1.0, the nominal strength approaches $F_y Z_x$ and the same problem occurs as for Model 1, where the available strength predicted by this model is lower than that predicted by the *Specification* approach for W-shapes with a shape factor below 1.2.

Model 3

A third approach is developed with the goal of increasing the design strength for those cases where the reduction for holes is small.

Since the flange in tension is controlled by tension rupture and the remainder of the shape is controlled by yielding in Model 2, Model 3 simply applies two different resistance/safety factors, $\phi = 0.75$ or $\Omega = 2.00$, for the tension flange contribution and $\phi = 0.9$ or $\Omega = 1.67$, for the compression flange and the web contributions. This multiple factor approach is similar to that used in connection design. For example, a bolted flange plate moment connection could have the tension flange plate controlled by rupture and the compression flange plate controlled by yielding. Thus, different resistance/safety factors would be applied in the design of each element yet they both participate in resisting the same connection moment. In the application here, using Equation 6, this approach yields directly the design strength as

$$\phi M_n = 0.9 F_y Z_x - \left(0.9 - 0.75\Psi\right)\left(F_y A_{fg}\right)\left(\frac{d-t_f}{2}\right) - \frac{0.9\left[\left(1-\Psi\right)F_y A_{fg}\right]^2}{4F_y t_w} \tag{7}$$

or the allowable strength as

$$\frac{M_n}{\Omega} = 0.6 F_y Z_x - \left(0.6 - 0.5\Psi\right)\left(F_y A_{fg}\right)\left(\frac{d-t_f}{2}\right) - \frac{0.6\left[\left(1-\Psi\right)F_y A_{fg}\right]^2}{4F_y t_w} \tag{8}$$

For this model, as Ψ approaches 1.0, the contribution of the tension flange is not fully restored to its yield strength since its contribution is always modified by the rupture resistance/safety factor when $\Psi = 1$. This amounts to a 15% reduction in the contribution of the tension flange to the design strength. For this model, the design strength and the allowable strength for all values of the ratio of flange-rupture strength to flange-yield strength results in available strengths greater than that obtained using Equation F13-1 from AISC 360-05.

SUMMARY

Figure 1 illustrates the LRFD results for the three models discussed earlier compared to the *Specification* equation for a W8×24. This particular shape was chosen because it is compact and has a shape factor close to the lowest of all W-shapes, 1.105. It can be seen that Model 3 predicts design strengths greater than those predicted by *Specification* Equation F13-1. Identical comparisons would result if ASD had been used for the figure.

The intent of this study was to confirm that Equation F13-1 with $\phi = 0.9$ or $\Omega = 1.67$ provides a prediction of flexural strength that is conservative. Models 1 and 2 show that for all W-shapes with a shape factor of 1.2 or greater, the prediction by the *Specification* equation is conservative. However, a more accurate model was needed for W-shapes with a shape factor less than 1.2. Model 3 is a reasonable analytical approach that can be considered conservative. Since Model 3 always provides an available flexural strength greater than that obtained using the *Specification* provisions, it is considered acceptable to use Equation F13-1 with $\phi = 0.9$ or $\Omega = 1.67$.

NOTATION

The notation used in this paper is consistent with that used in ANSI/AISC 360-05 with one symbol added, Ψ.

A_{fg} = gross flange area, in.^{2} (mm^2)

A_{fn} = net flange area, in.^{2} (mm^2)

F_{cr} = critical stress, ksi (MPa)

F_u = specified minimum tensile strength, ksi (MPa)

F_y = specified minimum yield stress, ksi (MPa)

M_n = nominal flexural strength, kip-in. (N-mm)

S_x = elastic section modulus about the *x*-axis, in.^{3} (mm^3)

Y_t = hole reduction coefficient

Z_x = plastic section modulus about the *x*-axis, in.^{3} (mm^3)

d = depth of section, in. (mm)

t_f = thickness of flange, in. (mm)

t_w = thickness of web, in. (mm)

ϕ = resistance factor

Ω = safety factor

Ψ = flange rupture to yield strength ratio

REFERENCES

AISC (2005), *Specification for Structural Steel Buildings*, ANSI/AISC 360, American Institute of Steel Construction, Chicago, IL.

Dexter, R.J., Alttstadt, S.A. and Gardner, C.A. (2002), *Strength and Ductility of HPS70W Tension Members and Tension Flanges with Holes*, University of Minnesota, Minneapolis, MN, March 2002.

APPENDIX

Derivations for the equations presented in this paper follow.

Model 1

In this case, both flanges are assumed to have the same reduction for the presence of holes. The nominal plastic moment strength is given by F_yZ_x when no holes are present. To account for the reduced strength of the flanges, the yield contribution of both flanges is deducted and replaced by the tension rupture contribution. This is clearly a conservative approach for determining the nominal flexural strength since it ignores the actual contribution of the compression flange. Thus,

$$M_n = F_yZ_x - 2F_yA_{fg}\left(\frac{d}{2}-\frac{t_f}{2}\right) + 2F_uA_{fn}\left(\frac{d}{2}-\frac{t_f}{2}\right) \qquad \text{(a)}$$

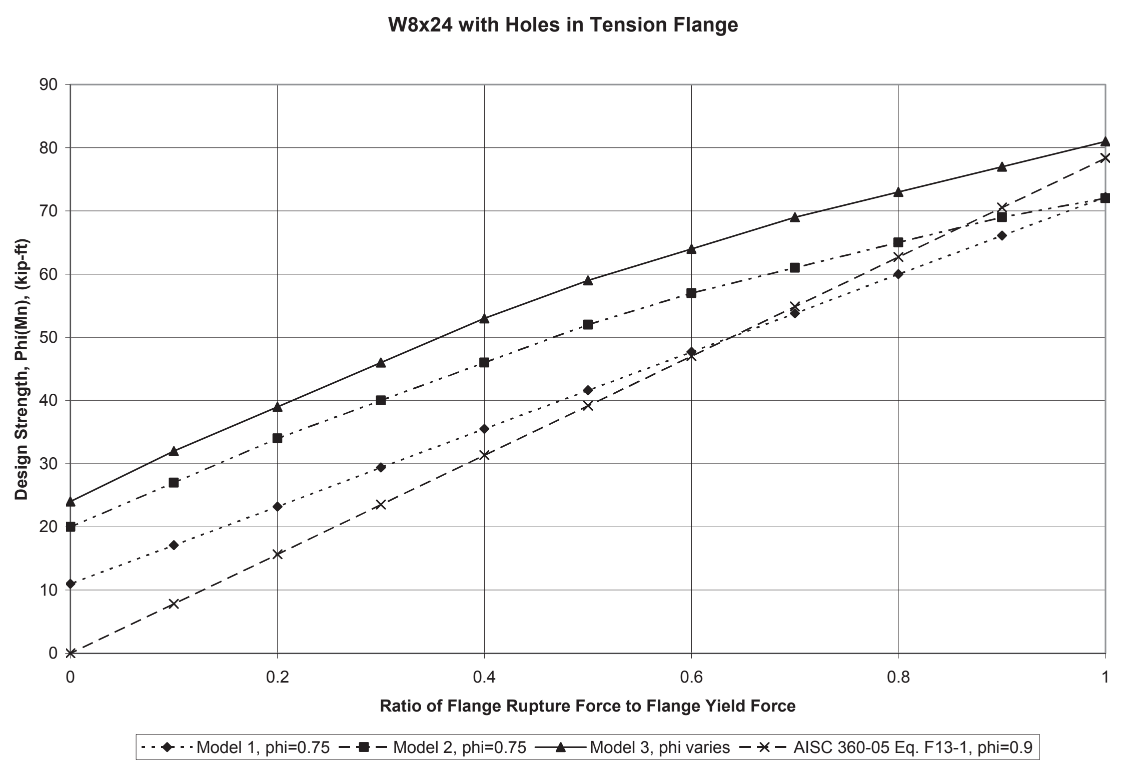

Fig. 1. Design strength for a W8×24 with holes in the tension flange.

Combining terms and multiplying the second term by $\left(\dfrac{F_yA_g}{F_yA_g}\right)$ yields

$$M_n = F_yZ_x + \frac{\left(F_uA_{fn} - F_yA_{fg}\right)}{F_yA_{fg}}\left(d - t_f\right)\left(F_yA_{fg}\right) \quad \text{(b)}$$

Defining $\Psi = \dfrac{F_uA_{fn}}{F_yA_{fg}}$ and substituting into Equation b gives

$$M_n = F_yZ_x - \left(1 - \Psi\right)\left(F_yA_{fg}\right)\left(d - t_f\right) \quad (5)$$

Model 2

For this model, only the holes in the tension flange are considered. First, the contribution of the web is determined by deducting the flanges from the nominal plastic moment strength of the W-shape.

$$M_{n1} = F_yZ_x - 2\left(F_yA_{fg}\right)\left(\frac{d}{2} - \frac{t_f}{2}\right) \quad \text{(c)}$$

Then the tension flange rupture and compression flange yield contributions are added.

$$\begin{aligned} M_{n2} = F_yZ_x &- 2\left(F_yA_{fg}\right)\left(\frac{d}{2} - \frac{t_f}{2}\right) + F_uA_{fn}\left(\frac{d}{2} - \frac{t_f}{2}\right) \\ &+ F_yA_{fg}\left(\frac{d}{2} - \frac{t_f}{2}\right) \end{aligned} \quad \text{(d)}$$

Finally, the last factor to consider is the impact of the shift in the plastic neutral axis into the compression zone of the web, defined as distance x from the centroid of the gross area. This results in a moment reduction based on the removal of some compression force and the addition of an equal tension force, captured through the multiplication by 2 in the last term. These forces are half of the difference between the flange yield force and flange rupture force. Thus,

$$\begin{aligned} M_{n3} = F_yZ_x &- 2\left(F_yA_{fg}\right)\left(\frac{d}{2} - \frac{t_f}{2}\right) + F_uA_{fn}\left(\frac{d}{2} - \frac{t_f}{2}\right) \\ &+ F_yA_{fg}\left(\frac{d}{2} - \frac{t_f}{2}\right) - 2\left[F_yt_w\left(\frac{x^2}{2}\right)\right] \end{aligned} \quad \text{(e)}$$

and the distance that the plastic neutral axis moves up into the compression zone, x, is

$$x = \frac{F_yA_{fg} - F_uA_{fn}}{2F_yt_w} \quad \text{(f)}$$

Substituting for x yields

$$\begin{aligned} M_{n3} = F_yZ_x &- 2\left(F_yA_{fg}\right)\left(\frac{d}{2} - \frac{t_f}{2}\right) + F_uA_{fn}\left(\frac{d}{2} - \frac{t_f}{2}\right) \\ &+ F_yA_{fg}\left(\frac{d}{2} - \frac{t_f}{2}\right) - \frac{\left(F_yA_{fg} - F_uA_{fn}\right)^2}{4F_yt_w} \end{aligned} \quad \text{(g)}$$

Combining terms and substituting $\Psi = \dfrac{F_uA_{fn}}{F_yA_{fg}}$ yields

$$M_n = F_yZ_x - \left(1 - \Psi\right)\left(F_yA_{fg}\right)\left(\frac{d - t_f}{2}\right) - \frac{\left[\left(1 - \Psi\right)F_yA_{fg}\right]^2}{4F_yt_w} \quad (6)$$

Model 3

The only difference between Model 3 and Model 2 is the application of the yielding and rupture resistance/safety factors. Based on Equation g, for LRFD the design strength becomes

$$\begin{aligned} \phi M_n = \phi_yF_yZ_x &- \phi_y2\left(F_yA_{fg}\right)\left(\frac{d}{2} - \frac{t_f}{2}\right) + \phi_rF_uA_{fn}\left(\frac{d}{2} - \frac{t_f}{2}\right) \\ &+ \phi_yF_yA_{fg}\left(\frac{d}{2} - \frac{t_f}{2}\right) - \frac{\phi_y\left(F_yA_{fg} - F_uA_{fn}\right)^2}{4F_yt_w} \end{aligned} \quad \text{(h)}$$

where ϕ_y is the resistance factor for yielding and ϕ_r is the resistance factor for rupture.

Combining terms and substituting $\Psi = \dfrac{F_uA_{fn}}{F_yA_{fg}}$, $\phi_y = 0.9$, and $\phi_r = 0.75$ yields

$$\begin{aligned} \phi M_n = 0.9F_yZ_x &- \left(0.9 - 0.75\Psi\right)\left(F_yA_{fg}\right)\left(\frac{d - t_f}{2}\right) \\ &- \frac{0.9\left[\left(1 - \Psi\right)F_yA_{fg}\right]^2}{4F_yt_w} \end{aligned} \quad (7)$$

For ASD the safety factors are applied to Equation g, yielding

$$\begin{aligned} \frac{M_n}{\Omega} = \frac{F_yZ_x}{\Omega_y} &- 2\frac{\left(F_yA_{fg}\right)}{\Omega_y}\left(\frac{d}{2} - \frac{t_f}{2}\right) + \frac{F_uA_{fn}}{\Omega_r}\left(\frac{d}{2} - \frac{t_f}{2}\right) \\ &+ \frac{F_yA_{fg}}{\Omega_y}\left(\frac{d}{2} - \frac{t_f}{2}\right) - \frac{\left(F_yA_{fg} - F_uA_{fn}\right)^2}{\Omega_y\left(4F_yt_w\right)} \end{aligned} \quad \text{(i)}$$

where Ω_y is the safety factor for yielding and Ω_r is the safety factor for rupture.

Combining terms and substituting $\Psi = \dfrac{F_uA_{fn}}{F_yA_{fg}}$, $\Omega_y = 1.67$, and $\Omega_r = 2.00$ yields

$$\begin{aligned} \frac{M_n}{\Omega} = 0.6F_yZ_x &- \left(0.6 - 0.5\Psi\right)\left(F_yA_{fg}\right)\left(\frac{d - t_f}{2}\right) \\ &- \frac{0.6\left[\left(1 - \Psi\right)F_yA_{fg}\right]^2}{4F_yt_w} \end{aligned} \quad (8)$$