DISCUSSION

# Designing Compact Gussets with the Uniform Force Method

Paper by Larry S. Muir
(First Quarter, 2008)

Discussion by Ramon F. Arias

The writer submits this discussion with some trepidation as he recognizes that the author of the paper is a well-known authority in the design of connections, and who has made valuable contributions to the AISC codes and to books on the subject.

Prior to discussing the substance of the author's paper, a typo should be corrected as follows:

The author's equation (8) reading $\Sigma F_x = 0 = H_c - H_c$ should read $\Sigma F_x = 0 = P\sin\theta - (H_b + H_c)$. This correction does not affect the author's analysis.

The first point of discussion is the author's approach. It appears that the author's solution to the compact gusset corresponds to one of two boundary conditions that would frame the actual solution.

In discussing this, two geometric points should be noted: point $B_o$ on the beam axis at the column flange and point $C_o$ on the column axis at the level of the top of the beam. In the traditional Uniform Force Method (UFM), the force ($H_b$, $V_b$) passes through point $B_o$ and the force ($H_c$, $V_c$) passes through point $C_o$. In other words, the moment $M_{bo}$ of the forces $H_b$ and $V_b$ relative to point $B_o$ and the moment $M_{co}$ of the forces $H_c$ and $V_c$ relative to point $C_o$ are both zero.

In the author's solution to the compact gusset, the force ($H_b$, $V_b$) passes through $B_o$ but the force ($H_c$, $V_c$) is left to drift away from $C_o$ and all of this in such a way that the equations of equilibrium are satisfied. In other words, $M_{bo} = 0$ and $M_{co}$ differs from zero. Under these conditions, expressions are derived for $H_c$, $V_b$, $H_b$ and $V_c$ as exhibited by the author's equations (10), (11), (12) and (13), respectively.

**Ramon F. Arias is a principal civil engineer with Babcock Hitachi K.K., Kure Division, Vancouver, BC.**

Additionally, the moments $M_{bo}$ and $M_{co}$ defined earlier are:

$$M_{bo} = 0 = H_b e_b - V_b \alpha \quad (13.1)$$

$$M_{co} = H_c \beta - V_c e_c \quad (13.2)$$

Equations 13.1 and 13.2 are not in the author's paper, but they can be easily derived from his analysis.

The other boundary condition consists of letting the force ($H_b$, $V_b$) drift away from point $B_o$ while keeping the force ($H_c$, $V_c$) through $C_o$ and satisfying the equations of equilibrium. That is to say, $M_{co} = 0$ while $M_{bo}$ differs from zero. Under these conditions the analysis gives the following results:

$$H_c = e_c P\{\cos\theta - e_b \sin\theta/(e_c + \alpha)\}/\beta \quad (10a)$$

$$V_b = e_b P\sin\theta/(e_c + \alpha) \quad (11a)$$

$$H_b = P\sin\theta - H_c \quad \text{(same as the author's equation 12)}$$

$$V_c = P\cos\theta - V_b \quad \text{(same as the author's equation 13)}$$

$$M_{bo} = H_b e_b - V_b \alpha \quad \text{(same as 13.1 above)}$$

$$M_{co} = 0 = H_c \beta - V_c e_c \quad \text{(same as 13.2 above)}$$

If the author's approach is correct, the solution of the problem created by the compact gusset should be located between these boundary conditions. In this solution, both $M_{bo}$ and $M_{co}$ are different from zero. These moments are relatively small. $M_{co}$ is usually neglected and $M_{bo}$ can be easily accommodated in the beam to column connection.

Table 1a shows a comparison between the two boundaries and includes a solution consisting on a weighted sum of the values from the boundary conditions. The weight factor for the boundary 1 values is $k_1 = d_c/(d_b + d_c)$ and for boundary 2 is $k_2 = d_b/(d_b + d_c)$, where $d_b$ and $d_c$ are the distances from points $B$ (centroid of the beam-to-column connection)

| Table 1a. Comparison between Muir's solution (Boundary 1), Boundary 2, and a Weighted Solution | | | |
|---|---|---|---|
| Parameters | Boundary 1 | Boundary 2 | Weighted Solution |
| $V_b$ | 50.3 | 46.0 | 47.9 |
| $H_b$ | 60.2 | 69.7 | 65.4 |
| $V_c$ | 7.09 | 11.4 | 9.46 |
| $H_c$ | 21.7 | 12.2 | 16.5 |
| $M_{bo}$ | 0.0 | 175.0 | 96.8 |
| $M_{co}$ | 91.4 | 0.0 | 40.8 |

and $C$ (centroid of the column-to-gusset connection) to the working point (WP) or point of intersection of the beam and column axes. Then,

$$V_b = k_1 V_{b1} + k_2 V_{b2}$$

$$H_b = k_1 H_{b1} + k_2 H_{b2}$$

and so on, where $V_{b1}$, $H_{b1}$, $V_{b2}$ and $H_{b2}$ indicate the reaction values corresponding to boundaries 1 and 2.

Of course, the two boundary conditions and the weighted solution satisfy $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$.

A second point of discussion is the approach adopted by the author and continued in this discussion, in which geometric constraints are imposed to determine the connection forces.

The solution to the compact gusset cannot be any of the two boundary conditions or the weighted solution because they contain geometric restrictions that are imposed by the designer (the forces must pass through certain points and must be oriented in certain directions). However, the behavior of the connection under the axial force does not have to follow arbitrary geometric constraints.

Since the forces are concurrent and their sum is zero, it follows as a result that the sum of their moments relative to any point is also zero. This approach is arbitrary as it gives solutions without having to make implicit or explicit reference to the moment caused by the local eccentricity of the brace axial force relative to the centroid $G$ of the connections of the gusset to beam and column. The fact that the sum of moments is zero should be a condition of the problem, not a result of imposed geometric constraints. In other words, the analysis must reflect the behavior of the connection, and the geometric constraints must be relaxed accordingly.

A third and final point of discussion is the adoption by the author of a force, $\Delta V_b$, to manipulate force, $V_b$, resulting in a moment, $M_b$, assigned to the beam to gusset connection and given by the author's equation (14). Actually, what is adopted is a couple, ($\Delta V_b$, $\alpha$), with forces, $\Delta V_b$, oriented vertically so that they only affect $V_b$ and $V_c$, acting at points $B$ and $C$. The results of this manipulation are shown in Table 1 of the author's paper (refer to the column entitled "Modified UFM with $\Delta V_b$" in this table). The adoption of a couple of forces that act in the vertical direction and applied at points that do not fall in a horizontal line is arbitrary. The forces should be oriented perpendicular to line $BC$ and it should affect all connection forces.

Additionally, the author does not explain why the moment, $M_b$, required to balance the couple, ($\Delta V_b$, $\alpha$), is applied to the beam-to-gusset interface. Presumably, the reason is that this connection is more rigid than the column-to-gusset connection. However, it could be argued that the design of a more compact gusset with a larger $\beta$, as in the author's example, results in an increased rigidity of the column-to-gusset connection compared to the concentric gusset connection case. Based on this increase, the moment should be assigned to the column-to-gusset connection. Furthermore, assigning the moment $M_b$ to the beam-to-gusset connection interface appears to defeat the purpose of the author's solution as this moment must also be applied to the beam to column connection.

The writer feels that if there is a need to manipulate the results, an alternative is to introduce a moment, $M_{bc}$, affecting all four connection forces. The moment, $M_{bc}$, should consist of a couple, ($\Delta F$, $d_{bc}$), where $d_{bc}$ is the distance between $B$ and $C$ ($d_{bc}^2 = \alpha^2 + \beta^2$). The force, $\Delta F$, would be selected so as to result in the desired values of the connection forces. Then, a balancing moment, $-\Delta M$, would have to be assigned to one of the two connections.

In summary, the author has identified a boundary condition to the solution of the compact gusset. However, this boundary condition has been reached by adopting geometric constraints that appear to invalidate it. This also applies to the second boundary condition and the weighted solution presented here. Some of the resulting connection forces in the boundary conditions, including the author's solution, could be underestimated. The introduction by the author of a force, $\Delta V_b$, to manipulate the results of his analysis also seems arbitrary.

A solution to the compact gusset design should (1) be based on the UFM, (2) with the equations of equilibrium as condition of its solution, (3) include in the analysis the local eccentricity of the brace force relative to the centroid $G$, (4) be reduced to the traditional UFM solution under the right circumstances, (5) not include arbitrary geometric constraints, and (6) be easily expanded to accept small eccentricities created by the brace axial force not passing through WP. This solution would have $M_{bo}$ and $M_{co}$ different from zero, except for the special case where the centroid $G$ is located on the axis of the brace.

## NOTATION

$d_b$ = distance from the beam-to-gusset connection to the working point

$d_c$ = distance from the column-to-gusset connection to the working point

$d_{bc}$ = distance between the beam-to-gusset and column-to gusset connection centroids

$e_b$ = one-half the depth of the beam

$e_c$ = one-half the depth of the column

$k_1$, $k_2$ = weight factors

$B$ = centroid of the beam-to-gusset connection

$B_o$ = centroid of the beam-to-column connection

$C$ = centroid of the column-to-gusset connection

$C_o$ = point on the column axis at the level of the top of the beam

$G$ = centroid of the combined beam-to-gusset and column-to-gusset connections

$H_b$, $H_{b1}$, $H_{b2}$ = shear force on the beam-to-gusset connection

$H_c$, $H_{c1}$, $H_{c2}$ = tension force on the column-to-gusset connection

$M_b$ = moment on the beam-to-gusset connection

$M_{bc}$ = moment introduced to manipulate the reactions

$M_{bo}$ = moment on point $B_o$

$M_{co}$ = moment on point $C_o$

$V_b$, $V_{b1}$, $V_{b2}$ = tension force on the beam-to-gusset connection

$V_c$, $V_{c1}$, $V_{c2}$ = shear force on the column-to-gusset connection

WP = point of intersection of the beam and column axes

$\alpha$ = distance from face of column to centroid of beam-to-gusset connection

$\beta$ = distance from face of beam flange to centroid of column-to-gusset connection

$\Delta F$ = force adopted to manipulate all reactions ($\Delta F = M_{bc}/d_{bc}$)

$\Delta V_b$ = change in the distribution of vertical reactions

($X$, $Y$) = force resulting from the vector addition of forces X an Y

($X$, $y$) = pair for forces X acting in opposite directions and at a distance