A Model Specification for Stability Design by Direct Analysis

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The 2005 AISC Specification for Structural Steel Buildings (AISC, 2005a), hereafter referred to as the AISC Specification, offers three alternatives for the design of structures for stability. The main body of the AISC Specification, in Chapter C, prescribes two methods: the Effective Length Method in Section C2.2a and the First-Order Analysis Method in Section C2.2b. (Neither method is identified by these names in the AISC Specification.) Appendix 7 presents the Direct Analysis Method. The Effective Length and First-Order Analysis Methods have limited applicability; the Direct Analysis Method is applicable to all structures.

Of the three approaches, the Effective Length Method will be most familiar to users of previous editions of the AISC *Specification* and that is why it was placed in the main body of the current edition. The Direct Analysis Method (now in an appendix) is, however, the most powerful and versatile of the available methods and, as noted, it is applicable to all structures, unlike the other approaches. It is likely that in time the Direct Analysis Method will become the "standard" method of design for stability.

This paper presents a model specification for stability design by direct analysis. It is based on the stability provisions of the 2005 AISC *Specification*, rewritten around the Direct Analysis Method alone. The material is presented in the language and format of the AISC *Specification*, including "User Notes" and the italicizing of terms listed in the glossary where they first appear in a section. The focus on a single method has offered the opportunity to expand some of the provisions beyond what is in the current AISC *Specification*, both to improve clarity and to address issues that have arisen from use of the document. Where this involved substantive changes, they are explained in an appendix to this paper (Appendix A). A second appendix (Appendix B) outlines the purpose or physical significance of each of the important steps in the Direct Analysis Method by showing the correlation of these steps to the basic requirements for design of structures for stability. The "traditional" Effective Length Method is included in the correlation to show how that method differs from the Direct Analysis Method.

A third appendix (Appendix C) provides guidance to the user on the modeling of structures for the application of the Direct Analysis Method.

This model specification is not an approved AISC specification or American National Standards Institute (ANSI) standard. In the author's judgment, however, a design that conformed to this model specification would also conform to the stability provisions of the 2005 AISC *Specification* (AISC, 2005a). Indeed, it is anticipated that Chapter C of the 2010 edition of the AISC *Specification* (which is in the ballot process as this paper goes to press) will be substantially similar to this model specification; the Effective Length Method and the First-Order Analysis Method (now in Chapter C) will be moved to Appendix 7.

MODEL SPECIFICATION: DESIGN FOR STABILITY

This specification addresses requirements for the design of structures for stability by the Direct Analysis Method. It is organized as follows:

- 1. General Stability Requirements
- 2. Calculation of Required Strengths
- 3. Calculation of Available Strengths

1. GENERAL STABILITY REQUIREMENTS

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on the stability of the structure and its elements shall be considered: (1) *second-order effects* (both *P*- Δ and *P*- δ *effects*); (2) flexural, shear, and axial deformations, and all other deformations that contribute to-displacements of the structure; (3) geometric imperfections; (4) stiffness reductions due to inelasticity; and (5) uncertainty in stiffness and strength. All load-dependent effects shall be calculated at a level of loading corresponding to *LRFD load combinations* or 1.6 times *ASD load combinations*.

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Any rational method of design for stability that considers all of the listed effects is permitted.

User Note: The term "design" as used in these provisions is the combination of analysis of the structure to determine the *required strengths* of components and the proportioning of the components to have adequate *available strength*.

The *Direct Analysis Method*, which consists of the calculation of required strengths in accordance with Section 2 and the calculation of available strengths of members and connections in accordance with Section 3, is permitted for all structures.

User Note: See Appendix B for an explanation of how requirements (1) through (5) of Section 1 are satisfied in the Direct Analysis Method.

2. CALCULATION OF REQUIRED STRENGTHS

The *required strengths* of components of the structure shall be determined from an analysis conforming to Section 2.1. The analysis shall include consideration of initial imperfections in accordance with Section 2.2 and adjustments to stiffness in accordance with Section 2.3.

2.1. General Analysis Requirements

The analysis of the structure shall conform to the following requirements:

(1) The analysis shall be an elastic *second-order analysis* that considers both $P-\Delta$ and $P-\delta$ *effects*.

User Note: The second-order analysis may consist of either a rigorous second-order analysis or a *first-order analysis* amplified to account for second-order effects. The impact of *P*- δ effects on structure response may be neglected where it can be shown to be negligible; however, it will still be necessary, in all cases, to consider the effects of *P*- δ on individual members.

- (2) The analysis shall consider flexural, shear and axial member deformations, and all other deformations that contribute to displacements of the structure. The analysis shall incorporate reductions in all stiffnesses that are considered to contribute to the stability of the structure, as specified in Section 2.3.
- (3) The analysis shall consider all gravity and other applied *loads* that may influence the stability of the structure.

User Note: It is important to include in the analysis all gravity loads, including loads on *leaning columns* and other elements that are not part of the *lateral load-resisting system*.

(4) For design by LRFD, the second-order analysis shall be carried out under *LRFD load combinations*. For design by ASD, the second-order analysis shall be carried out under 1.6 times the *ASD load combinations*, and the results shall be divided by 1.6 to obtain the required strengths of components.

2.2. Consideration of Initial Imperfections

The effect of initial imperfections on the stability of the structure shall be taken into account either by direct modeling of imperfections in the analysis as specified in Section 2.2a or by the application of *notional loads* as specified in Section 2.2b.

User Note: The imperfections considered in this section are imperfections in the locations of points of intersection of members. In typical building structures, the important imperfection of this type is the out-of-plumbness of columns. Initial out-of-straightness of individual members is not addressed in this section; it is accounted for in the compression member design provisions of Chapter E of the AISC *Specification* (AISC, 2005a) and need not be considered explicitly as long as it is within the limits specified in the AISC *Code of Standard Practice* (AISC, 2005b).

2.2a. Direct Modeling of Imperfections

In all cases and all types of structures, it is permissible to account for the effect of initial imperfections by including the imperfections in the analysis. The structure shall be analyzed with points of intersection of members displaced from their nominal locations.* The magnitude of the initial displacements shall be the maximum amount considered in the design; the pattern of the displacements shall be such that it provides the greatest destabilizing effect.

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^{*} As a logical extension of the Direct Analysis Method (beyond the explicit provisions of the 2005 *Specification*), imperfections may be modeled at additional locations in the analysis. When this is done, the effective unbraced lengths of members, for calculation of compressive strength for flexural buckling in the direction in which imperfections were included in the analysis, may be taken as the length between the points at which the imperfections were modeled.

User Note: Initial displacements similar in configuration to both displacements due to loading and anticipated buckling modes should be considered in the modeling of imperfections. The magnitude of the initial displacements should be based on permissible construction tolerances, as specified in the AISC *Code of Standard Practice* (AISC, 2005b) or other governing requirements, or on actual imperfections if known.

In the analysis of structures that support gravity loads primarily through nominally vertical columns, walls or frames, where the ratio of maximum second-order drift to maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations, with stiffnesses adjusted as specified in Section 2.3) in all stories is equal to or less than 1.7, it is permissible to neglect initial imperfections in the analysis for load combinations that include applied lateral loads.

2.2b. Use of Notional Loads to Represent Imperfections

For structures that support gravity loads primarily through nominally vertical columns, walls or frames, it is permissible to use notional loads to represent the effect of initial imperfections in accordance with the requirements of this section. The notional loads shall be applied to a model of the structure based on its nominal geometry.

User Note: The notional load concept is applicable to all types of structures, but the specific requirements in 2.2b(1) through 2.2b(4) are applicable only for the particular class of structure identified above.

Notional loads shall be applied as *lateral loads* at all levels. The notional loads shall be additive to other lateral loads and shall be applied in all load combinations, except as indicated in Section 2.2b(4). The magnitude of the notional loads shall be:

$$N_i = 0.002Y_i$$
 (2-1)

where

- N_i = notional load applied at level *i*, kips (N)
- Y_i = gravity load applied at level *i* from the LRFD load combination or 1.6 times the ASD load combination, as applicable, kips (N)

User Note: The notional loads can lead to additional (generally small) fictitious base shears in the structure. The correct horizontal reactions at the foundation may be obtained by applying an additional horizontal force at the base of the structure, equal and opposite in direction to the sum of all notional loads, distributed among vertical load-carrying elements in the same proportion as the gravity load supported by those elements. (2) The notional load at any level, N_i , shall be distributed over the level in the same manner as the gravity load at that level. The notional loads shall be applied in the direction that provides the greatest destabilizing effect.

User Note: For most building structures, the requirement regarding notional load direction may be satisfied as follows: For load combinations that do not include lateral loading, consider four alternative directions of notional load application, 90° apart, in the same direction at all levels; in load combinations that include lateral loading, apply all notional loads in the direction of the resultant of all lateral loads in the combination.

(3) The notional load coefficient of 0.002 in Equation 2-1 is based on a nominal initial story out-of-plumbness ratio of 1/500. Where the use of a different maximum out-of-plumbness is justified, it is permissible to adjust the notional load coefficient proportionally.

User Note: An initial out-of-plumbness of 1/500 represents the tolerance on column plumbness specified in the AISC *Code of Standard Practice* (AISC, 2005b).

(4) For frames in which the ratio of maximum second-order drift to maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations, with stiffnesses adjusted as specified in Section 2.3) is equal to or less than 1.7 in all stories, it is permissible to apply the notional load, N_i , only in gravity-only load combinations and not in combinations that include other lateral loads.

User Note: The specified drift ratio threshold of 1.7 is based on analyses using reduced stiffnesses. If the drift ratio is determined from analyses using nominal, unreduced stiffnesses, the equivalent drift ratio is 1.5.

2.3. Adjustments to Stiffness

The analysis of the structure to determine the *required strengths* of components shall use reduced stiffnesses, as follows:

(1) A factor of 0.8 shall be applied to all stiffnesses that are considered to contribute to the stability of the structure. It is permissible to apply this reduction factor to all stiffnesses in the structure.

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User Note: Applying the stiffness reduction to some members and not others can, in some cases, result in artificial distortion of the structure under load and possible unintended redistribution of forces. This can be avoided by applying the reduction to all members, including those that do not contribute to the stability of the structure.

(2) An additional factor, τ_b, shall be applied to the flexural stiffnesses of all members whose flexural stiffnesses are considered to contribute to the stability of the structure,

where

 $\begin{aligned} \tau_b &= 1.0 \text{ for } \alpha P_r / P_y \leq 0.5 \\ &= 4(\alpha P_r / P_y) [1 - (\alpha P_r / P_y)] \text{ for } \alpha P_r / P_y > 0.5 \end{aligned}$

$$\alpha = 1.0 (LRFD)$$
 $\alpha = 1.6 (ASD)$

and

 P_r = required axial compressive strength under LRFD or ASD load combinations, kips (N)

 P_v = axial yield strength, kips (N)

User Note: Taken together, sections (1) and (2) require the use of 0.8EA and $0.8\tau_b EI$ for structural steel members in the analysis instead of *EA* and *EI*.

(3) In structures to which Section 2.2b is applicable, in lieu of using $\tau_b < 1.0$ where $\alpha P_r/P_y > 0.5$, it is permissible to use $\tau_b = 1.0$ for all members if a notional load of $0.001Y_i$ [where Y_i is as defined in Section 2.2b(1)] is applied at all levels, in the direction specified in Section 2.2b(2), in all load combinations. These notional loads shall be added to those, if any, used to account for imperfections and shall not be subject to Section 2.2b(4). (4) Where components comprised of materials other than structural steel are considered to contribute to the stability of the structure and the governing codes and specifications for the other materials require greater reductions in stiffness, such greater stiffness reductions shall be applied to those components.

3. CALCULATION OF AVAILABLE STRENGTHS

When *required strengths* have been determined in accordance with Section 2, the *available strengths* of members and connections shall be calculated in accordance with the provisions of Chapters D, E, F, G, H, I and J, as applicable, of the 2005 AISC *Specification for Structural Steel Buildings* (AISC, 2005a), with no further consideration of overall structure stability. The *effective length factor*, *K*, of all members shall be taken as unity unless a smaller value can be justified by rational analysis.

Bracing intended to define the unbraced lengths of members shall have sufficient stiffness and strength to control member movement at the braced points. Methods of satisfying this requirement are provided in Appendix 6 of the 2005 AISC *Specification* (AISC, 2005a).

User Note: The requirements of Appendix 6 of the 2005 AISC *Specification* are not applicable to bracing that is included in the analysis of the overall structure as part of the overall load-resisting system.

REFERENCES

- AISC (2005a), Specification for Structural Steel Buildings, ANSI/AISC 360, American Institute of Steel Construction, Chicago, IL.
- AISC (2005b), *Code of Standard Practice for Steel Buildings and Bridges*, American Institute of Steel Construction, Chicago, IL.

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APPENDIX A

EXPLANATION OF CHANGES

The model specification for stability design presented in this paper is based on Chapter C and Appendix 7 of the 2005 AISC *Specification for Structural Steel Buildings* (AISC, 2005a). Where substantive technical changes have been made in AISC *Specification* provisions, they are explained in this appendix. The changes are conservative in that a design that conforms to the proposed model specification would also conform to the 2005 AISC *Specification*.

Type of Structure

Some of the provisions of the 2005 AISC *Specification*, specifically those related to the use of notional loads, are applicable only to conventional building structures that support gravity loads primarily through nominally vertical columns, walls or frames. They are not applicable, for instance, to arches or laterally unsupported compression chords of long-span trusses. This limitation is not noted explicitly in the AISC *Specification*.

The model specification is based on the very versatile Direct Analysis Method and is intended to be applicable to a broader range of structures than just conventional building frames. Therefore, those provisions that can only be used for typical building structures [Sections 2.2b and 2.3(3)] are clearly identified and alternatives usable with all structures are provided. [The notional load concept is broadly applicable, but the specific provisions in Sections 2.2b and 2.3(3) are intended only for the limited class of building structures.]

General Stability Requirements

Uncertainty in stiffness and strength has been added to the list of effects to be considered. All three of the stability design methods in the 2005 AISC *Specification* (as well as the method in the present work) include consideration of uncertainty in stiffness and strength, but this item was not included explicitly in the list of general requirements.

The requirement that second-order effects be considered at a level of load corresponding to LRFD load combinations or 1.6 times ASD load combinations is set forth now as a general requirement. The 2005 AISC *Specification* has this requirement only in the sections on specific methods, which could be taken to imply, incorrectly, that it applied only to those methods and was not a general requirement for all designs.

Exclusion of *P*-δ Effects

The 2005 AISC *Specification* permitted second-order analyses that neglected P- δ effects under certain conditions. This exclusion has been eliminated. Given that most structures would not have qualified for the exclusion and would have

required consideration of both *P*- Δ and *P*- δ effects, and therefore design offices would need the capability to handle both effects, there was little to be gained from the extra step of checking for applicability of the exclusion.

Inclusion of All Loads in the Analysis

Section 2.1(3) makes it clear that all loads on the structure, including loads on "leaner" columns and other components that are not part of the lateral load-resisting frame, must be included in the second-order analysis. This might appear obvious to engineers familiar with the fundamental principles of stability analysis, but there has been confusion on this point among some users of the 2005 AISC *Specification*.

Direct Modeling of Imperfections

The requirements for direct modeling of imperfections, covered in one sentence in the 2005 AISC *Specification*, are presented in greater detail.

Direct Modeling of Imperfections, Exclusion

When notional loads are used to simulate the effects of initial imperfections, the imperfections can, in effect, be neglected under certain conditions (when the ratio of second-order to first-order drift is below a certain threshold and the load combination includes applied lateral loads). The 2005 AISC *Specification* offers no analogous exclusion to consideration of imperfections when direct modeling of imperfections is used. The model specification offers the same exclusion for the direct-modeling approach as for the notional-load approach (see last paragraph of Section 2.2a).

Application of Notional Loads

Requirements regarding the distribution and direction of notional loads are specified in much greater detail in the present work [Section 2.2b(2)]. These requirements may have been implicit in the 2005 AISC *Specification*; they are now spelled out.

Drift Ratio Threshold for Excluding Notional Loads in Combination with Applied Lateral Loads

In the 2005 AISC *Specification*, the drift ratio (ratio of second-order drift to first-order drift) below which notional loads need not be combined with applied lateral loads is 1.5, based on analyses with nominal, unadjusted stiffnesses. Given that stiffnesses will always be reduced (by Section 2.3) in the Direct Analysis Method, this model specification defines the threshold value on the basis of analyses with reduced stiffnesses, which increases the value to 1.7.

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Table 1. Comparison of Basic Stability Requirements with Specific Provisions		
a 1 of This Model Specification	Provision in Direct Analysis Method (DAM)	Provision in Effective Length Method (ELM)
(both <i>P</i> - $∆$ and <i>P</i> - $δ$)	2.1(1). Consider second-order effects (P - Δ and P - δ)**	Consider second-order effects $(P-\Delta \text{ and } P-\delta)^{**}$
	2.1(2). Consider all deformations	Consider all deformations
Effect of joint-position imperfec- tions* on structure response	2.2a. Direct modeling or 2.2b. Notional loads	Apply notional loads
Effect of member imperfections on structure response	Included in the stiffness reduc- tion specified in 2.3	 All these effects are considered by using <i>KL</i> from a sidesway buckling analysis in the member strength check. Note that the only difference between DAM and ELM is that: DAM uses reduced stiffness in the analysis; <i>KL = L</i> in the member strength check ELM uses full stiffness in the analysis; <i>KL</i> from sidesway buckling analysis in the mem- ber strength check for frame members
Effect of member imperfections on member strength	Included in member strength formulas, with $KL = L$	
(4) Consider stiffness reduction due to inelasticityEffect of stiffness reduction on structure responseThis affects structure response and member strengthEffect of stiffness reduction on member strength(5) Consider uncertainty in strength and stiffness This affects structure response and member strengthEffect of stiffness/strength uncertainty on structure responseEffect of stiffness uncertainty on structure response and member strengthEffect of stiffness/strength uncertainty on structure response	Included in the stiffness reduc- tion specified in 2.3	
	Included in member strength formulas, with $KL = L$	
	Included in the stiffness reduc- tion specified in 2.3	
	Included in member strength formulas, with $KL = L$	
	the formation of the second structure response of the second structure response on structure response on structure response structure response for the structure response structure response structure response for the structure response structure response structure response for the structure response structure structure response structure str	Image: 1 of This Model SpecificationProvision in Direct Analysis Method (DAM)(both $P - \Delta$ and $P - \delta$)2.1(1). Consider second-order effects ($P - \Delta$ and $P - \delta$)**(both $P - \Delta$ and $P - \delta$)2.1(2). Consider all deformationsEffect of joint-position imperfec- tions* on structure response2.2a. Direct modeling or 2.2b. Notional loadsEffect of member imperfections on structure responseIncluded in the stiffness reduc- tion specified in 2.3Effect of member imperfections on member strengthIncluded in member strength formulas, with $KL = L$ Effect of stiffness reduction on structure responseIncluded in member strength formulas, with $KL = L$ Effect of stiffness reduction on member strengthIncluded in the stiffness reduc- tion specified in 2.3Effect of stiffness reduction on member strengthIncluded in the stiffness reduc- tion specified in 2.3Effect of stiffness/strength uncertainty on structure responseIncluded in member strength formulas, with $KL = L$ Effect of stiffness/strength uncertainty on structure responseIncluded in the stiffness reduc- tion specified in 2.3Effect of stiffness/strength uncertainty on structure responseIncluded in the stiffness reduc- tion specified in 2.3Effect of stiffness/strength uncertainty on structure responseIncluded in the stiffness reduc- tion specified in 2.3Effect of stiffness/strengthIncluded in member strength

* In typical building structures, the "joint-position imperfections" are the column out-of-plumbnesses. ** Second-order effects may be considered either by rigorous second-order analysis or by amplification of the results of first-order analysis (using the B1 and B2 amplifiers in the AISC *Specification*).

Adjustments to Stiffness

The 2005 AISC *Specification* requires analysis with reduced axial and flexural stiffnesses of members whose stiffnesses are considered to contribute to the lateral stability of the structure. It offers no explicit guidance, however, about member shear stiffnesses, diaphragm stiffnesses, column base rotational stiffnesses, etc. The present work takes the more conservative approach of applying the basic 0.8 reduction to all stiffnesses that contribute to the stability of the structure [see Section 2.3(1)].

Adjustments to Stiffness of Other Materials

The Direct Analysis Method is applicable to all structures including, for instance, combinations of concrete shear walls and steel frames. The model specification states that if the governing codes or specifications for other materials used in combination with structural steel require greater stiffness reductions than specified here for the steel, those greater reductions should be applied to the non-steel components in the analysis of the combined structure [see Section 2.3(4)]. This is not stated explicitly in the 2005 AISC *Specification*.

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APPENDIX B

RELATIONSHIP OF SPECIFICATION PROVISIONS TO GENERAL STABILITY REQUIREMENTS

The general requirements for design of structures for stability are listed as Items 1 through 5 in Section 1 of this model specification. Table 1 shows how these five requirements are addressed in the Direct Analysis Method (defined in Sections 2 and 3 of this specification). For comparison, the last column shows how the five requirements are addressed in the "traditional" Effective Length Method (Section C2.2a of the 2005 AISC *Specification*).

The First-Order Analysis Method (Section C2.2b of the 2005 AISC *Specification*) is not included in Table 1 because of its very indirect relationship to the five basic requirements. It uses mathematical manipulation to achieve roughly the same results as the Direct Analysis Method, as follows: The "additional lateral load" in Section C2.2b(2) of the 2005 AISC *Specification* is calibrated to achieve roughly the same result as a notional load for initial out-of-plumbness plus a B_2 multiplier for P- Δ plus a stiffness reduction; add a B_1 multiplier for P- δ as specified in Section C2.2b(3) of the 2005 AISC *Specification*, check member capacity using KL = L, and everything in the Direct Analysis Method is covered.

APPENDIX C

MODELING OF STRUCTURES FOR DESIGN BY THE DIRECT ANALYSIS METHOD

This appendix provides guidance and suggestions for the modeling of structures for the application of the Direct Analysis Method of design for stability. Though focused on the specification for stability design by direct analysis proposed in this paper, the modeling suggestions in this appendix are also applicable to stability design by direct analysis using Appendix 7 of the 2005 AISC *Specification*.

The Direct Analysis Method of design for stability is applicable to all types of structures; however, the following discussion on modeling is intended primarily for "typical" building structures made up of nominally vertical columns, walls or frames and horizontal floors and roofs. (The final section of this appendix discusses structures other than typical buildings.)

Components and Effects to Be Included

The specification contains the following requirements:

- The analysis shall consider flexural, shear and axial member deformations, and all other deformations that contribute to displacements of the structure.
- The analysis shall consider all gravity and other applied loads that may influence the stability of the structure.

It is important to note that "consider" is not synonymous with "include" in these provisions. Some of the listed effects could be considered and then, if judged to be insignificant (on the basis of a rational evaluation of their importance), be excluded from the analysis. Suggestions regarding the inclusion in the model of certain typical building components and their properties follow.

Lateral Load-Resisting Systems

Clearly, all lateral load-resisting systems and components must be included in the model. These might include braced frames, moment-resisting frames, shear walls, and other systems intended to provide lateral stability and resistance to lateral loads.

In general, it will be necessary to model these components at their correct locations in three-dimensional space. In symmetric structures with lateral load-resisting systems completely uncoupled in the two orthogonal directions, it may be possible in some cases to employ two-dimensional models. This should be attempted only when overall torsional instability is clearly not an issue (such as when the lateral load resisting components are well distributed through the building footprint or are located at or near the building perimeter). Braced frames may be represented in the model as either pin-connected or rigidly connected assemblies. What is important is that the analysis be consistent with the design: If rigid connections are assumed in the model, the design must account for the resulting moments; if pin connections are modeled, end moments may be neglected in the design of members.

Components Not Part of Lateral Load-Resisting Systems

The analysis must account for the destabilizing effect of all loads on the structure, including loads applied on components that are not part of the lateral load-resisting systems. This means that all vertical load-carrying components, including "leaning columns" (columns stabilized laterally through their connection to the rest of the building), and all the loads on these components, must be included in the model and the analysis.

It is not always necessary to model all leaning columns individually. A group of leaning columns that have equal lateral displacements may be modeled as a single column, with the load on the entire group applied to that single column. The single column should be located at the approximate centroid of the load on the group of columns it represents. (Where overall torsional instability of the building is of concern, leaning columns should not be grouped, since torsional displacements of floors and roof correspond to unequal lateral displacements in all columns.)

Beams and girders whose only function in the structure is to deliver floor loads to columns (with simple connections at the columns) need not, in general, be included in the model. The floor loads may be applied directly to the columns as concentrated loads.

Floor and Roof Diaphragms

Most of the computer programs in common use for the analysis of building structures allow floor and roof diaphragms to be modeled as being rigid in their own plane, which can greatly simplify the analysis. Even with general frame analysis software, modeling the floor and roof diaphragms as rigid (through the use of appropriate plate or beam elements) can simplify the analysis.

While the floor and roof diaphragms in real buildings are never perfectly rigid, the rigid-diaphragm assumption can often be justified. What is important is the in-plane deformation of the diaphragm relative to the interstory drift of the building. If the maximum diaphragm deformation is no more than a small fraction of the maximum interstory drift, the rigid-diaphragm idealization will cause little error in the results of the analysis.

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Most concrete slabs (formed or on steel deck) are stiff enough to be modeled as rigid in their own plane. Steel roof deck is much less rigid. Nevertheless, the flexibility of steeldeck roof diaphragms on multi-story residential and office buildings can often be neglected. These buildings do not typically have large distances between lateral load-resisting components (distances that the diaphragm must span horizontally); moreover, the stability demand at the top floor of a multi-story building is likely to be small.

Cold-formed steel roof diaphragms on industrial buildings with widely spaced lateral load-resisting components cannot usually be idealized as rigid. Nonrigid diaphragms may be modeled as horizontal beams of appropriate stiffness spanning between lateral load-resisting elements, supporting the tops of leaning columns. Or they could be modeled rigorously as plate elements.

Stiffness Adjustments

The specification (Section 2.3) requires adjustments to stiffness in the analysis model. This stiffness reduction is mandatory in analyses to be used for design for strength but is not necessary in analyses for serviceability. Instead of developing two separate models with different stiffnesses, it would be generally conservative to use the same model (with reduced stiffness) for serviceability as for strength, with the prescribed serviceability deformation limits increased appropriately.

Application of Notional Loads

The specification requires that the notional load at any level be distributed over the level in the same manner as the gravity load at that level. The most straightforward way to do this is to model the notional load as a fraction (typically 1/500) of the gravity load, applied horizontally rather than vertically, at the same locations as the gravity load. Thus, if the gravity loading is applied as concentrated loads at columns, the notional loads would also be applied that way. And if gravity load is applied as a distributed load on the floor, so too would the notional loads be applied. (If the program or modeling details do not permit distributed horizontal load on the floor diaphragms or framing, it will be necessary to convert these loads into concentrated loads at the columns.)

For gravity-only load combinations, four directions of notional load application, 90° apart, same direction at all floors, must be considered. For combinations that include wind or other lateral loading, the notional loads at all floors are applied in the direction of the resultant of all lateral loads in the combination.

Load Combinations

The codes that specify design loadings require consideration of numerous combinations of gravity and environmental loads. When different directions of the environmental loads are taken into account, the result can be a very large number of combinations. The number of loadings will be increased even further by the need to consider four different directions of notional load application for each gravity-only load combination.

For the second-order analysis that is required for stability design, the principle of superposition of loads does not apply; each load combination requires an independent analysis (and not just an adding together, with appropriate factors, of the results for each load component, as would be possible for a linear analysis).

The practice in many design offices is to apply all load combinations in every analysis. This approach has the benefit of simplicity (requiring no judgment in the selection of loads to be considered), and for linear analysis the penalty in computation time is small. But when the analysis is a second-order analysis, the time penalty can be significant.

To reduce computation time, designers using the Direct Analysis Method (or any method of design that requires second-order analysis) could attempt to identify the load combinations most likely to govern the design, and apply only those few combinations in all but the final cycles of the analysis, code-checking, resizing and re-analysis process.

Second-Order Analysis

Almost all computer programs that claim to do second-order analysis handle P- Δ effects adequately, but some do not consider P- δ effects. For many (if not most) real-world buildings, it is acceptable to use a program that neglects the effect of P- δ on the overall response of the structure: If the ratio of second-order drift to first-order drift is less than 1.5, and no more than one-third of the total gravity load on the building is on columns that are moment-connected in the direction of translation being considered, the error in a P- Δ -only analysis will be less than about 3% and may be considered negligible. (It is necessary in all cases to consider the effect of P- δ on individual compression members.)

The Direct Analysis Method of design for stability is compatible with approximate analysis procedures. While a second-order analysis is required, it does not have to be a rigorous large-deformation analysis using a sophisticated computer program. Linear, first-order analysis with results amplified to account for second-order effects, the " B_1 - B_2 procedure" specified as an option in the AISC *Specification* is an acceptable means of second-order analysis. In the B_1 - B_2 procedure, the B_1 multiplier, calculated for each member subject to combined compression and flexure and each direction of bending of the member, accounts for the increase in moment caused by P- δ effects on that individual member. The B_2 multiplier, calculated for each story of the structure and each direction of lateral translation of the story, accounts for the increase in member forces and moments caused by P- Δ effects.

Structures Other Than Typical Buildings

The Direct Analysis Method with direct modeling of imperfections can be a particularly powerful tool in the design of structures other than typical building frames. In the method as presented in this model specification (and, less explicitly, in the 2005 *Specification*), initial imperfections may be modeled directly in the analysis at points of intersection of members; individual members can then be designed assuming an effective length equal to the actual length between these points.

In a logical extension of the method, initial imperfections can be modeled directly in the analysis at additional points beyond the points of intersection of members; members can then be designed assuming an effective length equal to the length between these more closely-spaced points. (See footnote to Section 2.2a of the model specification.) For example, for a long-span truss with a laterally unbraced compression chord, initial lateral displacements could be included in the analysis model at each panel point. For an arch rib, initial lateral and vertical displacements could be modeled at a number of points along the rib.

In each case, the magnitude of the modeled initial displacements should be based on permissible construction tolerances and the pattern should reflect the anticipated buckling modes (typically single curve for lateral displacement of the truss chord or arch rib, double curve for in-plane displacement of the arch, and so on). Initial displacement patterns similar in shape to anticipated displacements due to loading should also be considered, if different from the buckling modes.

In the calculation of the available compressive strengths of individual members for comparison with required strengths determined from the analysis, the effective lengths of the members, for flexural buckling in the direction in which initial displacements were defined, should be taken as the lengths between the points at which the displacements were defined.

Thus, in the truss example, KL for lateral buckling of the chord would be the distance between panel points (even though the chord is not fully braced at these points). In the arch example, if initial displacements, in-plane and lateral, were applied to the rib at intervals of 20 ft in the analysis, KL for checking the available compressive strength of the rib may be taken as 20 ft, regardless of the actual spacing of "bracing" points on the rib.

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