Technical Note: Optimum Flexural Design of Steel Members Utilizing Moment Gradient and C_b

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Flexural strength of members based on the limit state of lateral-torsional buckling is a function of the moment gradient of the unbraced length under consideration. The bending modification factor, C_b, accounts for the shape of the moment gradient within the unbraced length and allows for adjustment of the member flexural strength, possibly increasing it by a considerable amount. Therefore, neglecting the impact of C_b on member strength may lead to overdesign. This paper discusses the application of C_b to the design of members subjected to bending including beams as well as members subjected to combined loading (compression and bending, tension and bending, or biaxial bending). The concept of C_b and the formula for determining the modification factor is similar when using Allowable Strength Design (ASD) or the Load and Resistance Factor Design (LRFD) method. However, designers used to the 1989 *Allowable Stress Design and Plastic Design Specification for Structural Steel Buildings* (AISC, 1989a), hereafter referred to as the 1989 ASD *Specification,* and the 9th edition of the AISC *Manual of Steel Construction* (AISC, 1989b) will find more updates in the new formula and method of calculating and using C_b. Numerical examples are presented using both ASD and LRFD methods.

Four limit states apply to bending of steel beams: yielding, lateral-torsional buckling, flange local buckling, and web local buckling. Yura, Galambos and Ravindra (1978) present a thorough discussion of behavior of steel beams under bending and the different controlling limit states. The modification factor, C_b, affects cases in which member unbraced length, L_b, is greater than L_p and thus lateral-torsional buckling (LTB) controls flexural strength.

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A fundamental assumption made in developing the relevant design aids included in the AISC *Steel Construction Manual* (AISC, 2005b) for flexural design when LTB controls is that the critical unbraced segment of the beam has a uniform moment diagram (gradient). If this is not the case, the modification factor, C_b, should be used to adjust the flexural strength of the member. It is noted that every unbraced segment of a beam has its own C_b, which may or may not be equal to that of other segments.

Calculation of C_b has gone through a number of evolutions over the years. Zoruba and Dekker (2005) offer a historic and technical overview of C_b in the AISC Specifications.

The 2005 AISC *Specification for Structural Steel Buildings* (AISC, 2005a), hereafter referred to as the AISC *Specification,* offers Equation 1 for calculation of C_b.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} R_m \leq 3.0 \qquad (1)$$

where

M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. (N-mm)
M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. (N-mm)
M_B = absolute value of moment at centerline of the unbraced beam segment, kip-in. (N-mm)
M_C = absolute value of moment at three-quarter point of the unbraced beam segment, kip-in. (N-mm)
R_m = cross-sectional monosymmetry parameter
= 1.0 for doubly symmetric members
= 1.0 for singly symmetric members subjected to single curvature bending
= $0.5 + 2(I_{yc}/I_y)$ for singly symmetric members subjected to reverse curvature bending
I_y = moment of inertia about the principal y-axis, in.^{4} (mm^4)
I_{yc} = moment of inertia about y-axis referred to the compression flange, or if reverse curvature bending, referred to the smaller flange, in.^{4} (mm^4)

Designers used to the 1989 ASD *Specification* (AISC, 1989a) will find more changes in the new formula and method of calculating and using C_b. This is due to the fact that the 1989 ASD *Specification* had not been updated between 1989 and 2005, while the LRFD *Specification* had gone through a number of revisions to reflect the state-of-the-art in research and practice.

Example 1 illustrates application of Equation 1 to determine C_b for a case not included in Table 3-1 of the AISC *Steel Construction Manual* (AISC, 2005b). It is noted that from this point on, all references in the following examples are made to the 13th Edition AISC *Steel Construction Manual*.

EXAMPLE 1

Given

Simply supported girder shown in Figure 1, laterally braced at ends and load point.
P: 3 kips DL + 10 kips LL
w: 0.35 kips/ft DL (including beam weight) + 1.15 kips/ft LL

Find

Calculate C_b using ASD and LRFD load combinations.

Solution

Using ASD load combinations:
P_a = 3 kips + 10 kips = 13.0 kips
w_a = 0.35 kip/ft + 1.15 kip/ft = 1.50 kip/ft

This beam and loading is a combination of cases 1 and 7 from Table 3-23 of the AISC *Steel Construction Manual* (pages 3-211 and 3-213).

Consider the case of beam with concentrated load at midspan. It can be shown that values of M_{max}, M_A, M_B and M_C are as follows.

M_A = 24.4 kip-ft

M_B = 48.8 kip-ft

M_C = 73.1 kip-ft

M_{max} = 97.5 kip-ft

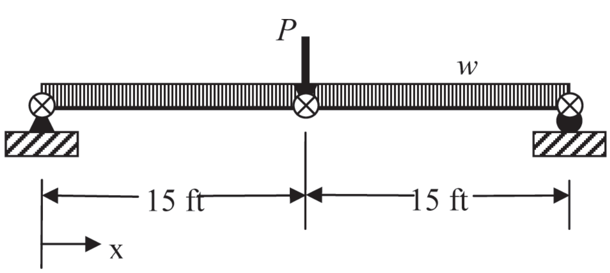

Fig. 1. Simply supported girder for Example 1.

Similarly, it can be shown that for the uniform load,

M_A = 73.8 kip-ft

M_B = 127 kip-ft

M_C = 158 kip-ft

M_{max} = 169 kip-ft

We now calculate C_b using Equation 1 with M_A, M_B, M_C and M_{max} totals from both loading conditions. Note that R_m = 1.0 (single curvature).

M_A = 24.4 kip-ft + 73.8 kip-ft
= 98.2 kip-ft

M_B = 48.8 kip-ft + 127 kip-ft
= 176 kip-ft

M_C = 73.1 kip-ft + 158 kip-ft
= 231 kip-ft

M_{max} = 97.5 kip-ft + 169 kip-ft
= 267 kip-ft

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}R_m$$

$$= \frac{12.5(267 \text{ kip-ft})}{2.5(267 \text{ kip-ft})+3(98.2 \text{ kip-ft})+4(176 \text{ kip-ft})+3(231 \text{ kip-ft})} \times (1.0)$$

$$= 1.41$$

Using LRFD load combinations:

P_u = (1.2)(3 kips) + (1.6)(10 kips)
= 19.6 kips

w_u = (1.2)(0.35 kip/ft) + (1.6)(1.15 kip/ft)
= 2.26 kip/ft

Note the relationship between loads calculated based on the ASD and LRFD load combinations.

$$\frac{P_u}{P_a} = \frac{19.6 \text{ kips}}{13.0 \text{ kips}} = 1.51$$

and

$$\frac{w_u}{w_a} = \frac{2.26 \text{ kip/ft}}{1.50 \text{ kip/ft}} = 1.51$$

Therefore, M_{max}, M_A, M_B and M_C for each loading case using the LRFD load combinations will be 1.51 times those found earlier based on the ASD load combinations. This ratio cancels out from top and bottom of the C_b equation and yields the same C_b value for the LRFD load combination as for the ASD approach.

Note that according to Table 3-1 in the AISC *Steel Manual* (page 3-10), C_b for the point load and uniformly distributed loads acting alone are 1.67 and 1.30, respectively.

The modification factor C_b has values ranging from 1.0 to 3.0. The case $C_b = 1.0$ is the most critical. Larger values of C_b may result in increased flexural strength. Section F1 of the AISC *Specification* states that "C_b is permitted to be conservatively taken as 1.0 for all cases." It further states that "For cantilevers or overhangs where the free end is unbraced, $C_b = 1.0$." The reader is encouraged to review the pertinent parts of the 2005 AISC *Specification* (AISC, 2005a) and its commentary for more details and exceptions to the above statements.

FLEXURAL STRENGTH FOR $C_b > 1.0$

Burgett and Tide (1980) offer a method for design of beams with $C_b > 1.0$ for the Allowable Stress Design (ASD) method using the 1989 *ASD Specification* (AISC, 1989a). He recommends using an effective unbraced length (L_e) instead of the original unbraced length (L_x) to select a trial section for $C_b > 1.0$. Burgett suggests use of the formula $L_e = L_x / C_b$ to determine L_e. Once a trial section is selected with L_e, M_u, and $C_b = 1.0$, it must be checked for compliance with the 1989 ASD *Specification* for the original L_x and C_b. This method works well for relatively large unbraced lengths and using the Allowable Stress Design method. The following discussion will cover procedures that are consistent with the unified 2005 AISC *Specification for Structural Steel Buildings* (AISC, 2005a) and apply to both Allowable Strength Design method as well as the Load and Resistance Factor Design methods.

According to the 2005 AISC *Specification* (AISC, 2005a), the nominal flexural strength of beams based on the limit state of lateral torsional buckling with $C_b > 1.0$ is determined using the formula given in Equation 2:

$$\left[M_n\right]_{C_b > 1.0} = \left(C_b\right)\left[M_n\right]_{C_b = 1.0} \leq M_p \text{ or } M_p' \tag{2}$$

ASD and LRFD versions of Equation 2 follow.

$$\left[\frac{M_n}{\Omega}\right]_{C_b > 1.0} = \left(C_b\right)\left[\frac{M_n}{\Omega}\right]_{C_b = 1.0} \leq \frac{M_p}{\Omega} \text{ or } \frac{M_p'}{\Omega} \quad \text{(2-ASD)}$$

$$\left[\phi_b M_n\right]_{C_b > 1.0} = \left(C_b\right)\left[\phi_b M_n\right]_{C_b = 1.0} \leq \phi_b M_p \text{ or } \phi_b M_p' \quad \text{(2-LRFD)}$$

FLEXURAL DESIGN FOR $C_b > 1.0$

Equation 2 and its ASD and LRFD versions form the basis for incorporating C_b in determining flexural strength of members based on the limit state of lateral torsional buckling. As noted earlier, C_b may be conservatively taken to be equal to 1.0 in all cases. However, by doing so the designer is likely to miss gains from $C_b > 1.0$, which may lead to smaller member sizes. Such gains may be quite significant, particularly for beams with relatively large unbraced lengths. Therefore, it makes sense to consider the impact of C_b for unbraced beams. Recall that values of C_b may range from 1.0 to 3.0. Thus, it is possible that member strength after incorporating C_b may be as much as three times that without considering C_b.

The amount of gain or benefit from C_b varies depending on section size, material properties, unbraced length, and value of C_b. Figure 2 graphically illustrates variation of nominal flexural strength versus the unbraced length for compact sections. The chart for non-compact sections would look similar to Figure 2, except that M_p', L_p' and L_m' will be used instead of M_p, L_p and L_m.

Two bending modification factors are utilized in the chart of Figure 2 with $C_{b2} > C_{b1} > 1.0$ to illustrate different possibilities. Note particularly that $C_b M_n$ may not exceed M_p for compact sections and M_p' for non-compact sections.

We will consider three cases in the following discussion on incorporating C_b in flexural design: beams with relatively small unbraced lengths, beams with relatively large unbraced lengths, and beams with moderate unbraced lengths. For ease of discussion, it is assumed here that all beam sections are compact. Since the AISC *Steel Construction Manual* lists M'_p for non-compact sections the same way in charts and tables as it does M_p for compact sections, there are no procedural differences between the two.

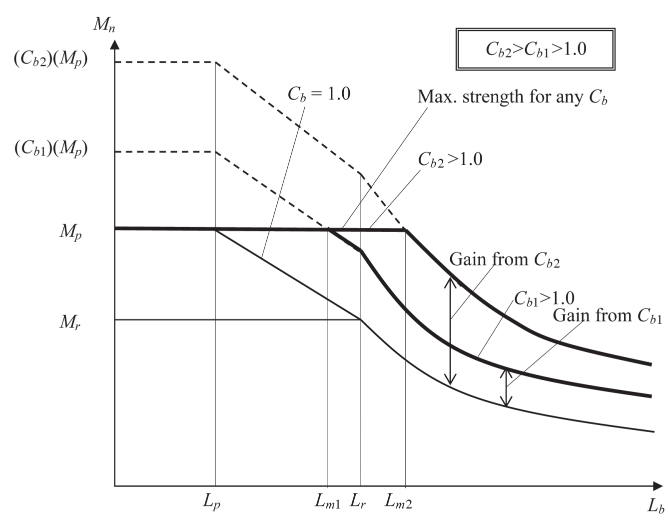

Fig. 2. Impact of C_b *on* M_n *for compact sections.*

For beams with relatively small unbraced length and L_b not exceeding L_p, either yielding or local buckling controls and C_b does not apply. In such cases, the nominal flexural strength is already at the maximum of M_p (M'_p for noncompact sections).

In cases of beams with relatively large unbraced lengths, it is very likely the beam will fully benefit from C_b and therefore Equation 2 may be used as follows for design.

$$M_u \le \left[\frac{M_n}{\Omega}\right]_{C_b > 1.0} = \left(C_b\right)\left[\frac{M_n}{\Omega}\right]_{C_b = 1.0} \le \left[\frac{M_p}{\Omega}\right] \quad \text{(3-ASD)}$$

$$M_u \le \left[\phi_b M_n\right]_{C_b > 1.0} = \left(C_b\right)\left[\phi_b M_n\right]_{C_b = 1.0} \le \phi_b M_p \quad \text{(3-LRFD)}$$

or

$$\frac{M_u}{C_b} \le \left[\frac{M_n}{\Omega}\right]_{C_b = 1.0} \quad \text{(4-ASD)}$$

$$\frac{M_u}{C_b} \le \left[\phi_b M_n\right]_{C_b = 1.0} \quad \text{(4-LRFD)}$$

As suggested by Equations 4, for design of beams with relatively large unbraced lengths and $C_b > 1.0$, the designer may select a trial section based on a fictitious required moment of $[M_u]_{C_b = 1.0} = (M_u / C_b)$. This section is then checked for strength using its original M_u and C_b.

For beams with moderate unbraced lengths, one may still follow the approach recommended for relatively large unbraced lengths, but use a smaller C_b to obtain a trial section. Alternatively, the designer may initially neglect the impact of C_b, obtain a trial section based on $C_b = 1.0$, and continue checking smaller sections until a desired section is found. In either case, the trial section must be checked for strength using its original M_u and C_b.

Note that in all cases, the requirement of $\phi_b M_p \ge M_u$ for LRFD and $M_p/\Omega \ge M_u$ for ASD must be checked for all trial sections before any additional work is done.

EXAMPLE 2

Given

Simply supported beam with $L = 50$ ft, $w_D = 0.24$ kip/ft (beam weight included), $w_L = 0.72$ kip/ft, A992 steel. Consider bending only.

Find

Use ASD and LRFD methods to select the lightest W-section if

(a) $L_b = 5$ ft
(b) $L_b = 25$ ft

Solution

ASD Method:

$$\begin{aligned} w_a &= w_D + w_L \\ &= 0.24 \text{ kip/ft} + 0.72 \text{ kip/ft} \\ &= 0.96 \text{ kip/ft} \end{aligned}$$

$$\begin{aligned} M_a &= \frac{w_a L^2}{8} \\ &= \frac{(0.96 \text{ kip/ft})(50 \text{ ft})^2}{8} \\ &= 300 \text{ kip-ft} \end{aligned}$$

Case (a)

$L_b = 5$ ft—relatively small, assume braced beam, C_b not applicable

From Table 3-2 (page 3-17) look for a section with $M_{px}/\Omega \ge M_a = 300$ kip-ft

Try W21×55

$L_b = 5.0$ ft $< L_p = 6.11$ ft, braced beam $\rightarrow$ assumptions about L_b and C_b correct

Therefore, $M_{nx}/\Omega = M_{px}/\Omega = 314$ kip-ft $> M_a = 300$ kip-ft

Use W21×55, A992 steel

Case (b)

$L_b = 25$ ft—relatively large $\rightarrow$ assume full benefit from C_b

Note from Table 3-1 of the AISC *Steel Construction Manual* (page 3-10) that $C_b = 1.30$ for a uniformly loaded simply supported beam braced at midpoint.

Select a trial section based on the following

$$\begin{aligned} \left[M_a\right]_{C_b = 1.0} &= \frac{\left[M_a\right]_{C_b = 1.30}}{C_b} \\ &= \frac{300 \text{ kip-ft}}{1.30} \\ &= 231 \text{ kip-ft} \end{aligned}$$

Go to page 3-122 of the AISC *Steel Construction Manual* with $L_b = 25$ ft, $M_a = 231$ kip-ft, and $C_b = 1.0$

Try W18×76

From Table 3-2 of the AISC *Steel Construction Manual* (page 3-16):

$M_{px}/\Omega = 407$ kip-ft $> M_a = 300$ kip-ft $\rightarrow$ section may work

$[M_{nx}/\Omega]_{C_b = 1.0} = 273$ kip-ft read directly off of the chart on page 3-122.

$[M_{nx}/\Omega]_{C_b = 1.3} = (C_b)[M_{nx}/\Omega]_{C_b = 1.0}$
$= (1.30)(273 \text{ kip-ft})$
$= 355 \text{ kip-ft} < M_{px}/\Omega = 407 \text{ kip-ft}$

Note that beam fully benefited from C_b since $[M_{nx}/\Omega]_{C_b = 1.3} < M_{px}/\Omega$

$[M_{nx}/\Omega]_{C_b = 1.3} = 355 \text{ kip-ft} > M_a = 300 \text{ kip-ft}$

W18×76 is adequate

For lightest, try W14×74

$M_{px}/\Omega = 314$ kip-ft $> M_a = 300$ kip-ft → may work

$[M_{nx}/\Omega]_{C_b = 1.0} = 212$ kip-ft read directly off of the chart on page 3-122.

$[M_{nx}/\Omega]_{C_b = 1.3} = (C_b)[M_{nx}/\Omega]_{C_b = 1.0}$
$= (1.30)(228 \text{ kip-ft})$
$= 296 \text{ kip-ft} < M_{px}/\Omega = 314 \text{ kip-ft}$

Therefore, $[M_{nx}/\Omega]_{C_b = 1.3} = 296$ kip-ft $< M_a = 300$ kip-ft → Not adequate

Use W18×76, A992 steel

LRFD Method:

$$
\begin{aligned}
w_u &= 1.2w_D + 1.6w_L \\
&= 1.2(0.24 \text{ kip/ft}) + 1.6(0.72 \text{ kip/ft}) \\
&= 1.44 \text{ kip/ft}
\end{aligned}
$$

$$
\begin{aligned}
M_u &= \frac{w_u L^2}{8} \\
&= \frac{(1.44 \text{ kip/ft})(50 \text{ ft})^2}{8} \\
&= 450 \text{ kip-ft}
\end{aligned}
$$

Case (a)

$L_b = 5$ ft—relatively small, assume braced beam, C_b not applicable

From Table 3-2 of the AISC *Steel Construction Manual* (page 3-17) look for a section with $\phi_b M_{px} \geq M_u = 450$ kip-ft

Try W21×55

$L_b = 5.0 \text{ ft} < L_p = 6.11$ ft, braced beam → assumptions about L_b and C_b correct

Therefore, $\phi_b M_{nx} = \phi_b M_{px} = 473$ kip-ft $> M_u = 450$ kip-ft

Use W21×55, A992 steel

Case (b)

$L_b = 25$ ft—relatively large → assume full benefit from C_b

Note from Table 3-1 of the AISC *Steel Construction Manual* (page 3-10) that $C_b = 1.30$ for a uniformly loaded simply supported beam braced at midpoints.

Select a trial section based on the following assumption.

$$
\begin{aligned}
\left[M_u\right]_{C_b = 1.0} &= \frac{\left[M_u\right]_{C_b = 1.30}}{C_b} \\
&= \frac{450 \text{ kip-ft}}{1.30} \\
&= 346 \text{ kip-ft}
\end{aligned}
$$

Go to page 3-122 of the AISC *Steel Construction Manual* and select a trial section for $L_b = 25$ ft, $M_u = 346$ kip-ft, and $C_b = 1.0$

Try W18×76

From Table 3-2 of the AISC *Steel Construction Manual* (page 3-16): $\phi_b M_{px} = 611$ kip-ft $> M_u = 450$ kip-ft → may work

$[\phi_b M_{nx}]_{C_b = 1.0} = 410$ kip-ft (read directly off of the chart on page 3-122)

$[\phi_b M_{nx}]_{C_b = 1.3} = (C_b)[\phi_b M_{nx}]_{C_b = 1.0}$
$= (1.30)(410 \text{ kip-ft})$
$= 533 \text{ kip-ft} < \phi_b M_{px}$
$= 611 \text{ kip-ft}$

Note that beam fully benefited from C_b since $[\phi_b M_{nx}]_{C_b = 1.3} < \phi_b M_{px}$

$[\phi_b M_{nx}]_{C_b = 1.3} = 533 \text{ kip-ft} > M_u = 450 \text{ kip-ft}$

W18×76 is adequate

It can be shown that a W14×74 is not adequate. Therefore, use W18×76, A992 steel

Note that if $C_b = 1.0$ were used, a W18×86 would have been selected instead.

It is noted that the focus of this paper and all example problems presented here is on the impact of C_b on member flexural strength. Other criteria, such as deflection, may control the design and must be checked.

Solid lines in the beam design charts of the AISC *Steel Construction Manual* are meant to obtain the lightest section for $C_b = 1.0$. Using them for selecting sections for $C_b > 1.0$ is not quite right, though reasonable. In order to find the lightest section for $C_b > 1.0$, at least two tries are necessary.

IMPACT OF C_b ON THE b_x COEFFICIENT

Part Six of the AISC *Steel Construction Manual* (AISC, 2005b) presents a method and aids for design of members subjected to combined loads (compression and bending, tension and bending, or biaxial bending). This approach, based on Aminmansour (2000 and 2006), utilizes three new coefficients p, b_x and b_y in analysis or design of such members. Following is a discussion on the impact of C_b on the b coefficient.

In addition to design of members subjected to combined loading, the method and aids presented in Part Six of the AISC *Steel Construction Manual* may prove to be helpful in design of beams in certain circumstances as illustrated in some of the examples that follow.

Values of b_x given in Part Six of the AISC *Steel Construction Manual* and the discussion that follows applies to beam-columns as well as members subjected to combined tension and bending, biaxial bending, or bending alone. Since the limit state of lateral-torsional buckling does not apply to bending of beams about their weak axis, our discussion here will be limited to the b_x coefficient only.

Values of the b_x coefficient given by Equations 5 and listed in Part 6 of the AISC *Steel Construction Manual* are based on C_b = 1.0.

$$b_x = \frac{8}{9\left(\dfrac{M_{nx}}{\Omega}\right)} \tag{5-ASD}$$

$$m = \frac{8}{9\left(\phi_b M_{nx}\right)} \tag{5-LRFD}$$

Combining Equations 2 and 5 yields:

$$\begin{aligned}\left[b_x\right]_{C_b>1.0} &= \frac{8}{9\left[\dfrac{M_n}{\Omega}\right]_{C_b>1.0}} \\ &= \frac{8}{9\left(C_b\right)\left[\dfrac{M_n}{\Omega}\right]_{C_b=1.0}} \\ &= \left(\frac{1}{C_b}\right)\left(\frac{8}{9\left[\dfrac{M_n}{\Omega}\right]_{C_b=1.0}}\right) \\ &= \frac{\left[b_x\right]_{C_b=1.0}}{C_b}\end{aligned} \tag{6 ASD}$$

$$\begin{aligned}\left[b_x\right]_{C_b>1.0} &= \frac{8}{9\left[\phi_b M_n\right]_{C_b>1.0}} \\ &= \frac{8}{9\left(C_b\right)\left[\phi_b M_n\right]_{C_b=1.0}} \\ &= \left(\frac{1}{C_b}\right)\left(\frac{8}{9\left[\phi_b M_n\right]_{C_b=1.0}}\right) \\ &= \frac{\left[b_x\right]_{C_b=1.0}}{C_b}\end{aligned} \tag{6 LRFD}$$

Now, apply the upper limit of Equations 2 to Equations 6 to obtain Equation 7 as follows:

$$\left[b_x\right]_{C_b>1.0} = \frac{\left[b_x\right]_{C_b=1.0}}{C_b} \geq \left[b_x\right]_{min} \tag{7}$$

where

$$\left[b_x\right]_{min} = \frac{8}{9\left(\dfrac{M_p}{\Omega} \text{ or } \dfrac{M'_p}{\Omega}\right)} \tag{8-ASD}$$

$$\left[b_x\right]_{min} = \frac{8}{9\left(\phi_b M_p \text{ or } \phi_b M'_p\right)} \tag{8-LRFD}$$

Values of $[b_x]_{\min}$ are listed in Table 6-1 of the AISC *Steel Construction Manual* at L_b = 0 ft for a large number of W-sections for both ASD and LRFD methods.

USING b_x IN ANALYSIS AND DESIGN FOR FLEXURE WITH C_b > 1.0

The following observations may be used in analysis and design of steel members subjected to bending, including combined loads, with C_b > 1.0. Keep in mind that values of b_x listed in Table 6-1 of the AISC *Steel Construction Manual* are for C_b = 1.0 and already account for compact / non-compactness.

For laterally braced beams the limit state of yielding (local buckling for non-compact sections) controls and C_b does not apply. Therefore, for such situations, simply use the listed values of b_x.

For beams with relatively large unbraced lengths, it is likely that the member will fully benefit from C_b. Therefore, note that

$$\left[b_x\right]_{C_b>1.0} = \frac{\left[b_x\right]_{C_b=1.0}}{C_b} \geq \left[b_x\right]_{min}$$

For beams with moderately long unbraced length where partial benefit from C_b may be realized, the designer may treat the beam as in the latter case, but use a smaller C_b to obtain a trial section.

In all preceding cases, the trial section must be analyzed exactly based on its L_b and original C_b values to obtain b_x.

Important Note: The reader is reminded that a section with a b_x value equal to or smaller than a required value will be adequate in bending for the conditions given. See Aminmansour (2000 and 2006) for more comprehensive information on this subject.

EXAMPLE 3

Given

W24×76, L_b = 16 ft, C_b = 1.67, A992 steel

Find

Calculate available flexural strength using both ASD and LRFD methods

Solution

ASD Method:

Method I:

From Table 3-2 of the AISC *Steel Construction Manual* (page 3-16):

M_p/Ω = 499 kip-ft
L_p = 6.78 ft
L_r = 19.6 ft
BF = 15.0 kips

L_p = 6.78 ft < L_b = 16 ft < L_r = 19.6 ft

$$\left[\frac{M_n}{\Omega}\right]_{C_b=1.0} = \left(\frac{M_n}{\Omega}\right) - BF\left(L_b - L_p\right)$$
$$= 499 \text{ kip-ft} - \left(15.0 \text{ kips}\right)\left(16.0 \text{ ft} - 6.78 \text{ ft}\right)$$
$$= 361 \text{ kip-ft}$$

$$\left[\frac{M_n}{\Omega}\right]_{C_b=1.0} = \left(C_b\right)\left[\frac{M_n}{\Omega}\right]_{C_b=1.0}$$
$$= (1.67)(361 \text{ kip-ft}) = 603 \text{ kip-ft} > \left(\frac{M_p}{\Omega}\right)$$
$$= 499 \text{ kip-ft}$$

Therefore, $[M_n/\Omega]_{C_b = 1.67} = [M_p/\Omega]$ = 499 kip-ft

Method II:

From Table 6-1 of the AISC *Steel Construction Manual* (page 6-42):
$b_x = 2.46 \times 10^{-3}$ (kip-ft)$^{-1}$ for C_b = 1.0
$[b_x]_{min} = 1.78 \times 10^{-3}$ (kip-ft)$^{-1}$

$$\left[b_x\right]_{C_b>1.0} = \frac{\left[b_x\right]_{C_b=1.0}}{C_b}$$
$$= \frac{2.46 \times 10^{-3} \text{ (kip-ft)}^{-1}}{1.67}$$
$$= 1.47 \times 10^{-3} \text{ (kip-ft)}^{-1} < \left[b_x\right]_{min}$$
$$= 1.78 \times 10^{-3} \text{ (kip-ft)}^{-1}$$

Use $[b_x]_{C_b = 1.67} = [b_x]_{min}$
$= 1.78 \times 10^{-3}$ (kip-ft)$^{-1}$

$$\left[\frac{M_n}{\Omega}\right]_{C_b=1.67} = \frac{8}{9\left[b_x\right]_{C_b=1.67}}$$
$$= \frac{8}{9\left[1.78 \times 10^{-3} \text{ (kip-ft)}^{-1}\right]}$$
$$= 499 \text{ kip-ft}$$

LRFD Method:

Method I:

From Table 3-2 of the AISC *Steel Construction Manual* (page 3-16):
$\phi_b M_p$ = 750 kip-ft
L_p = 6.78 ft
L_r = 19.6 ft
BF = 22.5 kips

L_p = 6.78 ft < L_b = 16 ft < L_r = 19.6 ft

$$\left[\phi_b M_n\right]_{C_b=1.0} = \phi_b M_p - BF\left(L_b - L_p\right)$$
$$= 750 \text{ kip-ft} - \left(22.5 \text{ kips}\right)\left(16.0 \text{ ft} - 6.78 \text{ ft}\right)$$
$$= 543 \text{ kip-ft}$$

$$\left[\phi_b M_n\right]_{C_b>1.0} = \left(C_b\right)\left[\phi_b M_n\right]_{C_b=1.0}$$
$$= (1.67)(543 \text{ kip-ft})$$
$$= 907 \text{ kip-ft} > \phi_b M_p = 750 \text{ kip-ft}$$

Therefore, $[\phi_b M_n]_{C_b = 1.67} = \phi_b M_p$ = 750 kip-ft

Method II:

From Table 6-1 of the AISC *Steel Construction Manual* (page 6-42):
$b_x = 1.64 \times 10^{-3}$ (kip-ft)$^{-1}$ for C_b = 1.0
$[b_x]_{min} = 1.19 \times 10^{-3}$ (kip-ft)$^{-1}$

$$\left[b_x\right]_{C_b>1.0} = \frac{\left[b_x\right]_{C_b=1.0}}{C_b}$$
$$= \frac{1.64 \times 10^{-3} \text{ (kip-ft)}^{-1}}{1.67}$$
$$= 0.982 \times 10^{-3} \text{ (kip-ft)}^{-1} < \left[b_x\right]_{min} = 1.19 \times 10^{-3} \text{ (kip-ft)}^{-1}$$

Use $[b_x]_{C_b = 1.67} = [b_x]_{min}$
$= 1.19 \times 10^{-3}$ (kip-ft)$^{-1}$

$$\left[\phi_b M_n\right]_{C_b=1.67} = \frac{8}{9\left[b_x\right]_{C_b=1.67}}$$
$$= \frac{8}{9\left[1.19 \times 10^{-3} \text{ (kip-ft)}^{-1}\right]}$$
$$= 747 \text{ kip-ft}$$

EXAMPLE 4

Given

Simply supported beam with $L = L_b = 30$ ft, $w_D = 0.30$ kip/ft, $w_L = 0.90$ kip/ft

Find

Select the lightest W14 of A992 steel. Consider bending only.

Solution

Since Table 6-1 of the AISC *Steel Construction Manual* has a more comprehensive list of W14s, we will use the b_x method in this design.

ASD Method:

$w_u = w_D + w_L = 0.30$ kip/ft + 0.90 kip/ft = 1.20 kip/ft

From Table 3-1 of the AISC *Steel Construction Manual* (page 3-10), $C_b = 1.14$.

$$M_u = \frac{w_u L^2}{8} = \frac{(1.20 \text{ kip/ft})(30 \text{ ft})^2}{8} = 135 \text{ kip-ft}$$

Use Equation 5-ASD and replace (M_{nx}/Ω) with M_u to obtain $(b_x)_{req}$

$$\left[(b_x)_{req}\right]_{C_b = 1.14} = \frac{8}{9(135 \text{ kip-ft})} = 6.58 \times 10^{-3} \text{ (kip-ft)}^{-1}$$

From Table 6-1 of the AISC *Steel Construction Manual* (page 6-71), try W14×61 with

$[b_x]_{C_b = 1.0} = 6.20 \times 10^{-3}$ (kip-ft)$^{-1}$ < 6.58×10^{-3} (kip-ft)$^{-1}$

This section is adequate for $C_b = 1.0$; therefore it is adequate for $C_b = 1.14$ as well. However, for illustration purposes, we will verify this fact.

$$\begin{aligned}\left[b_x\right]_{C_b = 1.14} &= \frac{\left[b_x\right]_{C_b = 1.0}}{C_b} \\ &= \frac{6.20 \times 10^{-3} \text{ (kip-ft)}^{-1}}{1.14} \\ &= 5.44 \times 10^{-3} \text{ (kip-ft)}^{-1} > \left[b_x\right]_{min} \\ &= 3.49 \times 10^{-3} \text{ (kip-ft)}^{-1}\end{aligned}$$

Since $[b_x]_{C_b = 1.14} = 5.44 \times 10^{-3}$ (kip-ft)$^{-1}$ < $[b_x]_{req}$ $= 6.58 \times 10^{-3}$ (kip-ft)$^{-1}$

W14×61 is adequate.

For lightest, try W14×53 with $b_x = 9.56 \times 10^{-3}$ (kip-ft)$^{-1}$

It can be shown that this section has a larger b_x than $[b_x]_{req} = 6.58 \times 10^{-3}$ (kip-ft)$^{-1}$ after incorporating the impact of C_b. Therefore, W14×53 is not adequate.

Use W14×61, A992 steel

LRFD Method:

$w_u = 1.2w_D + 1.6w_L = 1.2(0.30$ kip/ft$) + 1.6(0.90$ kip/ft$)$ $= 1.80$ kip/ft

From Table 3-1 of the AISC *Steel Construction Manual* (page 3-10), $C_b = 1.14$.

$$M_u = \frac{w_u L^2}{8} = \frac{(1.80 \text{ kip/ft})(30 \text{ ft})^2}{8} = 203 \text{ kip-ft}$$

Use Equation 5-LRFD and replace $\phi_b M_{nx}$ with M_u to obtain $(b_x)_{req}$

$$\left[(b_x)_{req.}\right]_{C_b = 1.0} = \frac{8}{9(203 \text{ kip-ft})} = 4.38 \times 10^{-3} \text{ (kip-ft)}^{-1}$$

From Table 6-1 of the AISC *Steel Construction Manual* (page 6-71), try W14×61 with

$[b_x]_{C_b = 1.0} = 4.12 \times 10^{-3}$ (kip-ft)$^{-1}$ < 4.38×10^{-3} (kip-ft)$^{-1}$

This section is adequate for $C_b = 1.0$; therefore it is adequate for $C_b = 1.14$ as well. However, for illustration purposes, we will verify this fact.

$$\begin{aligned}\left[b_x\right]_{C_b = 1.14} &= \frac{\left[b_x\right]_{C_b = 1.0}}{C_b} \\ &= \frac{4.12 \times 10^{-3} \text{ (kip-ft)}^{-1}}{1.14} \\ &= 3.61 \times 10^{-3} \text{ (kip-ft)}^{-1} > \left[b_x\right]_{min} \\ &= 2.32 \times 10^{-3} \text{ (kip-ft)}^{-1}\end{aligned}$$

Since $[b_x]_{C_b = 1.14} = 3.61 \times 10^{-3}$ (kip-ft)$^{-1}$ < $[b_x]_{req}$ $= 4.38 \times 10^{-3}$ (kip-ft)$^{-1}$

W14×61 is adequate.

For lightest, try W14×53 with $b_x = 6.36 \times 10^{-3}$ (kip-ft)$^{-1}$

It can be shown that this section has a larger b_x than $[b_x]_{req} = 4.38 \times 10^{-3}$ (kip-ft)$^{-1}$. Therefore, W14×53 is not adequate.

Use W14×61, A992 steel

Note that if $[b_x]_{C_b > 1.0} > [b_x]_{min}$, the beam fully benefits from $C_b > 1.0$. Otherwise, the benefit is partial.

EXAMPLE 5

Given

W18×86 member as shown in Figure 3, A992 steel, braced frame, bending about the *x*-axis only
$P_u = 630$ kips
$M_{ux} = 210$ kip-ft
$(KL)_x = 26$ ft
$(KL)_y = L_b = 13$ ft

Find

Determine if this section adequate per AISC *Specification* for the given conditions.

Solution

LRFD Method:

Obtain the following information for a W18×86 from Table 6-1 of the AISC *Steel Construction Manual* (page 6-54):
$[b_x]_{C_b = 1.0} = 1.37 \times 10^{-3}$ (kip-ft)$^{-1}$
$[b_x]_{min} = 1.27 \times 10^{-3}$ (kip-ft)$^{-1}$
$(r_x/r_y) = 2.95$

Determine $(KL)_{y\,eq}$

$$(KL)_{y\,eq} = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)} = \frac{26 \text{ ft}}{2.95} = 8.81 \text{ ft} < (KL)_y = 13.0 \text{ ft}$$

Use $KL = (KL)_y = 13.0$ ft

Therefore, $p = 1.14 \times 10^{-3}$ kips^{-1}

Note that in this case, $C_b = 1.30$ (braced at ends and midspan)

$$[b_x]_{C_b = 1.30} = \frac{[b_x]_{C_b = 1.0}}{1.30} = \frac{1.37 \times 10^{-3} \text{ (kip-ft)}^{-1}}{1.30} = 1.05 \times 10^{-3} \text{ (kip-ft)}^{-1} < [b_x]_{min} = 1.27 \times 10^{-3} \text{ (kip-ft)}^{-1}$$

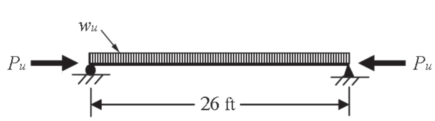

Fig. 3. Beam-column for Example 5.

Therefore, $[b_x]_{C_b = 1.30} = [b_x]_{min} = 1.27 \times 10^{-3}$ (kip-ft)$^{-1}$

$pP_u = (1.14 \times 10^{-3}$ kips$^{-1})(630$ kips$) = 0.718 > 0.200$

Use $bP_u + mM_{ux} + nM_{uy} \le 1.0$

$0.718 + [1.27 \times 10^{-3}$ (kip-ft)$^{-1}](210$ kip-ft$) + 0$
$= 0.718 + 0.267 + 0$
$= 0.985 < 1.00$

W18×86, A992 steel is adequate for the given conditions.

Note that the impact of $C_b = 1.3$ in this case is not significant. However, the example problem is presented as an illustration of the method.

Reader is encouraged to repeat this problem using the ASD method.

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