Simplified LRFD Design of Steel Members for Fire

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When a steel structure is exposed to fire, the steel temperatures increase and the strength and stiffness of the steel are reduced, leading to possible deformation and failure, depending on the applied loads and the support conditions. The increase in steel temperatures depends on the severity of the fire, the area of steel exposed to the fire, the amount of applied fire protection, the orientation and geometry of members, compartments, and properties/types of steel.

The control of structural behavior under fire conditions has historically been based on highly prescriptive building code requirements that specify hourly fire resistance ratings. A popular misconception concerning fire resistance ratings for walls, columns, floors and other building components is that the ratings imply the length of time that a building component will remain in place when exposed to an actual fire (FEMA, 2002). However, the traditional approach, based on standard testing methodologies such as ASTM E119 (ASTM, 2000), is often overly conservative and may not be realistic, since a number of factors, such as continuity, member interaction, restraint conditions and load intensity, are not accounted for. Furthermore, the characteristics and location of the structural member, as well as the actual nature of the fire scenario also influence the behavior and eventual failure of the structure and are often not accounted for, especially in the standard furnace test.

Modern structural steel design codes use an ultimate strength design format in which internal actions resulting from the maximum likely values of load are compared with the expected member strength using the short-term strength of the materials under ambient temperature. This design format is referred to as load and resistance factor design (LRFD) in North America (AISC, 2005a and 2005b), and limit states design in Europe (ECS, 1992).

Eurocode 3 provides design guidelines to follow when designing structural members to fire standards (ECS, 1995). These methods involve determining the fire loads imposed on a structure and analyzing the strength of each member at elevated temperatures. The structural design for fire in the Eurocode is conceptually similar to structural design for normal temperature conditions, but with reduction factors accounting for the strength loss at elevated temperatures. The methodologies for the calculation of the temperature of unprotected and protected steel members are given in lumped mass, time-step form, with the increase in temperature being based on energy transferred to the member.

Fire design has been recently stipulated in the AISC *Specification for Structural Steel Buildings,* hereafter referred to as the AISC *Specification* (2005). The new design stipulations, and fire design in general, remain relatively unfamiliar to most structural engineers. This paper proposes a simplified design methodology based on the AISC *Specification*. Design examples are presented and results are compared with experimental data.

LITERATURE REVIEW

A comprehensive overview of the fire resistance of building structures is described in *Structural Design for Fire Safety* (Buchanan, 2001). According to Buchanan, structural steel design for fire is conceptually similar to normal design. The three main differences are (1) the applied live loads are generally lower, (2) internal forces may be induced by thermal expansion and (3) steel strengths are reduced by elevated temperatures. Bailey and Moore (2000a) have developed a design method for calculating the performance of steel-framed structures subjected to fire, with composite flooring systems. Their companion paper (Bailey and Moore, 2000b) shows how the proposed design method can be applied to practical buildings. Usmani, Rotter, Lamont, Sanad and Gillie (2001) present theoretical descriptions of the key phenomena that govern the behavior of composite framed structures in fires. They discuss both thermal expansion and thermal bowing. Simplified fire design based on LRFD remain absent in the literature.

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LRFD DESIGN FOR FIRE

For LRFD, the AISC *Specification* requires that the design strength of each structural component, ϕR_n, equals or exceeds the required strength, R_u, determined on the basis of LRFD load combinations,

$$\phi R_n \geq R_u \tag{1}$$

where

R_u = required strength
R_n = nominal strength
ϕ = resistance factor corresponding to R_n
ϕR_n = design strength

The deterioration in strength and stiffness of structural members, components and systems shall be taken into account in accordance with Appendix 4 of the AISC *Specification*, Structural Design for Fire Conditions. For the purpose of design, the reduction factors k_E, k_y and k_u are defined as the ratio of, respectively, the elastic modulus, yield strength and tensile strength of steel, at elevated temperature to normal temperature. Normal or ambient temperature is assumed to be 20 °C (68 °F).

Tension Members

The design strength of tension members, $\phi_t P_n$, is the lower value obtained according to the limit states of yielding in the gross section and fracture in the net section (Chapter D of the AISC *Specification*). The design strength of a tension member for fire conditions (AISC Appendix 4.2.4.3b) assumes a uniform temperature over the cross section using the temperature equal to the maximum steel temperature.

(a) For yielding in the gross section

$$P_n = k_{y,max} F_y A_g \tag{2}$$

(b) For fracture in the net section

$$P_n = k_{u,max} F_u A_e \tag{3}$$

where

$k_{y,max}$ = reduction factor for the yield strength of steel at the maximum steel temperature
F_y = specified minimum yield strength
A_g = gross area of member
$k_{u,max}$ = reduction factor for the tensile strength of steel at the maximum steel temperature
F_u = specified minimum tensile strength
A_e = effective net area

Compression Members

The design strength of compression members whose elements have width-thickness ratios less than λ_y (compact and noncompact sections) is $\phi_c P_n$ (AISC *Specification* Chapter E). The nominal design strength of a compression member for fire conditions is

For $\dfrac{KL_{ub}}{r} \leq 4.71\sqrt{\dfrac{E}{F_y}} \times \sqrt{\dfrac{k_{E,max}}{k_{y,max}}}$

$$F_{cr} = \left[0.658^{\frac{k_{y,max}F_y}{k_{E,max}F_e}}\right] k_{y,max} F_y \tag{4}$$

For $\dfrac{KL_{ub}}{r} > 4.71\sqrt{\dfrac{E}{F_y}} \times \sqrt{\dfrac{k_{E,max}}{k_{y,max}}}$

$$F_{cr} = 0.877 k_{E,max} F_e \tag{5}$$

where

K = effective length factor
L_{ub} = laterally unbraced length of the member
r = governing radius of gyration about the axis of buckling
E = modulus of elasticity
$k_{E,max}$ = reduction factor for the elastic modulus of steel at the maximum steel temperature

F_e = $\dfrac{\pi^2 E}{\left(\dfrac{KL_{ub}}{r}\right)^2}$ = elastic buckling stress

Flexural Members

The flexural design strength of doubly-symmetric compact I-shaped members bent about their major axis is $\phi_b M_n$ (AISC *Specification* Chapter F). Under fire, the design strength of a flexural member is determined assuming that the bottom flange temperature is constant over the depth of the member.

Case (i)

$L_b \leq L_p \sqrt{\dfrac{k_{E,max}}{k_{y,max}}}$

$$M_n = k_{y,max} F_y Z_x \tag{6}$$

Case (ii)

$L_p \sqrt{\dfrac{k_{E,max}}{k_{y,max}}} < L_b \leq L_r \sqrt{\dfrac{k_{E,max}}{k_{y,max}}}$

$$M_n = C_b\left[k_{y,max}F_yZ_x - (k_{y,max}F_yZ_x - 0.7k_{y,max}F_yS_x) \times \left(\frac{L_b - L_p\sqrt{\dfrac{k_{E,max}}{k_{y,max}}}}{L_r\sqrt{\dfrac{k_{E,max}}{k_{y,max}}} - L_p\sqrt{\dfrac{k_{E,max}}{k_{y,max}}}}\right)\right] \le k_{y,max}F_yZ_x \tag{7}$$

Case (iii)

$$L_b > L_p\sqrt{\frac{k_{E,max}}{k_{y,max}}}$$

$$M_n = \frac{C_b\pi^2 k_{E,max}ES_x}{\left(\dfrac{L_b}{r_{ts}}\right)^2} \le k_{y,max}F_yZ_x \tag{8}$$

where

L_b = length between points which are either braced against lateral displacement of compression flange or braced against twist of the cross section

L_p = $1.76r_y\sqrt{\dfrac{E}{F_y}}$

Z_x = plastic section modulus about the major axis

L_r = $\pi r_{ts}\sqrt{\dfrac{E}{0.7F_y}}$

C_b = lateral-torsional buckling modification factor for nonuniform moment diagrams when both ends of the unsupported segment are braced and is permitted to be conservatively taken as 1.0 for all cases

S_x = elastic section modulus about the major axis

r_{ts} = $\sqrt{\dfrac{I_yh_o}{2S_x}}$

I_y = moment of inertia about the minor axis

h_o = distance between the flange centroids

PROPOSED SIMPLIFIED DESIGN METHOD FOR STEEL MEMBERS UNDER FIRE

As seen in the previous discussion, structural design for fire is conceptually similar to structural design for normal temperature conditions. Design of members is the same as normal temperature design but with degradation factors accounting for the strength and stiffness loss at elevated temperatures. This paper proposes a design methodology, suitable for preliminary design that correlates the design strength of the member at normal temperature. Reduction factors for design strengths under fire for tension, compression and flexural are presented in the following. The reduction factor is defined as the ratio of the design strength at elevated temperature to the design strength at normal temperature:

1. Tension members: The reduction factors, $k_{y,max}$ or $k_{u,max}$, in Equations 2 and 3, are plotted in Figure 1 against temperature.

2. Compression members: The reduction factors due to $k_{E,max}$ and $k_{y,max}$ in Equaitons 4 and 5 are plotted in Figure 2 with respect to

$$R = \frac{KL_{ub}/r}{\sqrt{E/F_y}}$$

R is a nondimensional stability factor. R = 4.71 is the transition point between elastic and inelastic buckling.

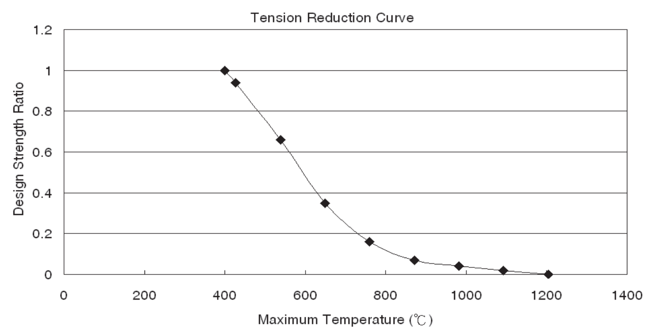

Fig. 1. Strength reduction curve of tension members.

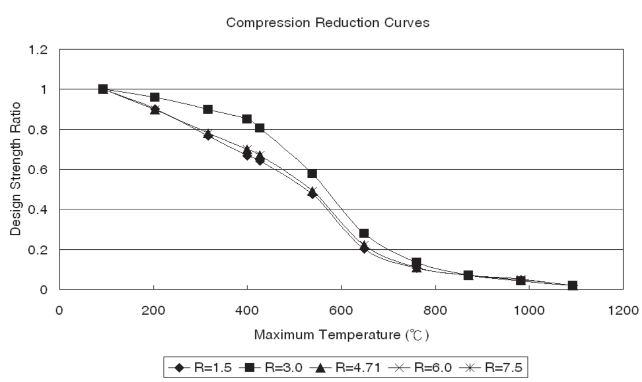

Fig. 2. Strength reduction curves of compression members.

3. Flexural members: The reduction factors due to $k_{E,max}$ and $k_{y,max}$, in Equations 6 through 8, are plotted in Figure 3 with respect to $RL = L_b/L_p$. It is assumed the area of the web is half the flange area, $A_w = 0.5A_f$. RL is a non-dimensional length factor. $RL = 1$ and $RL = L_y/L_p$ are transition points for the different design cases. The following additional simplifying and conservative assumptions are taken for determining the reduction factors and L_r/L_p.

$$S_x = Z_x = b_f t_f h_o$$

$$C_b = 1.0$$

$$r_{ts} = \sqrt{\frac{I_y h_o}{2S_x}} = \sqrt{\frac{I_y}{2b_f t_f}} = \sqrt{\frac{I_y}{2A_f}}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{I_y}{2A_f + A_w}}$$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76\sqrt{\frac{I_y}{2A_f + A_w}}\sqrt{\frac{E}{F_y}}$$

$$L_r = \pi r_{ts}\sqrt{\frac{E}{0.7F_y}} = \frac{\pi}{\sqrt{0.7}}\sqrt{\frac{I_y}{2A_f}}\sqrt{\frac{E}{F_y}}$$

Therefore,

$$\frac{L_r}{L_p} = \frac{\frac{\pi}{\sqrt{0.7}}\sqrt{\frac{I_y}{2A_f}}\sqrt{\frac{E}{F_y}}}{1.76\sqrt{\frac{I_y}{2A_f + A_w}}\sqrt{\frac{E}{F_y}}} = \frac{\pi}{1.76 \times \sqrt{0.7}}\sqrt{\frac{2 + (A_w/A_f)}{2}}$$

where

A_w = $(h_o - t_f)\, t_w$ = web area
t_w = web thickness
A_f = $b_f t_f$ = flange area

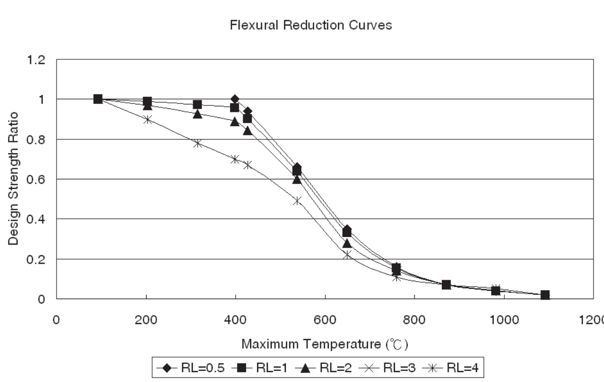

Fig. 3. Strength reduction curves of flexural members ($A_w = 0.5A_f$).

b_f = flange width
t_f = flange thickness

Load Combination

For checking the capacity of a structure or structural element to withstand the effect of an extraordinary event such as fire, the load combination from ASCE 7-02 (ASCE, 2002) is adopted:

$$1.2D + A_k + 0.5L \tag{9}$$

where

D = dead load
L = live load
A_k = load or load effect resulting from an extraordinary event

According to Section C2.5 of ASCE 7-02, extraordinary events arise from extraordinary service or environmental conditions that traditionally are not considered explicitly in design of ordinary buildings and structures. Such events are characterized by a low probability of occurrence and usually a short duration. Specific design provisions to control the effect of extraordinary loads and risk of progressive failure can be developed with a probabilistic basis. But often is the case that data available are limited to define the frequency distribution of the load. A_k must be specified by the authority having jurisdiction. Moreover, as discussed by Usmani et al. (2001), members may experience thermally induced axial force and moment due to thermal expansion and thermal bowing.

Derivation of the Required Design Strength Ratio under Fire

For fire conditions, the structural design follows the basic requirement,

$$\phi R_{nf} \geq R_{uf} \tag{10}$$

where

R_{uf} = required strength under fire
R_{nf} = nominal strength under fire
ϕR_{nf} = design strength under fire

Equation 10 may be rewritten as

$$\frac{\phi R_{nf}}{\phi R_n} \geq \frac{R_{uf}}{\phi R_n} \times \frac{R_u}{R_u} \tag{11}$$

or

$$\frac{\phi R_{nf}}{\phi R_n} \geq \frac{R_{uf}}{R_u} \times \frac{R_u}{\phi R_n} \tag{12}$$

The left side of Equation 12 can be referred to as the required design strength ratio. The required design strength ratio can be expressed as two independent factors,

$$\frac{\phi R_{nf}}{\phi R_n} \geq F_R \times F_{os} \tag{13}$$

where

$F_R = \dfrac{R_{uf}}{R_u}$ = required strength factor; ratio of strength under fire conditions over the strength under normal temperature conditions

$F_{os} = \dfrac{R_u}{\phi R_n}$ = overstrength factor of load demand over the provided capacity (less than 1.0)

Based on the preceding derivation, the required design strength ratio is defined as the product of F_R and F_{os}. The design criterion for fire is thus simplified to one that correlates the required design strength ratio with the reduction factors for fire conditions (see Figures 1, 2 and 3). The overstrength factor, F_o, is obtained from normal temperature design. The required strength factor, F_R, is discussed further.

Required Strength Factor, F_R

In most structural designs, the structure is analyzed elastically and the support and restraint conditions are assumed to remain unchanged. For the purpose of simplification, uniform thermal expansion and unrestrained boundary conditions are assumed in this paper. Because member internal forces are proportional to the applied loading combination, the required strength factor can be simplified as follows:

$$F_R = \frac{1.2D + 0.5L}{1.2D + 1.6L} \tag{14}$$

Equation 14 could be rewritten as

$$F_R = \frac{1.2 + 0.5\left(\dfrac{L}{D}\right)}{1.2 + 1.6\left(\dfrac{L}{D}\right)} \tag{15}$$

CRITICAL TEMPERATURE

Fire design, based on member strength, can be carried out using Equation 13 and Figures 1 through 3. But sometimes, instead of strength, member design based on the critical temperature is a more convenient indicator of structural performance under fire. Regression formulas are derived for the critical temperatures for different types of members.

Tension Members

For the design strength ratio, $\phi R_{nf}/\phi R_n$, given in Figure 1, the regression equation is plotted in Figure 4 (for T ≥ 400 °C).

$$\frac{\phi R_{nf}}{\phi R_n} \approx C_3 T^3 + C_2 T^2 + C_1 T + C_0 \tag{16}$$

where

$C_3 = -1.1497 \times 10^{-9}$

$C_2 = 5.0871 \times 10^{-6}$

$C_1 = -7.0096 \times 10^{-3}$

$C_0 = 3.0862$

Combining Equations 13, 15 and 16, the critical temperature can be obtained by solving the following equation:

$$C_3 T_{cr}^3 + C_2 T_{cr}^2 + C_1 T_{cr} + C_0 = \frac{1.2 + 0.5\left(\dfrac{L}{D}\right)}{1.2 + 1.6\left(\dfrac{L}{D}\right)} \times F_{os} \tag{17}$$

The critical temperatures under different live load and overstrength conditions are shown in Figure 5. Equation 17 may

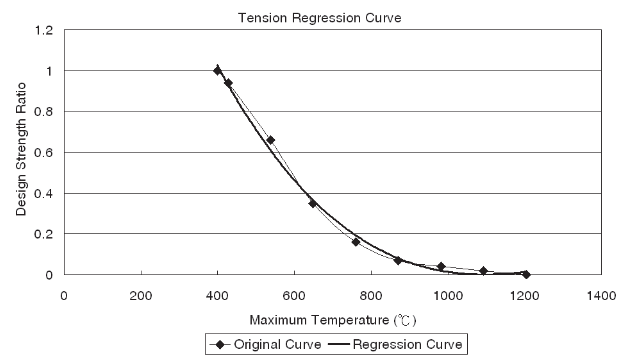

Fig. 4. Strength reduction regression curves of tension members.

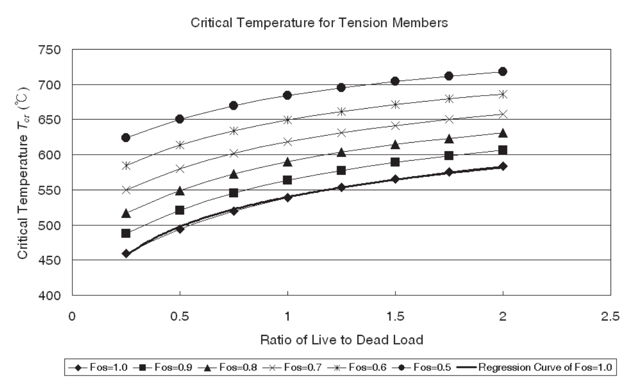

Fig. 5. Critical temperatures of tension members.

Table 1. Regression Coefficients for Critical Temperature of Tension Members (°C)		
F_{os}	$T_{cr} = A \times \ln\left(\frac{L}{D}\right) + B$	
	A	B
1.0	60.705	539.82
0.9	58.276	564.52
0.8	55.660	590.97
0.7	52.822	619.51
0.6	49.700	650.63
0.5	46.218	685.07

be expressed in the following simpler form for the critical temperatures of tension members (°C):

$$T_{cr} = A \times \ln\left(\frac{L}{D}\right) + B \tag{18}$$

where coefficients of A and B are listed in Table 1.

Alternatively, A and B may be approximated by these equations:

$$A = 29F_{os} + 32 \tag{19}$$

$$B = 826 - 290F_{os} \tag{20}$$

Compression Members

Referring to Figure 2, the design strength ratio, $\phi R_{nf}/\phi R_n$, does not vary greatly with the stability ratio R. To obtain a simplified formula, use R = 1.5 as the mean value, and a regression curve is obtained as shown in Figure 6 (T ≥ 93 °C):

$$\frac{\phi R_{nf}}{\phi R_n} \approx C_6T^6 + C_5T^5 + C_4T^4 + C_3T^3 + C_2T^2 + C_1T + C_0 \tag{21}$$

where

$C_6 = -1.2137 \times 10^{-18}$

$C_5 = -1.1832 \times 10^{-14}$

$C_4 = 3.9768 \times 10^{-11}$

$C_3 = -4.0404 \times 10^{-8}$

$C_2 = 1.6093 \times 10^{-5}$

$C_1 = -3.5752 \times 10^{-3}$

$C_0 = 1.2256$

Combining Equations 13, 15 and 21, the critical temperature can be obtained by solving the following equation:

$$C_6T_{cr}^6 + C_5T_{cr}^5 + C_4T_{cr}^4 + C_3T_{cr}^3 + C_2T_{cr}^2 + C_1T_{cr} + C_0 = \frac{1.2 + 0.5\left(\frac{L}{D}\right)}{1.2 + 1.6\left(\frac{L}{D}\right)} \times F_{os} \tag{22}$$

The critical temperatures under different dead and live load ratios are shown in Figure 7. Equation 22 may be expressed in the following simpler form for the critical temperatures of compression members (°C):

$$T_{cr} = C \times \ln\left(\frac{L}{D}\right) + F \tag{23}$$

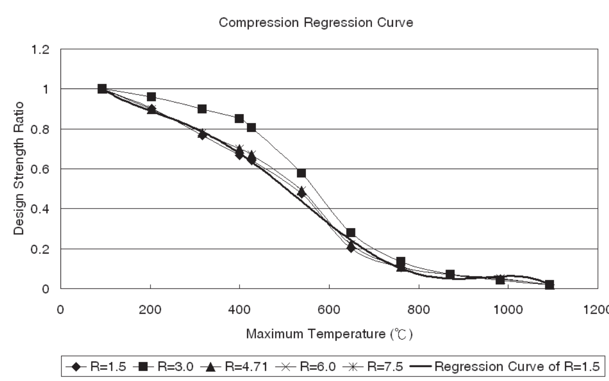

Fig. 6. Strength reduction regression curves of compression members.

Table 2. Regression Coefficients for Critical Temperature of Compression Members (°C)		
F_{os}	$T_{cr} = C \times \ln\left(\frac{L}{D}\right) + F$	
	C	F
1.0	109.94	436.79
0.9	87.342	475.14
0.8	72.330	510.27
0.7	61.308	543.98
0.6	52.604	577.47
0.5	45.371	611.82

Table 3. Regression Coefficients for Flexural Members Under Fire				
***RL* = 0.5**	C_6	C_5	C_4	C_3
	8.5426×10^{-17}	-3.5679×10^{-13}	5.8637×10^{-10}	-4.7694×10^{-7}
	C_2	C_1	C_0	
	1.9915×10^{-4}	-4.1899×10^{-2}	4.7078	
***RL* = 4**	C_6	C_5	C_4	C_3
	2.8344×10^{-18}	-2.8605×10^{-14}	6.7037×10^{-11}	-6.2053×10^{-8}
	C_2	C_1	C_0	
	2.4495×10^{-5}	-4.9401×10^{-3}	1.2953	

where coefficients of C and F are listed in Table 2.

Alternatively, C and F may be approximated by these equations:

$$C = 182F_{os}^2 - 148F_{os} + 75 \tag{24}$$

$$F = 787 - 347F_{os} \tag{25}$$

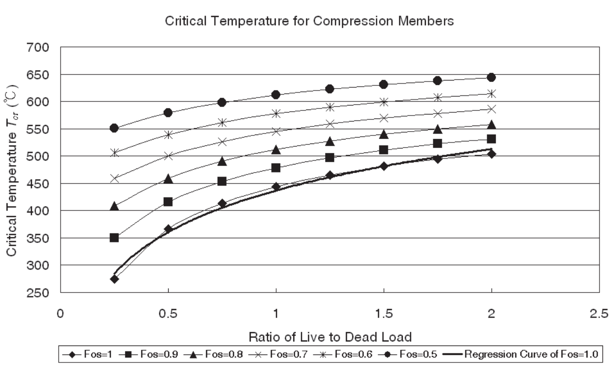

*Fig. 7. Critical temperatures of compression members (*R *=1.5).*

Flexural Members

Referring to Figure 8, the design strength ratios, $\phi R_{nf}/\phi R_n$, are plotted together with regression equations for the two extreme cases of braced length condition, RL = 0.5 and RL = 4.

$$\frac{\phi R_{nf}}{\phi R_n} \approx C_6T^6 + C_5T^5 + C_4T^4 + C_3T^3 + C_2T^2 + C_1T + C_0 \tag{26}$$

The coefficients of regression curves of flexural members are listed in Table 3. Combining Equations 13, 15 and 26, the critical temperature can be obtained by solving the following equation:

$$C_6T_{cr}^6 + C_5T_{cr}^5 + C_4T_{cr}^4 + C_3T_{cr}^3 + C_2T_{cr}^2 + C_1T_{cr} + C_0 = \frac{1.2 + 0.5\left(\frac{L}{D}\right)}{1.2 + 1.6\left(\frac{L}{D}\right)} \times F_{os} \tag{27}$$

Table 4. Regression Coefficients for Critical Temperature of Flexural Members (°C)				
F_{os}	$T_{cr} = G \times \ln\left(\frac{L}{D}\right) + H$ (°C) (RL = 0.5)		$T_{cr} = G \times \ln\left(\frac{L}{D}\right) + H$ (°C) (RL = 4)	
	G	**H**	**G**	**H**
1.0	56.954	553.74	109.03	451.55
0.9	51.019	575.52	84.463	489.23
0.8	46.157	597.59	69.618	523.15
0.7	42.042	620.46	59.048	555.65
0.6	38.490	644.71	50.807	587.99
0.5	35.406	671.20	44.005	621.28

The critical temperatures under different dead and live load ratios are shown in Figures 9 and 10. Equation 27 may be expressed in the following simpler form for the critical temperatures of flexural members (°C):

$$T_{cr} = G \times \ln\left(\frac{L}{D}\right) + H \quad (°C) \tag{28}$$

where coefficients of G and H are listed in Table 4.

Alternatively, G and H may be approximated by these equations:

For $RL = 0.5$

$$G = 35F_{os}^2 - 10F_{os} + 32 \tag{29}$$

$$H = 786 - 234F_{os} \tag{30}$$

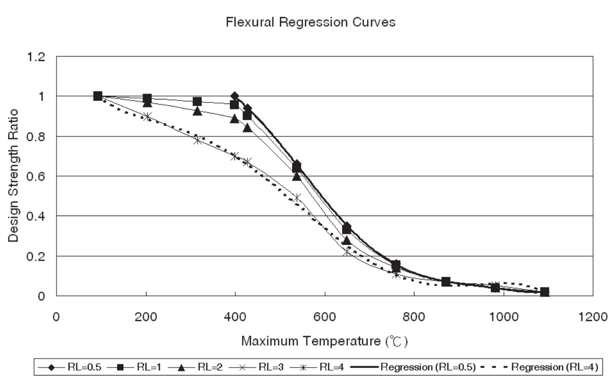

Fig. 8. Strength reduction regression curves of flexural members ($A_w = 0.5A_f$).

For $RL = 4$

$$G = 206F_{os}^2 - 184F_{os} + 86 \tag{31}$$

$$H = 791 - 336F_{os} \tag{32}$$

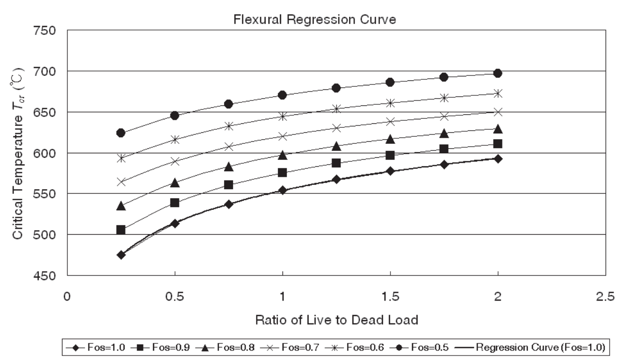

Fig. 9. Critical temperatures of flexural members (RL = *0.5*).

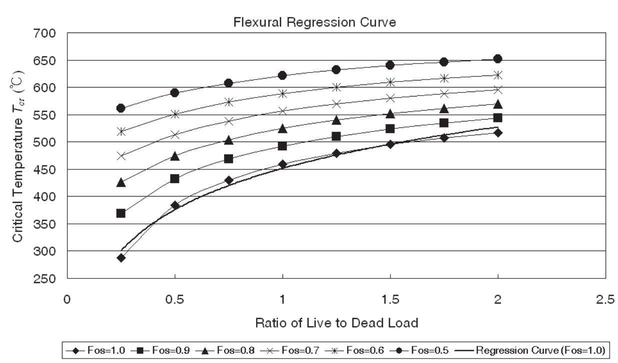

Fig. 10. Critical temperatures of flexural members (RL = *4*).

ILLUSTRATIVE EXAMPLE

The following example illustrates how the proposed methodology is applied for a simply supported steel beam with the given design data:

Dead load = 4 kN/m^2; Design live load = 2 kN/m^2
Span length (l) = 9000 mm; Width of the slab = 3000 mm
Adequate lateral restraint is assumed, i.e., $L_b < L_p$
Design mean temperature = 500 °C
Steel material is ASTM A36 (F_y = 250 MPa), modulus of elasticity (E) = 200000 MPa
Start out with a trial steel beam section W18×40

Normal temperature design

1. Check the local buckling of flange and web:

Flange: $\frac{b_f}{2t_f} = \frac{152.8}{2 \times 13.3} = 5.74 < \lambda_p$

where

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38 \times \sqrt{\frac{200000}{250}}$$

$$= 10.75 \quad \textbf{o.k.}$$

Web: $\frac{h}{t_w} = \frac{402.7}{8} = 50.34 < \lambda_p$

where

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76 \times \sqrt{\frac{200000}{250}}$$

$$= 106.35 \quad \textbf{o.k.}$$

This beam section is a compact one and the proposed methodology is applicable (doubly-symmetric compact I-shaped members bent about their major axis).

2. Calculate the moment capacity of the steel beam:

For W18 × 40, Z_x = 1293 cm^3

$$M_n = F_y Z_x = \frac{250 \times 1293}{1000} = 323.25 \text{ kN-m}$$

3. Combine factored dead and live loads and moments:

$w_u = 1.2w_D + 1.6w_L = 1.2 \times 4 + 1.6 \times 2 = 8$ kN/m^2

$$M_u = \frac{1}{8} w_u l^2 = \frac{1}{8} \times 8 \times 3 \times (9)^2 = 243 \text{ kN-m}$$

4. Compare the moment capacity against demand:

$\phi_b M_n = 0.9 \times 323.25 = 290.93$ kN-m $> M_u$ **o.k.**

Simplified LRFD fire design based on critical temperature

Calculate the moment overstrength ratio,

$$F_{os} = \frac{243}{290.93} = 0.835$$

Assume a braced length factor, RL = 0.5, since adequate lateral restraint is provided. Then, using Equations 28, 29 and 30, determine the critical temperature and compare with the design temperature of 500 °C.

$$T_{cr} = [35(0.835)^2 - 10(0.835) + 32] \times \ln\left(\frac{2}{4}\right) + [786 - 234(0.835)]$$

T_{cr} = 557 °C > 500 °C **o.k.**

Simplified LRFD fire design procedures based on member strength

1. Calculate the moment capacity of the steel beam:

Determine $k_{y,max}$ from linear interpolation. See Table 5.

$$k_{y,max} = 0.66 + (0.94 - 0.66)\left[\frac{(538 - 500)}{(538 - 427)}\right] = 0.76$$

The nominal moment capacity at 500 °C is

$M_{nf} = k_{y,max} F_y Z_x = 0.76 \times 323.25 = 246$ kN-m

2. The combined dead and live load moment for fire design:

$w_{uf} = 1.2w_D + 0.5w_L = (1.2 \times 4) + (0.5 \times 2) = 5.8$ kN/m^2

$$M_{uf} = \frac{1}{8} w_{uf} l^2 = \frac{1}{8}(5.8)(3)(9)^2 = 176 \text{ kN-m}$$

3. Compare with the moment capacity of the steel beam:

$\phi_b M_{nf} = 0.9(245.67) = 221$ kN-m $> M_{uf}$ **o.k.**

Cross-check critical temperature with the design strength

1. Determine, by linear interpolation, $k_{y,max}$ for the critical temperature = 557 °C. See Table 5.

$$k_{y,max} = 0.35 + (0.66 - 0.35)\left[\frac{(649 - 557)}{(649 - 538)}\right] = 0.6$$

Calculate the nominal moment strength,

$M_{nf} = k_{y,max} F_y Z_x = 0.6 \times 323.25 = 194$ kN-m

2. Compare the moment capacity with the combined dead and live load moment:

$\phi_b M_{nf} = 0.9(194) = 175$ kN-m

M_{uf} =176 kN-m

The two moment values are very close.

Table 5. Reduction Factor of Steel at Elevated Temperatures [from AISC *Specification* (AISC, 2005)]

Steel Temperature	$k_E = E_m/E$	$k_y = F_{ym}/F_y$	$k_u = F_{um}/F_u$
20 °C	*	*	*
93 °C	1.00	*	*
204 °C	0.90	*	*
316 °C	0.78	*	*
399 °C	0.70	1.00	1.00
427 °C	0.67	0.94	0.94
538 °C	0.49	0.66	0.66
649 °C	0.22	0.35	0.35
760 °C	0.11	0.16	0.16
871 °C	0.07	0.07	0.07
982 °C	0.05	0.04	0.04
1093 °C	0.02	0.02	0.02
1204 °C	0.00	0.00	0.00

*Use ambient properties

The main advantage of the proposed LRFD design methodology is that fire design is greatly simplified. The critical design temperature is obtained from one equation, e.g., Equation 28. At the preliminary design stage, structural designers can readily estimate the critical temperatures of steel members from two design parameters: the ratio of live to dead load and the overstrength factor, F_{os}.

EXPERIMENTAL VALIDATION

The simplified LRFD fire calculations are now compared with experimental results reported by Rubert and Schaumann (1986). Shown in Figure 11 are the simply supported I-section beams tested by Rubert and Schaumann. The beams were subjected to a constant load at mid-span and heated along the entire length. These beams were subject to various load ratios F_c/F_u ranging from 0.20 to 0.85, in which F_c and F_u are, respectively, the applied and ultimate concentrated loads. The experimental results are summarized in Table 6 and compared the simplified fire calculations. The self-weight of steel beams and applied concentrated load are dead loads; no live loads are included. Unrestrained boundary conditions are assumed. The predicted temperatures are calculated by Equation 27, assuming $RL = 0.5$ (span length is 1140 mm) and $F_R = 1.0$. It can be seen in Table 6 that the predicted critical temperatures, T_P, by the simplified fire calculations are in reasonable agreement with the critical temperatures reported experimentally, T_E. The predicted or calculated critical temperatures are lower than the experimental failure temperatures, thus, they are conservative.

CONCLUSIONS

This paper proposes a simplified LRFD design methodology for steel members under fire. By extending LRFD design under normal temperatures, the fire design criterion is derived that compares the required design strength ratio with the design strength reduction ratio. Design strength ratios are derived from the required strength factor and the overstrength factor. Simplified formulas for member strength at elevated temperatures, as well as their corresponding critical temperatures, are derived for steel members under tension, compression, and flexure (doubly-symmetric compact I-shaped members bent about their major axis). Sections may be compact or noncompact, but boundary conditions

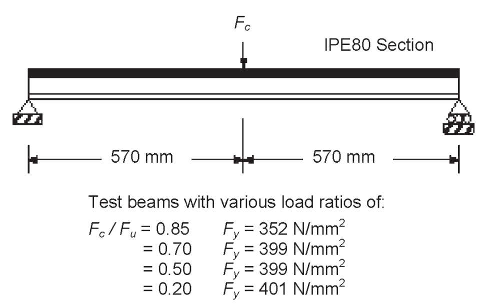

Fig. 11. Simply supported beam tested under fire (Rubert and Schaumann, 1986).

Table 6. Comparison of Calculated Results with Experimental Data from Rubert and Schaumann (1986)				
Specimen	**1**	**2**	**3**	**4**
F_y (N/mm^2)	352	399	399	401
F_c/F_u	0.85	0.70	0.50	0.20
1.2 M_D (kN-mm)	8338.63	7785.86	5564.61	2243.87
ϕM_n (kN-mm)	7356.10	8338.30	8338.30	8380.10
Normal Temperature Design	NG	OK	OK	OK
F_R	–	1.0	1.0	1.0
F_{os}	–	0.9337	0.6674	0.2678
Required $\dfrac{\phi_b M_{nf}}{\phi_b M_n}$	–	0.9337	0.6674	0.2678
Predicted T_P (°C)	–	431	533	688
Experimental T_E (°C)	520	540	600	730
T_P/T_E	–	0.80	0.89	0.94

are assumed to be unrestrained. Engineers and architects using the proposed methodology should be aware of the limitations of the member sections and boundary conditions. One set of experimental data was taken for comparison, and the calculated results predicted by the proposed simplified formulas compare reasonably well. However, the proposed methodology is best suited for preliminary design purposes, individual users should verify their specific design situation and final design should be carried out in conjunction with the AISC *Specification*.

NOTATION

A = regression coefficient

A_e = effective net area, mm^2

A_f = $b_f t_f$ = flange area, mm^2

A_g = gross area of member, mm^2

A_k = value of the load or load effect resulting from an extraordinary event

A_w = $(h_o - t_f)\, t_w$ = web area, mm^2

B = regression coefficient

b_f = width of flange, mm

C = regression coefficient

C_b = lateral-torsional buckling modification factor for nonuniform moment diagrams when both ends of the unsupported segment are braced and is permitted to be conservatively taken as 1.0 for all cases

C_i = regression coefficients, where i = 0, 1, 2, 3, 4, 5, 6

D = dead load

E = modulus of elasticity, MPa

F = regression coefficient

F_c = applied concentrated load

F_{cr} = critical stress, MPa

F_e = elastic buckling stress, MPa

F_{os} = $\dfrac{R_u}{\phi R_n}$ = overstrength factor

F_R = $\dfrac{R_{uf}}{R_u}$ = required strength factor

F_u = specified minimum tensile strength or ultimate concentrated load

F_y = specified minimum yield stress, MPa

G = regression coefficient

H = regression coefficient

h = clear distance between flanges less the fillet or corner radius at each flange, mm

h_o = distance between the flange centroids, mm

I_y = moment of inertia about the minor axis, mm^4

K = effective length factor

k_E = reduction factor for the elastic modulus of steel at the steel temperature

$k_{E,max}$ = reduction factor for the elastic modulus of steel at the maximum steel temperature

k_u = reduction factor for the tensile strength of steel at the steel temperature

$k_{u,max}$ = reduction factor for the tensile strength of steel at the maximum steel temperature

k_y = reduction factor for the yield strength of steel at the steel temperature

$k_{y,max}$ = reduction factor for the yield strength of steel at the maximum steel temperature

L = live load

L_b = distance between points braced against lateral displacement of the compression flange, or between points braced to prevent twist of the cross section

L_p = $1.76r_y\sqrt{\dfrac{E}{F_{yf}}}$ for I-shaped members including hybrid sections and channels, mm

L_r = $\pi r_{ts}\sqrt{\dfrac{E}{0.7F_y}}$, mm

L_{ub} = laterally unbraced length of the member

l = span of a simply supported steel beam, mm

M_n = nominal flexural strength

M_{nf} = nominal flexural strength for fire design

M_u = required flexural strength

M_{uf} = required flexural strength for fire design

P_n = nominal compressive or tension strength

R = $\dfrac{KL_{ub}/r}{\sqrt{E/F_y}}$

RL = $\dfrac{L_b}{L_p}$

R_n = nominal strength

R_{nf} = nominal strength under fire

R_u = required strength

R_{uf} = required strength under fire

r = governing radius of gyration about the axis of buckling, mm

r_{ts} = $\sqrt{\dfrac{I_y h_o}{2S_x}}$, mm

r_y = radius of gyration about y-axis, mm

S_x = elastic section modulus about the major axis, mm^3

T_{cr} = critical temperature, °C

T_E = critical temperature reported experimentally, °C

T_P = predicted critical temperature by the simplified fire calculations, °C

t_f = thickness of flange, mm

t_w = thickness of web, mm

w_D = design dead load, kN/m^2

w_L = design live load, kN/m^2

w_u = design required load, kN/m^2

w_{uf} = design required load for fire design, kN/m^2

x = subscript relating symbol to strong axis bending

y = subscript relating symbol to weak axis bending

Z_x = plastic section modulus about the major axis

ϕ = resistance factor corresponding to R_n

ϕ_b = resistance factor for flexure = 0.90

ϕ_c = resistance factor for compression = 0.90

ϕ_t = resistance factor for tension = 0.90 or 0.75

λ_p = limiting slenderness parameter for compact element

λ_y = limiting slenderness parameter for noncompact element

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