Reduced Beam Section Spring Constants

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A moment connection that includes a wide flange beam with trimmed flanges is commonly known as a reduced beam section (RBS) connection, and the beam of this connection is known as an RBS beam. RBS connections are widely used in steel buildings throughout the developed world, and those with radius cuts (Figure 1) have been designated a *prequalified* moment connection for special and intermediate moment frames by the American Institute of Steel Construction (AISC) and the American National Standards Institute (ANSI) (AISC, 2005). The key analytical components of a structure's stiffness are its elements' stiffness matrices. Hence, knowledge of the stiffness matrices for RBS beams is fundamental to the quantification of both the elastic and inelastic performance of structures that incorporate these nonprismatic beams. For example, a pushover analysis of a moment frame that includes RBS connections requires both the elastic stiffness matrices and plastic moment capacities of its RBS beams. The design procedures for RBS connections specified in AISC's *Prequalified Connections for Special and Intermediate Steel Moment Frames for Seismic Applications* (RBS Design Procedures) states, "in lieu of specific calculations, effective elastic drifts may be calculated by multiplying elastic drifts based on gross beam sections by 1.1 for flange reductions up to 50% of the beam flange width. Linear interpolation may be used for lesser values of beam width reduction" (AISC, 2005). Incorporation of the closed-form solution of RBS frame elements' stiffness matrices (RBS-CFS stiffness matrices) into structural analysis will give a more precise estimate of the elastic component of the drift of a frame that includes RBS connections than that recommended by the RBS Design Procedures. However, it may be difficult or impossible for some structural design firms to integrate RBS-CFS stiffness matrices into their structural analysis software. Methods to capture the effect of RBS cuts on the stiffness of beams by adjusting the moment of inertia of prismatic beams have been derived from approximate numerical integration applied to a limited sample of RBS beams (Grubbs, 1997; Dumonteil, 2006). Chambers, Almudhafar and Stenger (2003) were the first to present the closed-form solution of the stiffness matrix of an RBS beam. Recently Kim and Engelhardt (2007) extended the work of Chambers et al. (2003) to include transverse shear deformations in the RBS stiffness matrix and also offer a method to adjust the moment of inertia of prismatic beams to capture the affect of RBS cuts. The adjusted moment of inertia is referred to as an effective moment of inertia. No research has been published to date that offers a spring constant for semi-rigid connections, which can be easily implemented into commercial structural analysis software, to simulate the effect of RBS cuts on the stiffness of moment frames (Figure 2). This paper presents the derivation of the formula for the spring constants of an RBS beam using the RBS-CFS stiffness matrix derived by Chambers et al. (2003) and validates it. Further, from a study of the spring constants for a plethora of RBS beams, it was found that a strong linear relationship exists between the minimum plastic section modulus of RBS beams and their spring constants. Thus, this paper has direct applicability to the practical and accurate determination of the elastic drift of a moment frame with RBS connections.

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DERIVATION OF THE SPRING CONSTANTS OF AN RBS BEAM

The derivation of the RBS-CFS stiffness matrix is presented in detail in Chambers et al. (2003). The paper presents the following relationship between end moments, m_i and m_j, and rotations θ_i and θ_j

$$\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} = \frac{EI_g}{u(s+t)} \begin{bmatrix} t & -s \\ -s & t \end{bmatrix} \begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} \tag{1}$$

where

E = modulus of elasticity of the material

I_g = gross moment of inertia of the cross section about the major axis and

$$s = \frac{2a^3}{3L} - a^2 - \frac{g^3}{6L} + ga\left(\frac{a}{L} - 1\right) - \frac{gb}{L}\left(b + g\right) - \frac{1}{2Ln}\left\{2\gamma\left(R^2 - ef\right) + 4\Omega\left[ef + R^2\left(\frac{1}{\omega^2} - 1\right)\right] - \lambda\right\} \quad (2)$$

$$t = \frac{2a^3}{3L} + \frac{g^3}{3L} + \frac{ga}{L}\left(g + a\right) + \frac{gb}{L}\left(L - b\right) + a\left(L - a\right) + \frac{1}{4Ln}\left\{-2\gamma\left(2R^2 + e^2 + f^2\right) + 4\Omega\left[e^2 + f^2 + 2R^2\left(1 - \frac{1}{\omega^2}\right)\right] + 2\lambda\right\} \quad (3)$$

$$u = a + \frac{g}{2} + \frac{1}{n}\left(\Omega - \frac{\gamma}{2}\right) \quad (4)$$

where

a = distance from the end of the beam to the start of the radius cut as shown in Figure 1
b = length of the radius cut along the length of the beam as shown in Figure 1
b_f = beam flange width
c = depth of the radius cut normal to the beam length as shown in Figure 1
d = beam depth

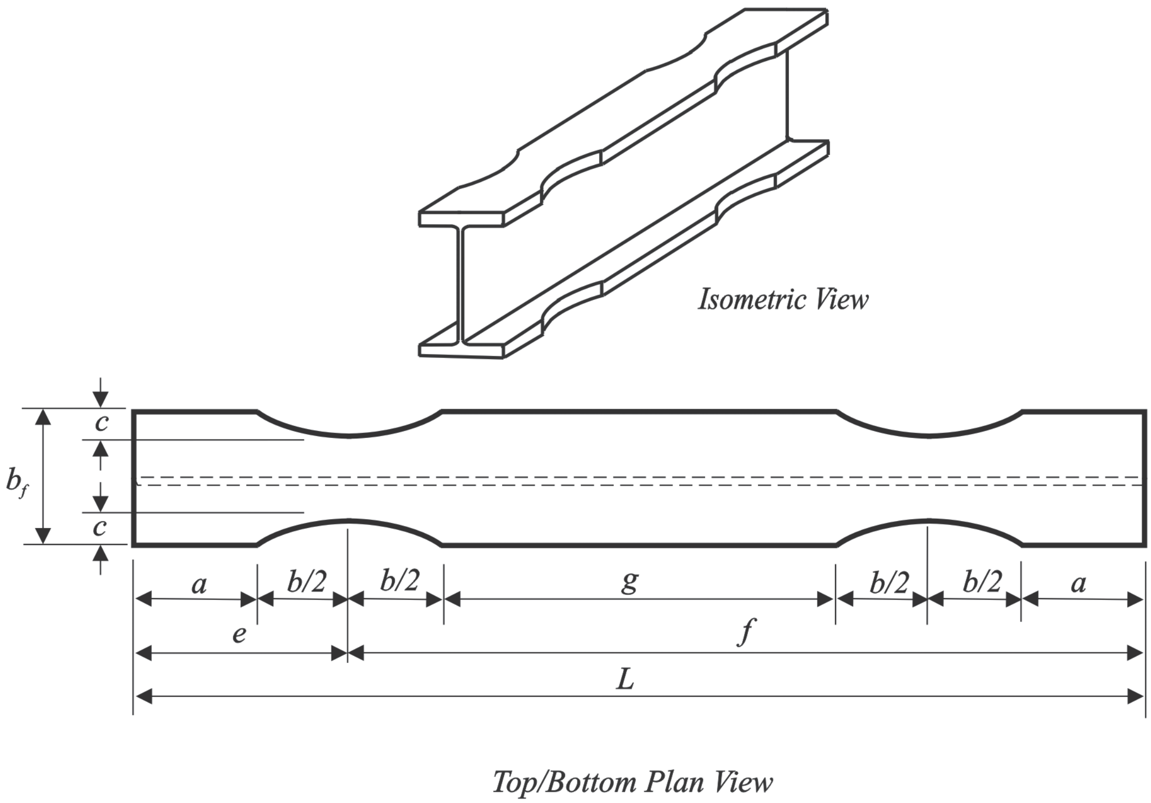


Fig. 1. RBS beam.

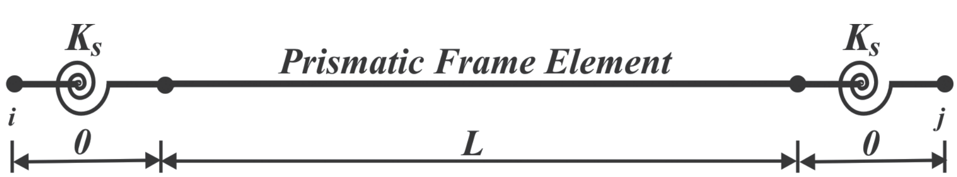


Fig. 2. Model of frame element with springs at its ends.

e = distance from the near end of the beam to the center of the radius cut as shown in Figure 1

f = distance from the far end of the beam to the center of the radius cut as shown in Figure 1

g = distance between the radius cuts at each end of the beam as shown in Figure 1

L = beam length

t_f = beam flange thickness

$$n = \frac{1}{I_g}\left[\frac{t_f^{\,3}}{12} + \left(\frac{d - t_f}{2}\right)^2 t_f\right]$$

$$R = \frac{c^2 + \dfrac{b^2}{4}}{2c}$$

$$\beta = 1 - 4n\left(c - R\right)$$

$$\gamma = \sin^{-1}\left(\right.$$

$$\lambda = R^2\left[\gamma + \frac{1}{2}\sin\left(2\gamma\right) + \frac{2}{\omega}\left(\sin\gamma + \frac{\gamma}{\omega}\right)\right]$$

$$\mu = 4Rn$$

$$\Omega = \frac{\tan^{-1}\left[\sqrt{\dfrac{1+\omega}{1-\omega}}\tan\left(\dfrac{\gamma}{2}\right)\right]}{\sqrt{1-\omega^2}}$$

$$\omega = \frac{\mu}{\beta}$$

Equation 1 may be abbreviated as

$$\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} = \begin{bmatrix} S_{ii} & S_{ij} \\ S_{ij} & S_{ii} \end{bmatrix}\begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} \tag{5}$$

where

$$S_{ii} = EI_g\left[\frac{t}{u\left(s+t\right)}\right]$$

$$S_{ij} = EI_g\left[\frac{-s}{u\left(s+t\right)}\right]$$

Solving Equation 5 for the rotations

$$\begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} = \frac{1}{S_{ii}^{\,2} - S_{ij}^{\,2}}\begin{bmatrix} S_{ii} & -S_{ij} \\ -S_{ij} & S_{ii} \end{bmatrix}\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} \tag{6}$$

Now, consider the relationship between Euler-Bernoulli beam end moments and rotations

$$\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} = \begin{bmatrix} \dfrac{4EI_g}{L} & \dfrac{2EI_g}{L} \\ \dfrac{2EI_g}{L} & \dfrac{4EI_g}{L} \end{bmatrix}\begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} \tag{7}$$

Solving Equation 7 for the rotations

$$\begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} = \begin{bmatrix} \dfrac{L}{3EI_g} & -\dfrac{L}{6EI_g} \\ -\dfrac{L}{6EI_g} & \dfrac{L}{3EI_g} \end{bmatrix}\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} \tag{8}$$

Referring to Figure 2, Equation 8 may be adjusted for an RBS beam by adding the flexibility of the rotational springs, $1/K_s$, at each end of the beam

$$\begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} = \begin{bmatrix} \dfrac{L}{3EI_g} + \dfrac{1}{K_s} & -\dfrac{L}{6EI_g} \\ -\dfrac{L}{6EI_g} & \dfrac{L}{3EI_g} + \dfrac{1}{K_s} \end{bmatrix}\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} \tag{9}$$

Considering only the rotational stiffness of the joints, the value of the rotational spring constant for an RBS frame element can be found by equating the diagonal terms of Equations 6 and 9

$$\frac{L}{3EI_g} + \frac{1}{K_s} = \frac{S_{ii}}{S_{ii}^{\,2} - S_{ij}^{\,2}} \tag{10}$$

Thus, the spring constant for an RBS connection is

$$K_s = \frac{3EI_g\left(S_{ii}^{\,2} - S_{ij}^{\,2}\right)}{3EI_g S_{ii} - L\left(S_{ii}^{\,2} - S_{ij}^{\,2}\right)} \tag{11}$$

SPRING CONSTANT VALIDATION

The design process for seismic moment frames usually begins with a target elastic story drift coefficient of approximately 0.25% and the base seismic shear (ASCE, 2005). The frame used to validate Equation 11, "the validation frame" is shown in Figure 3. The stiffness method was used to analyze the structure without flange reductions. The shear deformations and P-Delta effects were neglected. Thus, Euler-Bernoulli frame stiffness matrices were employed. The cross-sectional properties of the members that were used in the analysis are shown in Table 1. The modulus of elasticity was equal to 29,000 ksi. Due to axial deformations of the beams, the lateral displacement at the top of each column in a story of the validation frame was slightly different, and the average of these displacements was used to determine the story drift. The maximum story drift coefficient obtained from the frame analysis described earlier was 0.20%. The frame was then analyzed using RBS-CFS stiffness matrices for all the beams with $a = 0.5b_f$, $b = 0.85d$, and $c = 0.2b_f$, which are within the limits specified by the RBS Design Procedures. The frame was also analyzed using Euler-Bernoulli prismatic frame element stiffness matrices and rotational springs at each end of the frame elements (RBS-EBWS stiffness matrices). The moment-rotation spring constants of the springs were calculated from Equation 11 and are presented in Table 2. [The reader may refer to equation 13.31 of *Matrix Structural Analysis* (McGuire, Gallagher and Ziemian, 2000) to obtain the formula for stiffness matrix of a beam with rotational springs on each end.] A comparison between the analytical results for the validation frame is presented in Table 3. As can be seen from the table, the story drift coefficients using RBS-EBWS stiffness matrices range from 0.9% to 1.5% larger than the story drift coefficients obtained using RBS-CFS stiffness matrices. Differences exist because the spring constants only account for the RBS cuts' influence on the M/θ stiffness of the frame [i.e., compare Equation 1 with Equation 24 of Chambers et al. (2003)]. Thus, for practical purposes, Equation 11 is a valid equation for the rotational spring constants of an RBS beam.

TRANSVERSE SHEAR DEFORMATIONS

Because an industry-acceptable formula for the shear area, A_s, of a wide flange beam is not a function of the flange area ($A_s = dt_w$), spring stiffnesses obtained from Equation 11 may be used with prismatic frame elements that account for transverse shear deformations (i.e., Timoshenko beams) to obtain greater accuracy in the drifts of frames with RBS connections than that which would be obtained using Euler-Bernoulli beams.

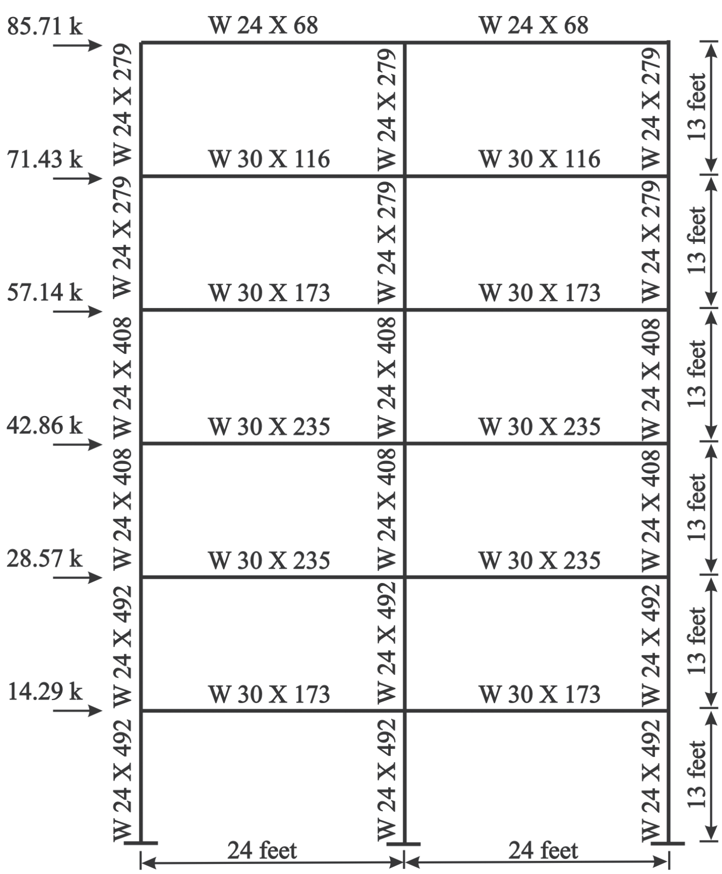


Fig. 3. Validation frame.

Table 1. Validation Frame Member Geometric Properties						
Section	d (in.)	b_f (in.)	t_f (in.)	t_w (in.)	A_g (in.^{2})	I_g (in.^{4})
W24×492	29.6	14.12	3.540	1.970	144.0	19100
W24×408	28.5	13.80	2.990	1.650	119.0	15100
W24×279	26.7	13.30	2.090	1.160	82.0	9600
W30×173	30.4	15.00	1.070	0.655	51.0	8230
W30×235	31.3	15.10	1.500	0.830	69.2	11700
W30×116	30.0	10.50	0.850	0.565	34.2	4930
W24×68	23.7	8.97	0.585	0.415	20.1	1830

Table 2. Validation Frame Beam Spring Constants for $a = 0.5b_f$, $b = 0.85d$, and $c = 0.2b_f$, (kip-in./rad)			
	S_{ii}	S_{ij}	K_s
W30×173	2996397.25	1426943.49	34025298.77
W30×235	4235845.34	2012390.79	45532313.05
W30×116	1812145.24	865520.46	23013555.73
W24×68	683142.46	328229.93	10610757.25

Table 3. Spring Constant Validation			
Story	% Drift		Relative Difference
	RBS Stiffness Matrix		
	CFS	EBWS	
1	0.12055	0.12168	0.94%
2	0.20625	0.20893	1.30%
3	0.21054	0.21367	1.49%
4	0.20481	0.20793	1.52%
5	0.21958	0.22242	1.29%
6	0.20566	0.20809	1.18%

RELATIONSHIP BETWEEN THE SPRING CONSTANT AND MINIMUM PLASTIC SECTION MODULUS OF RBS BEAMS

The minimum plastic section modulus of an RBS beam as defined in the RBS Design Procedures is

$$Z_e = Z_x - 2ct_f\left(d - t_f\right) \tag{12}$$

A strong linear relationship between K_s (Equation 11) and Z_e (Equation 12) was found from a study conducted on RBS beams. The wide flange shapes used for the study were obtained from the AISC Shapes Database v. 13.0. Only those RBS beams that satisfied criteria in the RBS Design Procedures were used. Beam lengths were limited to 25*d*. A plethora of data was generated—almost 200,000 springs (1,080 spring constant values for 177 wide flange shapes). Correlation coefficients squared (R^2) between K_s and Z_e for specific trim geometries (relative to the depth and flange width of the beam) and beam lengths (relative to the depth of the beam) were obtained for all the wide flange shapes used in the study. Figure 4 illustrates the exercises conducted. The maximum and minimum R^2 values were 0.9993 and 0.9966, respectively. Thus, in lieu of Equation 11, an excellent approximation for K_s may be found by multiplying Z_e by the slope of the best-fit line, *m*, for the given trim geometry and beam length

$$K_S = mZ_e \tag{13}$$

The values for *m* obtained from the study are presented in Table 4 (at the end of paper). Note that "% flange reduction" is equal to the total flange reduction, $2c/b_f$.

EXAMPLE

Consider a 30-ft W30×170 with $a = 0.7b_f$, $b = 0.75d$, and a 40% flange reduction. Pertinent geometric properties of a W30×170 are

$$d = 36.2 \text{ in.}, \; b_f = 12.0 \text{ in.}, \; t_f = 1.10 \text{ in.},$$

$$I_g = 10{,}500 \text{ in.}^4, \text{ and } Z_x = 668 \text{ in.}^3$$

Application of Equation 12

$$Z_e = 668 - \left(2\right)\left[\frac{\left(0.4\right)\left(12\right)}{2}\right]\left(1.10\right)\left(36.2 - 1.10\right) = 482.7 \text{ in.}^3$$

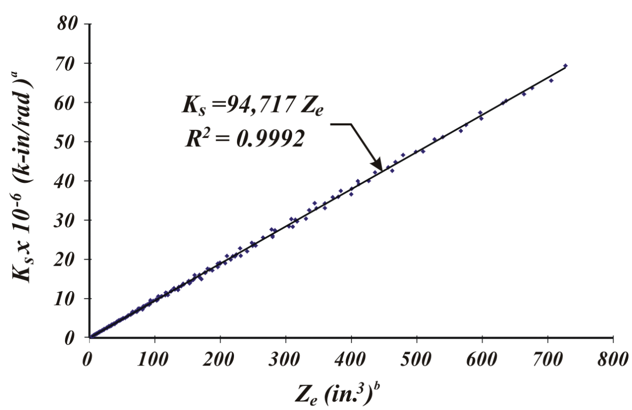
The length of the beam is 30(12)/36.2 = 9.945*d*. Linear interpolation within Table 4 gives

$$m = 90{,}932.9 \text{ kip/in.}^2\text{-rad}$$

Therefore, the spring constant for the beam is from Equation 13 approximately

$$K_S = mZ_e = \left(90{,}932.9\right)\left(482.7\right) = 43{,}893{,}214 \text{ kip-in./rad}$$

Equation 11 offers a more precise value of 45,449,330 kip-in./rad (3.5% larger than that value obtained using the table).



[a]1 k-in = 0.113 kN/m [b]1 in. = 25.4 mm

Fig. 4. Relationship between spring constants and minimum plastic section modulus of RBS beams (a = 0.75b_f, b = 0.65d, c = 0.25b_f, L = 5d).

CONCLUSION

Accurate analysis of frames incorporating RBS beams requires knowledge of the elastic stiffness matrix of RBS beams. This stiffness matrix is known. In lieu of using this stiffness matrix, an RBS beam can be modeled as an Euler-Bernoulli frame element with rotational springs at each end, which can be easily implemented in structural analysis software. The spring constants of these frame elements have been derived herein. The spring constants may also be added to prismatic frame elements that include transverse shear deformations for a more accurate analysis as shear deformations are predominantly a function of the web area. A strong linear relationship between the spring constants and the minimum plastic section modulus of RBS beams was found. Thus, to obtain a very accurate estimate of the rotational spring constant of an RBS beam, one may simply multiply the minimum plastic section modulus of an RBS beam by the coefficient provided in Table 4 of this paper.

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Table 4. Factors Relating the Minimum Plastic Section Modulus of RBS Beams to Their Rotational Spring Constants, *m* (kip/in.²-rad)

			Beam Length								
		Flange Reduction	5*d*	7.5*d*	10*d*	12.5*d*	15*d*	17.5*d*	20*d*	22.5*d*	25*d*
$a = 0.5b_f$	$b = 0.65d$	20%	223231	207906	200517	196181	193334	191321	189823	188665	187743
		30%	149807	139537	134585	131680	129771	128423	127419	126643	126025
		40%	112944	105214	101488	99301	97865	96850	96095	95511	95046
		50%	90707	84512	81526	79774	78623	77810	77205	76737	76364
	$b = 0.70d$	20%	209359	194423	187223	183000	180227	178268	176811	175684	174788
		30%	140611	130591	125761	122929	121069	119755	118777	118022	117420
		40%	106128	98577	94938	92803	91402	90412	89675	89106	88652
		50%	85353	79292	76371	74657	73533	72738	72147	71690	71326
	$b = 0.75d$	20%	197320	182722	175684	171559	168851	166938	165515	164416	163541
		30%	132612	122810	118086	115316	113499	112215	111260	110522	109935
		40%	100182	92787	89223	87134	85763	84795	84074	83518	83075
		50%	80664	74720	71856	70177	69075	68296	67718	67270	66914
	$b = 0.80d$	20%	186774	172472	165576	161535	158883	157011	155619	154543	153687
		30%	125592	115983	111350	108636	106855	105597	104662	103940	103365
		40%	94951	87694	84197	82147	80802	79853	79147	78601	78167
		50%	76526	70686	67872	66223	65141	64377	63809	63370	63021
	$b = 0.85d$	20%	177461	163420	156649	152682	150080	148243	146877	145822	144983
		30%	119383	109945	105393	102727	100978	99744	98826	98117	97553
		40%	90315	83181	79741	77727	76406	75473	74780	74244	73818
		50%	72849	67102	64332	62710	61646	60894	60336	59905	59562
$a = 0.55b_f$	$b = 0.65d$	20%	225214	209162	201430	196897	193922	191820	190257	189048	188086
		30%	151133	140376	135195	132158	130165	128756	127709	126899	126255
		40%	113939	105844	101946	99660	98160	97100	96312	95703	95218
		50%	91500	85015	81892	80061	78859	78010	77378	76890	76501
	$b = 0.70d$	20%	211212	195596	188075	183668	180776	178734	177215	176041	175107
		30%	141851	131376	126331	123375	121436	120066	119047	118260	117634
		40%	107060	99166	95366	93139	91678	90645	89878	89285	88813
		50%	86097	79763	76713	74926	73753	72925	72309	71833	71454
	$b = 0.75d$	20%	199060	183822	176483	172184	169364	167374	165893	164750	163840
		30%	133776	123547	118621	115735	113843	112506	111513	110745	110135
		40%	101058	93341	89626	87449	86022	85014	84265	83686	83225
		50%	81365	75163	72178	70429	69282	68472	67870	67405	67035
	$b = 0.80d$	20%	188414	173509	166328	162124	159367	157421	155974	154857	153968
		30%	126690	116677	111854	109030	107179	105872	104900	104150	103553
		40%	95778	88217	84576	82444	81046	80059	79326	78760	78309
		50%	77189	71105	68176	66461	65336	64543	63953	63497	63134
	$b = 0.85d$	20%	179011	164401	157360	153238	150536	148630	147213	146118	145248
		30%	120423	110602	105870	103100	101284	100003	99051	98316	97731
		40%	91097	83676	80100	78008	76636	75668	74949	74393	73952
		50%	73477	67499	64620	62935	61830	61051	60472	60025	59669

Table 4 (cont'd.). Factors Relating the Minimum Plastic Section Modulus of RBS Beams to Their Rotational Spring Constants, m (kip/in.^{2}-rad)											
			Beam Length								
		Flange Reduction	5*d*	7.5*d*	10*d*	12.5*d*	15*d*	17.5*d*	20*d*	22.5*d*	25*d*
$a = 0.6b_f$	$b = 0.65d$	20%	227209	210424	202347	197616	194512	192321	190691	189432	188430
		30%	152466	141220	135808	132639	130559	129091	127999	127156	126484
		40%	114939	106476	102405	100021	98456	97351	96530	95895	95390
		50%	92298	85520	82258	80348	79095	78210	77552	77044	76639
	$b = 0.70d$	20%	213076	196774	188930	184338	181326	179200	177619	176399	175427
		30%	143097	132164	126903	123824	121804	120378	119318	118499	117848
		40%	107996	99758	95796	93475	91954	90880	90081	89465	88974
		50%	86846	80236	77056	75195	73974	73112	72471	71977	71583
	$b = 0.75d$	20%	200808	184928	177285	172812	169880	167810	166272	165084	164139
		30%	134947	124287	119158	116156	114188	112799	111767	110969	110335
		40%	101938	93898	90029	87765	86281	85234	84455	83854	83376
		50%	82070	75609	72501	70682	69489	68648	68023	67540	67155
	$b = 0.80d$	20%	190061	174551	167084	162715	159851	157831	156330	155171	154249
		30%	127794	117375	112360	109426	107503	106147	105139	104360	103741
		40%	96608	88742	84956	82742	81290	80267	79506	78918	78451
		50%	77854	71526	68481	66699	65532	64709	64097	63624	63248
	$b = 0.85d$	20%	180569	165387	158074	153796	150994	149017	147549	146415	145514
		30%	121467	111262	106348	103474	101591	100263	99276	98514	97909
		40%	91884	84173	80461	78289	76867	75864	75119	74543	74086
		50%	74108	67898	64909	63161	62016	61208	60608	60145	59776
$a = 0.65b_f$	$b = 0.65d$	20%	229215	211692	203267	198337	195104	192823	191127	189817	188775
		30%	153807	142067	136423	133121	130955	129427	128290	127413	126715
		40%	115944	107112	102867	100382	98753	97603	96748	96088	95563
		50%	93101	86027	82627	80637	79332	78411	77726	77198	76777
	$b = 0.70d$	20%	214948	197958	189789	185010	181878	179668	178025	176757	175748
		30%	144350	132956	127478	124273	122173	120691	119590	118739	118063
		40%	108937	100353	96227	93813	92231	91115	90285	89645	89135
		50%	87598	80711	77401	75465	74196	73300	72634	72120	71712
	$b = 0.75d$	20%	202565	186039	178090	173442	170396	168248	166652	165420	164439
		30%	136123	125031	119697	116577	114534	113092	112021	111194	110536
		40%	102822	94457	90435	88082	86541	85454	84647	84023	83527
		50%	82778	76056	72825	70936	69698	68824	68176	67675	67276
	$b = 0.80d$	20%	191716	175597	167842	163307	160338	158243	156687	155486	154531
		30%	128903	118076	112868	109823	107829	106423	105378	104572	103930
		40%	97443	89270	85339	83041	81535	80474	79686	79077	78593
		50%	78523	71949	68787	66939	65728	64875	64241	63751	63362
	$b = 0.85d$	20%	182133	166377	158791	154356	151453	149406	147886	146712	145780
		30%	122515	111926	106829	103849	101899	100523	99502	98714	98087
		40%	92673	84673	80822	78572	77099	76060	75289	74693	74220
		50%	74741	68299	65199	63387	62201	61365	60744	60265	59884

Table 4 (cont'd.). Factors Relating the Minimum Plastic Section Modulus of RBS Beams to Their Rotational Spring Constants, m (kip/in.^{2}-rad)

			Beam Length								
		Flange Reduction	5*d*	7.5*d*	10*d*	12.5*d*	15*d*	17.5*d*	20*d*	22.5*d*	25*d*
$a = 0.7b_f$	$b = 0.65d$	20%	231231	212966	204192	199060	195698	193326	191563	190202	189120
		30%	155154	142919	137041	133604	131352	129763	128582	127670	126945
		40%	116954	107751	103330	100745	99051	97855	96967	96281	95736
		50%	93907	86536	82996	80926	79570	78612	77901	77352	76915
	$b = 0.70d$	20%	216830	199148	190652	185685	182431	180137	178432	177116	176069
		30%	145609	133752	128055	124725	122543	121005	119862	118979	118277
		40%	109882	100951	96660	94152	92509	91350	90490	89825	89296
		50%	88354	81189	77747	75736	74418	73488	72798	72265	71841
	$b = 0.75d$	20%	204330	187155	178899	174074	170915	168687	167033	165755	164740
		30%	137305	125778	120238	117000	114881	113386	112276	111419	110737
		40%	103711	95018	90841	88400	86802	85675	84838	84192	83678
		50%	83489	76506	73151	71190	69906	69001	68329	67810	67397
	$b = 0.80d$	20%	193378	176649	168603	163902	160825	158656	157045	155802	154814
		30%	130016	118781	113378	110222	108156	106699	105618	104783	104119
		40%	98280	89800	85722	83340	81781	80682	79866	79236	78735
		50%	79194	72373	69094	67179	65925	65042	64385	63879	63476
	$b = 0.85d$	20%	183703	167371	159510	154918	151913	149796	148223	147010	146046
		30%	123568	112592	107311	104226	102207	100784	99728	98913	98265
		40%	93466	85174	81185	78856	77331	76257	75459	74844	74354
		50%	75377	68701	65490	63615	62388	61523	60881	60386	59992
$a = 0.75b_f$	$b = 0.65d$	20%	233256	214247	205120	199786	196294	193831	192001	190589	189465
		30%	156508	143774	137661	134089	131750	130100	128875	127929	127176
		40%	117970	108392	103795	101109	99349	98108	97187	96475	95909
		50%	94717	87048	83368	81217	79808	78814	78076	77507	77053
	$b = 0.70d$	20%	218720	200343	191517	186361	182986	180607	178839	177475	176391
		30%	146873	134551	128634	125177	122914	121319	120134	119220	118493
		40%	110832	101551	97095	94492	92788	91587	90694	90006	89458
		50%	89113	81669	78095	76007	74641	73677	72962	72409	71970
	$b = 0.75d$	20%	206102	188276	179711	174708	171435	169127	167414	166092	165041
		30%	138491	126529	120781	117425	115229	113681	112531	111644	110939
		40%	104603	95583	91250	88720	87064	85897	85030	84361	83830
		50%	84203	76957	73478	71446	70116	69179	68483	67946	67519
	$b = 0.80d$	20%	195046	177705	169367	164499	161314	159070	157404	156118	155097
		30%	131134	119488	113890	110621	108483	106976	105858	104995	104309
		40%	99121	90332	86108	83641	82028	80891	80047	79396	78878
		50%	79868	72800	69403	67420	66123	65209	64530	64007	63590
	$b = 0.85d$	20%	185279	168370	160232	155482	152375	150186	148562	147309	146313
		30%	124624	113262	107795	104604	102517	101046	99955	99113	98444
		40%	94262	85678	81550	79140	77564	76454	75630	74994	74489
		50%	76015	69105	65782	63843	62575	61681	61018	60506	60100