Analysis of Cables as Equivalent Two-Force Members

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In RECENT YEARS, cable structures have become more widely used because of growing demand for long-span construction for aesthetic, economic, or other practical reasons. Aside from classical suspension bridges, examples of this type of structure are tall guyed towers, cable-stiffened bridges, and suspended roofs.

The methods of analysis of suspension bridges have been extensively explored, and will not be discussed herein. Instead, this paper presents a simplified analysis of cable members ordinarily encountered in the other types of structures mentioned above, by treating them as equivalent two-force members. The nonlinear behavior of such cables is accounted for by the use of an equivalent modulus of elasticity as well as equivalent strains. Problems commonly associated with cables, such as stresses (and change of sags) caused by a change of temperature or superimposed loads, or resulting from relative displacement of supports, can be readily solved by this method. It should be noted that the proposed procedure is also applicable to the analysis of transmission lines.

PRINCIPLE OF PROPOSED METHOD OF ANALYSIS

Consider an inclined cable as shown in Fig. 1. The proposed method of analysis is established on the basis that the static behavior of such a cable is analogous to that of a straight chord member having the same cross-sectional area A, but with a different modulus of elasticity, E', provided the horizontal reactions H can be always kept identical in the two systems (Fig. 2).

Thus, if the change of direct stress, ΔS, can be determined in the equivalent member when the cable is subjected to certain changes of external conditions, ΔH and consequently the cable stresses can be determined.

To establish a model for the equivalent member, consider a cable which is hinged at the lower end and is connected to a counterweight after passing over an upper frictionless pulley (Fig. 3). The counterweight is adjustable in weight so that the cable tension at the upper support may be regulated as required. That part of the cable beyond the pulley is assumed to be inextensible with respect to both stresses and temperature changes.

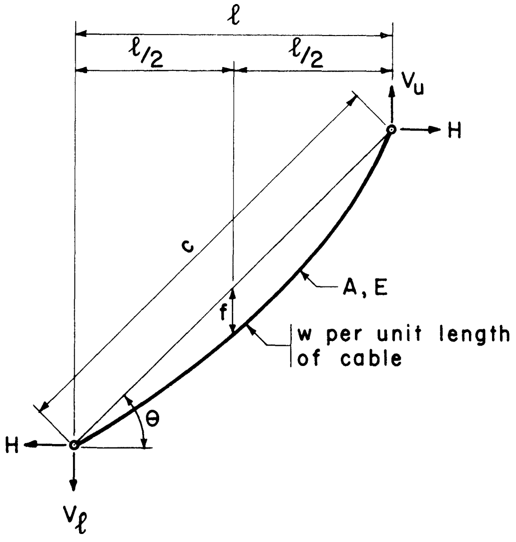

Fig. 1. Actual cable

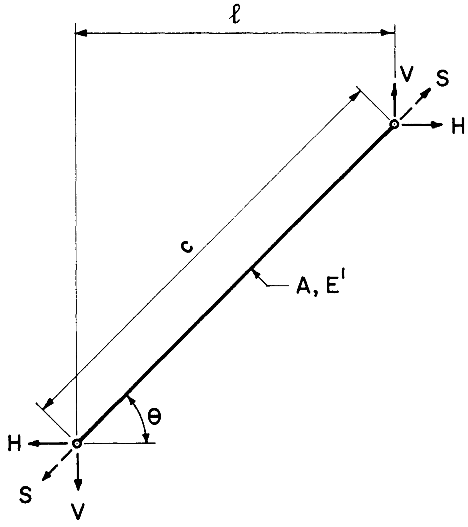

Fig. 2. Equivalent two-force member

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The span length l between the supports is to remain constant in the discussion immediately following. The initial horizontal reactions are equal to H.

Due to a change of temperature ΔT, for instance, the increase in cable length in the model is $\Delta L = \mu L\ (\Delta T)$, in which μ is the coefficient of thermal expansion, and L the length of cable between the supports. If the counterweight is allowed to be lowered by the same amount ΔL (Fig. 4a), H will remain unchanged because both the cable configuration and external loads have not varied from the initial condition. The problem, therefore, is to determine ΔH, the change of horizontal reaction which is required to raise the counterweight back to its original elevation.

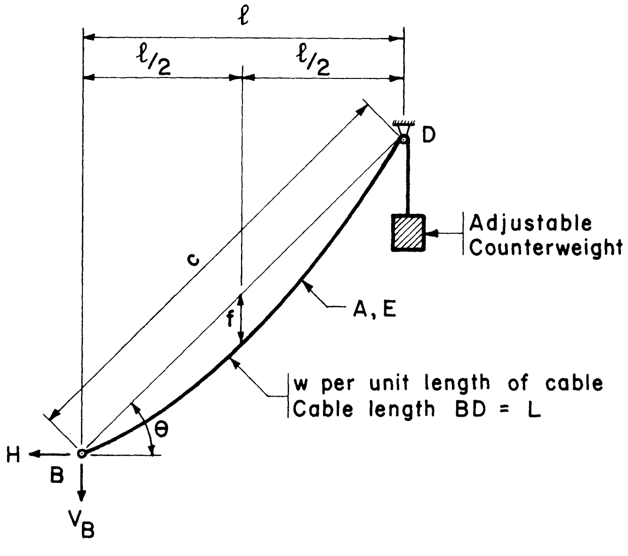


Fig. 3. Cable model

Consider now an equivalent two-force member with an initial direct stress S equal to $H/\cos\theta$ (Fig. 4b). If it is required to find the additional stress due to ΔT, the preceding analogy may be used by first maintaining S and allowing the upper end to move freely along the chord a distance $\Delta c = \mu' c(\Delta T)$, where μ' is the coefficient of thermal expansion of the equivalent member. (As will be shown later, although μ' is theoretically not identical to μ, they may be assumed to be equal for practical purposes.) Since there is no change of chord length in the final condition, a force ΔS must be applied subsequently to bring the upper end back to its initial position. When ΔS is found, $\Delta H = \Delta S \cos\theta$ can be computed, and with it, the final cable stresses. In order to determine ΔS, however, it is necessary to first establish the modulus of elasticity E' for the equivalent member, as well as the values of $\Delta c/c$ (equivalent strain) for various changes of external conditions to which the cables may be subjected.

MODULUS OF ELASTICITY OF THE EQUIVALENT TWO-FORCE MEMBER (E')

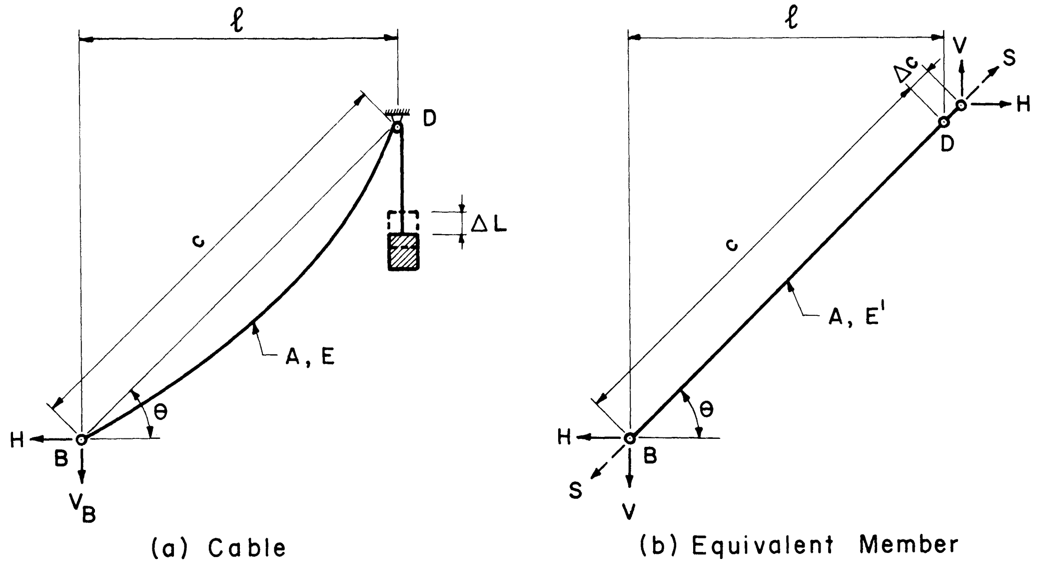
Except in unusual cases, the cable sag ratios (f/l) in the type of structures under consideration are sufficiently small so that the catenary curves may be approximated by parabolas. Referring to Fig. 3, if the load per unit length of cable is w, the total load acting on the cable is wL, L being the length of the cable. Thus,

$$Hf = wLl/8 \approx wl^2/(8\cos\theta) \tag{1}$$

or,

$$f/l = wl/(8H\cos\theta) \tag{2}$$

in which all the parameters are defined in Fig. 3.



(a) Cable (b) Equivalent Member

Fig. 4. Change of counterweight elevation in cable model and change of chord length in equivalent member

When the sag ratio is small, the length of cable may be expressed as

$$L = c\left[1 + \frac{8}{3}\left(\frac{f}{l}\right)^2 \cos^4\theta\right] \tag{3}$$

Substituting Equation (2) in Equation (3),

$$L = c\left[1 + \frac{1}{24}\left(\frac{wl\cos\theta}{H}\right)^2\right] \tag{4}$$

If the cable is inextensible, the change of cable length due to a change of H may be found by differentiating Equation (4) with respect to H, thus:

$$dL_i = -\frac{c}{12}\left(\frac{w^2l^2\cos^2\theta}{H^3}\right)dH \tag{5}$$

Physically speaking, Equation (5) implies that the cable length between the supports must be shortened by an amount dL_i for an increase of dH. In other words, the counterweight must move downward by a distance $|dL_i|$.

Since this discussion does not consider inextensible members, the elastic stretch of the cable must be taken into account; this is given by

$$\begin{aligned} L_e &= \frac{Hl}{AE}\left[\sec^2\theta + \frac{16}{3}\left(\frac{f}{l}\right)^2\right] \\ &= \frac{Hc}{AE\cos\theta}\left[1 + \frac{16}{3}\left(\frac{f}{l}\right)^2\cos^2\theta\right] \end{aligned} \tag{6}$$

where E and A are the modulus of elasticity and the cross-sectional area of the cable, respectively. Combining Equations (2) and (6), the latter may be written as

$$L_e = \frac{Hc}{AE\cos\theta}\left[1 + \frac{1}{12}\left(\frac{wl}{H}\right)^2\right] \tag{7}$$

Again, the change of L_e resulting from a change of H may be obtained by differentiating Equation (7) with respect to H, thus,

$$dL_e = \frac{c}{AE\cos\theta}\left[1 - \frac{1}{12}\left(\frac{wl}{H}\right)^2\right]dH \tag{8}$$

Therefore, in order to compensate for the elastic stretch, the counterweight must be lowered by a total amount

$$\begin{aligned} dL &= |dL_i| + dL_e \\ &= \left\{\frac{w^2l^2\cos^2\theta}{12H^3} + \frac{1}{AE\cos\theta}\left[1 - \frac{1}{12}\left(\frac{wl}{H}\right)^2\right]\right\}dH \end{aligned}$$

With H replaced by $S\cos\theta$, the preceding expression takes the following form:

$$dL = \frac{c}{AE\cos\theta}\left[1 + \frac{1}{12}\left(\frac{wl}{S}\right)^2\left(\frac{AE}{S} - \sec^2\theta\right)\right]dH \tag{9}$$

In practical cases, $\sec^2\theta$ is extremely small as compared with AE/S, and may be neglected without appreciable effects; therefore,

$$dL = \frac{c}{AE\cos\theta}\left[1 + \frac{1}{12}\left(\frac{wl}{S}\right)^2\left(\frac{AE}{S}\right)\right]dH \tag{10}$$

Consider now the equivalent two-force member (Fig. 4b). The change of chord length due to a change of S is

$$dc = \frac{d}{dS}\left(\frac{Sc}{AE'}\right)dS = \frac{c}{AE'}\,dS = \frac{c}{AE'\cos\theta}\,dH \tag{11}$$

If the static behavior of the equivalent member is to be analogous to that of the cable, the following condition must be satisfied:

$$\frac{dH}{dL} = \frac{dH}{dc} \tag{12}$$

From Equations (10), (11), and (12),

$$\frac{AE\cos\theta}{c}\left[\frac{1}{1 + (1/12)(wl/S)^2(AE/S)}\right] = \frac{AE'\cos\theta}{c}$$

or,

$$E' = \frac{E}{1 + (1/12)(wl/S)^2(AE/S)} \tag{13a}$$

In nondimensional form, Equation (13a) may be written as

$$\frac{E'}{E} = \frac{1}{1 + (1/12)(wl/S)^2(AE/S)} \tag{13b}$$

To facilitate computations, Fig. 5 shows (E'/E) plotted against (AE/S) for various values of (S/wl).

CORRECTION OF E' FOR THE EFFECT OF CHANGE OF CABLE STRESSES

Due to the nonlinear characteristics of cables, Equations (13a) and (13b) will no longer be correct if the change of cable stresses is appreciable. As demonstrated below, this effect can be accounted for by applying *one* simple correction to the value of E'.

Transposing Equation (13b),

$$\frac{1}{12}\left(\frac{wl}{S}\right)^2\left(\frac{AE}{S}\right) = \frac{1}{(E'/E)} - 1 = \frac{1 - (E'/E)}{(E'/E)} \tag{14}$$

On the other hand, if Equation (13b) is differentiated with respect to S,

$$d\left(\frac{E'}{E}\right) = \frac{1/12(wl/S)^2(EA/S)}{[1 + (1/12)(wl/S)^2(EA/S)]^2}\left(\frac{3}{S}\,dS\right)$$

By substituting Equations (14) and (13b) in the preceding expression, it can be shown that, in terms of finite differences,

$$\Delta\left(\frac{E'}{E}\right) = \left(\frac{E'}{E}\right)\left(1 - \frac{E'}{E}\right)\left(\frac{3\Delta S}{S}\right) \tag{15}$$

However, $\Delta(E'/E)$ is overcorrected in Equation (15) because ΔS will be changed as a result of the modification of E', which in turn will be affected because of the change of ΔS. A much improved result will be achieved if the average of the initial E'/E and the one after correction by Equation (15) is used; thus,

$$\begin{aligned}\frac{E'_r}{E} &= \frac{E'}{E} + \frac{E'}{E}\left(1 - \frac{E'}{E}\right)\left(\frac{3\Delta S}{2S}\right) \\ &= \frac{E'}{E} + \Delta\left(\frac{E'}{E}\right)_e \end{aligned} \tag{16a}$$

where

$$\Delta\left(\frac{E'}{E}\right)_e = \frac{E'}{E}\left(1 - \frac{E'}{E}\right)\left(\frac{3\Delta S}{2S}\right) \tag{17a}$$

Equations (16a) and (17a) may be alternately expressed in the following manner:

$$E'_r = E' + \Delta E'_e \tag{16b}$$

$$\Delta E'_e = E'\left(1 - \frac{E'}{E}\right)\left(\frac{3\Delta S}{2S}\right) \tag{17b}$$

The subscripts r and e in Equations (16) and (17) refer to "revised for effect of ΔS" and "effective" quantities, respectively. Figure 6 shows (E'_r/E) plotted against (E'/E) for various values of $(\Delta S/S)$.

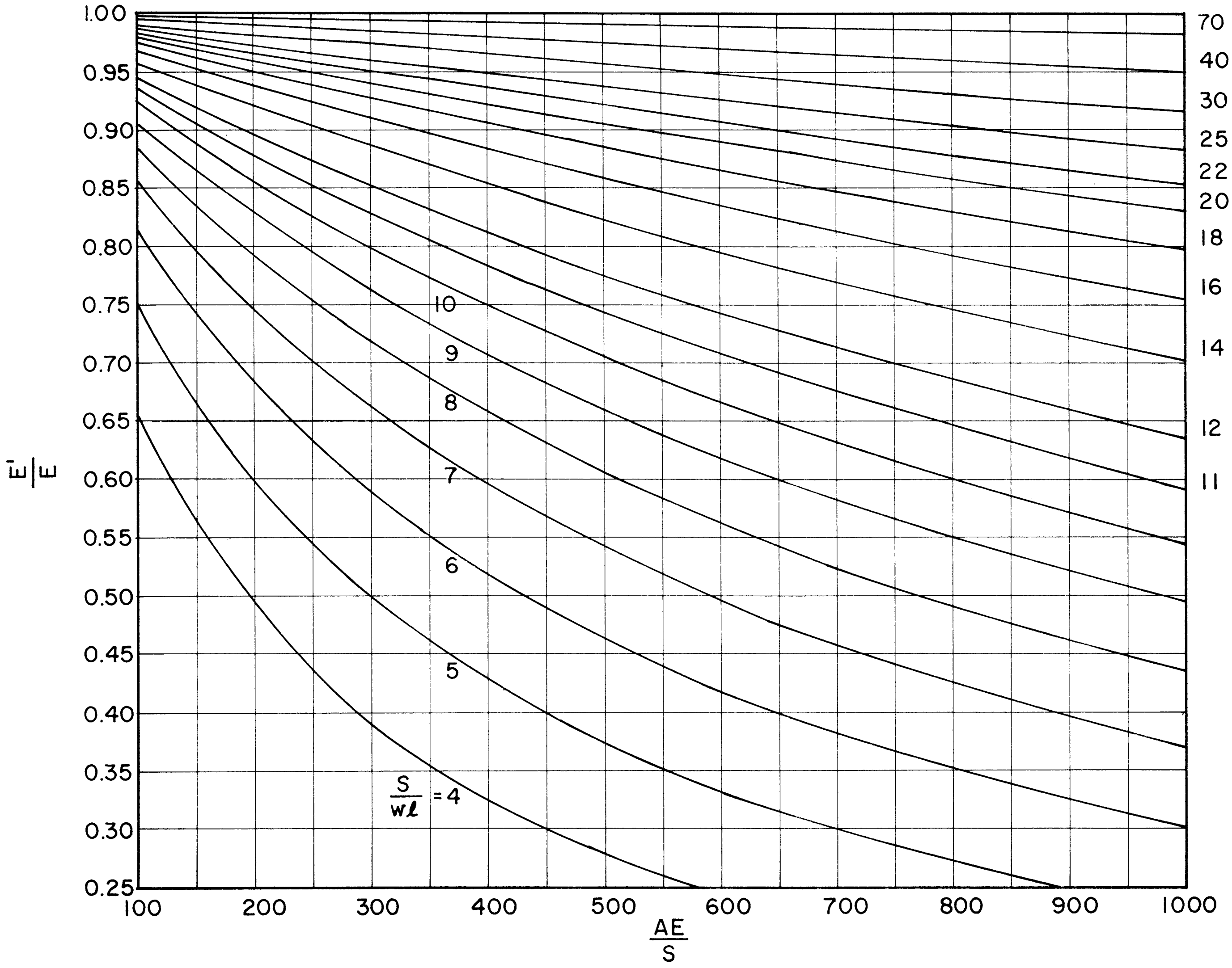


Fig. 5. (E'/E) plotted against (AE/S) for various values of (S/wl)

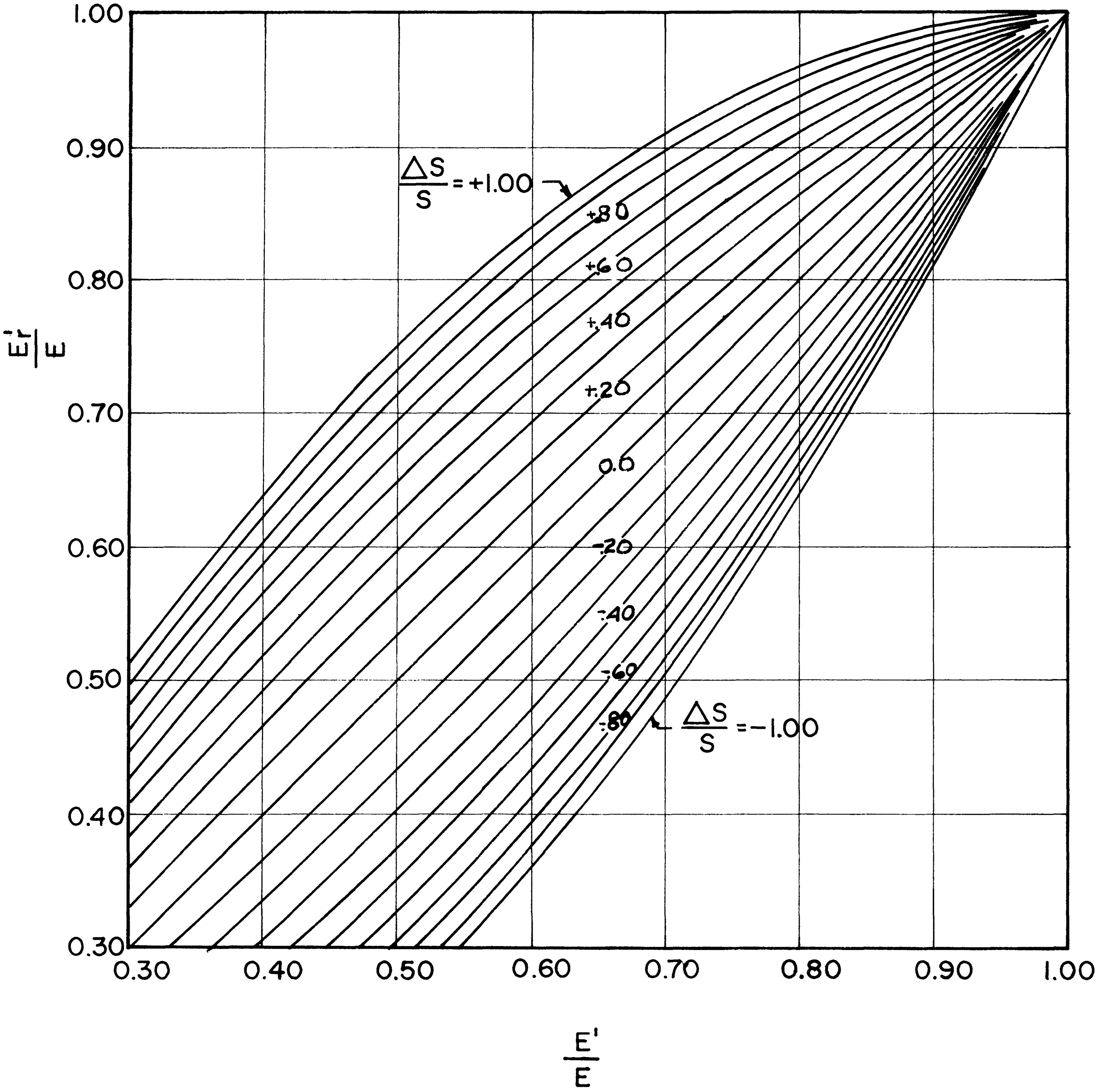


Fig. 6. *(E_r'/E) plotted against (E'/E) for various values of ($\Delta S/S$)*

CHANGE OF LENGTH OF THE EQUIVALENT TWO-FORCE MEMBER (Δc) DUE TO CHANGE OF EXTERNAL CONDITIONS OF CABLE

It can be seen from Equation (11) that, expressed in terms of finite differences, the change of stress in the equivalent member, ΔS, can be readily determined once Δc is known. The latter, however, is equal to ΔL according to Equations (10) and (12). It would be expedient, therefore, to establish the value ΔL, and consequently Δc, for a few common cases, where ΔL is defined as the change of counterweight elevation required to maintain either the initial horizontal reaction H, or the initial cable sag, as specified in the following derivations.

In the following, positive ΔL is directed downward, and positive Δc indicates that the equivalent member has lengthened in order to maintain the initial H or f values, as specified below.

Δc Due to Change of Temperature ΔT (H to Remain Constant)—This case has been discussed previously. Due to a change of temperature ΔT, the change of cable length is $\Delta L = \mu L(\Delta T)$. In order to maintain H, the counterweight must be lowered or raised by an amount ΔL to compensate for the difference in the length of the cable.

Thus,

$$\Delta c = \Delta L = \mu L(\Delta T) = \left(\frac{L}{c}\mu\right)(c)(\Delta T) = \mu' c(\Delta T) \tag{18a}$$

where L/c is defined by Equations (3) or (4), and μ' is the equivalent coefficient of thermal expansion of the straight member.

For the worst case, when $\theta = 0$, L/c equals 1.027 when $f/l = 1/10$, and 1.012 when $f/l = 1/15$. For practical cases, one may consider the following approximation by dropping the term L/c, so that

$$\Delta c = (\mu \cdot \Delta T)c \tag{18b}$$

Δc Due to Change of Distributed Load Δw_0—In order to arrive at a simplified procedure with sufficient accuracy, it will be necessary to group these types of problems into two general categories. In the first category, cables such as commonly encountered in guyed structures and parabolic suspended roofs, where the cable stresses are rather low and the sag-ratios relatively large, will be considered. In the second category, those cables which are highly stressed and extremely taut, such as those in cable-stiffened bridges, will be investigated.

However, if $\Delta w_0/w_0$ is small, the procedure given for the first category is applicable to the latter with satisfactory results.

For Low Stresses and Large Sag-Ratios (f to Remain Constant): Because of the initial large sag-ratio, if H is to be maintained constant, the sag could be increased to such an extent that it may no longer be permissible to approximate the cables by parabolas. For satisfactory results, therefore, the initial sag-ratio should instead be kept unchanged. Thus, assuming the initial values are w_0, H_0, and S_0, respectively, Equation (2) may be written as

$$f = \frac{w_0 l^2}{8H_0 \cos\theta} = \frac{(w_0 + \Delta w_0)l^2}{8(H_0 + \Delta H_0)\cos\theta} = \frac{wl^2}{8H\cos\theta}$$

from which

$$\Delta H_0 = \left(\frac{\Delta w_0}{w_0}\right)H_0; \quad \Delta S_0 = \left(\frac{\Delta w_0}{w_0}\right)S_0;$$

$$w = w_0 + \Delta w_0; \quad H = H + \Delta H_0; \quad S = S_0 + \Delta S_0 \tag{19}$$

Since the sag-ratio is being maintained constant, the change in elevation of the counterweight is equal to the elastic stretch of the cable due to ΔS_0. From Equation (7),

$$\Delta L = \frac{\Delta S_0 c}{AE}\left[1 + \frac{1}{12}\left(\frac{wl}{H}\right)^2\right]$$

Since $(wl/H) = (w_0 l_0/H_0) =$ constant,

$$\Delta L = \frac{\Delta S_0 c}{AE}\left[1 + \frac{1}{12}\left(\frac{w_0 l_0}{H_0}\right)^2\right]$$

Rearranging, and recalling that $\Delta c = \Delta L$,

$$\frac{\Delta c}{c} = \frac{\Delta S_0}{AE}\left[1 + \frac{1}{12}\left(\frac{wl}{S}\right)^2\left(\frac{1}{\cos^2\theta}\right)\right] \tag{20}$$

All the previous equations are still valid, but care must be taken to ascertain that in this case $w = w_0 + \Delta w_0$ and $S = S_0 + \Delta S_0$. It can be seen also that the second term in the bracket is usually small as compared with unity, and can be neglected in most cases.

For High Stresses and Extremely Small Sag-Ratios (H to Remain Constant): The method proposed in the preceding section cannot be applied in this case with sufficient accuracy, because ΔH is usually very large for very taut cables. Here, again, H may be maintained constant because of the small sag-ratios. Assuming that the initial values are w_0, f_0, H_0, and S_0, respectively, then from Equation (1),

$$H_0 = H = \frac{w_0 l^2}{8f_0\cos\theta} = \frac{(w_0 + \Delta w_0)l^2}{8(f_0 + \Delta f_0)\cos\theta}$$

Thus,

$$\frac{f_0 + \Delta f_c}{l} = \frac{f_0}{l}\left(1 + \frac{\Delta w_0}{w_0}\right) \tag{21}$$

The length of cable required to maintain H is

$$L' = c\left[1 + \frac{8}{3}\left(\frac{f_0 + \Delta f_0}{l}\right)^2\cos^4\theta\right]$$

$$= c\left[1 + \frac{8}{3}\left(\frac{f_0}{l}\right)^2\left(1 + \frac{\Delta w_0}{w_0}\right)^2\cos^4\theta\right]$$

$$= L + c\left\{\frac{8}{3}\left(\frac{f_0}{l}\right)^2\left[\frac{2\Delta w_0}{w_0} + \left(\frac{\Delta w_0}{w_0}\right)^2\right]\cos^4\theta\right\} \tag{22}$$

According to Equation (2),

$$\left(\frac{f_0}{l}\right)^2\cos^4\theta = \left(\frac{w_0 l}{8H_0\cos\theta}\right)^2\cos^4\theta = \left(\frac{w_0 l}{8S_0}\right)^2$$

Substituting the preceding expression into Equation (22), and noting $\Delta L = L - L'$,

$$\frac{\Delta c}{c} = \frac{\Delta L}{c} = -\frac{8}{3}\left(\frac{f_0}{l}\right)^2\left[\left(\frac{2\Delta w_0}{w_0}\right) + \left(\frac{\Delta w_0}{w_0}\right)^2\right]\cos^4\theta \tag{23a}$$

or,

$$\frac{\Delta c}{c} = -\frac{1}{24}\left(\frac{w_0 l}{S_0}\right)^2\left[\frac{2\Delta w_0}{w_0} + \left(\frac{\Delta w_0}{w_0}\right)^2\right] \tag{23b}$$

To be consistent, $w = w_0 + \Delta w_0$ must be used in the computations of E' and $\Delta E'_e$.

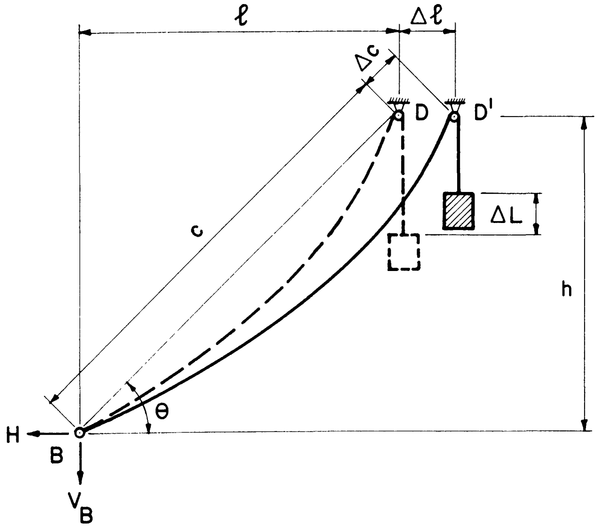


Fig. 7. Change of counterweight elevation due to Δl

Δc Due to Displacement of Supports or Change of Span or Chord Length (H to remain constant)—Referring to Fig. 7, and assuming that the upper support is displaced horizontally by an amount Δl,

$$H = \frac{wLl}{8f} = \frac{wL(l + \Delta l)}{8(f + \Delta f)}$$

Thus,

$$\frac{f + \Delta f}{l + \Delta l} = \frac{f}{l}$$

Consequently, if the effect of change of θ is neglected, which is permissible because only small displacements are being considered, the length of cable required to maintain H is

$$\begin{aligned} L' &= (c + \Delta c)\left[1 + \frac{8}{3}\left(\frac{f + \Delta f}{l + \Delta l}\right)^2 \cos^4 \theta\right] \\ &= (c + \Delta c)\left[1 + \frac{8}{3}\left(\frac{f}{l}\right)^2 \cos^4 \theta\right] \\ &= L + \left(\frac{L}{c}\right)\Delta c \\ &= L + \frac{L}{c}(\Delta l \cos \theta) \end{aligned}$$

Therefore,

$$\Delta c = \Delta L = L - L' = -\frac{L}{c}(\Delta l \cos \theta) \qquad (24a)$$

For the same reasons discussed previously, one may consider dropping the term L/c in Equation (24a) for practical cases. Then,

$$\frac{\Delta c}{c} = \frac{\Delta L}{c} = -\frac{\Delta l \cos \theta}{c} \qquad (24b)$$

Finally, for the general case when the upper support is displaced both horizontally and vertically,

$$\frac{\Delta c}{c} = \frac{\Delta L}{c} = -\frac{(\Delta l \cos \theta + \Delta h \sin \theta)}{c} \qquad (25)$$

in which Δh indicates upward vertical displacement.

It should be noted that, theoretically, if there is a change of Δl, the values of E' and $\Delta E'_e$ should be modified to reflect their effect in the analysis. However, since $\Delta l/l$ is usually very small, E' and $\Delta E'_e$ may simply be computed on the basis of l without appreciable errors.

PROCEDURE OF ANALYSIS

The proposed analysis is based on the principle of consistent displacements. The detailed procedure is outlined as follows:

1. Compute $\Delta c/c$ due to the change of external conditions. The expression of Δc for ordinary cases may be found in one of the following: Equations. (18), (20), (23), (24), (25).
2. Determine E'/E and E' from Fig. 5 or according to Equation (13). It should be noted that w = load per unit length of cable $\simeq$ load per unit length of chord. (When the cable is subjected to a change of superimposed load, the correct values of w and S ($w = w_0 + \Delta w_0$ and $S = S_0 + \Delta S_0$) are used in the calculations.
3. Compute ΔS in the equivalent two-force member: $\Delta S = -E'A(\Delta c/c)$.
4. Determine E'_r from Fig. 6 or according to Equations (17) and (16).
5. Compute $\Delta H = \Delta S_r \cos \theta = -E'_rA(\Delta c/c) \cos \theta$, where ΔS_r represents the final change of direct stress in the equivalent member.
6. The final horizontal reaction of the cable is equal to $H + \Delta H$, from which the final cable stresses and sags can be calculated.

NUMERICAL EXAMPLES

In the following examples (except in Example 4) the initial conditions are assumed as follows:

l = 1000 ft; $\theta = 55°$; $\cos \theta = 0.574$
c = 1743.447 ft; w = 9 lbs/ft of cable
H = 40 kips; E = 24,000 ksi; $A = 2.40$ in.^{2}
$\mu = 0.0000067/°F$

Example 1— $\Delta T = -50°F$

Solution:

Equation (18b): $(\Delta c/c) = (6.7)(10)^{-6}(-50)(c/c) = -3.35\ (10)^{-4}$ ft/ft

$S = 40/0.574 = 69.7$ kips

$(wl/S)^2 = (0.009 \times 1000/69.7)^2 = 0.01665$

$(AE/S) = 2.40\ (24)\ (10)^3/69.7 = 826$

From Fig. 5: $(E'/E) = 0.466$

or, from Eq. (13b):

$(E'/E) = [1 + (1/12)(0.01665)(826)]$

$= 0.466$

$E' = 0.466 (24) (10)^3 = 11.18 (10)^3$ ksi

$\Delta S = -11.18 (10)^3 (2.40) (-3.35) (10)^{-4}$

$= 8.99$ kips

From Fig. 6: $(E'_r/E) = 0.514$

$E'_r = 12.34 (10)^3$ ksi

or, from Eq. (17b):

$\Delta E'_e = 11.18(10)^3(1 - 0.466) [3(8.99)/2 (69.7)]$

$= 1.16 (10)^3$ ksi

Eq. (16b): $E'_r = (11.18 + 1.16) (10)^3$

$= 12.34 (10)^3$ ksi

$\Delta H = -12.34 (10)^3 (2.40) (-3.35) (10)^{-4} (0.574)$

$= 5.7$ kips

$H' = 40 + 5.7 = 45.7$ kips

Example 2— $\Delta w_0 = 3$ lbs/ft of cable

Solution:

Eq. (19): $\Delta S_0 = (3/9) (69.74) = 23.3$ kips

From Example 1: $(wl/S)^2 = (w_0 l_0/S_0)^2 = 0.01665$

Eq. (20):

$(\Delta c/c) = (23.2/EA) [1 + (1/12) (0.01665) (1/0.574)^2]$

$= 23.35/EA$ ft/ft

$S = S_0 + \Delta S_0 = 69.7 + 23.3 = 93.0$ kips

$H = 93.0 (0.574) = 53.3$ kips

$(AE/S) = 2.40 (24) (10)^3/93.0 = 619$

From Fig. 5: $(E'/E) = 0.538$

$E' = 0.538\, E$

$\Delta S = -0.538\, EA\, (23.35/EA) = -12.56$ kips

From Fig. 6: $(E'_r/E) = 0.488$

$\Delta H = -0.488\, EA\, (23.35/EA) (0.574) = -6.5$ kips

$H' = H + \Delta H = 53.3 - 6.5 = 46.8$ kips

Example 3— $\Delta l = 1.000$ ft

Solution:

From Example 1:

$S = 69.7$ kips; $(wl/S)^2 = 0.01665$;

$(E'/E) = 0.466$; $E' = 11.18 (10)^3$ ksi

Eq. (24b): $(\Delta c/c) = -1.000 (0.574)/1743.447$

$= -3.29 (10)^{-4}$ ft/ft

$\Delta S = -11.18 (10)^3 (2.40) (-3.290) (10)^{-4}$

$= 8.83$ kips

From Fig. 6: $(E'_r/E) = 0.513$

$E'_r = 12.31 (10)^3$ ksi

$\Delta H = -12.31(10)^3(2.40)(-3.29)(10)^{-4}(0.574)$

$= 5.6$ kips

$H' = 40 + 5.6 = 45.6$ kips

Example 4—

Given: $l = 700$ ft; $\theta = 0°$; $w_0 = 16$ lbs/ft of cable

$E = 24{,}000$ ksi; $A = 4.54$ in.^{2}; $H = 300$ kips

Required: Final horizontal reaction H'

Solution:

$\theta = 0$

$S = H = 300$ kips

$(w_0 l_0/S_0)^2 = (0.016 \times 700/300)^2 = 0.001394$

Eq. (23b):

$(\Delta c/c) = -(1/24) (0.001394) \times$

$[(2 \times 16/16) + (16/16)^2]$

$= -1.743 (10)^{-4}$ ft/ft

$w = 16 + 16 = 32$ lbs/ft of cable

$(wl/S)^2 = (0.032 \times 700/300)^2 = 0.00558$

$(AE/S) = 4.54 (24) (10)^3/300 = 363$

From Fig. 5:

$(E'/E) = 0.856$

$E' = 0.856 (24) (10)^3 = 20.5 (10)^3$ ksi

$\Delta S = -20.5 (10)^3 (4.54) (-1.743) (10)^{-4}$

$= 16.25$ kips

From Fig. 6: $(E'_r/E) = 0.864$

$E'_r = 20.74 (10)^3$ ksi

$\Delta H = -20.74 (10)^3 (4.54) (-1.743) (10)^{-4}$

$= 16.4$ kips

$H = 300 + 16.4 = 316.4$ kips

CONCLUSIONS

Cable structures have proved to be both economical and aesthetically attractive in the field of long-span construction. A sufficiently accurate analysis or design of such structures, however, is often found to be quite involved on account of the nonlinear behavior of cable members. This paper presents a simplified method by treating the cables as equivalent two-force members.

Their nonlinear behavior is accounted for by introducing the concept of equivalent modulus of elasticity and equivalent strains; thereby the stresses can be readily determined by using the principle of consistent displacements.

It is believed that this method will prove both useful and practical for office use, and will find its application in the analysis of guyed towers, cable-stiffened bridges, suspended roofs, and transmission lines.

ACKNOWLEDGMENT

The assistance given by Dr. Felix A. Wallace, Head of the Department of Civil Engineering and Applied Mechanics, in making the arrangements for typing of the manuscript is hereby acknowledged.