Designing Compact Gussets with the Uniform Force Method

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In 1991 an AISC task group endorsed the uniform force method (UFM) as the preferred method for determining the forces that exist at gusset interfaces. Since that time it has been included in the AISC *Manual of Steel Construction*. The UFM provides a standardized way to obtain economical, statically admissible force distributions for vertical bracing connections. One criticism of the method is that it sometimes results in oddly shaped or disproportionately large gusset plates. To overcome this perceived limitation of the UFM, designers have been seeking out alternate methods.

This paper demonstrates that removing one unnecessary geometrical constraint from the formulation of the UFM will allow greater freedom in gusset geometry, while maintaining the efficiencies that result from the method. A new formulation of the UFM is presented, and the strengths and weaknesses of other proposed design methods are also explored.

THE UNIFORM FORCE METHOD

The uniform force method has been included in the AISC *Manual of Steel Construction* since 1992. The UFM was originally proposed by Thornton (1991) and was based on observations from Richard's (1986) research. In the commonly accepted form, the UFM produces the following force distribution:

$$H_b = \frac{\alpha}{r} P \tag{1a}$$

$$V_b = \frac{e_b}{r} P \tag{1b}$$

$$H_c = \frac{e_c}{r} P \tag{1c}$$

$$V_c = \frac{\beta}{r} P \tag{1d}$$

where

$$r = \sqrt{\left(\alpha + e_c\right)^2 + \left(\beta + e_b\right)^2} \tag{2}$$

$$\alpha = \tan\theta \left(\beta + e_b\right) - e_c \tag{3}$$

In order to satisfy the relationship between α and β, the designer is often forced to use either an oddly shaped or disproportionately large gusset plate. Alternately, moments can be introduced at the connection interfaces. Neither approach is ideal.

ALTERNATIVES TO THE UNIFORM FORCE METHOD

Any viable alternative sought to replace the UFM should meet the following criteria: (1) it must provide a clear procedure to satisfy equilibrium and conform to the basic assumptions made during the analysis and design of the main members (the most important criterion); (2) since the UFM readily accommodates a wide range of geometries and boundary conditions, any alternate method should also be able to accommodate such situations; and (3) it must result in economical designs.

Several alternatives to the UFM have been proposed. Chief among the alternatives are the KISS Method, the parallel force method and the truss analogy method. None of these methods suffer from the constrictive relationship between α and β that exists in the UFM. In other words, these methods can be used with any gusset geometry and do not force the use of oddly shaped or large gusset plates. The strengths and weaknesses of these methods will be explored. In all of the discussions the work-point of the brace is assumed to be located at the intersection of the centerlines of the beam and the column, since this is the typical case.

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The KISS Method

KISS (Figure 1) is an acronym for "keep it simple stupid," and the method is simple, as the name implies, and fool-proof, though uneconomical. The method involves delivering the entire horizontal brace component directly to the beam through the beam-to-gusset connection and the entire vertical brace component directly to the column through the column-to-gusset connection. To satisfy equilibrium, moments must be introduced. At the beam-to-gusset the moment is equal to He_b, and at the column-to-gusset the moment is equal to Ve_c.

The KISS Method satisfies two of the three criteria for a viable alternative to the UFM. It satisfies equilibrium and the design and analysis assumptions, and it is universally applicable to all geometries and boundary conditions. However, the presence of the large moments at the connection interfaces makes it an uneconomical choice in practice.

The Parallel Force Method

In the parallel force method, sometimes referred to as the component method (Figure 2), the reactions of the gusset at the beam and column interfaces are assumed to act parallel to the brace force. Since the forces are parallel, they obviously do not intersect at a common point, as is the case with the UFM. Therefore, in order to maintain rotational equilibrium, two choices are available. Either the magnitude of the parallel forces are set so that they balance each other about the work-line of the brace, or moments are added at the beam and/or column interfaces. The additional moments, though lesser in magnitude than the KISS method, adversely impact the economy of the connection.

If the former approach is taken, rotational equilibrium of the beam and column will not generally be satisfied. A moment connection will then have to be added between the beam and the column. Since this will normally be a field-welded connection, this is considered to be an uneconomical alternative in most parts of the country.

In terms of applicability to a variety of boundary conditions, the parallel force method suffers from a major shortcoming. Since the forces at both the beam-to-gusset and the column-to-gusset interfaces are assumed parallel to the brace force, a horizontal component will always exist at the column-to-gusset connection. When framing to a column web, this presents a significant design challenge, which will usually be overcome by the addition of column stiffening local to the connection, further reducing the economy of the method.

The parallel force method only satisfies one of the three criteria for a viable alternative to the UFM. It satisfies equilibrium and the design and analysis assumptions, but it is not as economical as the UFM and is not suited to connections made to column webs.

The Truss Analogy Method

The truss analogy method (Figure 3) determines the force distribution on the gusset by modeling the interface forces as a pinned "truss" node located at the center of the brace-to-gusset connection. The truss analogy method suffers the same problem as the parallel force method when attaching to column webs. Additionally, the truss analogy method can result in counterintuitive and uneconomical force distributions. This is illustrated in Figure 3 where the gusset-

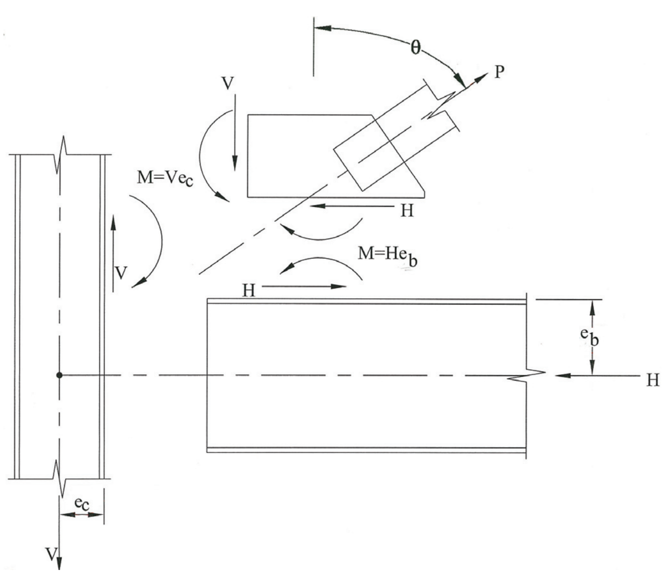

Fig. 1. KISS method.

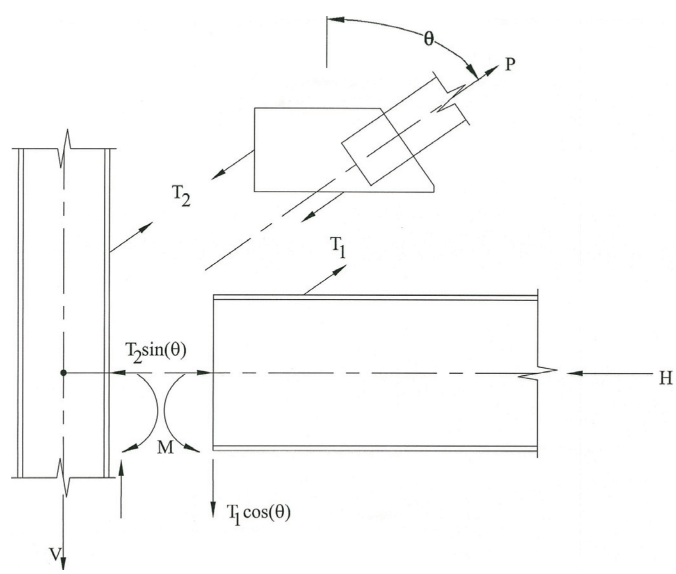

Fig. 2. Parallel force method.

to-column connection delivers only a horizontal component to the column. A formalized treatment of the equilibrium requirements for the beam and column has never been presented and is therefore left to the designer. Often moments are required at all of the connection interfaces in order to satisfy equilibrium.

The truss analogy method satisfies none of the criteria for a viable alternative to the UFM.

A GENERALIZED UFM

Since none of the alternatives investigated appear to provide better results than the UFM, it is advantageous to make adjustments to the formulation of the UFM to make it more applicable to compact gussets.

The goal of the UFM was to derive a procedure to obtain statically admissible force distributions, which would produce no moments at the connection interfaces and would be applicable to a wide range of geometries and boundary conditions. However, the procedure includes an additional constraint that unnecessarily limits its applicability. The force at the gusset-to-column interface, $\sqrt{V_c^2 + H_c^2}$, is forced to pass through a point that lies a distance, e_b, above the work-point.

Since there is a perceived problem with the UFM that can be overcome by removing this constraint, it is advantageous to eliminate it from the method. In order to do so, the problem must first be defined. There are essentially three elements involved: the beam, the column, and the gusset. The brace is neglected since it is assumed to carry only axial force and is not part of the indeterminate system. Each of the three members is subjected to three forces. In order for moments to be eliminated from the interfaces the forces applied to each element must intersect at a single point. These points of intersection are referred to as control points.

The Beam

It is easiest to begin with the beam (Figure 4), since the location of its control point is evident. The three forces applied to the beam are the horizontal component of the brace, H, the beam-to-gusset force, $\sqrt{V_b^2 + H_b^2}$, and the beam-to-column force, $\sqrt{V_b^2 + H_c^2}$. The horizontal component of the brace is resisted along the centerline of the beam and intersects the beam-to-column force at the point (e_c, 0). Therefore, the beam-to-gusset force must also pass through this point. From this we find that

$$\frac{V_b}{H_b} = \frac{e_b}{\alpha} \tag{4}$$

The Gusset

The three forces applied to the gusset (Figure 5) are the brace force, P, the beam-to-gusset force, $\sqrt{V_b^2 + H_b^2}$, and the gusset-to-column force, $\sqrt{V_c^2 + H_c^2}$. In order to eliminate moments at the interfaces, these three forces must intersect at a single point. Since the slope of the brace force, $1/\tan(\theta)$, and the slope of the beam-to-gusset force, e_b/α, are known, the intersection can be determined. The gusset control point is:

$$\left(\frac{e_b e_c \tan(\theta)}{e_b \tan(\theta) - \alpha}, \frac{e_b e_c}{e_b \tan(\theta) - \alpha} \right)$$

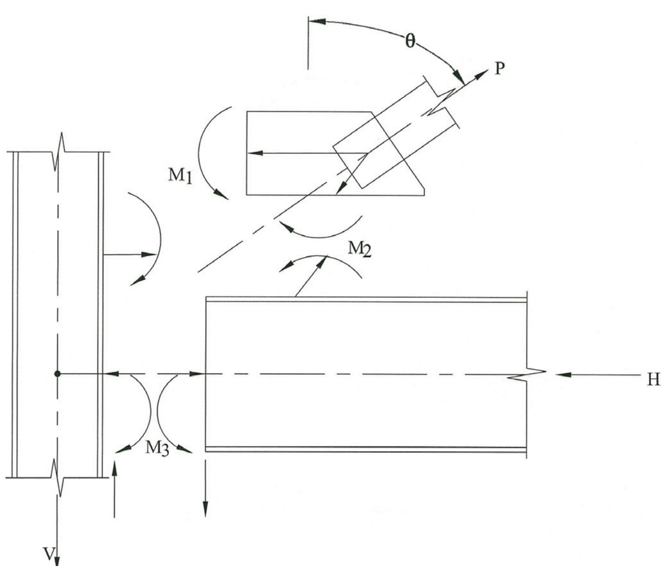

Fig. 3. Truss analogy method.

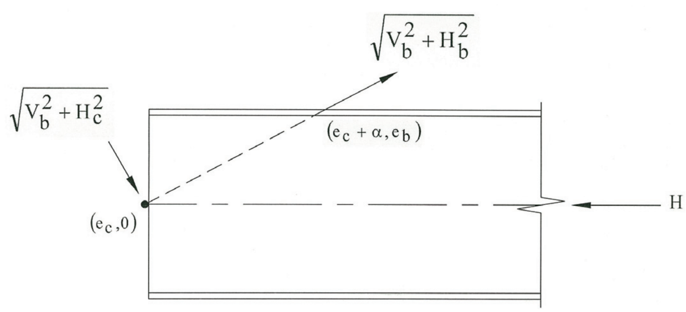

Fig. 4. Beam free body diagram.

The Column

The three forces applied to the column (Figure 6) are the vertical component of the brace, V, the column-to-gusset force, $\sqrt{V_c^2 + H_c^2}$, and the beam-to-column force, $\sqrt{V_b^2 + H_c^2}$. Knowing that the gusset-to-column force must pass through the gusset control point, the slope of the gusset-to-column force is:

$$\frac{V_c}{H_c} = \frac{\beta}{e_b}\left[\frac{e_b}{\beta}\left(1 - \frac{\tan(\theta)(e_b + \beta) - e_c}{\alpha}\right) + 1\right] \tag{5}$$

From this, since the column-to-gusset force and the beam-to-column force must intersect at the centerline of the column, the slope of the beam-to-column force is:

$$\frac{V_b}{H_c} = \frac{e_b}{e_c}\left(\frac{\tan(\theta)\left(e_b + \beta\right) - e_c}{\alpha}\right) \tag{6}$$

The point of intersection of the column-to-gusset force and the beam-to-column force, the column control point, is:

$$\left(0, e_b\left(\frac{\tan(\theta)\left(e_b + \beta\right) - e_c}{\alpha}\right)\right)$$

Force Distribution

Having established the geometrical constraints required to eliminate moments at all connection interfaces, the forces at the interfaces can be derived. Since the column must be in equilibrium, the following can be established:

$$\sum F_y = 0 = P\cos(\theta) - \left(V_b + V_c\right) \tag{7}$$

$$\sum F_x = 0 = H_c - H_c \tag{8}$$

$$\sum M = 0 = H_c\left(e_b + \beta\right) - P\cos(\theta)e_c \tag{9}$$

From this

$$H_c = \frac{\cos(\theta)e_c}{\left(e_b + \beta\right)}P \tag{10}$$

To satisfy the requisite geometry for the beam-to-gusset and beam-to-column forces, the following must be true:

$$V_b = \left[\frac{e_b\left(\sin(\theta)\left(e_b + \beta\right) - \cos(\theta)e_c\right)}{\alpha\left(e_b + \beta\right)}\right]P \tag{11}$$

The remaining forces are apparent:

$$H_b = P\sin(\theta) - H_c \tag{12}$$

$$V_c = P\cos(\theta) - V_b \tag{13}$$

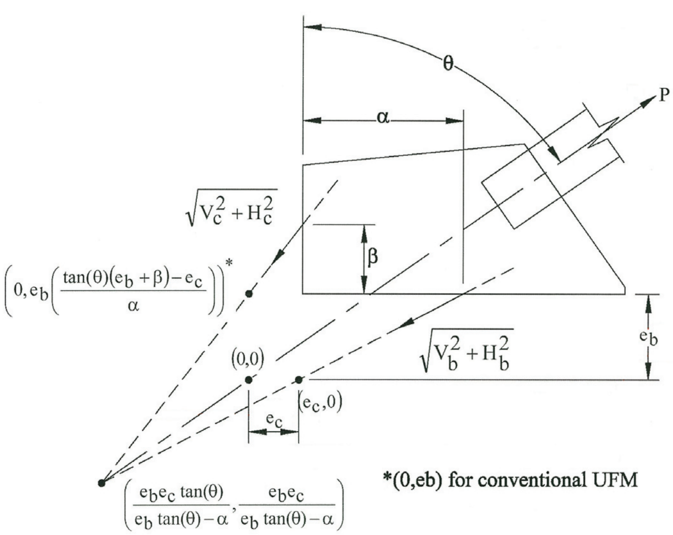

Fig. 5. Gusset free body diagram.

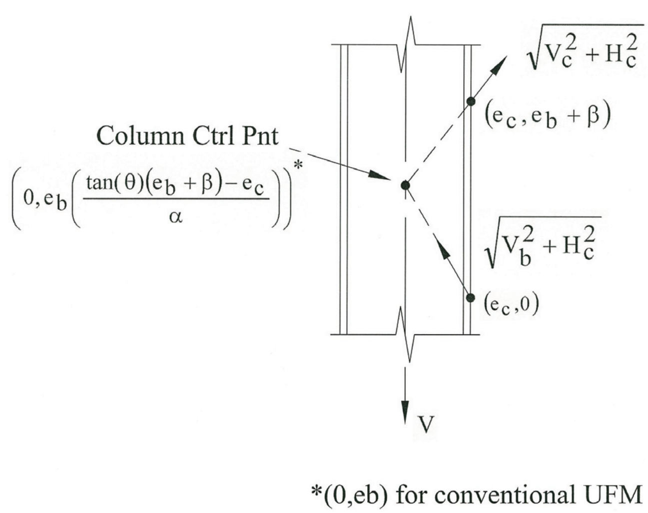

Fig. 6. Column free body diagram.

With the geometry and force distribution established, a new form of the UFM has been derived without the somewhat arbitrary constraint on the location of the column control point. Without this constraint, α and β can be set to any convenient values. This removes the need to consider the moments caused by $\bar{\alpha}$ and $\bar{\beta}$, where $\bar{\alpha}$ is the actual distance from the face of the column flange to the centroid of the gusset-to-beam connection, and $\bar{\beta}$ is the actual distance from the face of the beam flange to the centroid of the gusset-to-column connection.

However, there may still be a need to redistribute the vertical reaction delivered to the beam, V_b. This counteracting force is referred to as ΔV_b. ΔV_b can be introduced into this new formulation easily to produce the full spectrum of force distributions that can exist in the connection while maintaining column-to-gusset and beam-to-column connections free of moments. It is assumed that moments at the column-to-gusset and beam-to-column connections are uneconomical and therefore undesirable.

Of course the introduction of ΔV_b disrupts the established equilibrium and adjustments must be made. The adjustment involves introducing a moment at the beam-to-gusset interface. This moment can be calculated as:

$$M_b = H_b e_b - \left(V_b - \Delta V_b\right)\alpha \tag{14}$$

Column Moment

A moment gradient will exist in the column whether using the original formulation or the new formulation of the UFM presented in this paper. Using the original formulation, the moment will be zero at the intersection of the top of steel elevation and the centerline of the column. In the new formulation, the moment may be either positive or negative throughout the section of the column bounded by the connection or the moment may be zero at some section similar to the original formulation. In either case the maximum moment the column will be subjected to can be determined as:

$$M_c = \max\left\{V_c e_c, \left(V_c e_c - H_c\left(e_b + \beta\right)\right)\right\} \tag{15}$$

Since the choice of column section will usually be governed by buckling and the column is restrained from buckling local to the brace connection, it is normal practice to neglect this moment. For this reason, the moment internal to the column is not mentioned in the AISC *Steel Construction Manual* (AISC, 2005) discussion of the UFM.

An Example

The forces on the connection shown in Figure 7 will be calculated to demonstrate the new formulation.

$$\alpha = \frac{27.75}{2} + 0.5 = 14.375\text{ in.}$$

$$\beta = \frac{13}{2} = 6.5\text{ in.}$$

$$H_c = \frac{\cos\left(55^o\right)\left(7\right)}{\left(12+6.5\right)}100 = 21.7\text{ kips}$$

$$V_b = \left[\frac{\left(12\right)\left(\sin(55^o)\left(12+6.5\right) - \cos(55^o)\left(7\right)\right)}{\left(14.375\right)\left(12+6.5\right)}\right]\left(100\right)$$

$$= 50.3\text{ kips}$$

$$H_b = 100\sin\left(55^o\right) - 21.7 = 60.2\text{ kips}$$

$$V_c = 100\cos\left(55^o\right) - 50.3 = 7.06\text{ kips}$$

Summing moments on the beam about the beam control point produces:

$$V_b\alpha - H_b e_b = 50.3(14.375) - 60.2(12) \approx 0\text{ kip-in.}$$

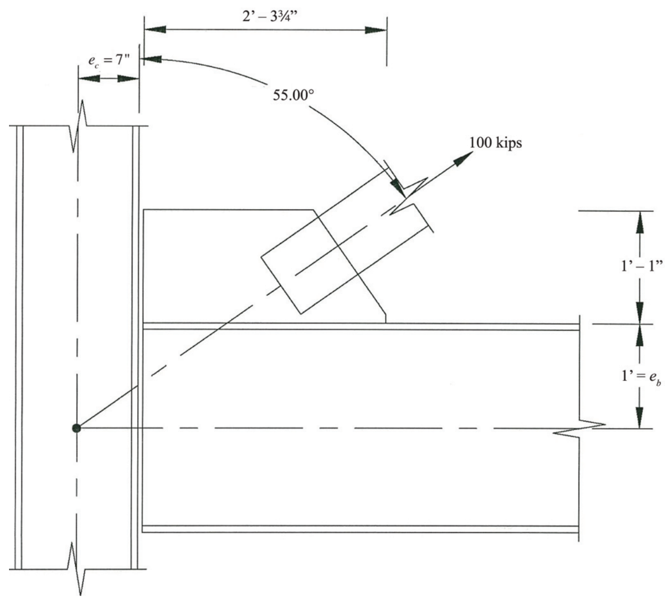

Fig. 7. Example.

Summing moments on the gusset about the work-point produces:

$$V_b\left(\alpha+e_c\right)-H_b e_b+V_c e_c-H_c\left(e_b+\beta\right)=50.3(14.375+7)-$$
$$60.2(12)+7.06(7)-21.7(12+6.5)\approx 0\text{ kip-in.}$$

Summing moments on the column about the beam-to-column connection produces:

$$P\cos(\theta)e_c-H_c\left(e_b+\beta\right)=100\cos(55^o)(7)-21.7(12+6.5)$$
$$\approx 0\text{ kip-in.}$$

Note that:

$$\tan(\theta)\left(\beta+e_b\right)-e_c=\tan(55^o)\left(6.5+12\right)-7=19.4\neq\alpha$$

For completeness the vertical coordinate of the column control point can be calculated as:

$$y_{ccp}=e_b\left(\frac{\tan(\theta)\left(e_b+\beta\right)-e_c}{\alpha}\right)$$
$$=(12)\left(\frac{\tan(55^o)\left(12+6.5\right)-7}{14.375}\right)=16.2\text{ in.}$$

It may be noted that for this case the term V_b is significantly larger than would be obtained using the traditional UFM. As is the case with the traditional UFM, a ΔV_b can be introduced to manipulate the distribution of vertical force. Taking ΔV_b equal to 13.1 kips produces the same distribution of vertical force that is obtained from the UFM when all parameters except α are held constant.

As can be seen from Table 1, which presents a comparison of the traditional UFM to the modified UFM, each can be modified to produce identical results. This is to be expected since each must satisfy equilibrium. The primary advantage to the new formulation is that it eliminates the need for the modifiers $\bar{\alpha}$ and $\bar{\beta}$. Also the new formulation makes it easier to overcome the perceived limitations of the UFM.

Table 1. Comparison of the Traditional and the Modified Uniform Force Methods

Parameters	Traditional UFM		Modified UFM	
	without $\bar{\alpha}$	with $\bar{\alpha}$	without ΔV_b	with ΔV_b
α	19.4	19.4	14.4	14.4
$\bar{\alpha}$	–	14.4	–	–
β	6.5	6.5	6.5	6.5
V_b	37.2	37.2	50.3	37.2
H_b	60.2	60.2	60.2	60.2
V_c	20.2	20.2	7.06	20.2
H_c	21.7	21.7	21.7	21.7
ΔV_b	–	–	–	13.1
M_b	–	188	–	188

OTHER PRACTICES THAT CAN REDUCE THE GUSSET PROFILE

Having eliminated the geometrical constraints on gusset size from the UFM, attention can be turned to other steps that can be taken to reduce the gusset profile.

The Whitmore Section

The Whitmore section is commonly accepted to be an area, which extends at a 30^o angle from the edges of the brace-to-gusset connection along the length of the connection. The area beyond this section is assumed to be ineffective in terms of gross tension yielding and compression buckling of the gusset. It is common practice to try to include all of the allowed Whitmore section within the gusset, but it is not a requirement to do so. By allowing the edges of the gusset plate to encroach on the Whitmore section, the profile of the gusset can be reduced.

Weld Size

It is common practice to attempt to limit fillet weld sizes to those that can be applied in a single pass, usually 5⁄16 in. This greatly enhances connection economy, since the number of passes required to complete a weld increases disproportionately with the leg size. To maintain a single pass weld, the gusset plate dimensions, particularly at the beam-to-gusset connection, are often increased. The gusset profile can be reduced by allowing multiple pass welds to be used, but only with increased fabrication costs.

Bolt Type

If reducing the gusset profile is of paramount concern, the strongest possible bolt configuration should be employed. Slip-critical connections should be avoided since they will require more bolts and therefore a larger gusset profile. Likewise, if the threads will be excluded from the shear plane, which is usually the case for heavily loaded bracing connections, then the "X-value" for the bolts should be used. Providing a detail that places the bolts in double shear at the brace-to-gusset connection also helps to reduce the gusset profile.

CONCLUSIONS

The UFM, as currently presented in the Manual, contains an unnecessary constraint on the location of the column control point. This constraint often gives designers the perception that the method is ill suited to the design of compact gusset plates.

By eliminating the unnecessary constraint in the new formulation, force distributions can be derived that consist of only shear and axial forces at the connection interfaces. The new formulation also simplifies the UFM by eliminating the need for $\bar{\alpha}$ and $\bar{\beta}$.

By manipulating the term ΔV_b, designers can obtain the full spectrum of force distributions that can exist in the connection while maintaining column-to-gusset and beam-to-column connections free of moments.

NOTATION

e_b = one-half the depth of the beam

e_c = one-half the depth of the column

y_{ccp} = vertical coordinate of the column control point

P = brace load

H = horizontal component of the brace load

H_b = shear force on the beam-to-gusset connection

H_c = axial force on the beam-to-column and gusset-to-column connections (assumes no transfer force)

M_b = moment on the beam-to-gusset connection

V = vertical component of the brace load

V_b = shear force on the beam-to-column connection and axial force on the beam-to-gusset connection

V_c = shear force on the gusset-to-column connection

ΔV_b = change in the distribution of vertical load

α = distance from the face of the column flange or web to the centroid of the gusset-to-beam connection

β = distance from the face of the beam flange to the centroid of the gusset-to-column connection

$\bar{\alpha}$ = actual distance from the face of the column flange to the centroid of the gusset-to-beam connection (This term is not required in the new formulation.)

$\bar{\beta}$ = actual distance from the face of the beam flange to the centroid of the gusset-to-column connection (This term is not required in the new formulation.)

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