Improved Flexural Stability Design of I-Section Members in AISC (2005)—A Case Study Comparison to AISC (1989) ASD

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The AISC *Specification for Structural Steel Buildings* (AISC, 2005) provisions for the flexural stability design of steel I-section members have been updated relative to previous Specifications to simplify their logic, organization and application, while also improving their accuracy and generality. White (2004, 2005) and White and Jung (2004) compare the updated AISC provisions to the provisions of the 1999 *Load and Resistance Factor Design (LRFD) Specification for Structural Steel Buildings* (AISC, 2000). White (2004) provides a detailed technical overview of the updated equations, including complete flowcharts of the resistance calculations. White and Jung (2004) and White and Kim (2004) validate the updated equations against more than 760 uniform bending and moment gradient experimental tests. This paper gives a brief overview of the updated provisions, and compares and contrasts their flexural resistance calculations with the corresponding calculations from the previous AISC Allowable Stress Design (ASD) Specification (AISC, 1989). The relative simplicity and accuracy of the AISC (2005) equations is highlighted. The nomenclature in this paper is consistent with AISC (2005) unless noted otherwise.

The next section outlines the key concepts associated with the updated AISC provisions. This is followed by a review of key concepts employed in the flexural resistance equations of the prior AISC (1989) ASD Specification. The paper closes with a case study comparison of the updated and the prior flexural resistance calculations.

KEY CONCEPTS WITHIN THE AISC (2005) PROVISIONS

All of the I-section member flexural stability resistance equations in AISC (2005) can be explained using the basic illustration shown in Figure 1. The flexural resistances in these Specifications involve two independent stability limit state calculations, one for flange local buckling (FLB) and the other for lateral-torsional buckling (LTB). The resistance equations for both FLB and LTB are based consistently on the logic of identifying the two anchor points shown in Figure 1 for the case of uniform major-axis bending. Anchor Point 1 is located at the effective length $KL_b = L_p$ for LTB, or the flange slenderness, $\lambda_{fc} = b_{fc}/2t_{fc} = \lambda_{pf}$ for FLB, corresponding to development of the maximum potential flexural resistance. This resistance is labeled in the figure as M_{max} (in terms of the bending moment) or F_{max} (in terms of the corresponding compression flange flexural stress), where $M_{max} = M_p$ for members with a compact web. However, it is generally less than M_p for members with noncompact or slender webs. Anchor Point 2 is located at the smallest effective length, $KL_b = L_r$, or flange slenderness $b_{fc}/2t_{fc} = \lambda_{rf}$, for which the LTB or FLB resistances are governed by elastic buckling. The ordinate of Anchor Point 2 is taken (in terms of the bending moment) as $R_{pg}F_LS_{xc} = 0.7R_{pg}F_yS_{xc}$, or $0.7R_{pg}M_{yc}$, for most I-shapes, where M_{yc} is the nominal yield moment

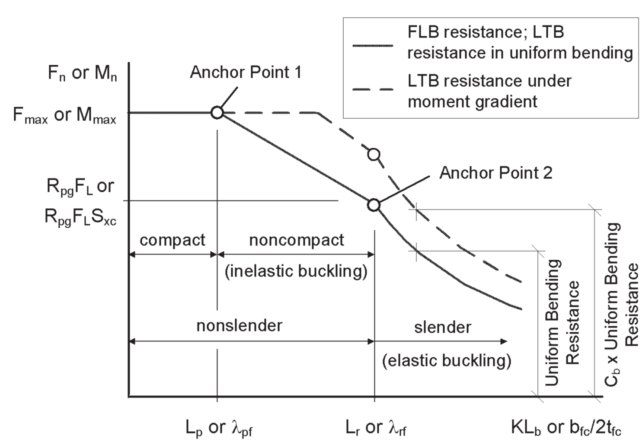

Fig. 1. Basic form of AISC (2005) FLB and LTB resistance equations.

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associated with the compression flange and R_{pg} is the web bend buckling strength reduction factor, equal to 1.0 for sections with compact or noncompact webs. The inelastic buckling resistance is expressed simply as a line between these two anchor points. For $KL_b > L_r$ or $b_{fc}/2t_{fc} > \lambda_{rf}$, the nominal resistance is defined explicitly as the theoretical elastic buckling moment or flange stress. The basic format shown in Figure 1, adopted largely from AISC (2000), greatly facilitates the definition of simple yet comprehensive flexural resistance equations.

For unbraced lengths subjected to moment gradient, AISC (2005) modifies the calculated LTB resistance by the moment gradient factor, C_b, as illustrated by the dashed line in Figure 1. In these cases, the uniform bending elastic and inelastic LTB strengths are scaled by C_b, with the exception that the resistance is capped by F_{max} or M_{max}. The calculated FLB resistance for moment gradient cases is the same as that for uniform bending, neglecting the relatively minor influence of moment gradient effects on the FLB limit state.

The coordinates of the anchor points shown in Figure 1 are (L_p, M_{max}) and (L_r, $R_{pg}F_LS_{xc}$) for LTB, and (λ_{pf}, M_{max}) and (λ_{rf}, $R_{pg}F_LS_{xc}$) for FLB, written in terms of the major-axis bending moment. The specific terms associated with these anchor points are discussed in detail below. Also, since the noncompact bracing limit, L_r, and the noncompact compression flange slenderness limit, λ_{rf}, are associated with the theoretical elastic buckling equations, the base AISC (2005) elastic buckling equations are presented.

Compact Bracing Limit, L_p

AISC (2005) specifies the equation

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_{yc}}} \tag{1}$$

as the compact bracing limit for doubly-symmetric compact-web members, while it specifies

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_{yc}}} \tag{2}$$

for all other I-section member types. Equation 1 uses r_y, the radius of gyration of the full cross-section about its minor axis, whereas Equation 2 uses r_t, which is essentially the radius of gyration of the compression flange plus one-third of the area of the web in compression. Equation 1 is taken from AISC (2000) whereas Equation 2 is based on the assessment of experimental data in White and Jung (2004) and White and Kim (2004). White and Jung (2004) show that Equation 2 gives the best correlation with experimental data for all types of I-section members. Equation 1 gives a larger value for the compact bracing limit; however, the largest increase in the LTB flexural resistance associated with this more liberal equation is never more than approximately 6%. The use of Equation 1 rather than Equation 2 for doubly-symmetric compact-web members is based on: (a) the LTB resistance is relatively insensitive to the unbraced length in the vicinity of Anchor Point 1; and (b) some additional restraint against LTB typically exists, particularly for these member types, beyond that typically considered in design.

Equation 2 is generally more restrictive than the prior AISC LRFD (AISC, 2000) and ASD (AISC, 1989) compact bracing limits for slender-web members. However, it is more liberal than the L_p equation recommended in the original LRFD research by Cooper, Galambos, and Ravindra (1978) for these member types. The tests considered by White and Jung (2004) show that in cases where the end restraint from adjacent unbraced segments is small (for example, when the adjacent unbraced lengths are also subjected to uniform bending), the true compact bracing limit is smaller than the AISC (2000) values. If Equation 2 is substituted into the original Column Research Council (CRC) based expression suggested by Basler and Thurlimann (1961) for the LTB resistance of slender-web members, a strength of $0.97M_y$ is obtained for members with R_{pg} equal to one. If L_p/r_t from Equation 2 is substituted as an equivalent slenderness ratio into the AISC (2005, 2000) inelastic column strength formula, a resistance of $0.95F_y$ is obtained. The conservatism of Equation 2 relative to AISC (1989) and AISC (2000) is offset somewhat by the larger value of F_L for slender-web members in AISC (2005), as discussed below.

Compact Flange Slenderness Limit, λ_{pf}

AISC (2005) defines the compact-flange slenderness limit by the equation

$$\lambda_{pf} = 0.38 \sqrt{\frac{E}{F_{yc}}} \tag{3}$$

for all types of I-section members. This equation is identical to the compact-flange limit in AISC (1989) and AISC (2000) and is based largely on the original research by Lukey, Smith, Hosain, and Adams (1969) as well as the subsequent studies by Johnson (1985).

Maximum Potential Flexural Resistance, M_{max}

The above equations define the extent of the plateau associated with the maximum flexural resistance, M_{max} (see Figure 1). As noted previously, for members with compact webs, M_{max} is equal to the cross-section plastic moment capacity, M_p. However, for members with noncompact or slender webs, the ordinate of Anchor Point 1, M_{max}, decreases as a function of the web slenderness, h_c/t_w, whereas the abscissa, L_p or λ_{pf}, is independent of the web slenderness. For noncompact-web members, M_{max} decreases linearly as a function of h_c/t_w between the compact-web limit, λ_{pw}, and the noncompact-web limit, λ_{rw}, as shown by Figure 2. The noncompact-web limit is given by the equation

$$\lambda_{rw} = 5.7\sqrt{\frac{E}{F_{yc}}} \tag{4}$$

For doubly-symmetric members, this equation is the same as the limit on h/t_w beyond which AISC (1989) classifies the member as a plate girder. Also, this equation is the noncompact-web limit specified in AISC (2000) in the case of doubly-symmetric I-sections. For singly-symmetric I-sections, AISC (2005) relates the noncompact-web limit to h_c/t_w rather than h/t_w. This is consistent with the handling of the web slenderness in the web bend buckling strength reduction factor, R_{PG}, in AISC (2000). White (2004) addresses the accuracy of this approximation.

AISC (2005) specifies the compact web limit as

$$\lambda_{pw} = 3.76\sqrt{\frac{E}{F_{yc}}} \tag{5a}$$

for doubly-symmetric I-section members, whereas it gives the equation

$$\lambda_{pw} = \frac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_{yc}}}}{\left(0.54\frac{M_p}{M_y} - 0.09\right)^2} \leq \lambda_{rw} \tag{5b}$$

for singly-symmetric I-section members. Equation 5b accounts for the larger demands on the web required to develop the cross-section plastic moment capacity in singly-symmetric I-sections. The term h_c/h_p in the numerator converts Equation 5b from its fundamental form associated with the plastic depth of the web in compression, h_p, to a form associated with the elastic depth of the web in compression, h_c. This is necessary so that a consistent web slenderness parameter, h_c/t_w, may be employed for the linear interpolation between the anchor points, (λ_{pw}, M_p) and (λ_{rw}, M_{yc}), in Figure 2. For a doubly-symmetric I-section with $h_c/h_p = 1.0$ and an assumed $M_p/M_y = 1.12$, Equation 5b reduces to Equation 5a.

For $h_c/t_w > \lambda_{rw}$, the web is defined as slender and M_{max} is given by the expression $R_{pg}M_{yc}$ as shown in Figure 2. In this case, the term $R_{pg} < 1$ accounts for the reduction in M_{max} due to the shedding of flexural stresses to the compression flange associated with the post-bend buckling response of the web. The web bend buckling strength reduction factor is written as

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - \lambda_{rw}\right) \leq 1.0 \tag{6}$$

where

$a_w = h_c t_w / A_{fc}$

This equation is a simplification of the equation for R_{PG} in AISC (2000) by using the web noncompact slenderness limit, λ_{rw}, given by Equation 4 rather than Equation 4 with F_{yc} replaced by a smaller compression flange stress accounting for the influence of LTB or FLB. The noncompact web limit, λ_{rw}, is the web slenderness, h_c/t_w, at which local web bend buckling starts theoretically at a compression flange major-axis bending stress equal to F_{yc}. Equation 6 is the original more accurate form proposed by Basler and Thurlimann (1961) with the above simplification. AISC ASD (1989) replaces the fraction involving the term a_w with $0.0005A_w/A_{fc}$ based on Basler and Thurlimann's research. This form is conservative for $A_w/A_{fc} < 2$, but gives unreasonable results for larger A_w/A_{fc} values. For noncompact and compact webs, R_{pg} is equal to one.

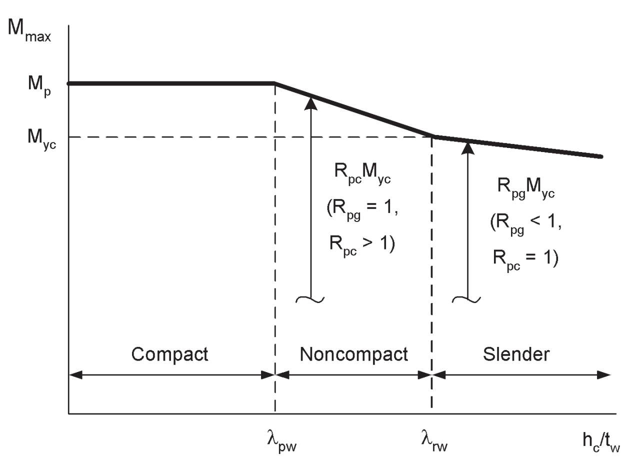

Fig. 2. M_{max} *versus the web slenderness* h_c/t_w.

Compression Flange Stress Corresponding to the Nominal Onset of Inelastic Buckling, F_L

AISC (2005) specifies $F_L = 0.7F_{yc}$ with the exception of highly monosymmetric compact-web and noncompact-web cross-sections with the larger flange in compression, where the neutral axis is so close to the compression flange that nominal tension flange yielding occurs prior to reaching a stress of $0.7F_{yc}$ at the compression flange. To address this latter case, AISC (2005) specifies

$$F_L = F_{yt}\frac{S_{xt}}{S_{xc}} \geq 0.5F_{yc} \tag{7}$$

when S_{xt}/S_{xc} < 0.7. The product $F_{yt}S_{xt}$ in this equation is the moment corresponding to nominal yielding at the tension flange. This value, divided by the section modulus to the compression flange, S_{xc}, is the compression flange stress corresponding to the onset of nominal yielding at the tension flange.

For slender-web members, F_L is taken equal to $0.7F_{yc}$ for all cases, including singly-symmetric sections with S_{xt}/S_{xc} < 0.7. This is because AISC (2005) specifies a separate tension flange yielding (TFY) limit state check for slender-web members. That is, the tension flange stress is limited to F_{yt} for these member types. AISC (2005) also specifies a limit state check associated with tension flange yielding for compact- and noncompact-web singly-symmetric I-section members with $S_{xt} < S_{xc}$. However, the TFY resistance for compact- and noncompact-web sections is generally larger than $M_{yt} = F_{yt}S_{xt}$. It varies linearly from M_{yt} to M_p as the web slenderness h_c/t_w varies from λ_{rw} to λ_{pw}. Therefore, Equation 7 is necessary for noncompact- and compact-web members, to avoid significant violation of the assumption of elastic member behavior when using the elastic LTB or FLB equations of the Specification.

The limit $F_L = 0.7F_{yc}$ is based on LTB and FLB experimental test data (White and Jung, 2004; White and Kim, 2004). This is a significant liberalization relative to the implicit use of $F_L = 0.5F_{yc}$ for slender-web members in prior Specifications.

Elastic LTB Stress, F_{cr}

The AISC (2005) elastic LTB resistance is based on a single equation applicable to all types of I-section members. This equation gives the exact beam-theory solution for LTB of doubly-symmetric I-section members, and it gives an accurate to somewhat conservative approximation for singly-symmetric noncomposite members and composite members in negative bending (White and Jung, 2003a and b; White, 2004). This equation may be written in terms of the compression flange flexural stress as

$$F_{cr} = C_b\frac{\pi^2 E}{(KL_b/r_t)^2}\sqrt{1+\frac{0.078}{X^2}(KL_b/r_t)^2} \tag{8}$$

where

$$X^2 = \frac{S_{xc}h_o}{J} \tag{9}$$

r_t = approximately the radius of gyration of the compression flange plus one-third of the area of the web in compression

S_{xc} = elastic section modulus to the compression flange

h_o = distance between the centroids of the flange elements

J = St. Venant torsion constant

Equation 9 is a simple ratio of the bending and torsional efficiencies of the cross-section. For a doubly-symmetric I-section, $X^2 \cong 2I_x/J$. This parameter ranges from 13 to 2,500 for the complete set of ASTM A6 W-shapes.

The radius of gyration, r_t, may be calculated exactly as

$$r_t = \frac{(I_yC_w)^{1/4}}{S_x^{1/2}} \tag{10a}$$

for doubly-symmetric I-sections (White and Jung, 2003a). AISC (2005) gives this equation, but refers to the corresponding radius of gyration as r_{ts}, to avoid its potential erroneous use for singly-symmetric I-section members. Alternately, r_t may be calculated generally for any rectangular flange I-section as

where

$$r_t \cong \frac{b_{fc}}{\sqrt{12\left(\frac{h_o}{d}+\frac{1}{6}a_w\frac{h^2}{h_o d}\left(1+12\frac{A_{fillet}}{h_c t_w}\right)\right)}} \tag{10b}$$

h_o = distance between the compression and tension flange centroids

d = total depth of the member

h = depth of the web

a_w = h_ct_w/A_{fc}

A_{fillet} = area of each of the web-to-flange fillets (White and Jung, 2003a)*

If one assumes $d \cong h_o \cong h$ and $A_{fillet} \cong 0$, Equation 10b becomes

$$r_t \cong \frac{b_{fc}}{\sqrt{12\left(1+\frac{1}{6}a_w\right)}} \tag{10c}$$

*A_{fillet} is commonly taken equal to zero for welded I-section members.

which is precisely the equation for the radius of gyration of the compression flange plus one-third of the depth of the web in compression. Equation 10b gives results that are within 1% of the exact Equation 10a for all rolled I-sections. Due to compensating effects in the approximation of Equation 10b by Equation 10c, Equation 10c also tends to give an accurate but slightly conservative approximation of Equation 10a.

For column-type I-sections with $h/b_{fc} \cong 1$, h/t_w less than about 50 and compact flanges, the second term under the radical in Equation 8 tends to be significantly larger than 1.0. Thus it would be quite uneconomical to discount this major contribution to the resistance to obtain a simpler form for Equation 8. However, in situations involving beam- or girder-type I-sections with h/b_{fc} greater than about 2.0 and $b_{fc}/2t_{fc}$ near the compact-flange limit, λ_{pf}, or larger, the contribution from the second term in Equation 8 is relatively small (White and Jung, 2003a). For slender-web members, the contribution from this term is neglected altogether, due to the reduction in the effective St. Venant torsional stiffness associated with web distortional flexibility (in other words, the deformation of the web into an S-shape upon twisting of the cross-section, and the corresponding reduction in the twist rotation of the flanges) (White and Jung, 2003c). In this case, Equation 8 reduces to the form

$$F_{cr} = C_b \frac{\pi^2 E}{(KL_b / r_t)^2} \tag{11}$$

used traditionally by AISC for slender-web members. Equation 11 is multiplied by the bend buckling strength reduction factor, R_{pg}, to obtain the elastic LTB flexural resistance for slender-web members in terms of the compression flange stress.

Noncompact Bracing Limit, L_r

The noncompact bracing limit, L_r, is obtained by equating the base elastic LTB resistance for uniform bending, $C_b = 1$, to the compression flange stress at the nominal onset of yielding, F_L. Equation 8 results in a more succinct expression for the noncompact lateral brace spacing than in AISC (2000),

$$L_r = \frac{1.95 r_t}{X} \frac{E}{F_L} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{F_L}{E}\right)^2 X^4}} \tag{12a}$$

applicable for all types of compact- and noncompact-web I-section members, whereas Equation 11 gives (White and Jung, 2003a),

$$L_r = \pi r_t \sqrt{\frac{E}{F_L}} \tag{12b}$$

Appendix F of AISC (2000) does not provide an L_r equation for compact- and noncompact-web singly-symmetric I-section members. Engineers often have assumed that this noncompact bracing limit must be calculated iteratively. White and Jung (2003b) give a closed-form alternative expression to Equation 12a for members with these section types, based on the rigorous application of thin-walled open-section beam theory. Unfortunately, this equation is significantly longer than Equation 12a. Also, due to the larger effects of web distortion in singly-symmetric members, the rigorous beam-theory equation does not necessarily give a better representation of the physical buckling resistance (White and Jung, 2003c).

Elastic FLB Stress, F_{cr}

AISC (2005) defines the base elastic FLB resistance by the equation

$$F_{cr} = \frac{0.9 E k_c}{\left(\frac{b_{fc}}{2t_{fc}}\right)^2} \tag{13}$$

where the parameter k_c is the flange local buckling coefficient, taken as $k_c = 0.76$ for rolled I-sections as in AISC (2000), and defined as

$$k_c = \frac{4}{\sqrt{h/t_w}}, \quad 0.35 \le k_c \le 0.76 \tag{14}$$

for other general I-shapes. Equation 13 is multiplied by the bend buckling strength reduction factor, R_{pg}, to obtain the elastic FLB resistance for slender web members. Equation 13 is the exact analytical expression for local plate buckling, given an exact calculation of the local buckling coefficient, k_c. Equation 14 defines a transition from a maximum k_c of 0.76 (corresponding to the assumed k_c for rolled I-shapes) to a minimum value of 0.35. The FLB coefficient for simply-supported edge conditions at the web-flange juncture is $k_c = 0.43$. Therefore, smaller values of k_c indicate that the web is tending to destabilize the flange. A value of h/t_w less than 28 is required to obtain $k_c = 0.76$, whereas k_c is equal to 0.35 for $h/t_w \ge 131$. Equation 14 was developed originally by equating the results from the AISC 1993 LRFD *Specification for Structural Steel Buildings* (AISC, 1993) resistance equations to measured experimental strengths for a number of tests in which the flexural resistance was governed by FLB, then back-solving for k_c (J.A. Yura, unpublished notes, 1992). The data used in these developments was predominantly from Johnson (1985). White and Jung (2004) and White and Kim (2004) discuss the correlation of the AASHTO (2004) and AISC (2005) equations with a larger updated set of experimental test results. Equation 14 may be considered as a simple but reasonable approximate lower-bound value for the FLB coefficient.

Noncompact Flange Slenderness Limit, λ_{rf}

Similar to the calculation of L_r, the noncompact flange slenderness limit, λ_{rf}, is obtained by equating the elastic FLB stress given by Equations 13 and 14 to the compression flange stress at the nominal onset of yielding, F_L. The resulting equation is

$$\lambda_{rf} = 0.95\sqrt{k_c E / F_L} \tag{15}$$

PRACTICAL CALCULATION OF EFFECTIVE LENGTH FOR LTB WHEN $KL_b < L_b$

AISC (2005) uses the unbraced length L_b with an implicit $K = 1$ in its presentation of Equations 8 and 11. This is based on the fact that in many practical situations, the benefits of continuity with adjacent unbraced lengths are rather minor. However, particularly for longer unbraced lengths, the influence of beam continuity on the lateral-torsional buckling resistance can be substantial. To account for this attribute of the behavior, the commentary of AISC (2005) recommends a simple design-oriented method for calculation of the elastic LTB effective length, KL_b. The recommended procedure was developed first by Nethercot and Trahair (1976) and is discussed in Galambos (1998). The Nethercot and Trahair (1976) procedure starts with the calculation of buckling resistances based on the actual unbraced length (in other words, using $K = 1$), and it uses the AISC sidesway-inhibited alignment charts with equivalent G values corresponding to the LTB behavior for the calculation of $K \leq 1$ in critical unbraced lengths. In cases where the adjacent unbraced segments have the same length as the segment under consideration and all three segments are subjected to the same uniform bending, the Nethercot and Trahair procedure gives $K = 1$. However, for other cases, the calculated K value for the critical unbraced length can be significantly smaller than 1. White and Jung (2004) and White and Kim (2004) utilize the Nethercot and Trahair (1976) method for calculation of K in their assessment of the AISC (2005) flexural resistance equations relative to extensive experimental test results. They note that when M_n is calculated simply based on $K = 1$, the LTB predictions are often substantially more conservative, leading to a substantially higher implicit reliability index β, although the dispersion in the test to the predicted flexural resistance ratio, M_{test}/M_n, is also substantially increased.

SAMPLE COMPARISONS TO EXPERIMENTAL TEST RESULTS

Figure 3 illustrates the correlation of the AISC (2005) equations with one set of focused experimental LTB tests on compact rolled members subjected to uniform major-axis bending. One can observe that the AISC (2005) equations overestimate the test resistances by a minor amount at smaller unbraced lengths. The AISC (2005) Specification assumes that some minor additional lateral restraint typically exists that compensates for this slightly liberal representation of the test data. The reader should note that the calculated K

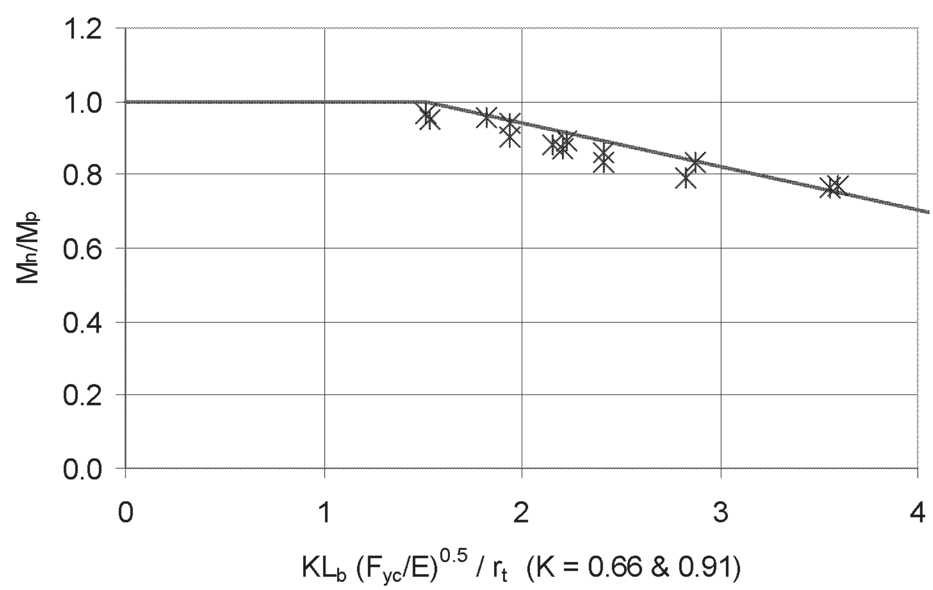

Fig. 3. Comparison of AISC (2005) predictions to compact rolled I-section member test results from Dux and Kitipornchai (1983) and Wong-Chung and Kitipornchai (1987) ($b_f/2t_f$ = 6.6 to 7.0, h/t_w = 34 to 36, h/b_f = 1.6, F_{yc} = 41.3 to 42.5 ksi).

values are 0.66 and 0.91 for the members considered in these tests. If $K = 1$ is used, the AISC (2005) equations are substantially conservative for these tests. The unbraced length is expressed in a normalized form in Figure 3, by multiplying the effective length KL_b by $(F_{yc}/E)^{0.5}/r_t$. Figure 4 shows a comparison of the calculated and test resistances for several doubly-symmetric compact-flange noncompact-web welded members. One can observe that again the AISC (2005) equations are slightly liberal relative to the test data for smaller unbraced lengths. Based on the full data set of 320 uniform bending flexural tests considered by White and Jung (2004), the use of Equation 2 for L_p gives mean values for M_{test}/M_n close to 1.0 for all types of I-section members throughout the inelastic LTB region of the response.

The reader should note how well the simple inelastic LTB equation (the line between Anchor Points 1 and 2 in Figure 1) represents the experimental data in Figures 3 and 4. The linear curve between Anchor Points 1 and 2 is clearly better than a multi-plateau representation with discontinuities in the flexural resistance at certain unbraced lengths, such as the prediction from the AISC (1989) ASD provisions. The recommended L_p and F_L equations are based on a total of more than 320 uniform bending and more than 440 moment-gradient experimental tests on rolled and welded I-section members (White and Jung, 2004; White and Kim, 2004). These references provide a detailed analysis of the professional bias factor M_{test}/M_n for the above data sets, and corresponding estimates of the notional reliability for statically determinate beams.

KEY CONCEPTS IN THE AISC (1989) ASD PROVISIONS

As noted in the introduction, a major focus of this paper is the illustration of the simplicity and accuracy of the AISC (2005) provisions relative to the prior AISC (1989) Specification. Therefore, it is useful to review a few of the key concepts associated with the flexural resistance calculations in AISC (1989). For purposes of comparison to AISC (2005), the AISC (1989) equations are written here in terms of the base nominal moment resistance. This is accomplished by multiplying the allowable stresses from AISC (1989) by the underlying factor of safety, 1.67 = 1/0.6, and by the section modulus to the flange under consideration, S_{xc} or S_{xt}.

Figure 5 illustrates the general approach for determining the LTB resistance in AISC (1989). For $L_b \le L_c$, the base nominal resistance is taken conservatively as $1.1M_y$ if the compression flange and the web are also compact, where 1.1 is the implicitly assumed shape factor M_p/M_y. The bracing limit L_c is the AISC ASD equivalent of L_p in AISC (2005) for members with a compact web and a compact compression flange. However, for $L_c < L_b \le L_u$, the base nominal resistance is taken as M_y. This results in a 10% discontinuity in the flexural resistance at $L_b = L_c$. Obviously, no such discontinuity exists in the physical flexural resistance.

For $L_b > L_u$, the nominal LTB resistance is smaller than M_y. AISC (1989) defines the LTB resistance in this range of the unbraced length using the traditional double formula approach. In the double formula approach, the elastic LTB equation is simplified by neglecting either the St. Venant

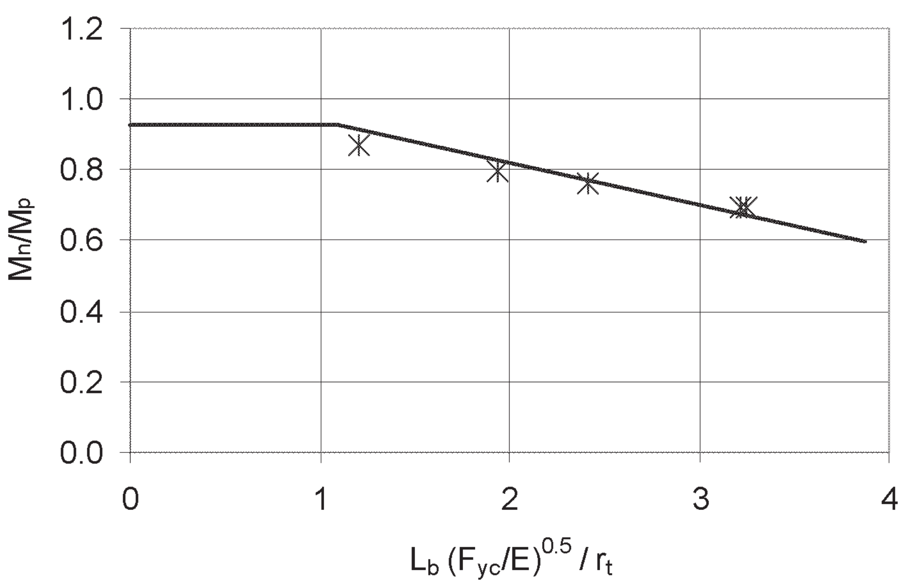

Fig. 4. Comparison of AISC (2005) predictions to doubly-symmetric compact-flange noncompact-web welded member test results from Richter (1998) ($b_f/2t_f$ = 8.0 to 8.1, h/t_w = 110, h/b_f = 3.6, F_{yc} = 48.4 ksi).

torsional stiffness (GJ) or the nonuniform torsional stiffness (EC_w). The larger of the two resulting elastic LTB resistances is taken as a conservative estimate of the physical elastic LTB strength. These two elastic LTB equations are labeled as Equations F1-7 and F1-8, respectively, in AISC (1989). In addition, AISC (1989) gives an inelastic LTB resistance equation, Equation F1-6, for the case where GJ is neglected. However, AISC (1989) uses only the resulting elastic LTB Equation F1-8 for the case where EC_w is assumed equal to zero. The LTB resistance is defined as the larger of the values from Equations F1-6 and F1-8 for smaller unbraced lengths where the inelastic LTB Equation F1-6 applies. Figure 5 shows one example solution for the complete LTB resistance of a compact I-section member. In this example, Equation F1-8 governs for $L_b \cong L_u$, Equation F1-6 governs for intermediate values of the unbraced length, Equation F1-8 governs for large L_b, and Equation F1-7 governs for a short range of the unbraced length just prior to the transition to Equation F1-8 for larger unbraced lengths. However, depending on the specifics of the cross-section, Equation F1-8 may tend to give larger or smaller strengths relative to the combination of Equations F1-6 and F1-7. For shallow cross-sections (small h/b_{fc}) with stocky plate elements, Equation F1-8 tends to give the larger governing resistance whereas for deep cross-sections (large h/b_{fc}) and thin plate elements, Equations F1-6 and F1-7 tend to give the larger governing resistance except at very large values of L_b, where Equation F1-8 may still govern.

For members in which the web slenderness, h/t_w, violates the AISC ASD compactness limit, which is equivalent to Equation 5a, AISC (1989) uses the same LTB resistance equations as shown in Figure 5 with the exception that the maximum flexural resistance is limited to M_y for h/t_w smaller than λ_{rw} from Equation 4 and to $R_{PG} M_y$ when h/t_w is larger than λ_{rw} such that the member is classified as a plate girder.

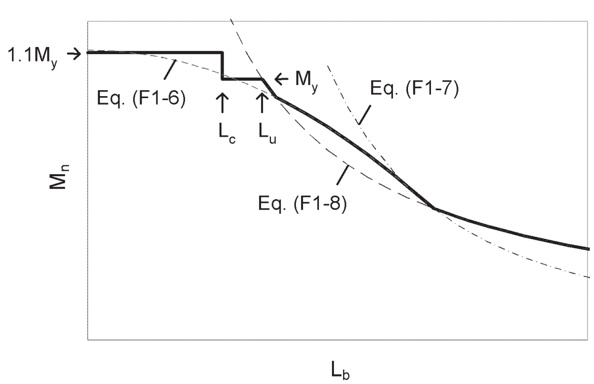

Fig. 5. Basic form of AISC (1989) LTB resistance equations, compact I-section members in uniform bending.

When expressed in terms of the nominal moment resistance, the base AISC (1989) LTB equations may be written as follows:

When $1.88\sqrt{\dfrac{EC_b}{F_{yc}}} \le \dfrac{L_b}{r_t} \le 4.19\sqrt{\dfrac{EC_b}{F_{yc}}}$

$$M_n = \left[1.11 - 0.0316\frac{F_{yc}}{E}\frac{1}{C_b}\left(\frac{L_b}{r_t}\right)^2\right] M_{yc} \le M_{yc} \qquad (16)$$

from (F1-6)

When $\dfrac{L_b}{r_t} \ge 4.19\sqrt{\dfrac{EC_b}{F_{yc}}}$

$$M_n = C_b\frac{0.99\pi^2 ES_{xc}}{\left(L_b / r_t\right)^2} \le M_{yc} \text{ from (F1-7)} \qquad (17)$$

For any value of $\dfrac{L_b}{r_t}$

$$M_n = C_b\frac{0.69ES_{xc}}{L_b d / A_{fc}} \le M_{yc} \text{ from (F1-8)} \qquad (18)$$

In addition, the unbraced length limits L_c and L_u may be expressed as

$$L_c = \min\left[0.45 b_f\sqrt{\frac{E}{F_{yc}}},\ \frac{0.69}{d / A_{fc}}\frac{E}{F_{yc}}\right] \qquad (19)$$

and

$$L_u = \max\left[1.88 r_t\sqrt{\frac{C_b E}{F_{yc}}},\ \frac{0.69}{(d / A_{fc})}\frac{C_b E}{F_{yc}}\right] \qquad (20)$$

One should note that AISC (1989) makes no mention of the use of an effective length factor for LTB. As noted previously, the use of $K < 1$ can lead to substantial gains in the economy for cases involving larger unbraced lengths.

For members in which the compression flange slenderness, $b_{fc}/2t_{fc}$, violates Equation 3, AISC (1989) defines the FLB resistance by an independent linear transition equation specified in Chapter F for $b_{fc}/2t_{fc}$ up to the noncompact limit,

$$0.56\sqrt{Ek_c^* / F_{yc}}$$

written in normalized form. This is followed by a second linear transition equation specified in Appendix B for $b_{fc}/2t_{fc}$ up to

$$1.15\sqrt{Ek_c^* / F_{yc}}$$

and by the equivalent of Equation 13, with k_c^* instead of k_c for larger values of the flange slenderness. The term k_c^* in the above expressions denotes the following flange local buckling coefficient in AISC (1989),

$$k_c^* = \frac{4.05}{\left(h/t_w\right)^{0.46}} \quad \text{for } h/t_w > 70\text{, otherwise 1.0} \tag{21}$$

This equation has a significant discontinuity at h/t_w = 70, where k_c^* changes from 0.57 to 1.0. Figure 6 compares the base AISC (1989) FLB resistance to the AISC (2005) resistance for a built-up I-section member with F_{yc} = 50 ksi, an assumed M_p/M_y = 1.16, and h/t_w = 69 or 71. One can observe that the above discontinuity in k_c^* results in a significant discontinuity in the AISC (1989) flexural resistance at h/t_w = 70. The FLB resistance in AISC (1989) is quite liberal for h/t_w = 69 relative to the ASD resistance for h/t_w = 71 as well as the AISC (2005) FLB resistance. However, for practical slenderness values where the flange is only marginally noncompact, the AISC (2005) resistance tends to be larger than that specified in AISC (1989).

CASE STUDY COMPARISONS

Compact Rolled Wide-Flange Members

Figures 7 through 13 illustrate the base nominal flexural resistances in AISC (2005) and AISC (1989) for three representative wide-flange sections: a W36×135, a W14×132 and a W18×55. The first case is a typical beam-type wide-flange section, where the cross-section aspect ratio, d/b_f, is relatively large (d/b_f = 2.97) and the torsional efficiency ratio is relatively high ($X^2 = S_x h_o/J$ = 2,180). Members composed of this type of section tend to be governed by Equations F1-6 and F1-7 in AISC (1989) (see Figure 5). The second case is a representative column-type wide-flange section, where the ratio d/b_f is close to 1.0 (d/b_f = 0.996) and X^2 is relatively small (= 231). Members composed of this type of section tend to be governed by Equation F1-8 in AISC (1989). That is, Equation F1-8 gives a larger resistance than the combination of Equations F1-6 and F1-7. The third case involves an intermediate wide-flange section with d/b_f = 2.40 and X^2 = 750. This section is selected for the case study comparisons because it gives a flexural resistance obtained from Equation F1-8 that is comparable to that obtained from the combination of Equations F1-6 and F1-7 for a wide range of unsupported lengths.

Figures 7, 9, and 11 illustrate the nominal flexural resistances in uniform bending for various members composed of the above sections. For AISC (2005), the resistances are shown both for an assumed K = 1 as well as for K = 0.8. In their assessment of inelastic beams under uniform bending moment, Lay and Galambos (1965) state that "K = 0.80 may be beyond [larger than] anything likely to occur in normal practice." They base this assessment in part on the consideration of beams loaded at the ⅓ span locations, with the center unbraced segment subjected to uniform bending and the outside unbraced lengths subjected to a linear variation in the moment. The Nethercot and Trahair (1976) method gives K = 0.83 for this case. Figures 8, 10, and 12 illustrate the magnitude of the flexural resistances obtained from the three different LTB equations in the AISC ASD Specification (AISC, 1989). The following conclusions may be drawn from the above figures:

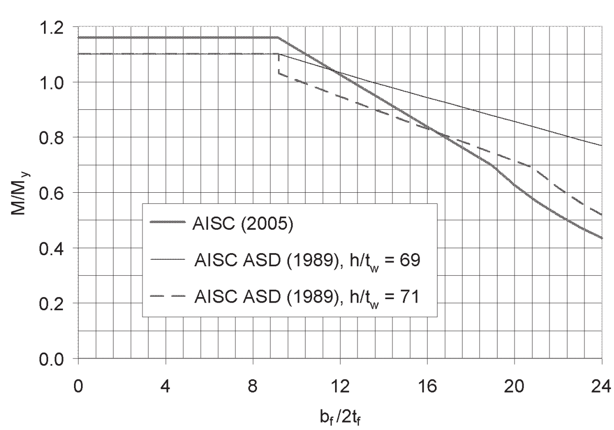

Fig. 6. Comparison of AISC (2005) flange local buckling flexural resistance to the corresponding AISC (1989) ASD resistance, built-up sections with F_{yc} *= 50 ksi, an assumed* M_p/M_y *= 1.16, and* h/t_w *= 69 or 71.*

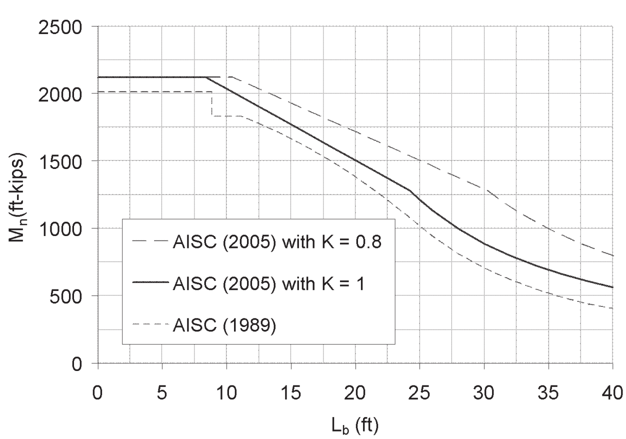

Fig. 7. Comparison of AISC (2005) and AISC ASD (1989) flexural resistances, W36×135 members in uniform bending ($X^2 = S_{xho}/J$ *= 2180),* F_y *= 50 ksi.*

- With the exception of the "corner" in the ASD-LTB resistance for the W14×132, where the elastic Equation F1-8 (Equation 18) gives $M_n = M_{yc}$, the AISC (2005) flexural resistances based on $K = 1$ are more liberal than the AISC (1989) ASD resistances. The nominal capacities realized in AISC (2005) are from zero to 40% larger than the AISC (1989) ASD nominal resistances, with the largest gains occurring for the W18×55 members at intermediate unbraced lengths, and the smallest gains occurring for the column-type W14×132 section members. For the W14×132 case, the AISC (2005) resistance is still up to 14% larger than the AISC (1989) ASD resistance. At the corner where Equation F1-8 gives a larger nominal capacity for the W14×132, the AISC (2005) resistance is only 9% smaller than the AISC (1989) prediction.

  It is not surprising that Equation 18 gives a liberal estimate at $M_n = M_{yc}$, since no inelastic transition curve is considered in AISC ASD for members in which the LTB resistance is governed by Equation F1-8. Johnston (1960) states, "This omission of a transition curve in design practice has proved satisfactory in application to rolled beams with riveted or bolted end framing connections. Such connections provide a partial end restraint about both the xx and yy axes, thus reducing the unsupported span and providing an additional, though undetermined, element of conservatism that tends to offset any lack of consideration of inelastic properties when no transition curve is used." However, for the case of three adjacent equal-length unbraced segments with the same uniform moment in each segment, these additional restraint conditions do not exist.

- The beneficial effects of calculating a $K < 1$ are quite dramatic, particularly for larger unbraced lengths. Even for the easily achievable $K = 0.8$ used here for purposes of illustration, the LTB resistance determined using AISC (2005) is as much as 80% larger than the corresponding resistance using $K = 1$ with AISC (1989). Even for the W14×132 members, where Equation F1-8 gives a reasonably accurate representation of the true elastic LTB resistance for $K = 1$, the elastic LTB resistance using AISC (2005) and $K = 0.8$ is as much as 41% larger than that obtained using $K = 1$ with AISC (1989).

- The AISC (2005) characterization of the uniform bending LTB resistance illustrated in Figure 1 is significantly simpler and more straightforward than the multiple equations and multiple plateaus in AISC (1989) (see Figures 5, 8, 10, and 12). The ASD (AISC, 1989) Equations F1-6, F1-7 and F1-8 were developed during the slide rule era, when the simpler algebraic form of these equations had significant advantages. However, the use of Equations 8 and 12a should not present any problem at the present time (2005), even for manual calculations, particularly since the parameters $X^2 = S_{xc}h_o/J$ and r_t can be tabulated for standard I-shapes or easily calculated for built-up I-shapes. For doubly-symmetric I-section members, Equation 8 is a particularly useful and understandable form for the elastic LTB resistance, expressed in terms of the compression flange major-axis bending stress. All of the variables in this equation are well known in terms of their physical significance, and are readily available or can be easily calculated during the design process. Equation 8 shows that the fundamental elastic LTB resistance is simply a function of the elastic modulus, E, the LTB slenderness, L_b/r_t, the torsional efficiency ratio, $X^2 = S_{xc}h_o/J$, and the moment gradient modifier, C_b.

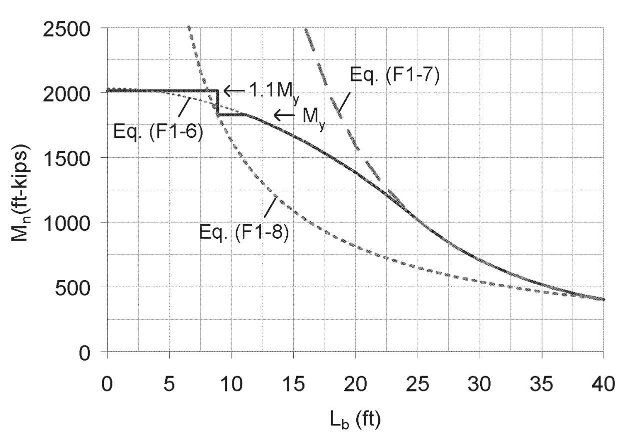

Fig. 8. AISC (1989) ASD flexural resistance equations, W36×135 members in uniform bending, $F_y = 50$ *ksi.*

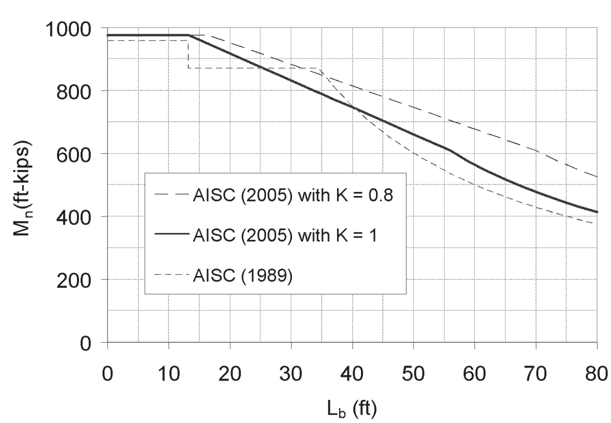

Fig. 9. Comparison of AISC (2005) and AISC ASD (1989) flexural resistances, W14×132 members in uniform bending ($X^2 = S_{xho}/J = 231$*),* $F_y = 50$ *ksi.*

Other interesting results are obtained if one compares the AISC (2005) and AISC (1989) estimates for members composed of a heavier Group 4 or 5 rolled column shape. These results have some similarity to those shown for the W14×132 in Figures 9 and 10. In particular, due to the large value of J relative to $S_{xc}h_o \cong I_x/2$ and the correspondingly small value for X in Equations 8 and 11, column-type I-section members are able to develop large compression flange stresses at relatively large unbraced lengths (see Figure 9). However, there are some differences compared to the results shown here for the intermediate weight W14×132 column section, mainly:

1. The shape factor M_p/M_y for heavy column-type I-sections is much larger than the implicit value of 1.1 assumed in AISC ASD. This raises the AISC (2005) M_{max} or F_{max} relative to the AISC ASD maximum potential resistances, and can lead to larger AISC (2005) strengths even at the knee in the AISC (1989) LTB resistance.

2. The resistances given by the AISC (2005) Equation 8 can be much larger than those given by the AISC (1989) Equation F1-8. For the heaviest W14×808 wide-flange section ($X^2 = 13$), the AISC (2005) elastic LTB resistance is uniformly 1.54 times the corresponding AISC (1989) resistance using $K = 1$. For cases with $K = 0.8$, the AISC (2005) LTB resistance is up to 78% larger.

Figure 13 compares the AISC (2005) and AISC (1989) LTB resistances for a range of W18×55 members and a

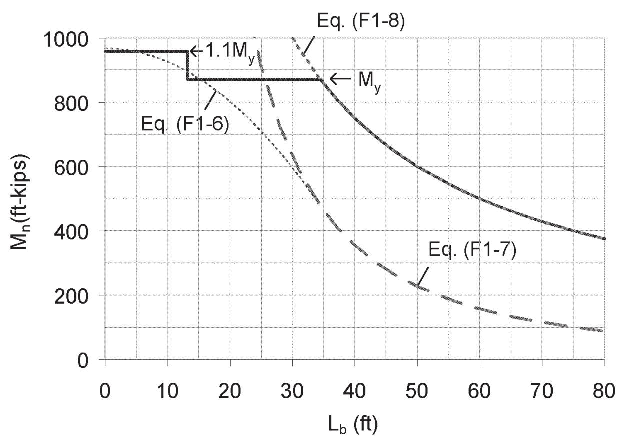

Fig. 10. AISC (1989) ASD flexural resistance equations, W14×132 members in uniform bending, F_y *= 50 ksi.*

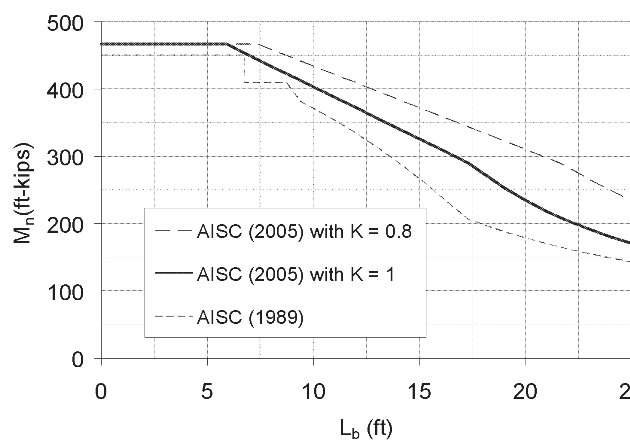

Fig. 11. Comparison of AISC (2005) and AISC (1989) ASD flexural resistances, W18×55 members in uniform bending ($X^2 = S_{xho}/J$ *= 750),* F_y *= 50 ksi.*

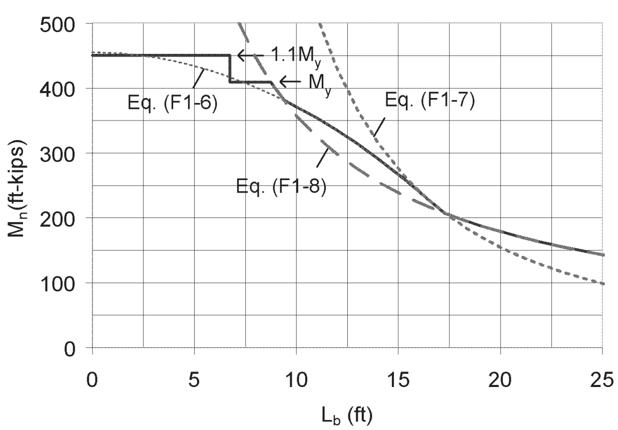

Fig. 12. AISC (1989) ASD flexural resistance equations, W18×55 members in uniform bending, F_y *= 50 ksi.*

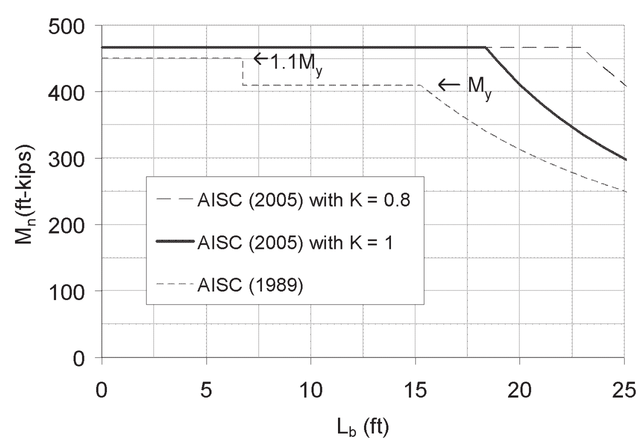

Fig. 13. Comparison of AISC (2005) and AISC (1989) ASD flexural resistances, W18×55 members with C_b *= 1.75,* F_y *= 50 ksi.*

typical moment gradient case involving $C_b = 1.75$. One can observe from this figure that AISC (2005) offers substantial benefits for $L_b > L_c$, since AISC (1989) conservatively reduces its base nominal resistance to M_y for unbraced lengths larger than L_c. Also, AISC (1989) incorporates the C_b modifier into the inelastic LTB Equation F1-6 in the fashion shown in Equation 16, whereas AISC (2005) simply scales the inelastic buckling resistance by C_b but with a cap of M_{max} ($= M_p$ for this example) on the resistance. The experimental test results collected by White and Kim (2004) clearly justify the use of $M_n = M_p$ for compact I-section members for relatively large unbraced lengths such as those shown in the figure. Gains as large as 72% are obtained above the AISC (1989) resistance based on $C_b = 1.75$ and $K = 1$ for the above W18×55 members.

Also, AISC (1989) ASD uses the same LTB resistance equations regardless of the web slenderness. The noncompact web slenderness only influences the plateau of the ASD resistance (by disallowing the use of $1.1M_{yc}$). In AISC (2005), the inelastic LTB transition curve is influenced by the corresponding maximum potential moment level, M_{max}, as shown in Figure 1, rather than M_{max} simply acting as a cap on an independently calculated LTB resistance. White and Jung (2004) show that this leads to an improved characterization of the inelastic LTB strengths.

Similar to the previous W18×55 example, Figure 14 shows that for $C_b = 1.75$, AISC (2005) provides a further liberalization of the computed resistance relative to AISC (1989). This is due to the different manner in which AISC (2005) utilizes C_b.

Singly-Symmetric Compact-Flange Noncompact-Web Example

Figures 14 and 15 compare the AISC (2005) and AISC (1989) flexural resistances for a representative singly-symmetric compact-flange noncompact-web I-section with the larger flange in compression, and show the corresponding relationships between the AISC (1989) flexural resistance equations. The ratio S_{xt}/S_{xc} is equal to 0.866 for this example, and therefore $F_L = 0.7F_{yc}$ in AISC (2005). However, $h_c/t_w = 118$ for this cross-section versus $\lambda_{rw} = 131$ and $\lambda_{pw} = 72$. Therefore, the web is nearly slender. Nevertheless, the AISC (2005) resistance at small unbraced lengths, which is governed by the tension flange yielding (TFY) limit state, is 5.3% larger than M_{yt}. AISC (1989) limits the maximum potential flexural resistance of these types of members to the yield moment, M_{yt}.

Doubly-Symmetric Noncompact-Flange Slender-Web Example

Figure 16 compares the AISC (2005) and AISC (1989) flexural resistances for a representative doubly-symmetric slender-web I-section with a noncompact compression flange. The respective FLB equations govern the maximum resistances in this plot both for the AISC (2005) and the AISC (1989) Specifications. For members with $C_b = 1$, the AISC (1989) ASD Specification tends to give a larger estimated resistance for unbraced lengths close to where its LTB and FLB resistances are the same. This is again due to the fact that the ASD LTB equations are independent of the web slenderness whereas the AISC (2005) inelastic LTB transition curve is influenced by the value of M_{max}, which is equal to $R_{pg} M_{yc} < M_{yc}$ for this slender-web cross-section.

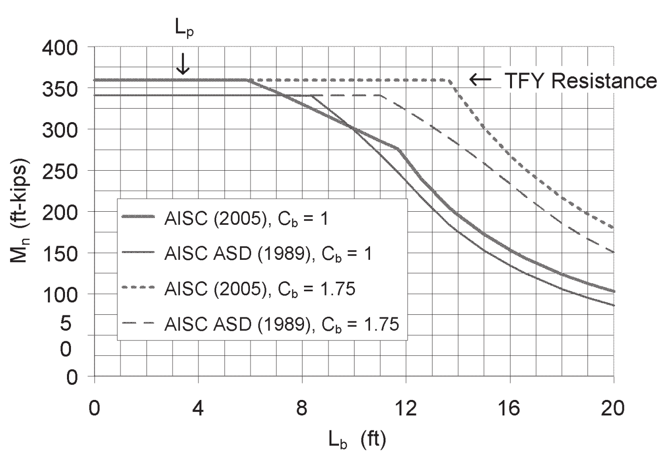

Fig. 14. Comparison of AISC (2005) and AISC (1989) ASD flexural resistances, singly-symmetric compact-flange noncompact-web section members ($D \times t_w = 24$ in. $\times$ 0.1875 in., $b_{fc} \times t_{fc} = 6$ in. $\times$ 0.5 in., $b_{ft} \times t_{ft} = 6$ in. $\times$ 0.375 in., $F_y = 55$ ksi).

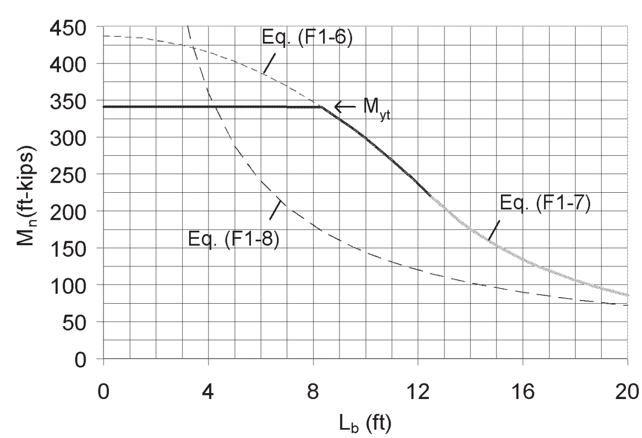

Fig. 15. AISC (1989) ASD flexural resistance equations, singly-symmetric compact-flange noncompact-web section members ($D \times t_w = 24$ in. $\times$ 0.1875 in., $b_{fc} \times t_{fc} = 6$ in. $\times$ 0.5 in., $b_{ft} \times t_{ft} = 6$ in. $\times$ 0.375 in., $F_y = 55$ ksi).

Also, the AISC (2005) elastic LTB resistance is slightly smaller than the AISC ASD LTB resistance for this example. This is due to the conservative use of F_{yc} in the λ_{rw} term of Equation 6. AASHTO (2004) allows M_n (R_{pg} = 1) / S_{xc} in place of F_{yc} in this term to correctly account for the fact that the strength reduction due to web bend buckling is not as great when LTB or FLB occurs at a lower stress level. The AISC (1989) ASD Specification also allows this refinement in the calculation of its bend buckling strength reduction factor. One can observe that this refinement makes little difference in the computed results for this example, even though h_c/t_w = 200. Lastly, as in the previous example, the different handling of the term C_b in AISC (2005) results in a more liberal calculation in the vicinity of the "knee" of the flexural resistances for C_b = 1.75.

CONCLUDING REMARKS

The AISC (2005) flexural resistance provisions represent the synthesis of the best research and practice from more than 50 years of developments. This paper provides a brief outline of these provisions and compares them to the flexural resistance provisions of the prior AISC (1989) ASD Specification. Emphasis is placed on how the AISC (2005) resistances provide improvements both in the accuracy as well as the simplicity of the design calculations. The paper illustrates the results not only for compact rolled I-section members, but also for several other more general cases. More information on the background and usage of the AISC (2005) provisions can be found in the Commentary to the new Specification and the references listed in this paper and therein.

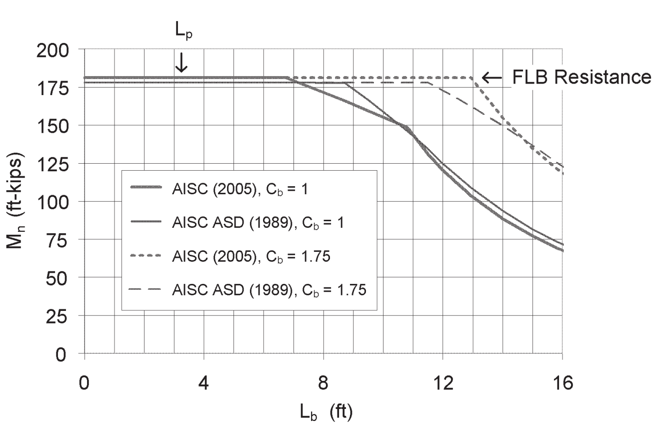

Fig. 16. Comparison of AISC (2005) and AISC (1989) ASD flexural resistances, doubly-symmetric noncompact-flange slender-web section members ($D \times t_w$ = *25 in.* $\times$ *0.125 in.,* $b_f \times t_f$ = *6 in.* $\times$ *0.25 in.,* F_y = *55 ksi).*

ACKNOWLEDGMENTS

The authors acknowledge the members of AISC Task Committee 4 (Louis Geschwindner, Chair) for their many efforts in the updating of the AISC (2005) flexural resistance provisions. The opinions, findings and conclusions expressed in this paper are the authors' and do not necessarily reflect the views of the above individuals, groups and organizations.

REFERENCES

AASHTO (2004), *AASHTO LRFD Bridge Design Specifications*, 3rd Ed., with *2005 Interim Provisions*, American Association of State and Highway Transportation Officials, Washington, DC.

AISC (2005), *Specification for Structural Steel Buildings*, American Institute of Steel Construction, Inc., March 9, Chicago, IL.

AISC (2000), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, December 27, 1999, American Institute of Steel Construction, Inc., Chicago, IL.

AISC (1993), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, December 1, American Institute of Steel Construction, Inc., Chicago, IL.

AISC (1989), *Specification for Structural Steel Buildings: Allowable Stress Design and Plastic Design*, June 1, American Institute of Steel Construction, Inc., Chicago, IL.

Barth, K.E. and White, D.W. (1997), "Finite Element Evaluation of Pier Moment-Rotation Characteristics in Continuous-Span Steel I-Girders," *Engineering Structures*, Vol. 20, No. 8, pp. 761–778.

Basler, K. and Thurlimann, B. (1961), "Strength of Plate Girders in Bending," *Journal of the Structural Division*, ASCE, Vol. 87, No. ST6, pp. 153–181.

Bradford, M.A. and Gao, Z. (1992), "Distortional Buckling Solutions for Continuous Composite Beams," *Journal of Structural Engineering*, ASCE, Vol. 118, No. 1, pp. 73–89.

Climenhaga, J.J. and Johnson, R.P. (1972), "Local Buckling in Continuous Composite Beams," T*he Structural Engineer*, Vol. 50, No. 9, pp. 367–374.

Cooper, P.B., Galambos, T.V. and Ravindra, M.K. (1978), "LRFD Criteria for Plate Girders," *Journal of the Structural Division*, ASCE, Vol. 104, No. ST9, pp. 1389–1407.

Dekker, N.W., Kemp, A.R., and Trinchero, P. (1995), "Factors Influencing the Strength of Continuous Composite Beams in Negative Bending," *Journal of Constructional Steel Research*, Vol. 34, No. 2-3, pp. 161–186.

Dux, P.F. and Kitipornchai, S. (1983), "Inelastic Beam Buckling Experiments," *Journal of Constructional Steel Research*, Vol. 3, No. 1, pp. 3–9.

Galambos, T.V. (1998), *Guide to Stability Design Criteria for Metal Structures*, T.V. Galambos (ed.), Structural Stability Research Council, Wiley Interscience, New York, NY.

Grubb, M.A. and Carskaddan, P.S. (1981), *Autostress Design of Highway Bridges, Phase 3: Moment Rotation Requirements*, Research Laboratory Report, United States Steel Corporation, Monroeville, PA.

Grubb, M.A. and Carskaddan, P.S. (1979), *Autostress Design of Highway Bridges, Phase 3: Initial Moment Rotation Test*, Research Laboratory Report, United States Steel Corporation, Monroeville, PA.

Johnson, D.L. (1985), "An Investigation into the Interaction of Flanges and Webs in Wide Flange Shapes," *Proceedings, Annual Technical Session*, Structural Stability Research Council, pp. 395–405.

Johnston, B.G. (1960), *Guide to Design Criteria for Metal Compression Members*, B.G. Johnston (ed.), Column Research Council, Engineering Foundation, New York, NY.

Kemp, A.R. (1996), "Inelastic Local and Lateral Buckling in Design Codes," *Journal of Structural Engineering*, Vol. 122, No. 4, pp. 374–381.

Lay, M.G. and Galambos, T.V. (1965), "Inelastic Steel Beams Under Uniform Moment," *Journal of the Structural Division*, ASCE, Vol. 91, No. 6, pp. 67–93.

Leon, R.T., Hoffman, J.J., and Staeger, T. (1996), AISC *Design Guide No. 8 Partially Restrained Composite Connections*, American Institute of Steel Construction, Inc., Chicago, IL.

Lukey, A.F., Smith, R.J., Hosain, M.U., and Adams, P.F. (1969), "Experiments on Wide-Flange Beams under Moment Gradient," *Welding Research Council Bulletin, No. 142*, July, pp. 1–19.

Nethercot, D.A and Trahair, N.S. (1976), "Lateral Buckling Approximations for Elastic Beams," *The Structural Engineer*, Vol. 54, No. 6, pp. 197–204.

Oehlers, D.J. and Bradford, M.A. (1999), *Elementary Behavior of Composite Steel and Concrete Structural Members*, Butterworth Heinemann, Oxford.

Richter, J.F. (1998), "Flexural Capacity of Slender Web Plate Girders," M.S. Thesis, University of Texas, Austin, May.

White, D.W. (2004), "Unified Flexural Resistance Equations for Stability Design of Steel I-Section Members–Overview," Structural Engineering, Mechanics and Materials Report No. 24a, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

White, D.W. (2005), "Unified Approach for the Flexural Stability Design of Steel I-Section Members," *Proceedings*, ASCE-SEI Structures Congress.

White, D.W. and Jung, S.-K. (2004), "Unified Flexural Resistance Equations for Stability Design of I-Shaped Members – Uniform Bending Tests," Structural Engineering, Mechanics and Materials Report No. 28, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

White, D.W. and Kim, Y.D. (2004), "Unified Flexural Resistance Equations for Stability Design of I-Shaped Members–Moment Gradient Tests," Structural Engineering, Mechanics and Materials Report No. 29, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

White, D.W. and Jung, S.-K (2003a), "Simplified Lateral-Torsional Buckling Equations for I-and Channel-Section Members," Structural Engineering, Mechanics and Materials Report No. 24a, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, 23 pp.

White, D.W. and Jung, S.-K (2003b), "Simplified Lateral-Torsional Buckling Equations for Singly-Symmetric I-Section Members," Structural Engineering, Mechanics and Materials Report No. 24b, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

White, D.W. and Jung, S.-K. (2003c), "Effect of Web Distortion on the Buckling Strength of Noncomposite Discretely-Braced I-Beams," Report No. 24c, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

White, D.W. and Barth, K.E. (1998), "Strength and Ductility of Compact-Flange I Girders in Negative Bending," *Journal of Constructional Steel Research*, Vol. 45, No. 3, pp. 241–280.

Wong-Chung, A.D. and Kitipornchai, S. (1987), "Partially Braced Inelastic Beam Buckling Experiments," *Journal of Constructional Steel Research*, Vol. 7, No. 3, pp. 189–211.