Direct Analysis and Design Using Amplified First-Order Analysis Part 1: Combined Braced and Gravity Framing Systems

DONALD W. WHITE, ANDREA SUROVEK, and SANG-CHEOL KIM

In 1976 and 1977, LeMessurier published two landmark papers on practical methods of calculating second-order effects in frame structures. LeMessurier addressed the proper calculation of second-order displacements and internal forces in general rectangular framing systems based on first-order elastic analysis. He also addressed the calculation of column buckling loads or effective length factors using the results from first-order analysis. Several important facts are emphasized in LeMessurier's papers:

1. In certain situations, braced-frame structures can have substantial second-order effects.
2. The design of girders in moment-frame systems must account for second-order moment amplification.
3. Control of drift does not necessarily prevent large second-order effects.
4. In general, second-order effects should be considered both in the assessment of service drift as well as maximum strength.

Furthermore, LeMessurier discussed the influence of nominal out-of-plumbness in rectangular frames as well as inelastic stiffness reduction in members subjected to large axial loads, although the handling of these factors has matured in the time since his seminal work.

Donald W. White is professor, structural engineering, mechanics and materials, Georgia Institute of Technology, Atlanta, GA.

Andrea Surovek is assistant professor, civil and environmental engineering, South Dakota School of Mines and Technology, Rapid City, SD.

Sang-Cheol Kim is structural engineer, AMEC Americas, Tucker, GA.

In spite of the significant contributions from LeMessurier and others during the past 30 years, there is still a great deal of confusion regarding the proper consideration of second-order effects in frame design. Engineers can easily misinterpret and incorrectly apply analysis and/or design approximations due to an incomplete understanding of their origins and limitations. For instance, in braced frames, it is common to neglect second-order effects altogether. Although this practice is acceptable for certain structures, it can lead to unconservative results in some cases. LeMessurier (1976) presents an example that provides an excellent illustration of this issue.

The 2005 AISC *Specification for Structural Steel Buildings* (AISC, 2005a), hereafter referred to as the 2005 AISC *Specification*, provides a new method of analysis and design, termed the Direct Analysis Method (or DM). This approach is attractive in that:

- It does not require any K factor calculations,
- It provides an improved representation of the internal forces throughout the structure at the ultimate strength limit state,
- It applies in a logical and consistent fashion for all types of frames including braced frames, moment frames and combined framing systems.

The DM involves the use of a second-order elastic analysis that includes a nominally reduced stiffness and an initial out-of-plumbness of the structure. The 1999 AISC *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 1999), hereafter referred to as the 1999 AISC *Specification* and the 2005 AISC *Specification* permit this type of analysis as a fundamental alternative to their base provisions for design of stability bracing. In fact, the base AISC (1999) and AISC (2005a) stability bracing requirements are obtained from this type of analysis.

This paper demonstrates how a form of LeMessurier's (1976) simplified second-order analysis equations can be combined with the AISC (2005a) DM to achieve a particularly powerful analysis-design procedure. In this approach, $P\Delta$

shears associated with the amplified sidesway displacements are applied to the structure using a first-order analysis. This removes the need for separate analyses for "no lateral translation" (NT) and "with lateral translation" (LT), which are required in general for accurate amplification of the internal forces in the AISC (2005a) B_1-B_2 (or NT-LT) approach. The combination of the DM with the proposed form of LeMessurier's equations for the underlying second-order analysis streamlines the analysis and design process while focusing on the following important system-related attributes:

- Ensuring adequate overall sidesway stiffness.
- Accounting for second-order $P\Delta$ effects on *all* the lateral load-resisting components in the structure, when these effects are significant, including the influence of reductions in stiffness and increases in displacements as the structure approaches its maximum strength.

The first part of the paper gives an overview of the AISC (2005a) DM in the context of braced frames, or combined braced and gravity framing systems. This is followed by a step-by-step outline of the combined use of the DM with LeMessurier's (1976) second-order analysis approach. The paper closes by presenting analysis results and load and resistance factor design (LRFD) checks for a basic braced column subjected to concentric axial compression, and for an example long-span braced frame from LeMessurier (1976). A companion paper, Part 2 (White, Surovek, and Chang, 2007), discusses an extension of the above integrated approach to general framing systems including moment frames and moment frames combined with gravity and braced framing.

OVERVIEW OF THE DIRECT ANALYSIS METHOD FOR BRACED-FRAME SYSTEMS

For simply-connected braced structures, the DM requires two modifications to a conventional elastic analysis:

1. A uniform nominal out-of-plumbness of $\Delta_o = L/500$ is included in the analysis, to account for the influence of initial geometric imperfections, incidental load eccentricities and other related effects on the internal forces under ultimate strength loadings. This out-of-plumbness effect may be modeled by applying an equivalent notional lateral load of

$$N_i = 0.002Y_i \tag{1}$$

   at each level in the structure, where Y_i is the factored gravity load acting at the i^{th} level. Alternatively, the nonverticality may be modeled explicitly by altering the frame geometry. The above nominal out-of-plumbness is equal to the maximum tolerance specified in the AISC *Code of Standard Practice* (AISC, 2005b).

2. The nominal stiffnesses of all the components in the structure are reduced by a uniform factor of 0.8. This factor accounts for the influence of partial yielding of the most critically loaded component(s), as well as uncertainties with respect to the overall displacements and stiffness of the structure at the strength limit states.

These adjustments to the elastic analysis model, combined with an accurate calculation of the second-order effects, provide an improved representation of the second-order inelastic forces in the structure at the ultimate strength limit. Due to this improvement, the AISC (2005a) DM bases the member axial resistance, P_n, on the actual unsupported length not only for braced and gravity frames, but also for all types of moment frames and combined framing systems.

The above modifications are for the assessment of strength. In contrast, serviceability limits are checked using the ideal geometry and the nominal (unreduced) elastic stiffness. Also, it should be noted that the uniform factor of 0.8, applied to all the stiffness contributions, influences only the second-order effects in the system. That is, for structures in which the second-order effects are small, the stiffness reduction has a negligible effect on the magnitude and distribution of the system internal forces. The rationale for the above modifications is discussed in detail by White, Surovek-Maleck, and Kim (2003a), White, Surovek-Maleck, and Chang (2003b), Surovek-Maleck and White (2004), and White, Surovek, Alemdar, Chang, Kim, and Kuchenbecker (2006). The reader is referred to Maleck (2001), Martinez-Garcia (2002), Deierlein (2003, 2004), Surovek-Maleck, White, and Ziemian (2003), Maleck and White (2003), Nair (2005), and Martinez-Garcia and Ziemian (2006) for other detailed discussions as well as for validation and demonstration of the DM concepts.

STEP-BY-STEP APPLICATION: COMBINATION OF THE DIRECT ANALYSIS METHOD WITH LEMESSURIER'S SECOND-ORDER ANALYSIS PROCEDURE

The proposed second-order analysis and design procedure involves a combination of the DM with a specific form of the approach for determining second-order forces in braced-frame systems originally presented by LeMessurier (1976). A succinct derivation of the key equations is provided in Appendix A. For a given load combination, the basic steps of the proposed combined procedure are as follows:

1. Perform a first-order elastic analysis of the structure.
2. Obtain the total first-order story lateral displacement(s).
3. Calculate the story sidesway displacement amplification factor(s).

4. Calculate the story $P\Delta$ shears based on the amplified story sidesway displacement(s).

5. Apply the $P\Delta$ shears in a separate first-order analysis to determine the second-order portion of the internal forces.

6. Calculate the required forces by summing the appropriate first- and second-order contributions and check against the corresponding design resistances.

In the context of the DM, the above $P\Delta$ shears include the effects of a nominal reduction in the structure stiffness and a nominal initial out-of-plumbness, Δ_o. However, for checking of service deflection limits, or for conventional strength analysis and design using the Effective Length Method (ELM), the same approach can be applied using $\Delta_o = 0$ and zero stiffness reduction, in other words, with a stiffness reduction factor of 1.0.

The following is a more detailed description of the steps:

1. Perform a first-order analysis to obtain the first-order internal member forces and story sidesway displacements for each of the load types that need to be considered. The authors recommend the use of separate analyses for each load type (*D, L, W,* L_r, *E,* etc.) at the nominal (unfactored) load levels. By arranging the analyses in this way, the results can be factored and combined using superposition for each of the required load combinations.

2. Obtain $\bar{\Delta}_1$, the total first-order story lateral displacement(s) for a given load combination, by summing the analysis results from step 1 multiplied by the appropriate load factors. Note that throughout this paper, the over bar on a variable means that it is obtained by applying a stiffness reduction factor of 0.8 whenever the DM is used.

3. Calculate the story sidesway amplification factor(s) associated with a given load combination using the equation

$$\bar{B}_{lt} = \frac{1}{1 - \dfrac{\beta_i}{\bar{\beta}}} \quad (2)$$

where

$$\beta_i = \frac{\Sigma P_r}{L} \quad (3)$$

is referred to as the *ideal stiffness* (Galambos, 1998) and

$$\bar{\beta} = \frac{\Sigma H}{\bar{\Delta}_{1H}} \quad (4)$$

is the actual story sidesway stiffness of the lateral load resisting system in the first-order analysis model. The term ΣP_r in Equation 3 is the total factored vertical load supported by the story and L is the story height. The term ΣH in Equation 4 is the total story shear force due to the applied loads and $\bar{\Delta}_{1H}$ is the first-order horizontal displacement due to ΣH. If $\Sigma H = 0$ (and therefore $\bar{\Delta}_{1H} = 0$), $\bar{\beta}$ may be determined by applying any fraction of ΣP_r as an equal and opposite set of shear forces at the top and bottom of all the stories and then dividing by the corresponding $\bar{\Delta}_{1H}$. For combined braced and gravity frames, the sidesway amplification factor $\bar{B}_{lt}$ is the same as the term B_2 in AISC (2005a). The symbol $\bar{B}_{lt}$ is used in this paper, since the subscripts 0, 1, and 2 are reserved to denote the initial, first-order and second-order forces and/or displacements. The notation "*lt*" stands for "lateral translation."

4. For each of the load combinations, calculate the story $P\Delta$ shears, $\bar{H}_{P\Delta}$, using the equation

$$\bar{H}_{P\Delta} = \Sigma P_r \bar{B}_{lt} \left(\frac{\Delta_o + \bar{\Delta}_1}{L} \right) \quad (5)$$

and apply to each story of the frame in a separate first-order analysis to determine the second-order component of the internal forces. Figure 1 illustrates the application of these forces in a multi-story frame.†

5. Add the second-order forces from step 4 to the first-order forces from step 1 times the load factors for the strength load combination under consideration. This gives the required strengths throughout the structure.

6. For axially loaded members subjected to transverse loads, amplify the internal moments using the traditional NT amplification factor, denoted by B_1 in AISC (2005a). The calculation of internal moments in general framing systems is addressed by White, Surovek, and Chang (2007).

7. Check the required forces from steps 5 and 6 versus the corresponding design resistances.

†White et al. (2003a) discuss an appropriate reduction in the Δ_o values, or the corresponding notional loads, N_i, from Equation 1, for tall multi-story frames. The reader should note that the resulting lateral loads shown in Figure 1 include the effect of both the amplified initial out-of-plumbness, $\bar{B}_{lt}\,\Delta_o$, as well as the amplified sidesway deflections due to the applied loads, $\bar{B}_{lt}\bar{\Delta}_1$.

It should be noted that the above procedure may be applied for design by either LRFD or ASD. However, if applied for ASD, AISC (2005a) requires that the applied loads must be multiplied by a parameter $\alpha = 1.6$ to account for the second-order effects at the ultimate load level. The resulting internal forces are subsequently divided by α to obtain the required ASD forces.

To use the above procedure for strength analysis and design by the DM, the nominal initial out-of-plumbness

$$\Delta_o = 0.002L \quad (6a)$$

and the reduced stiffness

$$\bar{\beta} = 0.8\beta \quad (6b)$$

are employed. Also, since the stiffness is reduced, the corresponding first-order story sidesway displacements are

$$\bar{\Delta}_1 = \frac{\Delta_1}{0.8} \quad (6c)$$

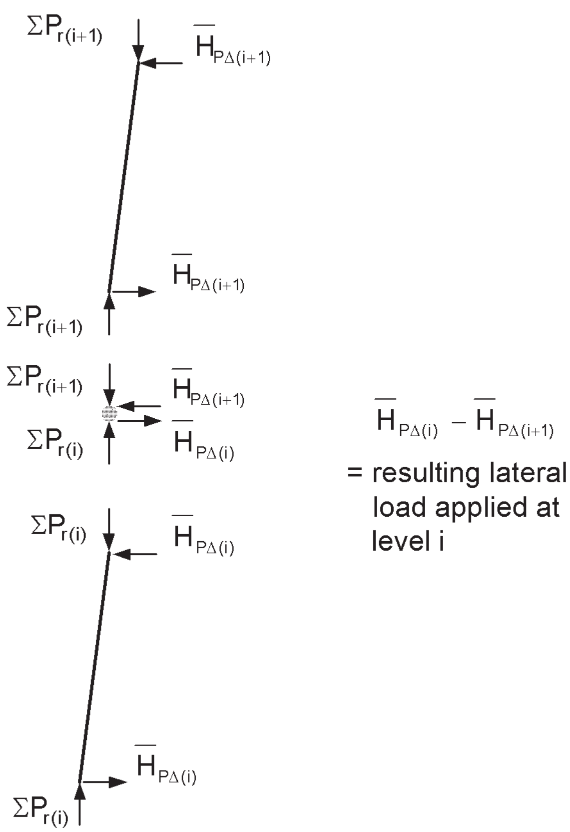

Fig. 1. Application of the PΔ *shear forces* $\bar{H}_{P\Delta}$ *in a multi-story frame.*

where Δ_1 is the first-order story displacement, obtained using the nominal (unreduced) stiffness values. The out-of-plumbness, Δ_o, may be modeled explicitly by canting the frame geometry or it may be represented by the equivalent notional lateral loads given by Equation 1. If a notional lateral load is used, this load is handled the same as any other applied lateral load in the above procedure. Alternately, for service load analysis, or for strength analysis and design by the conventional Effective Length Method (ELM), where the analysis is conducted on the idealized nominally-elastic initially-perfect structure, the above terms are

$$\Delta_o = 0 \quad (7a)$$

$$\bar{\beta} = \beta \quad (7b)$$

and

$$\bar{\Delta}_1 = \Delta_1 \quad (7c)$$

The above analysis approach gives an exact solution for the second-order DM, ELM, or service level forces and displacements within the limits of:

- the idealization of the lateral load resisting system as a truss,
- the approximation $\cos\left(\bar{\Delta}_{tot}/L\right) \cong 1$, where $\bar{\Delta}_{tot} = \bar{B}_{lt}\left(\Delta_o + \bar{\Delta}_1\right)$,
- the assumption that the stiffness $\bar{\beta}$ for any story is the same value for both of the loadings ΣH and $\bar{H}_{P\Delta}$, and
- the approximation of equal sidesway displacements throughout each floor or roof level.

That is, with these qualifications, LeMessurier's procedure is an "exact" noniterative second-order analysis (Vandepitte, 1982; Gaiotti and Smith, 1989; White and Hajjar, 1991) for combined gravity and braced-frame systems. The assumption of $\cos\left(\bar{\Delta}_{tot}/L\right) \cong 1$ is certainly a reasonable one, since any structure that violates this limit will likely have objectionable sidesway deflections under service loads. For multistory structures, the interactions between the stories, for example, the rotational restraint provided at the top and bottom of the columns in a given story as well as the accumulation of story lateral displacements due to overall cantilever bending deformations of the structure, strictly are different for different loadings and load effects. However, the differences in the $\bar{\beta}$ values for the different loadings are typically small. The handling of unequal lateral displacements at a given story level, for example, due to thermal expansion or due to flexible floor or roof diaphragms, is addressed by White et al. (2003a).

Since the influence of member axial deformations, including differential column shortening, as well as other contributions to the displacements can be included in the first-order analysis, the above approach is applicable to all types of combined gravity and braced building frames. For a multi-story frame, sidesway due to column elongation and shortening in the stories below the one under consideration, in other words, sidesway due to cantilever deformations of the building, is addressed by analyzing the full structure. In general, it is not appropriate to determine $\bar{\beta} = \Sigma H / \bar{\Delta}_{1H}$ by just applying equal and opposite lateral loads at the top and bottom of a single story.

The proposed approach is particularly amenable to preliminary analysis and design. Typically, it is desired for structures to satisfy a certain drift limit under service load conditions. The story $P\Delta$ shear forces, $\bar{H}_{P\Delta}$, can be calculated for preliminary design by scaling the target service load drift limit to obtain the corresponding $\bar{\Delta}_1 / L$ for use in Equation 5. Part 2 of the paper illustrates this process (White et al., 2007).

For structures with a large number of stories and/or complex three-dimensional geometry, the proposed analysis approach can be programmed to avoid excessive manual calculations. The above approach can be implemented for general 3D analysis of building frames by incorporating the concepts discussed by Wilson and Habibullah (1987) and White and Hajjar (1991) to include the $P\Delta$ effects associated with overall torsion of the structural system. However, in the view of the authors, the use of general-purpose second-order analysis software is often preferable for complex 3D frames. The most important benefit of the proposed approach is that it facilitates preliminary analysis and design (using an estimated $\bar{\Delta}_1$ based on target service drift limits). Also, it is a useful aid for understanding second-order responses and checking of computer results. For instance, if $\bar{H}_{P\Delta}$ from Equation 5 is smaller than a certain fraction of the story shear due to the applied lateral loads ΣH (say 5%), the Engineer may choose to exercise his or her judgment and assume that the second-order sidesway effects are negligible.

BRACED COLUMN EXAMPLE

Figure 2 shows the results obtained using the proposed combination of the DM with LeMessurier's second-order analysis equations for one of the most basic analysis and design solutions addressed in the AISC (2005a) *Specification*—determination of the required bracing forces for simply-supported columns. For this problem, the stability bracing provisions of AISC (2005a) also apply. These provisions require a minimum brace stiffness of

$$\beta_{br} = \frac{2}{\phi}\frac{P_{r(LRFD)}}{L} = 2.67\frac{P_u}{L} = 2.67\beta_i \qquad (8a)$$

in LRFD, where $\phi = 0.75$ and $P_{r(LRFD)} = P_u$ is the required axial compressive strength of the column obtained from the LRFD load combinations, such as $1.2D + 1.6L$. They require

$$\beta_{br} = 2\Omega\frac{P_{r(ASD)}}{L} \qquad (8b)$$

in ASD, where $\Omega = 2.0$ and $P_{r(ASD)}$ is the required axial compressive strength of the column obtained using the ASD load combinations, for example, $D + L$.

If one assumes a live-to-dead load ratio, $L/D = 3$, then the ratio, $P_{r(LRFD)}/P_{r(ASD)}$ is 1.5 for the above dead and live load combinations. By substituting $P_{r(ASD)} = P_u/1.5$ into Equation 8b, one can observe that this equation is equivalent to Equation 8a at $L/D = 3$. However, for other live-to-dead load ratios and/or other load combinations, the required minimum brace stiffness is slightly different in ASD and LRFD.

At the β_{br} limit given by Equation 8a, the sidesway amplification obtained from an explicit second-order analysis without any reduction in the stiffness is $B_{lt} = 1.6$. When the analysis of the system shown in Figure 2 is conducted using the DM, $\bar{\beta} = 0.8\beta_{br}$ and the sidesway amplification

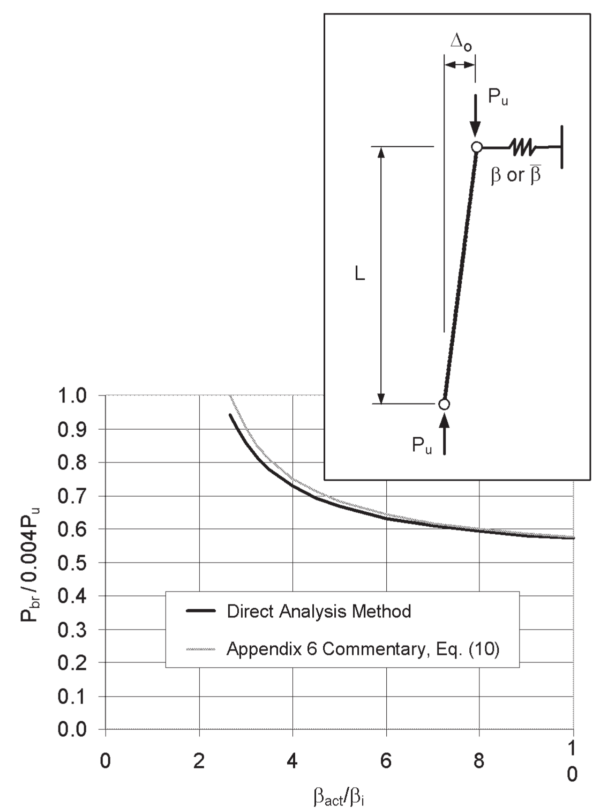

Fig. 2. Braced column example.

is $\bar{B}_{lt} = 1.88$. As a result, the brace force induced by the axial force, P_r, acting through the amplified nominal initial out-of-plumbness of $\bar{B}_{lt}\,\Delta_o = \bar{B}_{lt}\,(0.002L)$ is

$$P_{br} = P_u\ \bar{B}_{lt}\ (0.002) = 0.00376P_u \tag{9a}$$

from Equation 5 (note that $\bar{\Delta}_1$ in this problem). The corresponding base AISC (2005a) Appendix 6 brace force requirement is

$$P_{br} = 0.004P_u \tag{9b}$$

in LRFD (6% higher). The above 6% reduction in the brace force may be considered as a benefit allowed by AISC (2005a) for the explicit use of a second-order analysis for LRFD. Figure 2 shows the brace force requirements for the above basic column case as a function of increasing values of β_{act}/β_i, where β_{act} is the nominal (unreduced) brace stiffness. The requirements are plotted using the AISC (2005a) DM with LeMessurier's second-order analysis approach as well as the using the equation

$$P_{br} = \frac{0.004}{2 - \dfrac{\beta_{br}}{\beta_{act}}} P_u \tag{10}$$

which is specified as a refinement of the brace force requirement in the AISC (2005a) Appendix 6 Commentary. One can observe that the force requirements from the DM and from Equation 10 are increasingly close to one another for larger values of β_{act}/β_i. At $\beta_{act}/\beta_i = 10$, the DM requires $P_{br} = 0.00228P_u$ whereas Equation 10 gives $P_{br} = 0.00231P_u$, only 57% and 58% of the base AISC (2005a) brace force requirement, respectively. It should also be noted that if the above analyses are conducted using the AISC (2005a) ASD provisions, the results from the corresponding form of Equation 10 and the corresponding DM solution match exactly.

LONG-SPAN BRACED FRAME EXAMPLE

Figure 3 shows a long-span roof structure originally considered by LeMessurier (1976). The frame consists of 165-ft-long trusses at 20 ft on center, supporting a total (unfactored) gravity load of 100 psf. A live-to-dead load ratio of 3 is assumed for the purposes of checking this frame by LRFD. The structure is 18 ft high and is required to resist a nominal wind load of 15 psf. These nominal loadings are the same as in (LeMessurier, 1976); however, the LRFD factored loadings obtained using the above assumptions are different than LeMessurier's factored loads. The nominal column axial loads are 0.5(165 ft)(2 kips/ft) = 165 kips. A W8×48 with F_y = 50 ksi is selected for the column size based on the LRFD load combinations. LeMessurier selected an ASTM A572 Grade 50 W14×48 section in the original design. A brace is provided on the left side of the structure; for architectural reasons this member is an HSS 4.5×4.5×3⁄16 with F_y = 46 ksi. The original brace selected by LeMessurier was an

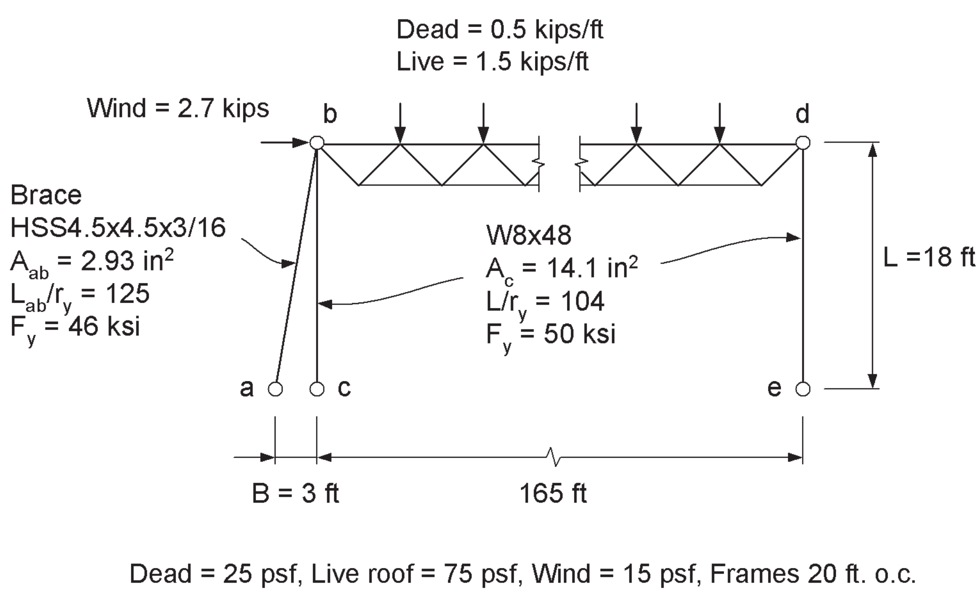

Fig. 3. LeMessurier's (1976) example frame, designed by LRFD.

HSS 3.5×3.5×3⁄16 with the same yield strength. The increase in size of the brace is predominantly a result of the LRFD wind load factor in SEI/ASCE 7-05 (ASCE, 2005) compared to that originally assumed by LeMessurier. This brace is sized to act in both tension and compression.

The load combinations considered in this example include the service cases (ASCE, 2005),

$$D + L_r$$

and

$$D + 0.5L_r + 0.7W$$

where

D = dead load

L_r = roof live load

W = wind load

as well as the strength combinations,

$$1.2D + 1.6L_r + 0.8W,$$

$$0.9D + 1.6W$$

and

$$1.2D + 0.5L_r + 1.6W$$

The size of column bc and the tension force requirement in the brace ab are governed by the strength load combination $1.2D + 1.6L_r + 0.8W$ with the wind acting to the right. However, the size of the diagonal ab is governed by its compressive strength under the load combination $0.9D + 1.6W$ with the wind acting to the left.

In the following, separate first-order analyses of the above structure under the nominal (unfactored) gravity and wind loads are presented first. Then the calculation of the required axial strengths by the DM is illustrated for the load combination $1.2D + 1.6L_r + 0.8W$ with the wind acting to the right. The DM results for the other load combinations are summarized at the end of this presentation. Finally, the results from the following analysis models are compared and contrasted with the DM analysis results:

1. First- and second-order analysis using the nominal (unreduced) elastic stiffness and $\Delta_o = 0$. These solutions are appropriate for analysis of service load conditions, but are not always appropriate for calculation of the force requirements at strength load levels. The proposed form of LeMessurier's (1976) equations is used for these second-order analyses with zero elastic stiffness reduction and zero initial out-of-plumbness. Since all the strength load combinations considered here involve a lateral wind load, these second-order analyses satisfy all the requirements of the Effective Length Method (ELM) of the 2005 AISC Specification.

2. Summation of the stability bracing forces obtained from the refined equations specified in the AISC (2005a) Appendix 6 Commentary with the bracing forces obtained from a first-order structural analysis. This is the approach specified in AISC (1999) for calculation of the required strength of braced-frame systems. The AISC (2005a) Specification no longer uses this approach. Instead, it specifies the use of either the DM (in its Appendix 7) or the use of a second-order elastic analysis with nominal stiffness and perfect frame geometry, but with a minimum lateral load included in gravity-only load combinations (in the ELM procedure of its Chapter C). The stability bracing provisions of the AISC (2005a) Appendix 6 are specified solely to handle "bracing intended to stabilize individual members," in other words, cases where bracing forces due to the applied loads on the structure are not calculated. However, it is useful to understand the close relationship between the DM and the AISC (1999) and AISC (2005a) Appendix 6 stability bracing provisions.

3. Distributed Plasticity Analysis. This method of analysis is useful for evaluation of all of the analysis and design methods, since it accounts rigorously for the effects of nominal geometric imperfections and member internal residual stresses. The details of the Distributed Plasticity Analysis solutions are explained in Appendix B.

Base First-Order Analysis, Nominal (Unfactored) Loads

LeMessurier (1976) derived base first-order elastic analysis equations for the structure in Figure 3. These equations are summarized here for purposes of continuity. As noted above, the nominal $(D + L)$ column load in this example is $P = 165$ kips. The corresponding total story vertical load is $\Sigma P = 330$ kips. The axial compression in column bc causes it to shorten. Consequently, compatibility between the diagonal ab and the top of the column at b requires a story drift ratio of

$$\frac{\Delta_{1P}}{L} = \frac{P\left(\dfrac{L}{B}\right)}{A_c E} = 0.00242 = \frac{1}{413} \tag{11}$$

where

L = story height

B = horizontal distance between the base of column bc and the base of the diagonal ab

A_c = column area = 14.1 in.^{2}

E = 29,000 ksi

The nominal elastic sidesway stiffness of the structure is given by

$$\beta = \frac{A_c E}{L\left[(1+\lambda)\left(\frac{L}{B}\right)^2 + \lambda\right]} = 8.743\ \frac{\text{kips}}{\text{in.}} \tag{12}$$

$$\lambda = \frac{L_{ab} A_c}{A_{ab} L} \tag{13}$$

L_{ab} = length of the diagonal = $\sqrt{L^2 + B^2}$ = 18.25 ft

A_{ab} = area of the brace = 2.93 in.^{2}

Equations 11 and 12 are rather simple to derive for the example frame. In cases where the framing is more complex, it is often more straightforward to determine the story stiffness β as $\Sigma H/\Delta_{1H}$. Based on Equation 12, the drift ratio due to the nominal horizontal load ΣH = 2.7 kips is

$$\frac{\Delta_{1H}}{L} \quad \frac{\Sigma H}{\beta L} \quad 0.00143 = \frac{1}{699} \tag{14}$$

It is important to note that the story drift ratio under the nominal gravity load alone (Δ_{1P}/L) is significantly larger than 1/500. Furthermore, the drift due to the nominal gravity load is 1.7 times that due to the nominal wind load. This attribute of the response, as well as the magnitude of the total vertical load, makes this example a severe test of any stability analysis and design procedure for braced frames. The first-order drift under the nominal wind load itself is rather modest.

Strength Analysis Under 1.2D + 1.6L_r + 0.8W, with the Wind Acting to the Right, Using the Direct Analysis Method

The ideal stiffness of the structure for the load combination (1.2D + 1.6L_r + 0.8W) is

$$\beta_{i(1.2D+1.6L)} = \frac{1.5\Sigma P}{L} = 2.292\ \frac{\text{kips}}{\text{in.}} \tag{15}$$

where the factor 1.5 is obtained from [1.2D +1.6(3D)] / [D + (3D)]. Furthermore, the sidesway amplification based on the reduced stiffness model is

$$\bar{B}_{lt(1.2D+1.6L)} = \frac{1}{1-\dfrac{\beta_{i(1.2D+1.6L)}}{0.8\beta}} = 1.487 \tag{16}$$

The total story drift at the maximum strength limit is therefore

$$\left(\frac{\bar{\Delta}_{tot}}{L}\right)_{(1.2D+1.6L+0.8W)} = \bar{B}_{lt(1.2D+1.6L)} \times\left[0.002+\left(1.5\frac{\Delta_{1P}}{L}+0.8\frac{\Delta_{1H}}{L}\right)\left(\frac{1}{0.8}\right)\right] = 0.0120 = \frac{1}{83} \tag{17}$$

This results in a story $P\Delta$ shear of

$$\bar{H}_{P\Delta(1.2D+1.6L+0.8W)} = 1.5\Sigma P \times\left(\frac{\bar{\Delta}_{tot}}{L}\right)_{(1.2D+1.6L+0.8W)} = 5.87\text{ kips} \tag{18}$$

a tension force requirement in the diagonal ab of

$$\bar{F}_{ab(1.2D+1.6L+0.8W)} = \left(0.8\Sigma H + \bar{H}_{P\Delta(1.2D+1.6L+0.8W)}\right) \times\left(\frac{L_{ab}}{B}\right) = 48.8\text{ kips} \tag{19}$$

and a maximum compressive strength requirement in column bc of

$$\bar{F}_{bc(1.2D+1.6L+0.8W)} = 1.5P + \left(0.8\Sigma H + \bar{H}_{P\Delta(1.2D+1.6L+0.8W)}\right) \times\left(\frac{L}{B}\right) = 296\text{ kips} \tag{20}$$

Synthesis of Results

Table 1 summarizes the key results from the service and the LRFD strength load combinations considered for the example long-span braced frame. The results for the service load combinations are presented first, followed by the results for the strength load combinations. The procedures utilized for the analysis calculations are listed in the second column of the table. The third through sixth columns include the following information: the forces in the brace ab (F_{ab} based on the nominal stiffness or $\bar{F}_{ab}$ based on the reduced stiffness, as applicable); the comparable forces in column bc (F_{bc} or $\bar{F}_{ab}$); the drift values (Δ_{tot}/L based on the nominal stiffness or $\bar{\Delta}_{tot}/L$ based on the reduced stiffness, as applicable); and the sidesway amplification factor (B_{lt} based on the nominal stiffness or $\bar{B}_{lt}$ based on the reduced stiffness, as applicable).

The second-order sidesway amplification is 1.21 and 1.12 under the service load combinations, (D + L_r) and (D + 0.5L_r + 0.7W), respectively. Therefore, although the structure is braced, its second-order effects are significant even under service loading conditions. Engineers often assume that second-order effects are small in braced structures. This is true in many situations, but this assumption can lead to unacceptable service load performance in some cases. The maximum total drift (including the second-order $P\Delta$ effects) for the two service load combinations considered here is 1/341. This drift is acceptable for many types of structures (West, Fisher and Griffis, 2003). However, the corresponding first-order drift is 1/413, 21% smaller than

Table 1. Summary of Analysis Results					
Load Combination	**Analysis Procedure**	**F_{ab} or $\bar{F}_{ab}$ (kips)**	**F_{bc} or $\bar{F}_{bc}$ (kips)**	**Δ_{tot}/L or $\bar{\Delta}_{tot}/L$**	**B_{lt} or $\bar{B}_{lt}$**
Service $D + L_r$	First-order, nominal stiffness, $\Delta_o = 0$			1/413	
	Second-order, nominal stiffness, $\Delta_o = 0$			1/341	1.212
Service $D + 0.5L_r + 0.7W$ W acting to right	First-order, nominal stiffness, $\Delta_o = 0$			1/398	
	Second-order, nominal stiffness, $\Delta_o = 0$			1/354	1.123
Strength $1.2D + 1.6L_r + 0.8W$ W acting to right	First-order, nominal stiffness, $\Delta_o = 0$	13.1	–260	1/209	
	AISC (2005a) ELM[(a)]	32.6	–280	1/154	1.355
	AISC (2005a) Appendix 6	44.5	–291	1/96	1.537
	AISC (2005a) DM[(b)]	48.8	–296	1/83	1.487
	Distributed Plasticity Analysis[(c)]	40.8	–272	1/98	
	Distributed Plasticity Analysis[(d)]	44.2	–291	1/97	
Strength $0.9D + 1.6W$ W acting to left	First-order, nominal stiffness, $\Delta_o = 0$	–26.3	–11.2	–1/574	
	AISC (2005a) ELM	–27.1	–10.4	–1/551	1.041
	AISC (2005a) Appendix 6	–28.1	–9.4	–1/253	1.055
	AISC (2005a) DM	–28.3	–9.2	–1/228	1.052
	Distributed Plasticity Analysis	–28.1	–9.3	–1/243	
Strength $1.2D + 0.5L_r + 1.6W$ W acting to right	First-order, nominal stiffness, $\Delta_o = 0$	26.3	–137	1/255	
	AISC (2005a) ELM	32.3	–143	1/225	1.134
	AISC (2005a) Appendix 6	35.8	–147	1/142	1.187
	AISC (2005a) DM	37.2	–148	1/124	1.173
	Distributed Plasticity Analysis	36.2	–147	1/137	

[(a)]The ELM analysis is based on the nominal (unreduced) elastic stiffness and $\Delta_o = 0$.
[(b)]DM analysis is based on a reduced elastic stiffness of 0.8 of the nominal stiffness, and $\Delta_o = 0.002L$.
[(c)]Maximum load capacity reached due to a compression failure of column bc at $P = 272$ kips at 0.936 of $1.2D + 1.6L_r + 0.8W$. The AISC (2005a) LRFD column strength is $\phi_c P_n = 288$ kips.
[(d)]Analysis conduced up to $1.2D + 1.6L_r + 0.8W$ with distirbuted yielding neglected such that the structure remains elastic at this load level.

the actual drift calculated including the $P\Delta$ effects under the service loading, $D + L_r$.

The influence of unavoidable geometric imperfections tends to be relatively small at service load levels, where these effects tend to be offset somewhat by incidental contributions to the stiffness from connection rotational stiffnesses, cladding, etc. Also, yielding effects are usually negligible at service load levels. Therefore, the reduction in the elastic stiffness and the out-of-plumbness utilized in the DM solution at strength load levels are not necessary and should not be employed for service load analysis. However, as discussed by LeMessurier (1976 and 1977), the second-order effects at service (or working) load levels generally should not be neglected. Neglecting the second-order effects can lead to a violation of the service deflection limits and inadequate structural performance in cases where these effects are significant.

As noted previously, the strength load combination ($1.2D + 1.6L_r + 0.8W$) gives the largest required tension force in the brace ab and the largest required compression force in the column bc of the example frame. Therefore, most of the following discussions are focused on the results for this load combination. A conventional first-order analysis for this load combination gives a tension force in the brace ab of only 13.1 kips. Equilibrium of the deflected geometry requires a brace force of $F_{ab} = 32.6$ kips based on the AISC (2005a) ELM model, which assumes no initial geometric imperfections and no reduction in the effective stiffness of the structure at the strength limit state. This is due largely to the significant drift of the structure under the gravity loads, $1.2D + 1.6L_r$, in other words, $1.5\Delta_{1P}$, plus the significant sidesway amplification of B_{lt} = 1.355. LeMessurier (1976) notes a similar significant increase in the diagonal brace tension in his discussions (but in the context of allowable stress design). The column bc axial compression is also increased from –260 to –280 kips due to the $P\Delta$ effects.

If one assumes a nominal $\Delta_o = 0.002L$ in the direction of the sidesway and conducts a Distributed Plasticity Analysis to rigorously account for the effects of early yielding due to residual stresses (see Appendix B), a diagonal brace tension of 40.8 kips is obtained when the frame reaches its maximum strength. In this example, the structure's maximum strength determined from this type of analysis occurs at 0.936 of $(1.2D + 1.6L_r + 0.8W)$. The maximum strength is governed by a failure of column bc at P = 272 kips [6% smaller than the AISC (2005a) LRFD column design strength]. If yielding is delayed such that the frame remains fully elastic at $(1.2D + 1.6L_r + 0.8W)$ (for example, if the actual F_y is sufficiently larger than the specified minimum value such that column bc remains elastic), the Distributed Plasticity Analysis model gives F_{ab} = 44.2 kips and F_{bc} = –291 kips at this load level. This is shown as a second entry in Table 1 for the Distributed Plasticity Analysis [also see footnote (d)]. The reader should note that none of the Distributed Plasticity results shown in Table 1 account for attributes such as connection slip, connection elastic deformations or yielding, or foundation flexibility. These attributes can increase the diagonal brace tension further. In addition, the influence of axial elongation in the long-span roof system, causing a larger sidesway of the leaning column on the right-hand side of the frame, is not considered here. White et al. (2003a) address the handling of this effect, including the effect of axial deformation in member bd due to changes in temperature. The DM provides a reasonable estimate of the frame deflections and internal forces from the above Distributed Plasticity solutions, giving $\bar{F}_{ab}$ = 48.8 kips (tension) and $\bar{F}_{bc}$ = –296 kips (compression) as illustrated in the previous section.

Interestingly, for this example the calculation using the stability bracing equations in AISC (1999) and the AISC (2005a) Commentary to Appendix 6 gives the closest estimate of the second Distributed Plasticity results for the load combination $1.2D + 1.6L_r + 0.8W$. These calculations are summarized as follows:

1. The designer must recognize that the drift of the frame under the above gravity plus wind load combination substantially violates the assumption of $\Delta_{o.total} = \Delta_o + \Delta_1 = 0.002L$ in the base Appendix 6 equation for the stability bracing force, in other words, the horizontal force component in the bracing system due to second-order effects. Based on an assumed nominal initial out-of-plumbness of $\Delta_o = 0.002L$ and the first-order analysis calculations summarized in Table 1, $\Delta_{o.total}/L = (\Delta_o + \Delta_1)/L = 0.002 + 1/209 = 0.00678 = 1/148$.

2. The Commentary to Appendix 6 of (AISC, 2005a) and the Commentary to Chapter C of (AISC, 1999) indicate, "for other Δ_o and θ_o values, use direct proportion to modify the brace strength requirements." Note that the above Commentaries use the term Δ_o to represent $\Delta_{o.total}$. The notation $\Delta_{o.total}$ is used here to clarify the *two* sources of relative transverse displacement between the brace points. Therefore, the appropriately modified base equation for the stability bracing force is

$$P_{br} = 0.004\Sigma P_u\left(\frac{0.00678}{0.002}\right) = 0.0136\Sigma P_u \qquad (21)$$

3. In addition to the above modification, the AISC (2005a and 1999) Commentaries specify, "If the brace stiffness provided, β_{act}, is different from the requirement, then the brace force or brace moment can be multiplied by the following factor:

$$\frac{1}{2-\dfrac{\beta_{br}}{\beta_{act}}}\text{"} \qquad (22)$$

where

$$\beta_{br} = \frac{2\Sigma P_u}{\phi L} \qquad (23)$$

and $\phi = 0.75$. By applying this factor to the above calculation of the stability bracing force, one obtains

$$P_{br} = 0.0136\Sigma P_u \frac{1}{2-\dfrac{2(1.5)(330\text{ kips})}{0.75(216\text{ in.})\left(8.743\dfrac{\text{kips}}{\text{in.}}\right)}} \qquad (24)$$

$$= 0.0104\Sigma P_u = 0.0104(1.5)(330\text{ kips}) = 5.16\text{ kips}$$

It is important to note that in this example, the above coefficient of 0.0104 is substantially larger than the base Appendix 6 coefficient of 0.004.

4. Finally, the above stability bracing force must be applied to the bracing system along with the horizontal force determined from a first-order elastic analysis for the subject applied load combination. That is, the bracing system must be designed for a total horizontal force of

$$P_{br} + 0.8\Sigma H = 5.16\text{ kips} + 2.16\text{ kips} = 7.32\text{ kips}$$

in addition to the vertical load of $1.5P$ = 1.5(165 kips) = 247.5 kips applied to the left-hand column. This is specified in Section C3.2 of AISC (1999) by the clause, "These story stability requirements shall be combined with the lateral forces and drift requirements from other sources, such as wind or seismic loading." AISC (2005a) no longer includes this clause in its Appendix 6, since Appendix 6 is intended only for cases where the bracing is not subjected to any forces determined from a structural analysis.

The resulting calculations are

$$F_{ab(1.2D+1.6L+0.8W)} = (P_{br} + 0.8\Sigma H)\left(\frac{L_{ab}}{B}\right) \qquad (25)$$
$$= 44.5 \text{ kips}$$

and

$$F_{bc(1.2D+1.6L+0.8W)} = 1.5P + (P_{br} + 0.8\Sigma H)\left(\frac{L}{B}\right) \qquad (26)$$
$$= 291 \text{ kips}$$

Note that the result from Equation 24 can be obtained more directly by applying LeMessurier's (1976) approach as follows:

$$P_{br} = \Sigma P_u \frac{1}{1 - \dfrac{\beta_i}{0.75\beta_{act}}}\left(\frac{\Delta_o + \Delta_1}{L}\right)$$
$$= 1.5(330 \text{ kips})\left[\frac{1}{1 - \dfrac{1.5(330 \text{ kips})}{0.75(216 \text{ in.})\left(8.743\dfrac{\text{kips}}{\text{in.}}\right)}}\right] \qquad (27)$$
$$\times\left(0.002 + \frac{1}{209}\right)$$
$$= 5.16 \text{ kips}$$

Equation 27 is simply Equation 5 but with a stiffness reduction factor of 0.75 used in the sidesway amplifier and with zero stiffness reduction assumed in the calculation of Δ_1. The corresponding sidesway amplification and total drift ratio are shown in the last two columns of Table 1. Obviously, the use of a reduced stiffness in the sidesway amplifier and the use of a nominal (unreduced) stiffness in the calculation of the deflections due to the applied loads is inconsistent. The DM provides a consistent second-order analysis calculation based on $\Delta_o = 0.002L$ and an elastic stiffness reduction factor of 0.8.

As noted previously, AISC (2005a) Chapter C also allows the use of a second-order elastic analysis with the nominal elastic stiffness and idealized perfect geometry, as long as notional lateral loads of $0.002Y_i$ are included in all gravity-only load combinations. The approach is referred to as the Effective Length Method (ELM) in Table 2-1 of AISC (2005c). For the load combinations considered in Table 1, this approach amounts to the use of a conventional second-order analysis with the nominal elastic stiffness and Δ_o = 0 (since all the load combinations considered in the table include wind load). As a result, the AISC (2005a) Chapter C provisions give required strengths for members ab and bc of only 32.6 kips and –280 kips, respectively. The relatively large difference between F_{ab} = 32.6 kips and the DM value of $\overline{F}_{ab}$ = 48.8 kips is due to: (1) the small angle of the brace relative to the vertical orientation; (2) the large gravity load supported by the structure; (3) the relatively small wind load; and (4) the correspondingly small lateral stiffness of the bracing system. If a symmetrical configuration of the bracing were introduced, the second-order $P\Delta$ effects in this frame are dramatically reduced. Also, Figure 2 shows that the second-order internal story shears approach $0.002\Sigma P_r$ when the bracing system has substantial stiffness relative to the ideal stiffness β_i. In many cases, particularly if ΣP_r is relatively small, these internal shear forces are only a small fraction of the lateral load resistance of the bracing system. However, in sensitive stability critical frames such as LeMessurier's example in Figure 3, the application of the AISC (2005a) DM provisions is considered prudent.

The required strengths for the other load combinations shown in Table 1 are less sensitive to the design approach. For example, the governing axial compression in brace ab is determined as –28.3 kips using the DM for the load combination $0.9D + 1.6W$ with the wind acting to the left. A basic first-order analysis for this load combination gives -26.3 kips, only 7.1% smaller. The AISC (2005a) Chapter C approach gives a compressive force of –27.1 kips, only 3.0% smaller.

SUMMARY

This paper presents an analysis-design approach based on a combination of the AISC (2005a) Direct Analysis Method (DM) with a form of LeMessurier's (1976) simplified second-order analysis equations. The results from several DM analysis solutions are compared and contrasted with the results from other analysis solutions including second-order service drift calculations, AISC (2005) Effective Length Method (ELM) solutions, refined calculations based on the Commentaries to AISC (1999) Chapter C and AISC (2005a) Appendix 6, and benchmark Distributed Plasticity Analysis solutions. The DM is attractive in that:

- It does not require any K factor calculations,
- It provides an improved representation of the internal forces throughout the structure at the ultimate strength limit state,

- It applies in a logical and consistent fashion for all types of frames including braced frames, moment frames and combined framing systems.

LeMessurier's (1976) second-order analysis equations are particularly useful in that they capture the second-order effects in rectangular frame structures by explicitly applying the $P\Delta$ shears associated with the amplified sidesway displacements in a first-order analysis. LeMessurier's approach also can be used for analysis of service deflections and for conventional strength analysis and design by the Effective Length Method (ELM) using the nominal elastic stiffness and the idealized perfect structure geometry. However, the combination of the DM with LeMessurier's equations for the underlying second-order analysis streamlines the analysis and design process, while also focusing the Engineer's attention on the importance of:

- Ensuring adequate overall sidesway stiffness, and
- Accounting for second-order $P\Delta$ effects on all the lateral load-resisting components in the structural system at the strength load levels.

ACKNOWLEDGMENTS

The concepts in this paper have benefited from many discussions with the members of AISC Task Committee 10 and the former AISC-SSRC Ad hoc Committee on Frame Stability in the development of the Direct Analysis Method. The authors express their appreciation to the members of these committees for their many contributions. The opinions, findings and conclusions expressed in this paper are those of the authors and do not necessarily reflect the views of the individuals in the above groups.

REFERENCES

AISC (2005a), *Specification for Structural Steel Buildings*, ANSI/AISC 360-05, American Institute of Steel Construction, Inc., Chicago, IL.

AISC (2005b), *Code of Standard Practice for Steel Buildings and Bridges*, American Institute of Steel Construction, Inc., Chicago, IL.

AISC (2005c), *Steel Construction Manual*, 13th Ed., American Institute of Steel Construction, Inc., Chicago, IL.

AISC (1999), *Load and Resistance Factor Design Specification for Steel Buildings*, American Institute of Steel Construction, Inc., Chicago, IL.

Alemdar, B.N. (2001), "Distributed Plasticity Analysis of Steel Building Structural Systems," Doctoral dissertation, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

ASCE (1997), "Effective Length and Notional Load Approaches for Assessing Frame Stability: Implications for American Steel Design," Task Committee on Effective Length, Technical Committee on Load and Resistance Factor Design, Structural Engineering Institute, American Society of Civil Engineers, p. 442.

ASCE (2005), *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-05, American Society of Civil Engineers, Reston, VA.

Deierlein, G. (2003), "Background and Illustrative Examples on Proposed Direct Analysis Method for Stability Design of Moment Frames," AISC Task Committee 10 and AISC-SSRC Ad Hoc Committee on Frame Stability, Department of Civil and Environmental Engineering, Stanford University, p. 17.

Deierlein, G. (2004), "Stable Improvements: Direct Analysis Method for Stability Design of Steel-Framed Buildings," Structural Engineer, November, pp. 24–28.

Galambos, T.V. (1998), *Guide to Stability Design Criteria for Metal Structures*, 5th E, T.V. Galambos (ed.), Structural Stability Research Council, Wiley.

Galambos, T.V. and Ketter, R.L. (1959), "Columns Under Combined Bending and Thrust," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 85, No. EM2, pp. 135–152.

Giaotti, R. and Smith, B.S. (1989), "P-Delta Analysis of Building Structures," *Journal of Structural Engineering*, ASCE, Vol. 115, No. 1, pp. 755–770.

LeMessurier, W.J. (1976), "A Practical Method for Second Order Analysis. Part 1: Pin Jointed Systems," *Engineering Journal*, AISC, Vol. 12, No. 4, pp. 89–96.

LeMessurier, W.J. (1977), "A Practical Method for Second Order Analysis. Part 2: Rigid Frames," *Engineering Journal*, AISC, Vol. 14, No. 2, pp. 49–67.

Maleck, A. (2001), "Second-Order Inelastic and Modified Elastic Analysis and Design Evaluation of Planar Steel Frames," Doctoral dissertation, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, p. 579.

Martinez-Garcia, J.M. and Ziemian, R.D. (2006), "Benchmark Studies to Compare Frame Stability Provisions," *Proceedings*, SSRC Annual Technical Sessions, pp. 425–442.

Martinez-Garcia, J.M. (2002), "Benchmark Studies to Evaluate New Provisions for Frame Stability Using Second-Order Analysis," M.S. Thesis, School of Civil Engineering, Bucknell University, Lewisburg, PA, p. 757.

Maleck, A. and White, D.W. (2003), “Direct Analysis Approach for the Assessment of Frame Stability: Verification Studies,” *Proceedings*, SSRC Annual Technical Sessions, pp. 423–441.

Nair, R.S. (2005), “Stability and Analysis Provisions of the 2005 AISC Specification for Steel Buildings,” *Proceedings*, Structures Congress 2005, SEI/ASCE, p. 3.

Surovek-Maleck, A. and White, D.W. (2004), “Alternative Approaches for Elastic Analysis and Design of Steel Frames. I: Overview,” *Journal of Structural Engineering*, ASCE, Vol. 130, No. 8, pp. 1186–1196.

Surovek-Maleck, A., White, D.W., and Ziemian, R.D. (2003), “Validation of the Direct Analysis Method,” *Structural Engineering, Mechanics and Materials Report No. 35*, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA.

Vandepitte, D. (1982), “Non-Iterative Analysis of Frames Including the P-Δ Effect,” *Journal of Constructional Steel Research*, Vol. 2, No. 2, pp. 3–10.

West, M., Fisher, J., and Griffis, L.G. (2003), *Serviceability Design Considerations for Low-Rise Buildings, Steel Design Guide Series 3*, 2nd Ed., American Institute of Steel Construction, Inc., Chicago, IL, p. 42.

Wilson, E.L. and Habibullah, A. (1987), “Static and Dynamic Analysis of Multi-Story Buildings, Including P-Delta Effects,” *Earthquake Spectra*, Vol. 3, No. 2, pp. 289–298.

White, D.W. and Hajjar, J.F. (1991), "Application of Second-Order Elastic Analysis in LRFD: Research To Practice," *Engineering Journal*, AISC, Vol. 28, No. 4, pp. 133–148.

White, D.W., Surovek-Maleck, A.E., and Kim, S.-C. (2003a), “Direct Analysis and Design Using Amplified First-Order Analysis, Part 1: Combined Braced and Gravity Framing Systems,” *Structural Engineering, Mechanics and Materials Report No. 42*, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA., p. 34.

White, D.W., Surovek-Maleck, A.E., and Chang, C.-J. (2003b), “Direct Analysis and Design Using Amplified First-Order Analysis, Part 2: Moment Frames and General Framing Systems,” *Structural Engineering, Mechanics and Materials Report No. 43*, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA., p. 39.

White, D.W., Surovek, A.E., Alemdar, B.N., Chang, C.-J., Kim, Y.D., and Kuchenbecker, G.H. (2006), “Stability Analysis and Design of Steel Building Frames Using the 2005 AISC Specification,” *International Journal of Steel Structures*, Vol. 6, No. 2, pp. 71–91.

White, D.W., Surovek, A.E., and Chang, C-J. (2007), “Direct Analysis and Design Using Amplified First-Order Analysis, Part 2: Moment Frames and General Framing Systems,” *Engineering Journal*, AISC, Vol. 44, No. 4, pp. 323–340.

APPENDIX A

DERIVATION OF AMPLIFIED FIRST-ORDER ELASTIC ANALYSIS EQUATIONS

This appendix derives a specific form of the method for determining second-order forces in braced-frame systems originally presented by LeMessurier (1976). The heavily idealized model shown in Figure 4 represents the essential attributes of a story in a rectangular frame composed of gravity framing combined with a braced-frame system. Gravity framing is defined by AISC (2005a) as a "portion of the framing system not included in the lateral load resisting system." This type of framing typically has simple connections between the beams and columns and assumed to provide zero lateral load resistance. The braced-frame system is designed to transfer lateral loads and some portion of the vertical loads to the base of the structure, as well as to provide lateral stability for the full structure. The structure's sidesway stiffness is assumed to be provided solely by the braced-frame system, which AISC (2005a) allows to be idealized as a vertically-cantilevered simply-connected truss.

The vertical load carrying members in the physical structure are represented by a single column in the Figure 4 model. The bracing system is represented by a spring at the top of this column. This spring controls the relative displacement between the top and bottom of the story. An axial load of ΣP_r is applied to the model, where ΣP_r is the total *required* vertical load supported by the story. The model has a nominal initial out-of-plumbness of Δ_o, taken equal to a base value of 0.002L in the DM but taken equal to zero in conventional analysis and design by the Effective Length Method (ELM). This nominal out-of-plumbness represents the effects of unavoidable sidesway imperfections. A horizontal load, ΣH, is also applied to the model, where ΣH represents the story shear due to the applied loads on the structure. The first-order shear force in the bracing system, (F_1), is equal to ΣH. Figure 4 may be considered as a representation of a single-story braced-frame structure, or as an idealized free-body diagram of one level in a multi-story system. In the latter case, a portion of ΣH and ΣP_r is transferred from the story above the level under consideration.

Based on a first-order elastic analysis of the above model, the load ΣH produces a story drift of $\bar{\Delta}_{1H} = \Sigma H/\bar{\beta}$, where $\bar{\beta}$ is the reduced lateral stiffness of the bracing system as specified for the DM.‡ In general, the vertical loads also produce a drift of the story whenever the loadings and/or the frame geometry are not symmetric. This first-order lateral displacement is denoted by $\bar{\Delta}_{1P}$. The net lateral force in the bracing system is zero due to this displacement.§ The total first-order inter-story drift is denoted by $\bar{\Delta}_1 = \bar{\Delta}_{1H} + \bar{\Delta}_{1P}$. Summation of moments about point A gives

$$\Sigma HL + \Sigma P_r\left(\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2\right) = \bar{\beta}\left(\bar{\Delta}_{1H} + \bar{\Delta}_2\right)L \qquad \text{(A1)}$$

where the displacement $\bar{\Delta}_2$ is the additional drift due to second-order ($P\Delta$) effects. Since

$$\Sigma H = \bar{\beta}\bar{\Delta}_{1H} \qquad \text{(A2)}$$

the terms ΣHL and $\bar{\beta}\bar{\Delta}_{1H}L$ cancel on the left and right-hand sides of Equation A1, and therefore this equation becomes

$$\Sigma P_r\left(\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2\right) = \bar{\beta}\bar{\Delta}_2 L \qquad \text{(A3)}$$

If the columns are initially plumb (in other words, $\Delta_o = 0$) and the horizontal load ΣH is such that $\bar{\Delta}_1$ is also equal to zero, then one finds that either $\bar{\Delta}_2$ must be zero, or the system is in equilibrium under an arbitrary sidesway displacement $\bar{\Delta}_2$ at a vertical load

$$\Sigma P_r = \Sigma \bar{P}_{cr} = \bar{\beta}L \qquad \text{(A4)}$$

where $\Sigma \bar{P}_{cr}$ is the sidesway buckling load of the combined system. One can observe that this load is proportional to the bracing system stiffness, $\bar{\beta}$. The minimum bracing stiffness required to prevent sidesway buckling under the load ΣP_r (with Δ_o and ΣH equal to zero for a symmetrical structure

‡ The symbol β is selected for the lateral stiffness of the structural system in this paper and in White et al. (2007), consistent with the use of this term for the lateral stiffness of a bracing system in AISC (1999 & 2005a). This is different from the definition of β in LeMessurier (1976). Also, in this paper, an over bar "‾" is shown on all quantities that are influenced by the stiffness reduction employed within the DM. The equations presented are equally valid for a service load analysis or a conventional strength load analysis with zero stiffness reduction, in other words, with a stiffness reduction factor of 1.0.

§ In the context of the bar-spring model of Figure 4, the displacement $\bar{\Delta}_{1P}$ is equivalent to a lateral movement of the horizontal spring's support. See the example of Figure 3 for an illustration of the source of this displacement.

subjected to symmetrical vertical load) is defined as the *ideal stiffness* (Galambos, 1998)

$$\beta_i = \frac{\Sigma P_r}{L} \tag{A5}$$

whereas the sidesway stiffness *provided* by the bracing system is denoted by the symbol $\bar{\beta}$. Based on Equation A4, the provided stiffness can be written in terms of the sidesway buckling load as

$$\bar{\beta} = \frac{\Sigma \bar{P}_{cr}}{L} \tag{A6}$$

By solving for the displacement $\bar{\Delta}_2$ of the imperfect laterally loaded system from Equation A3, one obtains the following after some algebraic manipulation,

$$\bar{\Delta}_2 = \frac{\Sigma P_r\left(\Delta_o + \bar{\Delta}_1\right)}{\bar{\beta}L - \Sigma P_r} = \frac{\left(\Delta_o + \bar{\Delta}_1\right)}{\frac{\Sigma \bar{P}_{cr}}{\Sigma P_r} - 1} = \frac{\left(\Delta_o + \bar{\Delta}_1\right)}{\frac{\bar{\beta}}{\beta_i} - 1} \tag{A7}$$

This displacement induces a second-order lateral force in the bracing system of

$$\bar{F}_2 = \bar{\beta}\bar{\Delta}_2 = \frac{\bar{\beta}}{\beta_i}\frac{\Sigma P_r}{L}\left[\frac{\left(\Delta_o + \bar{\Delta}_1\right)}{\frac{\bar{\beta}}{\beta_i} - 1}\right] = \frac{1}{1 - \frac{\beta_i}{\bar{\beta}}}\left[\frac{\left(\Delta_o + \bar{\Delta}_1\right)}{L}\Sigma P_r\right]$$
$$= \bar{B}_{lt}\left[\frac{\left(\Delta_o + \bar{\Delta}_1\right)}{L}\Sigma P_r\right] \tag{A8}$$

where

$$\bar{B}_{lt} = \frac{\frac{\bar{\beta}}{\beta_i}}{\frac{\bar{\beta}}{\beta_i} - 1} = \frac{1}{1 - \frac{\beta_i}{\bar{\beta}}} = \frac{1}{1 - \frac{\Sigma P_r / L}{\Sigma H / \bar{\Delta}_{1H}}}$$
$$= \frac{1}{1 - \frac{\Sigma P_r}{\Sigma \bar{P}_{cr}}} \tag{A9}$$

is the sidesway displacement amplification factor. The last two forms shown in Equation A9 are the same as the expressions provided for the sidesway amplification factor, B_2, in AISC (2005a) for braced-frame systems. The symbol $\bar{B}_{lt}$ is used in this paper, since the subscripts 0, 1 and 2 are reserved to denote the initial, first-order and second-order force and/or displacement quantities in this work. The notation "*lt*" stands for "lateral translation."

Summation of all the contributions to the total drift in Figure 4 gives

$$\bar{\Delta}_{tot} = \left(\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2\right)$$
$$= \left(\Delta_0 + \bar{\Delta}_1\right) + \frac{\left(\Delta_0 + \bar{\Delta}_1\right)}{\frac{\bar{\beta}}{\beta_i} - 1} \tag{A10}$$
$$= \bar{B}_{lt}\left(\Delta_0 + \bar{\Delta}_1\right)$$

or in other words, the total drift of the story $\bar{\Delta}_{tot} = (\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2)$ is equal to the total first order displacement $(\Delta_o + \bar{\Delta}_1)$ multiplied by the sidesway displacement amplification factor, $\bar{B}_{lt}$. The total horizontal shear force developed in the

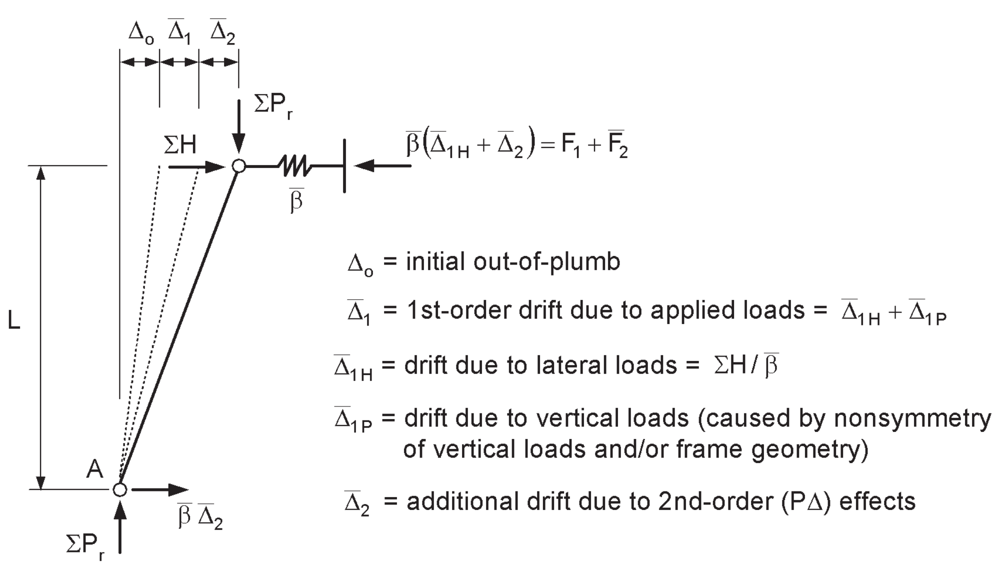

Fig. 4. Idealized model of a story in a general rectangular frame composed of gravity framing combined with a braced-frame system.

bracing system is in turn

$$\begin{aligned} F_1 + \bar{F}_2 &= \Sigma H + \bar{B}_{lt}\left(\frac{\Delta_o + \bar{\Delta}_1}{L}\right)\Sigma P_r \\ &= \Sigma H + \bar{H}_{P\Delta} \end{aligned} \tag{A11}$$

where

$$\bar{H}_{P\Delta} = \Sigma P_r \bar{B}_{lt}\left(\frac{\Delta_o + \bar{\Delta}_1}{L}\right) \tag{A12}$$

or

$$\begin{aligned} F_1 + \bar{F}_2 &= \Sigma H + \bar{B}_{lt}\left(\Delta_o + \bar{\Delta}_{1P} + \bar{\Delta}_{1H}\right)\beta_i \\ &= \Sigma H + \bar{B}_{lt}\left(\Delta_o + \bar{\Delta}_{1P} + \frac{\Sigma H}{\bar{\beta}}\right)\beta_i \\ &= \bar{B}_{lt}\left[\Sigma H + \Sigma P_r\left(\frac{\Delta_o + \bar{\Delta}_{1P}}{L}\right)\right] \end{aligned} \tag{A13}$$

From Equation A13, it is apparent that the total horizontal force in the bracing system is equal to the sidesway amplifier $\bar{B}_{lt}$ times the sum of the following internal shear forces: (1) the first order force due to the applied lateral loads, ΣH; (2) the $P\Delta$ shear force due to the initial out-of-plumbness, Δ_o; and (3) the $P\Delta$ shear force due to the lateral deflection caused by the vertical loads on the story. In the traditional AISC (2005a) no translation-lateral translation (NT-LT) analysis approach, the last of the above horizontal forces is captured by artificially restraining the structure against sidesway in an NT analysis. The reverse of the artificial reactions is then applied to the structure along with any other horizontal forces in an LT analysis. An estimate of the corresponding total second-order internal forces is then obtained by multiplying the results from the LT analysis by the corresponding story $\bar{B}_{lt}$ amplifier. White et al. (2003a) show the derivation of this procedure in the context of the above fundamental equations. However, one can see from Equation A11 that the total internal shear force in the lateral load resisting system is also simply equal to the primary (or first-order) applied load effect ΣH plus the effect of ΣP_r acting through the total amplified story drift

$$\frac{\bar{\Delta}_{tot}}{L} = \bar{B}_{lt}\frac{\left(\Delta_o + \bar{\Delta}_1\right)}{L} = \frac{\left(\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2\right)}{L}$$

Stated most directly, the total story internal shear force is simply equal to ΣH plus the $P\Delta$ shear force $\bar{H}_{P\Delta}$ given by Equation A12. Therefore, Equation A11 points to a more straightforward procedure than the traditional B_1-B_2 or NT-LT analysis approach. This equation shows that the second-order shear forces may be obtained by a first-order analysis in which equal and opposite $P\Delta$ shears $\bar{H}_{P\Delta}$ (Equation A12) are applied at the top and bottom of each story of the structure. The Engineer never needs to consider the subdivision of the analyses into artificial NT and LT parts. Additional considerations associated with $P\delta$ amplification of internal moments in moment-frame systems are addressed by White et al. (2007).

APPENDIX B

DISTRIBUTED PLASTICITY ANALYSIS RESULTS FOR LEMESSURIER'S (1976) EXAMPLE FRAME

It is informative to compare the elastic analysis and design solutions presented in the paper for LeMessurier's example frame to the results from a Distributed Plasticity Analysis. Distributed Plasticity Analysis is a useful metric for evaluation of all of the analysis and design methods, since it accounts rigorously for the effects of nominal geometric imperfections and member internal residual stresses. The load combination ($1.2D + 1.6L_r + 0.8W$) with the wind applied to the right is considered here (refer to Figure 3), since this combination gives the governing axial force requirement in the most critically loaded member, column bc. As noted previously, this load combination also produces the largest tension in brace ab. For the Distributed Plasticity Analysis, a nominal out-of-plumbness of $0.002L$ to the right is assumed throughout the frame and a nominal out-of-straightness of $0.001L$ is assumed in column bc. Also, the Lehigh residual stress pattern (Galambos and Ketter, 1959), which has a maximum residual compression of $0.3F_y$ at the flange tips and a linear variation over the half-flange width to a constant self-equilibrating residual tension in the web, is taken as the nominal residual stress distribution for the wide-flange columns. These are established parameters for calculation of benchmark design strengths in LRFD using a Distributed Plasticity Analysis (ASCE, 1997; Martinez-Garcia, 2002; Deierlein, 2003; Surovek-Maleck et al., 2003; Surovek-Maleck and White, 2004; White et al., 2006). Columns bc and de are assumed to have their webs oriented in the direction normal to the plane of the frame. The tension at the maximum load level in brace ab is significantly less than its yield load; therefore, the residual stresses in the brace are not a consideration in the Distributed Plasticity Analysis for the above load combination. A resistance factor of $\phi = 0.90$ is applied to both the yield strength, F_y, and the elastic modulus, E, including the occurrence of F_y in the above description of the nominal residual stresses. The steel is assumed to be elastic-plastic, with a small inelastic modulus of $0.0009E$ for numerical purposes. The gravity and the lateral loads are applied proportionally to the frame in the Distributed Plasticity solution. Two mixed elements (Alemdar, 2001), which are capable of accurately capturing the inelastic $P\delta$ moments in column bc as this member approaches its maximum strength, are employed to model all the members. All the members are modeled as ideally pin-connected. The roof system is modeled by a strut between the two columns, and the gravity loads from the roof system are applied as concentrated vertical loads at the top of each of the columns. The Engineer should note that it is essential to include column de in any second-order analysis model. Otherwise, the second-order effects caused by the "leaning" of this column on the lateral load resisting system are missed.

The Distributed Plasticity solution predicts a maximum load capacity of the frame at 0.936 of the maximum roof live load combination. The predominant failure mode is the flexural buckling of column bc at an axial compression of 272 kips. This load is 0.944 of the AISC (1999) column strength of $\phi_c P_n = 288$ kips based on $\phi_c = 0.90$. There is some yielding in the middle of the column unbraced length at the predicted maximum load level, but the amount of yielding is relatively minor and the frame deformations are still predominantly elastic. Approximately 10% of the column area is yielded resulting in a reduction in the effective elastic weak-axis moment of inertia of 29% at the mid-length of the column. Column bc is still fully elastic over a length of 4.5 ft at each of its ends. The above solution for the column strength is within the expected scatter band for the actual-to-predicted column strengths based on the single AISC (1999) column curve formula. The total drift of the frame at the maximum load level is $\Delta_{tot}/L = 0.0102$, including the initial out-of-plumbness of $\Delta_o/L = 0.002$. It should be noted that this drift is only slightly larger than $\Delta_{tot}/L = 0.00958$ obtained by a second-order *elastic* analysis of the structure at 0.936 of $(1.2D + 1.6L_r + 0.8W)$ using 0.9 of the nominal elastic stiffness. The tension force in brace ab is 40.8 kips at the maximum strength limit in the Distributed Plasticity Analysis, versus 38.8 kips in the above corresponding second-order elastic analysis.

If the maximum load capacity of column bc is assumed to be greater than or equal to that required to reach the design load level of $(1.2D + 1.6L_r + 0.8W)$, and if the effects of minor yielding at this load level are assumed to be negligible, the above inelastic analysis gives the second-order elastic solution (based on 0.9 of the structure nominal elastic stiffness) of $F_{bc} = 291$ kips, $F_{ab} = 44.2$ kips and $\Delta_{tot}/L = 0.0103$. The Engineer should note that this solution is only slightly less conservative than the recommended DM values of $\bar{F}_{bc} = 296$ kips from Equation 20, $\bar{F}_{ab} = 48.8$ kips from Equation 19, and $\bar{\Delta}_{tot}/L = 0.0120$ from Equation 17. The DM values account approximately for the potential additional sidesway deflections and $P\Delta$ effects associated with yielding at the maximum strength limit. The reader is referred to Martinez-Garcia (2002) for other DM and Distributed Plasticity Analysis examples involving truss framing and using the established parameters employed in the above study.

The results from the Distributed Plasticity Analysis solutions for the load combinations $0.9D + 1.6W$ and $1.2D + 0.5L_r + 1.6W$ are summarized in Table 1. The member forces for these load combinations are sufficiently small such that no yielding occurs at the strength load levels (including the consideration of residual stress effects). Therefore, these solutions are the same as obtained using a second-order elastic analysis with a nominal stiffness reduction factor of 0.9 and $\Delta_o = 0.002L$.

APPENDIX C

NOMENCLATURE

B_1 = Nonsway moment amplification factor in AISC (1999)

$B_{lt}, \bar{B}_{lt}$ = Sidesway displacement amplification factor given by Equation 2 or Equation A9

E = Modulus of elasticity

F_1 = First-order story shear force in the bracing system, equal to ΣH

$\bar{F}_2$ = Second-order shear force in the bracing system; second-order contribution to the force in a component of the lateral load resisting system

F_y = Yield stress

$H_{P\Delta}, \bar{H}_{P\Delta}$ = Story shear due to $P\Delta$ effects

L = Story height

$N_i, \bar{N}_i$ = Notional load at i^{th} level in the structure

P = Column axial load

P_{br} = Stability bracing shear force required by AISC (1999) & (2005a)

P_n = Nominal axial load resistance

P_r = Required axial load resistance

Y_i = Total factored gravity load acting on the i^{th} level

$\beta, \bar{\beta}$ = Total story sidesway stiffness of the lateral load resisting system

β_i = Story sidesway destabilizing effect, or ideal story stiffness, given by Equation 3

ϕ = Resistance factor

Δ_o = Initial story out-of-plumbness

$\Delta_1, \bar{\Delta}_1$ = First-order interstory sidesway displacement due to applied loads $= \Delta_{1H} + \Delta_{1P}$ or $\bar{\Delta}_{1H} + \bar{\Delta}_{1P}$

$\Delta_2, \bar{\Delta}_2$ = Additional interstory sidesway displacement due to second-order ($P\Delta$) effects

$\Delta_{1H}, \bar{\Delta}_{1H}$ = First-order interstory sidesway displacement due to ΣH

$\Delta_{1P}, \bar{\Delta}_{1P}$ = First-order interstory sidesway displacement due to vertical loads

$\Delta_{tot}, \bar{\Delta}_{tot}$ = Total interstory sidesway displacement

Σ = Summation

ΣH = Story shear due to the applied loads on the structure

ΣP = Total story vertical load

ΣP_r = Total required story vertical load

$\Sigma \bar{P}_{cr}$ = Story sidesway buckling load, given by Equation A4

$\bar{\ }$(over bar) = Indicates quantities that are influenced by the stiffness reduction employed in the Direct Analysis Method