Direct Analysis and Design Using Amplified First-Order Analysis Part 2: Moment Frames and General Framing Systems

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In Part 1 of this paper (White, Surovek and Kim, 2007), a method is presented for practical analysis and design of combined braced and gravity framing systems using the Direct Analysis Method (referred to in short as the DM). For braced frames, the DM involves two modifications to a conventional elastic analysis:

1. A uniform nominal out-of-plumbness of $L/500$ is included in the analysis. If desired, this nominal geometric imperfection may be modeled by a notional lateral load of

$$N_i = 0.002Y_i \qquad (1)^{\dagger}$$

at each level in the structure, where Y_i is the total factored gravity load acting at the i^{th} level.

2. The nominal stiffnesses of all the components in the structural system are factored by a uniform value of 0.8.

The rationale for these modifications is explained in Part 1. These adjustments to the elastic analysis model, combined with an accurate calculation of second-order effects, provide an improved representation of the second-order inelastic forces in the structure at the ultimate strength limit. Due to this improvement in the calculation of the internal forces, the AISC (2005a) DM bases the member axial resistance, P_n, on the actual unsupported length for all types of framing systems.

Part 1 proposes a specific underlying second-order elastic analysis approach that is particularly straightforward to apply for rectangular framing systems, thus making it simple to assess when second-order effects are or are not important and to account for these effects. Instead of amplifying the calculated first-order internal forces (including the moments in the case of moment-frame systems), the method focuses on calculating the amplified first-order sidesway interstory displacements, $\bar{\Delta}_{tot} = \bar{B}_{lt}\,(\Delta_o + \bar{\Delta}_1)$. It then applies the story $P\Delta$ shears associated with these displacements as equal and opposite forces at the top and bottom of each story to determine the second-order component of the internal forces throughout the structure. This proposed second-order elastic analysis approach is based on a form of the equations originally proposed by LeMessurier (1976). Within the limits of a number of practical approximations explained in Part 1, this approach is an exact noniterative $P\Delta$ analysis in the context of braced frames. The equation for $\bar{B}_{lt}$ in Part 1 is the same form as the sidesway amplifier for braced frames in AISC (2005a).

This paper addresses an extension of the procedures from Part 1 to moment frames and general combined framing systems. The next section of the paper explains one additional modification required by the AISC (2005a) DM for moment frames. This is followed by a generalization of the amplified first-order elastic analysis equations from Part 1 to account for $P\delta$ (P-small delta) effects in moment-frame systems. The development focuses on a simplification of the equations originally presented by LeMessurier (1977). The paper closes by demonstrating various attributes of the proposed analysis and design calculations using one of LeMessurier's (1977) example frames.

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[†]Equations 1 through 27 and A1 through A13, and Figures 1 through 4, are from Part 1 (White et al., 2007).

OVERVIEW OF THE DIRECT ANALYSIS METHOD FOR MOMENT FRAMES

Part 1 (White et al., 2007) explains the application of the AISC (2005a) DM for combined gravity and braced-frame systems. For moment frames, the DM requires one additional modification to a conventional elastic analysis: for members where the axial force, αP_r, exceeds $0.5P_y$, an additional inelastic stiffness reduction of

$$\tau = 4\left(1 - \frac{\alpha P_r}{P_y}\right)\frac{\alpha P_r}{P_y} \quad (28)$$

is applied to the member flexural rigidity, EI, or in other words,

$$E\bar{I}_e = 0.8\tau EI \quad (29a)$$

where

$E\bar{I}_e$ = member effective flexural rigidity

This modification is required to account for the more severe impact of distributed yielding on the flexural (versus the axial) deformations in certain situations. Distributed yielding has a greater effect on the flexural than on the axial rigidity particularly in cases such as weak-axis bending of I-shapes. The rationale for the above modifications is discussed in detail by White, Surovek-Maleck, and Kim (2003a), White, Surovek-Maleck, and Chang (2003b), Surovek-Maleck and White (2004), and White, Surovek, Alemdar, Chang, Kim, and Kuchenbecker (2006). The reader is referred to Maleck (2001), Martinez-Garcia (2002), Deierlein (2003 and 2004), Surovek-Maleck, White, and Ziemian (2003), Maleck and White (2003), Nair (2005), and Martinez-Garcia and Ziemian (2006) for other detailed discussions as well as validation and demonstration of the DM concepts.

AMPLIFIED FIRST-ORDER ELASTIC ANALYSIS EQUATIONS

With two minor modifications, the amplified first-order elastic analysis procedure presented in Part 1 can be applied to general rectangular frames composed of any combination of moment, braced, and gravity systems. These modifications are:

1. An additional term must be included in the sidesway displacement amplifier, $\bar{B}_{lt}$, to account for the influence of $P\delta$ (P-small delta) moments in moment-frame columns on the sidesway response.

2. The "no-translation" (or NT) moment amplifier, B_1, in AISC (2005a) based on the reduced elastic stiffness, should be applied to the total column moments.

The next section explains how $P\delta$ moments influence the sidesway response when moment frames provide part of the lateral load resistance. A simplified form of LeMessurier's (1977) equations is developed that accounts for these effects. This is followed by an explanation of the concepts behind the above application of the NT moment amplifier to the total column moments.

In some cases where the axial compression in the beams is particularly large, for example in certain portal frames due to the horizontal thrust from the foundation, one other modification is necessary to account for the corresponding reduction in the flexural stiffness of the beams. The reduction in the beam flexural stiffnesses due to axial compression is generally smaller then 5% and may be neglected whenever $\alpha P_r \leq 0.05\ P_{e(L)}$,

where

P_r = the required axial force in the member for the load combination being considered either under LRFD or ASD

α = the factor by which the applied loads are increased above the LRFD or ASD load combination levels to reach the ultimate load level, equal to 1.0 for LRFD and 1.6 for ASD

$\bar{P}_{e(L)}$ = the Euler buckling load $\pi^2 E\ \bar{I}_e/L^2$ based on the unsupported length of the member in the plane of bending

Otherwise, the influence of the axial compression may be accounted for conservatively by using an effective flexural rigidity of

$$E\bar{I}_e' = [1 - \alpha P_r / \bar{P}_{e(L)}]\ E\bar{I}_e \quad (29b)$$

for the beam members (AISC, 2005a; King, 2001).

The suggested form of LeMessurier's equations may be applied as an approximate solution for nonrectangular gable and monoslope portal frames. To the authors' knowledge, Eurocode 3 (CEN, 2003) provides the only published guidelines for the applicability of rectangular frame idealizations to these structure types. CEN (2003) suggests that the rectangular frame idealizations associated with Equations 2 through 5 (see Part 1) are sufficient without the additional stiffness reduction of Equation 29b as long as: (1) the roof slope is not steeper than 1:2 (26°); and (2) $\alpha P_r \leq 0.09\bar{P}_{e(L)}$ in the rafters, with L taken as the full system length along the rafters. Furthermore, Equation 5 should be applied on a column-by-column basis at each story or level in frames containing sloping members, unequal height columns, and/or flexible diaphragms. This accounts for the fact that the displacements, $\bar{\Delta}_1$, can be significantly different for the different columns in these types of frames. The authors suggest further study of the above Eurocode limits to quantify their implications fully with respect to the accuracy of the structural responses in gable and monoslope portal frames. For general geometries and loadings, the authors recommend the use of a general-purpose second-order analysis that accurately accounts for both $P\Delta$ and $P\delta$ effects.

Modification of the Sidesway Amplification Factor to Account for $P\delta$ Effects in Moment-Frame Columns on the Sidesway Response

Figure 5 shows a conceptual model similar to the one for combined gravity and braced frames in Figure 4. This figure represents a hypothetical case in which the moment-frame system, represented by the cantilever column on the right-hand side with an elastic spring at its base, supports zero gravity (vertical) load. The only change from the model of Figure 4 is that the linear spring of stiffness $\bar{\beta}$ is replaced by a flexural system of the same lateral stiffness. The single lateral load resisting column in Figure 5 represents the contributions from all the columns of the physical moment-frame system to the structure's sidesway stiffness (or flexibility). The rotational spring at the base of this column represents the contributions from the beams and connections of the moment-frame system. The amplified first-order analysis equations presented in Part 1 apply to this conceptual model without any modification. In other words, in the hypothetical case that the lateral load resisting system supports zero gravity load, it does not matter whether this system is composed of a truss, a shear panel, or a combination of flexural members. Sidesway stiffness is sidesway stiffness regardless of the source.

The reader familiar with LeMessurier (1977) should note that the symbol β in this paper represents the total sidesway stiffness of the lateral load resisting system, consistent with the AISC (1999 and 2005a) term for the lateral stiffness of a bracing system. This is different from LeMessurier's (1977) notation for β. LeMessurier's β parameter is denoted in this paper by the term β_L. In the context of the model shown in Figure 5,

$$\beta = \frac{\beta_L E I_c}{L^3} \tag{30}$$

where $\beta_L = 3$ for the cantilever column shown, if the elastic spring at its base is effectively rigid, and $\beta_L = 12$ for a column with rigid rotational restraint at both ends. Also, the relationship between the story stiffness, β, and LeMessurier's P_L parameter, in other words, the contribution from a column to the story stiffness in terms of the rotational displacement, Δ_1/L, is

$$\beta L = P_L = \frac{\beta_L E I_c}{L^2} \tag{31}$$

In the general case where the moment-frame system also supports significant gravity load, a free-body diagram such as that shown in Figure 6 must be considered. The term, $\Sigma_m P_r$, in this figure represents the total gravity load supported by

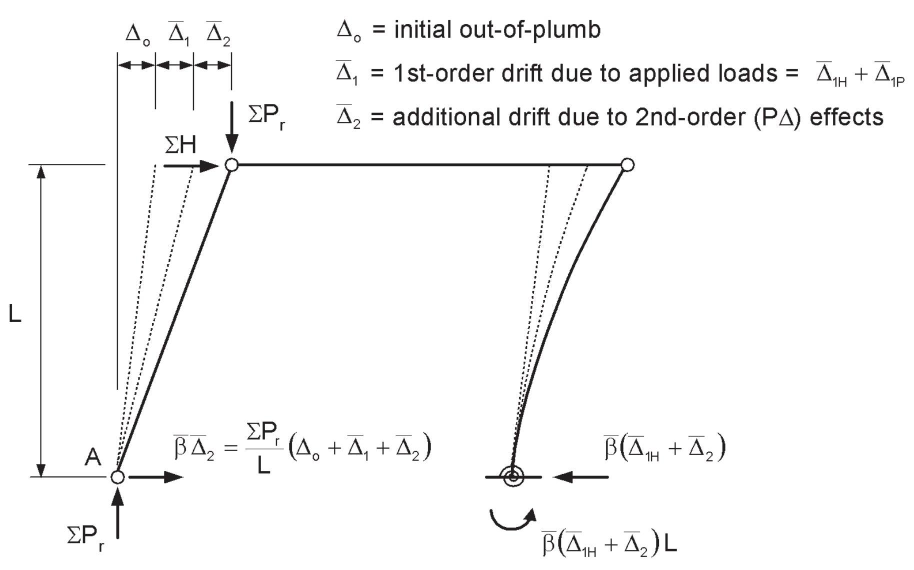

Fig. 5. Idealized model of a story in a general rectangular frame composed of gravity framing combined with a moment-frame system—zero vertical load applied to the moment-frame system.

the moment-frame system, whereas $\Sigma_g P_r$ represents the vertical load supported by gravity columns. LeMessurier (1977) provides a useful approximate method for second-order analysis of this type of structure. In the limit that the lateral load resisting columns are relatively rigid and the flexibility of the moment frame is predominantly due to deformations in its beams and connections, the lateral load resisting columns remain essentially straight under the drift of the story. Therefore, the solution presented in Part 1 also applies to this problem without modification. However, if the flexural rigidities of the beams and connections in the moment-frame system are relatively large, additional $\Sigma_m P_r \delta$ moments exist in the lateral load resisting columns as illustrated by the diagram on the right-hand side of Figure 6. These $P\delta$ (P-small delta) moments are the effect of the column axial forces acting through the transverse displacements relative to the member chord. These moments cause sidesway displacements in addition to the first-order displacements associated with the shear, ΣH, and the second-order displacements associated with the $P\Delta$ moments, $\Sigma P_r\ (\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2)\ x/L$. That is, they cause an additional amplification of the first-order lateral displacements, $(\Delta_o + \bar{\Delta}_1)$.

Based on the developments presented by LeMessurier (1977), the above $P\delta$ effects may be accounted for by the following modified form of Equation A10,

$$\bar{\Delta}_{tot} = \left(\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2\right) = \bar{B}_{lt}\left(\Delta_o + \bar{\Delta}_1\right) \tag{32}$$

$$= \frac{1}{1 - \dfrac{\Sigma P_r + \Sigma_m C_L P_r}{\beta L}}\left(\Delta_o + \bar{\Delta}_1\right)$$

The additional term, $\Sigma_m C_L P_r$, accounts for the $P\delta$ effects on the sidesway response of the columns, and is a summation over all the moment-frame columns of the individual member parameter, C_L, multiplied by the axial compression in each member. LeMessurier (1977) shows that C_L depends on the ratio of the column stiffness relative to the components that provide rotational restraint at its ends. That is, C_L depends on the joint stiffness factor

$$G = \frac{\Sigma EI_c / L}{\Sigma EI_b / L_{beff}} \tag{33}$$

at the top and bottom of each column, where $\Sigma EI_b/L_{beff}$ is the sum of the effective stiffnesses provided by the beams and their connections at the beam-column joints. For fully-restrained (FR) systems,

$$L_{beff} = L_b\left(2 - \frac{M_F}{M_N}\right) \tag{34}$$

where

M_F = moment at the far end of the beam being considered

M_N = moment at the near end, obtained from a first-order lateral load analysis of the frame (AISC, 2005a)

See ASCE (1997) for calculation of $\Sigma EI_b/L_{beff}$ for other cases, such as PR frames.

Generally, C_L varies from a value of 0.216, for columns with both ends rigidly restrained (in other words, $G = 0$ at both ends) or with one end fixed and rotation unrestrained at

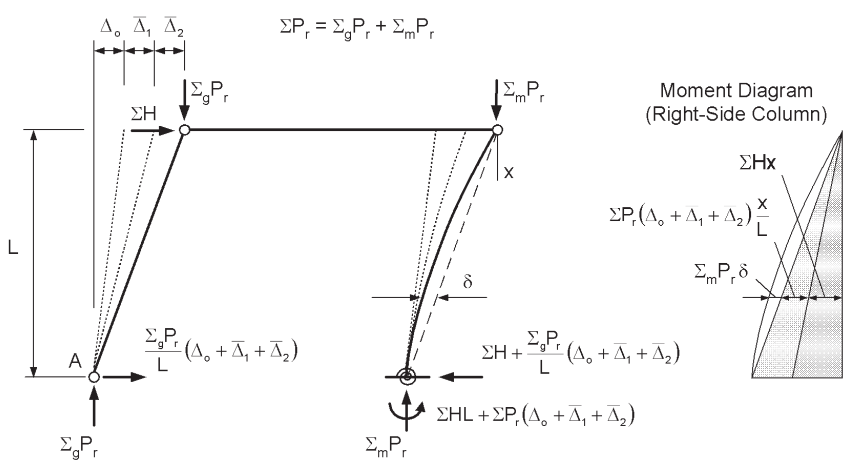

Fig. 6. Idealized model of a story in a rectangular frame composed of gravity framing combined with a moment-frame system—vertical loads of $\Sigma_m P_r$ *supported by the moment-frame system and* $\Sigma_g P_r$ *by the gravity framing system.*

the other end, to a value of zero when both ends of the member are ideally pinned. In the latter case, the column is of course no longer part of the lateral load resisting system. It is a pin-ended gravity column (or a bracing system member) in which δ (the deflection relative to the rotated chord) is zero along the entire member length. Therefore, the $P\delta$ moment is zero and $C_L = 0$ in this type of column.

LeMessurier's (1977) procedure requires some effort to calculate C_L for each of the moment-frame columns. Although simpler methods of determining C_L have been developed (LeMessurier, 1993), these methods still involve a calculation for each of the column members. A more efficient approach can be devised by using a conservative "average" C_L. This is accomplished by rewriting the sidesway amplification factor in Equation 32 as

$$\bar{B}_{lt} = \frac{1}{1-\dfrac{\Sigma P_r + \Sigma_m C_L P_r}{\bar{\beta} L}} = \frac{1}{1-\dfrac{\Sigma P_r (1+C_{Lavg})}{\bar{\beta} L}} = \frac{1}{1-\dfrac{\Sigma P_r / L}{\bar{\beta} R_M}} = \frac{1}{1-\dfrac{\beta_i}{\bar{\beta} R_M}} \tag{35}$$

where

$$\frac{1}{R_M} = \left(1 + C_{Lavg}\right) \tag{36}$$

By using LeMessurier's (1977) equations, one can show that if $G_A \le G_B$ and $G_A \ge 0.2$, where G_A and G_B are the joint stiffness factors at the two ends of a column, C_L is always less than or equal to 0.18 and can in fact be as small as 0.11. Therefore, if $G = 0.2$ is assumed as a representative lower bound for the joint stiffness factor in practical moment-frame systems, C_{Lavg} can be taken conservatively as 0.18 and thus R_M in Equation 36 can be taken as 0.85. AISC (2005a) specifies this R_M value for all systems that include moment frames for part of the lateral load resistance. In cases where one or both ends of all the members are ideally fixed, the maximum unconservative error in the effective sidesway buckling resistance, $\bar{\beta} R_M$, associated with the use of $C_{Lavg} = 0.18$ and $R_M = 0.85$, is 3%.

The above approximation ignores the fact that $C_L = 0$ in any columns belonging to gravity or braced-frame systems. A better approximation for general systems involving combined gravity, moment and braced frames is

$$R_M = 1 - 0.15 \frac{\Sigma_m P_r}{\Sigma P_r} \tag{37}$$

where

$\Sigma_m P_r$ = sum of the vertical loads supported by the columns in the moment frame(s)

Based on Equation 37, R_M varies from 0.85 when all the columns in the story are moment-frame columns to a value of 1.0 for a story that contains only gravity and braced-frame columns. This equation is related to Equation C-C2-7 for the parameter R_L, as it appears in the Commentary to the AISC *Specification for Structural Steel Buildings* (AISC, 2005a), that is used in the calculation of sidesway buckling loads.

One exception can be made to the calculation of R_M by Equation 37. Based on LeMessurier's (1977) equations, if $G_A \le G_B$ and $G_A \ge 4$, C_L is always less than 0.03, and in fact can be smaller than 0.01. Therefore, assuming that 3% maximum underestimation of the sidesway buckling load [implicit in the use of $R_M = 0.85$ in AISC (2005a)] is acceptable, then in stories where all the moment-frame G values are greater than or equal to 4, C_{Lavg} may be taken as zero. This rule can be stated as

$$R_M = 1.0 \text{ for } G_{min} \ge 4 \tag{38}$$

where

G_{min} = smallest joint stiffness factor in the moment-frame system(s) of the story

In summary, based on the above developments, the total sidesway displacements in a story may be calculated using the simplified equation,

$$\bar{\Delta}_{tot} = \left(\Delta_o + \bar{\Delta}_1 + \bar{\Delta}_2\right) = \bar{B}_{lt}\left(\Delta_o + \bar{\Delta}_1\right) = \frac{1}{1-\dfrac{\beta_i}{\bar{\beta} R_M}}\left(\Delta_o + \bar{\Delta}_1\right) \tag{39}$$

where R_M is calculated from Equations 37 and 38.

Given the total sidesway displacement calculated from Equation 39, the total internal lateral bending moment in the story may be expressed as

$$\Sigma \bar{M} = \Sigma M_{1H} + \Sigma P_r \bar{\Delta}_{tot} = \Sigma M_{1H} \left[\frac{1 + \dfrac{\beta_i}{\bar{\beta}}\left(1 - \dfrac{1}{R_M}\right)}{1 - \dfrac{\beta_i}{\bar{\beta}} \dfrac{1}{R_M}} \right] + \Sigma P_r \Delta_0 \frac{1}{1 - \dfrac{\beta_i}{\bar{\beta} R_M}} \tag{40}$$

after substituting Equation 39 for $\bar{\Delta}_{tot}$ and some algebraic simplification, where $\Sigma M_{1H} = \Sigma HL$. If the initial out-of-plumbness is taken equal to zero, or if the "first-order" $P\Delta$ shears due to the nominal out-of-plumbness, $\Sigma P_r \Delta_0 / L$, are applied to the structure, producing the notional lateral loads specified by Equation 1 in addition to the applied lateral loads, ΣH, Equation 40 may be written simply as

$$\Sigma\bar{M} = \Sigma M_{1H} + \Sigma P_r \bar{\Delta}_{tot}$$

$$= \Sigma M_{1H} \left[\frac{1 + \frac{\beta_i}{\bar{\beta}}\left(1 - \frac{1}{R_M}\right)}{1 - \frac{\beta_i}{\bar{\beta}} \frac{1}{R_M}} \right] \quad (41)$$

where, in this case, ΣM_{1H} includes the effect of any nonzero notional lateral loads.

Equation 41 shows that in general the total story internal sidesway moment is amplified differently than the story sidesway displacement (see Equation 39). LeMessurier (1977) also shows this result, but in the context of his parameters β_L and P_L. Since R_M is generally less than or equal to 1.0, the internal moment amplification is always smaller than the sidesway displacement amplification. However, at the limit of $R_M = 1.0$, the amplification of the internal moments (Equation 41) and the amplification of the sidesway interstory displacements (Equation 39) are identical. At this limit, the sidesway amplifier reduces to Equation 2, derived for braced-frame systems in Appendix A of Part 1 (White et al., 2007).

The proposed amplified first-order elastic analysis method does not amplify the first-order internal sidesway moments directly, as in Equations 40 and 41. Rather, the total sidesway displacements are calculated from Equation 39. Then the corresponding $P\Delta$ shears,

$$\bar{H}_{P\Delta} = \Sigma P_r \bar{B}_{lt} \left(\frac{\Delta_o + \bar{\Delta}_1}{L} \right) \quad (5)$$

are applied to the structure as equal and opposite forces at the top and bottom of each story to determine the second-order components of all the internal forces throughout the structure. This avoids the need for separate artificial "no lateral translation" (NT) and "with lateral translation" (LT) analyses, and leads to a more general, straightforward and intuitive calculation of the second-order internal forces than the application of story-based amplification factors such as in Equations 40 and 41.

Application of the NT Amplification Factor to the Total Column Moments

In practically all cases where moment frames are used to provide a portion of the lateral load resistance, and where the moment-frame columns are idealized as not having any transverse loads along their length, the traditional AISC (2005a) "no-translation" moment amplifier is equal to 1.0. In the context of the DM, this amplifier can be expressed as

$$\bar{B}_{nt} = \frac{C_m}{1 - \frac{P_r}{\bar{P}_{e.nt}}} \geq 1.0 \quad (42)$$

where

C_m = equivalent uniform moment factor defined in AISC (2005a)

$\bar{P}_{e.nt}$ = member elastic buckling load calculated assuming that sidesway is prevented (but using the reduced elastic flexural rigidity given by Equation 29a)

In structures where $\bar{B}_{nt} = 1.0$ in all of the moment-frame columns, no further modification of the step-by-step procedures outlined in Part 1 is necessary. Furthermore, in cases where $\bar{B}_{nt} \geq 1.0$ in some of the moment-frame columns, the additional $P\delta$ effects on the column moments can be approximated conservatively by applying Equation 42 to the *total* member moments, including the sidesway $P\Delta$ moments caused by the loads, $\bar{H}_{P\Delta}$ (Equation 5). The authors consider this approximation to be acceptable since $\bar{B}_{nt}$ is often relatively close to 1.0 even in extreme cases where the NT moment amplifier is greater than one. Also, in cases where B_{nt} (calculated using the nominal elastic stiffness) is greater than 1.2, the Commentary of AISC (2005a) recommends the use of a rigorous (in other words, general-purpose) second-order elastic analysis. In such cases, the $P\delta$ moments in the columns significantly increase the column NT deformations, and therefore reduce the effective column flexural stiffnesses. This reduction in the column stiffnesses influences the end restraint provided to the beams by the column members, as well as the distribution of the internal moments throughout the moment frames. A typical result is that the column NT moments are actually reduced, while the positive bending moments in the beams are increased. Simplified moment amplification procedures, such as the AISC (2005a) NT-LT method, do not have any chance of providing accurate estimates of the internal moments in these extreme cases. In unusual cases where B_{nt} or $\bar{B}_{nt}$ is significantly greater than 1.0, the authors suggest that it is prudent for the Engineer to use a carefully tested and validated general-purpose second-order analysis program that accurately captures both $P\Delta$ and $P\delta$ effects. The AISC (2005a) Appendix 7 Commentary suggests a few simplified benchmark problems for testing of second-order analysis software. Other more general second-order elastic analysis benchmark problems are available from references such as Clarke, Bridge, Hancock, and Trahair (1993). White et al. (2003b) provide additional discussions regarding the handling of $P\delta$ effects in frames containing members with B_{nt} or $\bar{B}_{nt}$ significantly greater than 1.0.

LEMESSURIER'S DESIGN EXAMPLE 4

Figure 7 shows the framing plan and elevation of a 30-story apartment building that LeMessurier (1977) presented as his fourth design example. In this system, story deep staggered trusses span the 60 ft width of the building. Since these trusses fully brace the columns in the N-S direction, the column webs are oriented in the E-W direction. This example focuses on the approximate analysis and preliminary design of the interior columns between the third and fourth floors as well as the spandrels at these levels (see Figure 7c). LeMessurier (1977) specifies a distributed gravity load of w_g =125 psf = $D + L$ at each level of the building (including the roof, for simplicity) as well as a spandrel load of q_g = 0.5 kip/ft = $D + L$. A live-to-dead load ratio of 1.5 is assumed for both of these loadings in this paper, thus giving D = 50 psf and L = 75 psf as the specific distributed gravity loads. The wind load, p_w, is taken as 24 psf as specified by LeMessurier. All the structural framing is ASTM A36 steel and the girders are assumed to be fully braced, as in the original design. The load combinations considered for this example are the service combination (ASCE, 2005),

$$D + 0.5L + 0.7W$$

as well as the strength combinations (ASCE, 2005),

$$1.2D + 1.6L \text{ and } 1.2D + 0.5L + 1.6W$$

Since the columns in this example are subjected to full reversed-curvature bending, $B_{nt} = \bar{B}_{nt} = 1.0$ from Equation 42 in all cases (under all service and strength loadings).

Base First-Order Analysis, Nominal (Unfactored) Loads

This section summarizes the approximate analysis for preliminary as well as potentially final analysis and design of LeMessurier's Example 4. The approximate analysis is based on isolating a typical subassembly around an interior column in the story under consideration (see Figure 7c). The subassembly is isolated by making traditional simplifying assumptions about the manner in which the gravity and wind loads are distributed, as well as how the subassembly participates with the rest of the structure in providing lateral load resistance and stability for the full system.

Since the geometry of the frame is symmetrical and the gravity loads are applied symmetrically in this example,

$$\frac{\Delta_{1P}}{L} = 0 \tag{43}$$

The gravity axial load on an interior column just below the fourth floor comes from 28 levels (including the roof) and is calculated as

$$P = 28L_b \frac{B}{2} w_g = 2{,}625 \text{ kips} \tag{44}$$

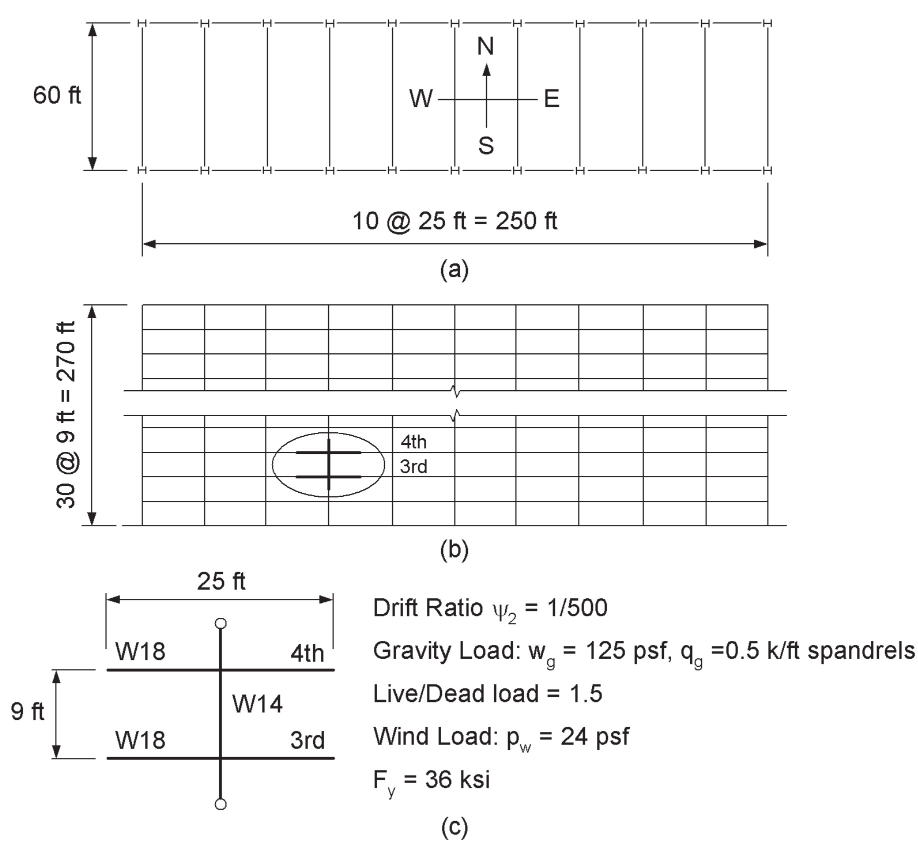

Fig. 7. LeMessurier's (1977) Example 4 frame.

The spandrels have a maximum nominal gravity moment estimated as

$$M_g = q_g L_b^2/12 = 26.0 \text{ kip-ft} \quad (45)$$

The nominal $(D + L)$ moment in the interior columns is taken as zero, as in LeMessurier (1977).

The nominal wind shear in each interior column between the third and fourth floors is

$$H = 27.5L\frac{B}{2}p_w\frac{1}{10} = 17.8 \text{ kips} \quad (46)$$

by the portal method, where 27.5 levels contribute to the wind shear, and the column shear forces are assumed to be distributed two parts to the interior columns and one part to the exterior members. The resulting nominal wind moment in the interior columns is

$$M_w = HL/2 = 80.2 \text{ kip-ft} \quad (47)$$

which is also the average wind moment in the third and fourth floor spandrels. The interior column axial force due to wind is zero based on the portal method idealization. The changes in the column axial forces due to lateral loading or general sidesway can be included in the proposed analysis approach (by performing an analysis of the full structure), but are expected to be small for this frame. Similarly, the sidesway deflections due to cantilever action are expected to be small for this 30-story frame, since its overall height-to-width ratio is close to 1.0.

Design Requirements for Acceptable Drift at Service Load Levels

LeMessurier (1977) makes a compelling argument that in general, second-order $P\Delta$ effects should be considered when checking service drift limit states. The authors suggest that at the least, when the drift limit is established using specific rational criteria such as preventing damage to nonstructural elements, it is imperative to compare the calculated second-order drifts to the rationally determined drift limit. Conversely, in cases where the drift limits are more arbitrary, the engineer should consider what actual service drifts can be tolerated. LeMessurier develops an equation for a required first-order drift limit, ψ_1, given the following:

1. A specified limit on the actual second-order drift, ψ_2.
2. The distributed wind load used for the analysis, p_w.
3. An average gravity load distributed over the interior volume of the structure, γ_g.
4. The width over which γ_g acts in the plane of the framing system(s) being considered, ΣL_b.

The following equation is the same as LeMessurier's, except the factor R_M from Equations 37 and 38 is included in the development,

$$\psi_1 = \frac{p_w}{\dfrac{p_w}{\psi_2} + \dfrac{\gamma_g \Sigma L_b}{R_M}} \quad (48)$$

This equation accounts for $P\Delta$ effects at the specified service load level in converting from the specified second-order drift limit, ψ_2, to the corresponding first-order limit, ψ_1‡. The term R_M is included in Equation 48 since its effect on the sidesway displacement amplification can be significant at service load levels in some cases§. Nevertheless, the subsequent calculations for the example frame show that $R_M = 1.0$ is sufficient based on Equation 38. Therefore, if a second-order drift limit of $\psi_2 = 0.002 = 1/500$ is selected for the load case, $D + 0.5L + 0.7W$, one obtains

$$\begin{aligned}\psi_1 &= \frac{0.7p_w}{\dfrac{0.7p_w}{0.002} + \dfrac{0.7w_g}{L}\dfrac{\Sigma L_b}{R_M}} \\ &= \frac{0.7(24 \text{ psf})}{\dfrac{0.7(24 \text{ psf})}{0.002} + \dfrac{0.7(125 \text{ psf})}{9 \text{ ft}}\dfrac{250 \text{ ft}}{1.0}} \\ &= 0.00155 \\ &= \frac{1}{645}\end{aligned} \quad (49)$$

where the 0.7 factor on w_g is based on the service dead load factor of 1.0, the live load factor of 0.5, and the assumed

‡ Also, Equation 41 is based on the assumption that the tributary widths perpendicular to the plane of the frame are the same for both the gravity and the wind loads.

§ LeMessurier (1977) suggests that the effect of C_L, and thus the effect of R_M, can be neglected when B_{lt} with $R_M = 1.0$ is less than 1.50. This is because the resulting maximum unconservative error in the amplification of the sidesway moments is 3% at this value of B_{lt}. However, the maximum error in the amplification of the sidesway displacements is 10% at this value of B_{lt}.

live-to-dead load ratio of 1.5, in other words, [1(1) + 0.5(1.5)] / [1 + 1.5] = 0.7. The limit in Equation 49 corresponds to a first-order drift under the nominal wind loading of

$$\frac{\Delta_{1H}}{L} = \frac{\psi_1}{0.7} = \frac{1}{451} \tag{50}$$

This is approximately the same as the first-order drift limit selected by LeMessurier (1977) to restrict the second-order drift resulting from application of the load combination, $D + L + W$, to 1/300.

Given the above ψ_1 limit, the minimum nominal sidesway stiffness required from each interior column subassembly of the moment frame (assuming each interior column contributes two parts to the total story stiffness and each exterior column contributes one part) is

$$\beta = \frac{0.7H}{\psi_1 L} = 74.5\,\frac{\text{kips}}{\text{in.}} \tag{51}$$

In the following, a trial size is selected for the columns of the example frame based on the required strength resulting from application of the load combination, $1.2D + 1.6L$, assuming $\beta = 74.5$ kip/in. and a column inelastic stiffness reduction factor, $\tau = 1.0$ (Equation 28). Once this preliminary column size is determined, a trial size is then determined for the spandrels to achieve the required nominal sidesway stiffness of Equation 51. Subsequently, final strength checks are performed for the columns and the spandrels.

Trial Column Size for (1.2*D* + 1.6*L*)

Based on the DM, the column strength term, $\phi_c P_n$, for the example frame is always governed by weak-axis flexural buckling with $KL_y = L = 9$ ft, since K is taken equal to 1.0 in both the strong and weak-axis directions. Furthermore, in the DM, the framing system is analyzed with a nominal initial out-of-plumbness of $\Delta_o = 0.002L$ and a reduced elastic stiffness. As a result, a more rational estimate is obtained for the required internal forces than in traditional analysis and design methods.

The ideal story stiffness per interior column under the maximum live load combination is

$$\beta_i = \frac{1.44P}{L} = 35.0\,\frac{\text{kips}}{\text{in.}} \tag{52}$$

where 1.44 = [1.2(1) + 1.6(1.5)] / [1 + 1.5]. Therefore, based on $R_M = 1.0$, assuming that this is an accurate value for the example frame, and using $\beta = 74.5$ kip/in. from Equation 51 as a target,

$$\bar{B}_{lt} = \frac{1}{1 - \dfrac{\beta_i}{0.8R_M\beta}} = 2.42 \tag{53}$$

This gives a total story drift at the maximum strength limit of

$$\frac{\bar{\Delta}_{tot}}{L} = \bar{B}_{lt}\frac{\Delta_o}{L} = 0.00485 = \frac{1}{206} \tag{54}$$

and a story $P\Delta$ shear per column of

$$\bar{H}_{P\Delta} = 1.44P\frac{\bar{\Delta}_{tot}}{L} = 18.3 \text{ kips} \tag{55}$$

The resulting required column flexural strength is

$$M_r = \bar{H}_{P\Delta}L\frac{1}{2} = 82.5 \text{ kip-ft} \tag{56}$$

Given this required moment and the required axial load capacity of $P_r = 1.44P = 3{,}780$ kips, a W14×455 can be selected using a beam-column design aid such as in AISC (2005b), but with $F_y = 36$ ksi. The applicable beam-column interaction check is

$$\frac{P_r}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_r}{\phi_b M_n}\right) = 0.899 + \frac{8}{9}(0.033) = 0.928 \le 1.0 \tag{57}$$

where $\phi_c P_n = 4{,}205$ kips for $KL_y = 9$ ft and $\phi_c = 0.9$, and $\phi_b M_n = \phi_b M_p = 2{,}527$ kip-ft for $L_b = 9$ ft. The column $P\Delta$ moment due to the initial nominal out-of-plumbness does not influence the column size selection, but increases the interaction equation value from 0.899 to 0.928.

At this point, it is apparent that the ratio of the required axial strength to the yield load of the selected column cross-section is

$$\frac{P_r}{P_y} = \frac{P_r}{A_g F_y} = \frac{3{,}780 \text{ kips}}{4{,}824 \text{ kips}} = 0.784 \tag{58}$$

Therefore, the columns in the example frame experience substantial distributed yielding due to the applied load plus residual stress effects under the loading ($1.2D + 1.6L$). As a result, the inelastic stiffness reduction factor, τ, (Equation 28) needs to be considered. This is accomplished after preliminary sizes are selected for the spandrels. The column inelastic stiffness reduction is also neglected in the preliminary design of the spandrels.

Selection of the Spandrels Based on Drift Control

Based on the trial column size W14×455 ($I_c = 7{,}190$ in.^{4}) and the required β from Equation 51, LeMessurier's stiffness factor, βL, is

$$\beta_L = \frac{\beta L^3}{EI_c} = 0.451 \tag{59}$$

LeMessurier (1977) provides the following equation for the required joint stiffness factor at the top and bottom of a column when the end restraints are equal and points of inflection exist at the middle of the column and the beam lengths,

$$\left[G = \frac{\Sigma I_c / L}{\Sigma I_b / L_b}\right] = \left(\frac{12}{\beta_L} - 1\right) \tag{60}$$

At this stage, if one assumes equal I_c/L for the columns above and below the story as in LeMessurier (1977), all of the quantities in Equation 60 are established except I_b. By solving Equation 60 for I_b, one obtains the minimum girder moment of inertia necessary to provide the required β from Equation 51, or to satisfy the drift constraint of Equation 49. This value is $I_b = 780$ in.^{4} Based on the assumption that it is desirable to restrict the girder nominal depths to 18 in. within the 9 ft story height, the most economical section that satisfies this service drift-based requirement is a W18×50 with $I_x = 800$ in.^{4} LeMessurier (1977) also selects this member in his final design.

It is informative to check the drift of the frame under the service load combination, $D + 0.5L + 0.7W$, using the above W14×455 column and W18×50 spandrels. The ideal stiffness for this load case is

$$\beta_i = \frac{0.7P}{L} = 17.0 \text{ kips} \tag{61}$$

where the 0.7 factor in this equation is based on $(D + 0.5L)$ and $D/L = 1.5$. Furthermore,

$$G = \frac{\Sigma I_c / L}{\Sigma I_b / L_b} = 25.0 \tag{62}$$

for the above preliminary design sections, assuming the same I_c/L in the adjacent stories. This gives

$$\beta_L = \frac{12}{(1+G)} = 0.462 \tag{63}$$

(LeMessurier, 1977), and

$$\beta = \frac{\beta_L E I_c}{L^3} = 76.5 \frac{\text{kips}}{\text{in.}} \tag{64**}$$

Given the above values, and using R_M = 1.0 based on Equation 38,

$$B_{lt} = \frac{1}{1 - \dfrac{\beta_i}{\beta}} = 1.29 \tag{65}$$

and

$$\frac{\Delta_{tot}}{L} = B_{lt} \frac{0.7H}{\beta} \frac{1}{L} = 0.00194 = \frac{1}{515} \tag{66}$$

Therefore, the selected design satisfies the maximum second-order drift limit of 1/500.

At this point, preliminary sections have been selected for the interior columns and the girders based on an estimate of the required strength of the columns under $(1.2D + 1.6L)$ plus the subsequent spandrel requirements for drift control. The next section shows the preliminary design of the spandrels for strength. This is followed by the final strength checks for the column and spandrel members. These strength checks are compared to values obtained using the traditional Effective Length Method (ELM), amended as specified in AISC (2005a). Appendix A compares the elastic analysis-design solutions from both the DM and the ELM to the results from a Distributed Plasticity Analysis. Distributed Plasticity Analysis is a useful metric for evaluation of all of the analysis and design methods, since it accounts rigorously for the effects of nominal geometric imperfections and member internal residual stresses.

Selection of Spandrels Based on Strength

The above column moment calculations for $(1.2D + 1.6L)$ and the use of the trial β of 74.5 kips/in. also apply to the calculation of the average maximum spandrel moments in the third and fourth floor levels. That is, the required spandrel moments due to $P\Delta$ effects are $\bar{H}_{P\Delta}L/2$ = 82.5 kip-ft at this load level from Equation 56. By adding this moment to the moment at the beam ends due to the gravity load, one obtains

$$\begin{aligned} M_r &= 1.44M_g + \bar{H}_{P\Delta}L/2 \\ &= 37.5 \text{ kip-ft} + 82.5 \text{ kip-ft} = 120 \text{ kip-ft} \end{aligned} \tag{67}$$

The W18×50 spandrels easily satisfy these required moments.

The required girder moments for the load combination, $1.2D + 0.5L + 1.6W$, are based on $P_r = 0.78P$ = 2,048 kips, where 0.78 = [1.2(1) + 0.5(1.5)] / (1 + 1.5), and the corresponding ideal story stiffness per interior column is

$$\beta_i = \frac{P_r}{L} = 19.0 \frac{\text{kips}}{\text{in.}} \tag{68}$$

** In lieu of Equations 55 through 57, β can be calculated directly from the wind analysis as $\beta = \Sigma H / \Delta_{1H}$.

Based on the DM, using R_M = 1.0 and the actual β = 76.5 kips/in. from Equation 64, the sidesway amplification factor for this load case is

$$\bar{B}_{lt} = \frac{1}{1 - \dfrac{\beta_i}{0.8 R_M \beta}} = \frac{1}{1 - \dfrac{19.0 \text{ kips/in.}}{0.80(1.0)(76.5 \text{ kips/in.})}} = 1.45 \quad (69)$$

This gives a total story drift of

$$\frac{\bar{\Delta}_{tot}}{L} = B_{lt}\left(\frac{\Delta_o}{L} + 1.6\frac{\Delta_{1H}}{L}\frac{1}{0.8}\right) \quad (70)$$

$$= 1.45\left[0.002 + 1.6\left(\frac{17.8 \text{ kips} / 76.5 \text{ kips/in.}}{(9 \text{ ft})(12 \text{ in./ft})}\right)\left(\frac{1}{0.8}\right)\right]$$

$$= 0.00915 = \frac{1}{109}$$

and a story $P\Delta$ shear per column of

$$\bar{H}_{P\Delta} = 0.78P\frac{\bar{\Delta}_{tot}}{L} = 18.7 \text{ kips} \quad (71)$$

The resulting maximum moment requirement in the girders is

$$M_r = 0.78M_g + 1.6M_w + \bar{H}_{P\Delta}\, L/2 = 233 \text{ kip-ft} \quad (72)$$

assuming that the column end moments above and below the third story are equal to the end moments in the column being considered [or taking the moments calculated in Equation 72 as the average required moments in the spandrels as in LeMessurier (1977). The W18×50 section selected for drift control easily satisfies this requirement.

Column Strength Check Under (1.2*D* + 1.6*L*) Including the τ Reduction

Since the value of P_r/P_y in the W14×455 columns selected in the preliminary calculations is greater than 0.5 at (1.2*D* + 1.6*L*), the AISC (2005a) DM requires that the inelastic stiffness reduction factor, τ, be included in calculating their flexural rigidity. The ideal stiffness for this load combination is still equal to 35.0 kips/in. as given by Equation 52. Given $P_r/P_y = 0.784$ from Equation 58, the AISC (2005a) τ factor for the DM is

$$\tau = 4\left(1 - \frac{P_r}{P_y}\right)\frac{P_r}{P_y} = 0.678 \quad (73)$$

This in turn gives

$$G_\tau = \frac{\tau I_c / L}{I_b / L_b} = 16.94 \quad (74)$$

$$\beta_{L\tau} = \frac{12}{1 + G_\tau} = 0.669 \quad (75)$$

$$\beta_\tau = \frac{\tau \beta_{L\tau} E I_c}{L^3} = 75.1\frac{\text{kips}}{\text{in.}} \quad (76)$$

$$\bar{B}_{lt} = \frac{1}{1 - \dfrac{\beta_i}{0.8 R_M \beta_\tau}} = \frac{1}{1 - \dfrac{35 \text{ kips/in}}{0.8(1.0)(75.1 \text{ kips/in.})}} = 2.39 \quad (77)$$

$$\frac{\bar{\Delta}_{tot}}{L} = \bar{B}_{lt}\frac{\Delta_o}{L} = 0.00479 \quad (78)$$

$$\bar{H}_{P\Delta} = 1.44P\frac{\bar{\Delta}_{tot}}{L} = 18.1 \text{ kips} \quad (79)$$

and

$$M_r = \bar{H}_{P\Delta}\, L/2 = 81.5 \text{ kip-ft} \quad (80)$$

This moment is slightly smaller than that of the previous calculation (see Equation 56) due to the fact that the W18×50 sections selected for the spandrels provide slightly more stiffness than required to satisfy the target drift limit $\psi_2 \le 1/500$. The moment would evaluate to equal 79.5 kip-ft in Equation 56 if the actual β of 76.5 kips/in. were used (see Equation 64). The interaction equation check (see Equation 57) is dominated by the axial load term, and therefore the interaction value is still 0.928 when this equation is re-evaluated using M_r from Equation 80.

If the AISC (2005a) Chapter C ELM is used with a calculated in-plane *inelastic* effective length factor of $K = 2.75$ for this frame, the governing beam-column check is

$$\frac{P_r}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_r}{\phi_b M_n}\right) = 0.949 + \frac{8}{9}(0.025) = 0.972 \quad (81)$$

It is important to note that the axial capacity ratio, $P_r/\phi_c P_n = 0.949$, based on in-plane sidesway buckling of the frame, is larger than the corresponding value of 0.899 in Equation 57, which is based on out-of-plane column buckling. AISC (2005a) requires the use of a minimum lateral load of $N_i = 0.002Y_i$ (the same as the notional lateral load in Equation 1) for gravity-only load combinations when the ELM is employed. The minimum lateral load gives a rather inconsequential increase in the beam-column strength check in Equation 81. In general, this minimum lateral load is necessary to ensure that geometric imperfection effects are accounted for in some way in determining the internal forces required for strength. The corresponding additional internal moments are strictly not necessary for the design of the beam-columns for in-plane stability. However, these additional moments can be an important consideration in the design of the beam-column members for out-of-plane stability, and in the design of the beams, the beam-to-column connections, and the column bases.

One can observe that the governing beam-column interaction value of 0.928 from Equation 57, using the DM, is slightly less conservative than the above value of $P_r/\phi_c P_n$ = 0.949, using the traditional AISC (1999) ELM (where no minimum lateral load is required in the analysis and design calculations). Also, the calculation of the inelastic effective length factor of K = 2.75 involves significantly more effort than the above DM calculations. The simpler elastic effective length Equation C-C2-5 of AISC (2005a), K = 4.62 for the example frame, leads to a substantially larger $P_r/\phi_c P_n$ of 1.11. The reader is referred to White et al. (2003b) for the details of these calculations.

The amplification factor for the sidesway displacements is B_{lt} = 1.84 under the load combination, 1.2D + 1.6L, in the example frame, determined using the ELM (in other words, no stiffness reduction). AISC (2005a) requires the use of the DM in all cases where B_{lt} is greater than 1.5, since the approximations associated with the ELM can be rather significant in certain cases having large sidesway amplification. In LeMessurier's Example 4, these approximations lead to a conservative strength assessment for the column members. However, in some cases, for example see Deierlein (2004), Kuchenbecker, White, and Surovek-Maleck (2004), and White et al. (2006), these approximations can lead to somewhat unconservative results for the beam and connection moments, the column base moments, and/or the moments for checking the out-of-plane strength of the beam-columns.

All the calculations illustrated thus far utilize the base beam-column strength interaction curve of AISC (2005a) Section H1.1. This curve is a single bilinear form for checking the combined in-plane and out-of-plane resistances. However, AISC (2005a) Section H1.3 also allows a separate in-plane and out-of-plane resistance check for doubly-symmetric members loaded by axial compression and strong-axis bending. This gives a more accurate (less conservative) estimate of the beam-column capacities in cases where axial capacity is governed by out-of-plane failure. Within the context of the DM, Section H1.3 requires a check of the in-plane strength using the interaction equation shown previously but with $\phi_c P_n$ based on the in-plane L/r_x (= 14.7 for this example) as follows,

$$\frac{P_r}{\phi_c P_{nx}} + \frac{8}{9}\left(\frac{M_r}{\phi_b M_n}\right) = \frac{3{,}780 \text{ kips}}{4{,}292 \text{ kips}} + \frac{8}{9}\left(\frac{81.5 \text{ kip-ft}}{2{,}527 \text{ kip-ft}}\right) = 0.909 \tag{82}$$

and it provides a separate interaction equation [Equation H1-2 of AISC (2005a)] that gives an enhanced representation of the out-of-plane strength using $\phi_c P_n$ based on L/r_y as follows,

$$\frac{P_r}{\phi_c P_{ny}} + \left(\frac{M_r}{\phi_b M_n}\right)^2 = \frac{3{,}780 \text{ kips}}{4{,}205 \text{ kips}} + \left(\frac{81.5 \text{ kip-ft}}{2{,}527 \text{ kip-ft}}\right)^2 = 0.900 \tag{83}$$

Based on the more refined separate in-plane and out-of-plane beam-column strength checks from AISC (2005a), both the DM and the ELM predict that the in-plane strengths govern for the critical (1.2D + 1.6L) load combination. The DM gives the above beam-column interaction value of 0.909 whereas the ELM gives a value of 0.972 based on Equation 81. Also, the DM streamlines the design of the beam-column members, since it uses K = 1 throughout in calculating the column axial strengths, $\phi_c P_n$.

Spandrel Strength Check Under (1.2*D* + 1.6*L*) Including the τ Reduction

The spandrel moments are also influenced by the inelastic stiffness reduction in the columns. Based on the approximate analysis used in this study, the $P\Delta$ contribution to these moments is also 81.5 kip-ft, from Equation 80, which is smaller than the 82.5 kip-ft used in the preliminary calculations (see Equation 56). Therefore, the spandrels have sufficient strength under the load combination, 1.2D + 1.6L, as shown here,

$$\begin{aligned} M_r &= 1.44M_g + \bar{H}_{P\Delta}L/2 = 37.5 \text{ kip-ft} + 81.5 \text{ kip-ft} \\ &= 119 \text{ kip-ft} \le \phi_b M_p = 273 \text{ kip-ft} \end{aligned} \tag{84}$$

The spandrel required moment under (1.2D + 1.6L) is M_r = 37.5 + 62.7 ft-kips = 100 kip-ft using the AISC (2005a) ELM. The DM gives a more rational calculation of the internal moments at the strength limit for this type of frame, which has a significant second-order sidesway amplification.

It should be noted that although the column axial loads and inelastic stiffness reduction are relatively large at the maximum strength level in the example frame, the influence of the stiffness reduction on the internal forces and on the strength checks, is relatively minor.

Column Strength Check under (1.2*D* + 0.5*L* + 1.6*W*)

The final check necessary for LeMessurier's Example 4 is the column strength under the maximum wind load combination. As stated previously, P_r = 2,048 kips for this load case, and the ideal stiffness is β_i = 19.0 kips/in. from Equation 68. Also, P_r/P_y = 0.424 for this load combination. Therefore, τ = 1 and β = 76.5 kips/in. from Equation 64. The key calculations are thus

$$\bar{B}_{lt} = \frac{1}{1 - \dfrac{\beta_i}{0.8R_M\beta_\tau}} = \frac{1}{1 - \dfrac{19 \text{ kips/in.}}{0.8(1.0)(76.5 \text{ kips/in.})}} = 1.45 \quad (85)$$

$$\frac{\bar{\Delta}_{tot}}{L} = \bar{B}_{lt}\left[\frac{\Delta_o}{L} + 1.6\left(\frac{\Delta_{1H}}{L}\right)\left(\frac{1}{0.8}\right)\right]$$
$$= 1.45\left[0.002 + 1.6\left(\frac{1}{464}\right)\left(\frac{1}{0.8}\right)\right] \quad (86)$$
$$= 0.00915 = \frac{1}{109}$$

where $\Delta_{1H}/L = 1/464$ is taken from Equation 70,

$$\bar{H}_{P\Delta} = 0.78P\frac{\bar{\Delta}_{tot}}{L} = 18.7 \text{ kips} \quad (87)$$

and

$$M_r = 1.6M_w + \bar{H}_{P\Delta}L/2 = 213 \text{ kip-ft} \quad (88)$$

This results in an interaction equation value of

$$\frac{P_r}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_r}{\phi_b M_n}\right) = \frac{2{,}048 \text{ kips}}{4{,}205 \text{ kips}} + \frac{8}{9}\left(\frac{213 \text{ kip-ft}}{2{,}527 \text{ kip-ft}}\right) \quad (89)$$
$$= 0.562$$

The interaction value for this check using the AISC (2005a) ELM is 0.662 (White et al., 2003b).

Comparison to Other First-Order Elastic Analysis Methods

It should be readily apparent that the proposed second-order analysis procedure has significant advantages relative to the AISC (2005a) B_1-B_2 or NT-LT analysis method in that it does not require separate NT and LT analyses. One determines the amplified sidesway displacements directly, then calculates and applies the associated $P\Delta$ shears to the structure in a separate first-order analysis to determine the second-order internal forces. This avoids questions such as which story B_2 factor should be applied to the beam moments at a given floor level (the one for the story above or the one for the story below?). The proposed procedure generally does a better job of estimating the distribution of second-order internal forces between the different framing systems in combined gravity, braced and moment frames. Also, the application of the above story $P\Delta$ shears to determine the second-order forces is more intuitive, in other words, it is more representative of the true second-order behavior, than the application of amplification factors to the member axial forces and moments as specified in AISC (2005a).

AISC (2005a) also states in a user note that "it is conservative to apply the sum of the non-sway and sway moments … by the B_2 amplifier, in other words, $M_r = B_2(M_{nt} + M_{lt})$." This approach of amplifying the total member moments by B_2 leads to significant conservatism in the calculation of the spandrel moments for the ($1.2D + 1.6L$) load combination of the example frame. The required moments using this approximation with the DM are 171 and 242 kip-ft for the loadings ($1.2D + 1.6L$) and ($1.2D + 0.5L + 1.6W$), respectively [versus 119 kip-ft from Equation 80 (81.5 kip-ft) plus $1.44M_g$ (1.44×26.0 kip-ft), and 233 kip-ft from Equation 72]. They are 132 and 198 kip-ft for these loadings using this approximation with the ELM (versus 100 and 191 kip-ft using the recommended procedure). Nevertheless, the girder size is still governed by the drift requirements for this example. In addition, since the column strength checks are dominated by the axial capacity ratio (see Equations 57, 81 and 89), the use of the above conservative approach has a negligible effect on these checks for the example frame. White et al. (2006) show an example frame in which the approach suggested in the above AISC (2005a) user note results in an overestimation of the beam and column design moments sufficient to impact the selection of the member sizes.

SUMMARY

This paper presents an application of the AISC (2005a) Direct Analysis Method (DM) for moment and general combined framing systems. The DM accounts explicitly for nominal initial out-of-plumbness of the framing as well as the reduction in the stiffness of the structure at the maximum strength limit of its most critical member or members. As a result, this approach provides a more rational estimate of the internal forces at the maximum strength limit. Also, the column and beam-column strength checks in moment frames may be based on $K = 1$ by using this method. One additional modification to a conventional elastic analysis is required in general for beam-columns in moment frames, in other words, the flexural rigidity must be reduced by an additional column inelastic stiffness reduction factor, τ, for columns loaded by axial forces in excess of $0.5P_y$.

This paper proposes two modifications to the underlying amplified first-order elastic analysis approach presented in Part 1 to extend this procedure to general rectangular framing involving any combination of moment, braced and gravity systems. These modifications are:

1. An additional term is included in the sidesway displacement amplifier, $\bar{B}_{lt}$, to account for the influence of $P\delta$ (P-small delta) moments in moment-frame columns on the sidesway response.

2. The traditional "no-translation" (or NT) moment amplifier, B_1 in AISC (2005a) based on the use of a reduced elastic stiffness, is applied to the total column moments.

Suggestions are also provided for approximate handling of frames with large axial compression in the beams or rafters and/or nonrectangular geometry.

The paper presents analysis and design calculations using the above combined procedures for an example from LeMessurier (1977). This example illustrates a number of important stability design considerations. The preliminary design of LeMessurier's frame starts with the calculation of a target story sidesway stiffness necessary to hold the second-order drift under service loads to a specified limit. The DM, combined with the proposed amplified first-order elastic analysis procedure for calculation of the second-order internal forces, provides a more intuitive, straightforward and accurate set of analysis and design calculations than the traditional AISC ELM and/or the AISC NT-LT analysis procedures. The combination of the AISC (2005a) DM with the proposed amplified first-order elastic analysis equations is generally applicable to all types of rectangular framing systems.

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APPENDIX A

COMPARISON OF MAXIMUM STRENGTHS BY THE AISC (2005A) DIRECT ANALYSIS AND EFFECTIVE LENGTH METHODS TO THE MAXIMUM STRENGTH OBTAINED BY DISTRIBUTED PLASTICITY ANALYSIS

It is informative to compare the elastic analysis and design solutions presented in the body of the paper to the results from a refined Distributed Plasticity Analysis for the governing strength load combination, $1.2D + 1.6L$. Distributed Plasticity Analysis is a useful metric for evaluation of all the analysis and design methods, since it accounts rigorously for the effects of nominal geometric imperfections and member internal residual stresses. For the Distributed Plasticity Analysis, a nominal out-of-plumbness of $0.002L$ is assumed. Although modeling of column out-of-straightness is necessary in general for a rigorous Distributed Plasticity Analysis, column out-of-straightness is not included here since the columns are subjected to fully-reversed curvature bending and their L/r values are quite small. Also, the Lehigh residual stress pattern (Galambos and Ketter, 1959) is used. This pattern has a maximum residual compression of $0.3F_y$ at the flange tips and a linear variation over the half-flange width to a constant self-equilibrating residual tension in the web. A resistance factor of $\phi = 0.90$ is applied to both the yield strength, F_y, and the elastic modulus, E. These are established parameters for calculation of benchmark strengths in LRFD using a Distributed Plasticity Analysis (ASCE, 1997; Martinez-Garcia, 2002; Deierlein, 2003; Surovek-Maleck et al., 2003; Surovek-Maleck and White, 2004; White et al., 2006). The steel is assumed to be elastic-plastic, with a small inelastic modulus of 0.0009E. The gravity and lateral loads are applied proportionally to the frame in the Distributed Plasticity solution. The FE++ software system (Alemdar, 2001) is used for the analysis. Two flexibility-based elements, which include an exact equilibrium based description of the moments along their length including the moments from the transverse distributed loads, are employed to represent the beams. Two mixed elements, which are capable of accurately capturing the elastic and inelastic $P\Delta$ and $P\delta$ beam-column moments, are employed to model the columns.

For the purpose of the Distributed Plasticity Analysis, all ten bays of the frame shown in Figure 7 are modeled. Inflection points are assumed at the mid-height of the second and fourth stories to obtain an isolated subassembly. The additional story gravity load at levels 3 and 4, beyond the distributed loads applied to the spandrels, is applied as concentrated loads at the beam-column joints, two parts to the interior columns and one part to the exterior columns. The exterior columns are taken as W14×233 sections in the third and fourth stories, the W14×455 section is assumed for the interior columns in the fourth story, and the interior and exterior columns in the second story are taken as W14×500 and W14×257 members, respectively. The larger sizes in the second story are necessary such that the maximum strength in the Distributed Plasticity Analysis is not governed by the second-story columns.

The above Distributed Plasticity Analysis predicts a maximum strength of 1.105 of the maximum live load combination, $1.2D + 1.6L$. Correspondingly, if the design load is scaled until a beam-column unity check of 1.0 is obtained for the $(1.2D + 1.6L)$ load combination, the maximum strengths predicted by the AISC (2005a) Direct Analysis Method (DM) and Effective Length Method (ELM) are 1.102 and 1.033 of this load combination, respectively. Interestingly, if the minimum lateral load requirement of the AISC (2005a) ELM is neglected and the column capacity is determined using an inelastic effective length factor, the axial capacity ratio, $P_r/\phi_c P_n$, reaches 1.0 also at 1.102 of the load combination, $1.2D + 1.6L$. However, the total story drift at the frame capacity in the Distributed Plasticity Analysis is $\Delta_{tot}/L = 0.00587$, compared to a drift of 0.00379 in the AISC (2005a) ELM analysis (nonzero due to the required minimum lateral load) and 0.00479 by the DM (from Equation 78). The maximum internal moments in the third floor spandrels range from 0.573 to 0.554 of $\phi_b M_p = 273$ kip-ft in the Distributed Plasticity Analysis, whereas they are

- 0.436 of $\phi_b M_p$ at 1.0 times (1.2D + 1.6L) in the DM (see Equation 84) and
- 0.367 of $\phi_b M_p$ at 1.0 times (1.2D + 1.6L) using the AISC (2005a) ELM.

If the minimum lateral load requirement is neglected in the AISC (2005a) ELM, the maximum girder moment at 1.0 times $(1.2D + 1.6L)$ is only 0.137 of $\phi_b M_p$. The drift of the frame is $\Delta_{tot}/L = 0.00470$ at 95% of the maximum load capacity in the Distributed Plasticity Analysis. The moments in the spandrels calculated by the Distributed Plasticity Analysis and by the DM match closely at this load level.

The failure mode in this structure is inelastic sidesway "buckling" of the third story columns, since the girders are still completely elastic at the maximum load limit. The effective elastic moment of inertia in the W14×455 interior columns varies along their length and ranges from 0.28 to 0.52 of the elastic moment of inertia at the limit load in the Distributed Plasticity Analysis. The reductions in the flexural rigidities are similar but are slightly smaller in the exterior columns.

One can see that both the ELM and the DM give a reasonable estimate of the Distributed Plasticity solution, with the DM calculations being somewhat simpler to perform and giving comparable or slightly better estimates of the behavior observed in the Distributed Plasticity Analysis. The conventional ELM in AISC (1999) determines the internal forces in the fictitious geometrically-perfect nominally-elastic structure, but then compensates for this idealization in the design of the beam-columns by basing $\phi_c P_n$ on the buckling strength of the perfect structure, typically via a story sidesway buckling K factor (Surovek-Maleck and White, 2004; Deierlein, 2004; White et al., 2006). This approach generally underestimates the M_r values in the beam, connection, and out-of-plane beam-column strength checks [hence, the requirement of a minimum lateral load with gravity-only load combinations in AISC (2005a)]. For lightly-loaded laterally-stiff structures, these errors are small. However, for more heavily-loaded and/or laterally-flexible structures, these errors can be significant. The DM provides a more rational estimate of the internal forces in the structural system at the strength limit of the most critical member or member(s) for all types of frames.

APPENDIX B

NOMENCLATURE

A_g = Gross area of cross section

$\bar{B}_{lt}$ = Sidesway displacement amplification factor given by Equation 35

B_1, $\bar{B}_{nt}$ = Non-sway moment amplification factor

C_L = Factor from LeMessurier (1977) that accounts for the influence of individual member $P\Delta$ effects on the amplification of the sidesway displacements

C_{Lavg} = Weighted average, C_L, over all the columns in a story $= \dfrac{\Sigma_m (C_L P_r)}{\Sigma P_r}$

C_m = Beam-column equivalent uniform moment factor

E = Modulus of elasticity

F_y = Yield stress

G = Joint stiffness factor given by Equation 33

G_A, G_B = Joint stiffness factor at column ends A and B

H, ΣH = Story shear due to the applied loads on the structure

$H_{P\Delta}$, $\bar{H}_{P\Delta}$ = Story shear due to $P\Delta$ effects

I = Moment of inertia

I_b, I_c = Moment of inertia of beam and column

$\bar{I}_e$ = Effective elastic moment of inertia

K = Column effective length factor

L = Story height

L_b = Length of a beam

L_{beff} = Length of an equivalent prismatic elastic beam subjected to fully-reversed curvature bending

M_F, M_N = Sidesway moment at the far and near end of a beam, respectively

M_g = Nominal (unfactored) moment due to gravity load

M_L, M_S = Larger and smaller end moment, respectively

M_n = Nominal moment resistance

M_p = Plastic moment resistance

M_r = Required moment resistance

M_w = Nominal (unfactored) moment due to wind load

N_i = Notional load at i^{th} level in the structure

P = Column axial load

P_{cr} = Column buckling load

$\bar{P}_{e(L)}$ = Euler buckling load $\pi^2 E \bar{I}_e / L^2$ based on the unsupported length of the member in the plane of bending

$\bar{P}_{e.nt}$ = Column non-sway elastic buckling resistance

P_L = Contribution from a column to the story stiffness in terms of the rotational displacement Δ_1/L, in other words, column shear force required to obtain a first-order drift of $\Delta_1/L = 1$.

P_n = Nominal axial load resistance

P_r = Required axial load resistance

ΣP_r = Total required story vertical load

P_y = Yield load $= A_g F_y$

$P\Delta$ = P-large delta effect, equal to the moment caused by the member axial force acting through the relative transverse displacement between its ends, or equal to the total story vertical load acting through the total interstory sidesway displacement

$P\delta$ = P-small delta effect, equal to the moment caused by the member axial force acting through the transverse displacement relative to a chord between its ends

R_M = Story factor that accounts for the influence of $P\delta$ effects in moment-frame columns on the amplification of the sidesway displacements

Y_i = Total factored gravity load acting on the i^{th} level

p_w = Wind load in pounds per square foot

q_g = Uniformly distributed gravity loading on spandrels

r_x, r_y = Radius of gyration about x- and y-axis, respectively

w_g = Uniform floor distributed gravity load

β, $\overline{\beta}$ = Total story sidesway stiffness of the lateral load resisting system

β_i = Story sidesway destabilizing effect, or ideal story stiffness = $\Sigma P_r / L$

β_L = LeMessurier's column sidesway stiffness coefficient, shown in Equation 30

δ = Transverse deflection of a member relative to a chord between its ends

γ_g = Average gravity load per unit interior volume of the structure

ϕ = Resistance factor

ϕ_b, ϕ_c = Resistance factor for flexure and for compression

τ = Column inelastic stiffness reduction factor

τ (subscript) = Indicates quantities calculated including the influence of column inelastic stiffness reduction

ψ_1 = First-order service drift limit, given by Equation 48

ψ_2 = Specified second-order service drift limit

Δ = Inter-story sidesway displacement

Δ_0 = Initial story out-of-plumbness

Δ_1, $\overline{\Delta}_1$, = First-order interstory sidesway displacement due to applied loads = $\Delta_{1H} + \Delta_{1P}$ or $\overline{\Delta}_{1H} + \overline{\Delta}_{1P}$

Δ_2, $\overline{\Delta}_2$, = Additional interstory sidesway displacement due to second-order ($P\Delta$) effects

Δ_{1H} , $\overline{\Delta}_{1H}$ = First-order interstory sidesway displacement due to ΣH

Δ_{1P} , $\overline{\Delta}_{1P}$ = First-order interstory sidesway displacement due to vertical loads

Δ_{tot} , $\overline{\Delta}_{tot}$ = Total interstory sidesway displacement

Σ = Summation

Σ_c, Σ_g, Σ_m = Summation over the connected columns, gravity columns and moment frame columns, respectively

$\overline{\ }$(over bar) = Indicates quantities that are influenced by the stiffness reduction employed in the Direct Analysis Method