Modified Slenderness Ratio for Built-up Members

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The effect of shear on the buckling strength of latticed members is well known (Bleich, 1952; Timoshenko and Gere, 1961). This type of compression member features widely spaced individual shapes (for example, angles and channels) interconnected by laces (see Figure 1a). The AISC Specification has been providing specific design and detailing requirements of this type of member for many decades, but the shearing effect has been ignored because the reduction in compression strength for the built-up members thus designed is insignificant (Salmon and Johnson, 1996).

Figure 1(b) shows the second type of built-up member that is commonly used. It features closely spaced individual components intermediately connected by either bolts or welds. When buckled about the axis shown in the figure, connectors between individual components are subject to shear, and the need to consider such shearing effects was pointed out by Libove (1985). Based on the study of Zahn and Haaijer (1987), AISC introduced for the first time an empirical version of the modified slenderness ratio, $(KL/r)_m$, in the first edition of the *LRFD Specification for Structural Steel Buildings* (AISC, 1986). Further study on this subject by Aslani and Goel (1989, 1991a, 1991b) led to a theory based version of $(KL/r)_m$ in the subsequent editions of the LRFD Specification and the 2005 AISC *Specification for Structural Steel Buildings* (AISC, 2005). More complicated in its format, it was believed that this improved version would lead to a better prediction of the compressive strength of this type of built-up member.

As more test data became available after the work of Aslani and Goel, the objective of this study is to re-examine the issue of modified slenderness ratio based on both theory and available test data. In this paper, theory and models proposed by previous researchers are first presented. Calibrating to an enhanced database of test results, a recommendation aiming for accuracy and simplicity is made.

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MODIFIED SLENDERNESS RATIO

Theory

Bleich (1952) derived the elastic buckling strength of built-up latticed columns based on an energy approach. The transition from stable to unstable equilibrium of any elastic system is characterized by the energy condition,

$$V - W = 0 \tag{1}$$

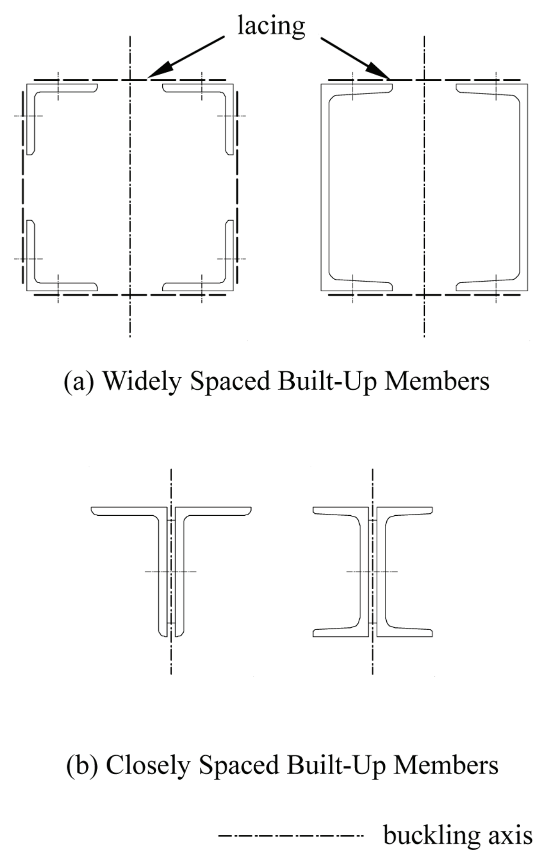


Fig. 1. Classification of built-up members.

where

V = strain energy

W = work done by the external axial force

Solving the energy balance for external force, the buckling load is,

$$P_{cr} = \frac{\pi^2 E I_0}{(KL)^2 + \left(\frac{\pi^2}{24}\right)\left(\frac{I_0}{I_{ib}}\right)a^2} \tag{2}$$

See the Notation section for definitions of variables. Dividing P_{cr} by the cross-sectional area of the column gives the following buckling stress equation,

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\left\{\sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{\pi^2}{12}\right)\left(\frac{I_0}{I}\right)\left(\frac{a}{r_{ib}}\right)^2}\right\}^2}\left(\frac{I}{I_0}\right) \tag{3}$$

Note that I is the moment of inertia of the integral section, and I_0 is the moment of inertia of the integral section neglecting the moment of inertia of individual components. The ratio I_0/I can be expressed as a function of the separation ratio, α, as follows,

$$\frac{I_0}{I} = \frac{\alpha^2}{1+\alpha^2} \tag{4}$$

Express F_{cr} as a function of the modified slenderness ratio, $(KL/r)_m$,

$$F_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)_m^2} \tag{5}$$

$(KL/r)_m$ can be derived by equating Equations 3 and 5 as follows,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{1+\alpha^2}{\alpha^2}\right)\left(\frac{KL}{r}\right)_o^2 + 0.82\left(\frac{a}{r_{ib}}\right)^2} \tag{6}$$

where

$$\frac{\pi^2}{12} = 0.82$$

Equation 6 represents the exact form of $(KL/r)_m$.

To apply to the case of latticed member with widely spaced individual components, Bleich (1952) assumed that $I/I_0 = 1.0$. This leads to a simplified form of Equation 3 as follows:

$$F_{cr} = \frac{\pi^2 E}{\left\{\sqrt{\left(\frac{KL}{r}\right)_o^2 + \frac{\pi^2}{12}\left(\frac{a}{r_{ib}}\right)^2}\right\}^2} \tag{7}$$

and the corresponding $(KL/r)_m$ is

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\left(\frac{a}{r_{ib}}\right)^2} \tag{8}$$

AISC Approaches

Intended for the design of built-up members with closely spaced individual components (Figure 1b), formulae first introduced by AISC in the first edition of the *LRFD Specification for Structural Steel Buildings* (AISC, 1986) were based on the work of Zahn and Haaijer (1987).

a. For intermediate connectors that are snug-tight bolted,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \tag{9}$$

b. For intermediate connectors that are welded or pretensioned bolted,

with $a/r_i > 50$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i} - 50\right)^2} \tag{10}$$

with $a/r_i \leq 50$

$$\left(\frac{KL}{r}\right)_m = \left(\frac{KL}{r}\right)_o \tag{11}$$

Note that the connector types mentioned above refer to intermediate connectors. According to the AISC *Specification for Structural Steel Buildings* (AISC, 2005), end connectors always have to be welded or pretensioned bolted.

The above expressions were empirically derived based on 28 data points, with the majority of tests conducted by Zandonini (1985) and some by Astaneh, Goel, and Hanson (1982, 1985).

Note that the I/I_0 term appears twice on the right-hand side of Equation 3, and Bleich assumed that this ratio be equal to

Table 1. Experimental Database					
Reference	Cross Section	Boundary Condition K	Intermediate Connector Type	Number of Connectors	Number of Specimens[(d)]
Lue et al., 2004	•Double Channels Back-to-Back •Double Channels Toe-to-Toe	Knife Edge $K = 1.0$	Snug Tight Welded	2, 3, 4	14 (7) 28 (10)
Sherman and Yura, 1998	•Double Angles Back-to-Back	Knife Edge $K = 1.0$	Snug Tight Pretensioned	0, 1, 2, 5	4 (3) 4 (3)
Temple and Elmahdy, 1995, 1996	•Double Channels Toe-to-Toe	Knife Edge $K = 0.65$[(a)]	Welded	1, 2, 3	6 (0)
Temple and Elmahdy, 1993	•Rectangular Bars	Knife Edge $K = 1.0$	Welded	0, 1, 3, 7	24 (10)
Aslani and Goel, 1989, 1991a	•Double Angles Back-to-Back •Double Angles Toe-to-Toe	Four Hinged Frame $K = 0.869, 0.941$[(b)]	Welded	1, 2, 3, 5	8 (5)
Astaneh et al., 1982, 1985	•Double Angles Back-to-Back	Four Hinged Frame $K = 0.80$ to 0.94[(c)]	Snug Tight Pretensioned Welded	1, 2	2 (1) 1 (1) 6 (1)
Zandonini, 1985	•Double Channels Back-to-Back	Elastic Hinge Evaluated Length	Snug Tight Welded	2, 3, 4	14 (14) 14 (14)

[(a)] assumed in this research
[(b)] per Aslani and Goel (1989, 1991a)
[(c)] per Astaneh et al. (1982, 1985)
[(d)] Number in parentheses represents the number of specimens satisfying $a/r_i \leq \frac{3}{4}\,(KL/r)$.

1.0 in both places in deriving Equation 8 for applications in widely spaced latticed members. In an attempt to introduce the separation factor, α, in the $(KL/r)_m$ expression for closely spaced built-up members, Aslani and Goel (1989, 1991b) assumed that only the I/I_0 term at the end of Equation 3 be equal to 1.0. Substituting Equation 4 for the I_0/I term under the radical in Equation 3 gives

$$F_{cr} = \frac{\pi^2 E}{\left\{\sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{\pi^2}{12}\right)\left(\frac{\alpha^2}{1+\alpha^2}\right)\left(\frac{a}{r_{ib}}\right)^2}\right\}^2} \tag{12}$$

The corresponding $(KL/r)_m$ is,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{1+\alpha^2}\left(\frac{a}{r_{ib}}\right)^2} \tag{13}$$

Aslani and Goel (1989, 1991b) reported good correlation with four data points, which were obtained from testing of double-angle members. Equations 10 and 11 were replaced by Equation 13 in the second and third editions of the LRFD Specification; the same equation is also adopted in the 2005 *Specification for Structural Steel Buildings* (AISC, 2005).

Duan and Chen Model

With the same data set used by Zahn and Haaijer (1987), Duan and Chen (1988) also proposed a simpler empirical formula for $(KL/r)_m$ as follows,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_m a}{r_i}\right)^2} \tag{14}$$

where

K_m = 1.0 for snug-tight bolted connectors

K_m = 0.65 for welded and pretensioned bolted connectors

Temple and Elmahdy (1995, 1996) evaluated Equation 14 based on the test results of double channels in a toe-to-toe welded configuration. It was found that $K_m = 0.65$ would give unconservative predictions, especially when the member is slender. The researchers recommended a value of 1.0 instead, which converges to Equation 9 for the case of snug-tight bolted connectors.

ANALYSIS OF EXPERIMENTAL TEST RESULTS

Database

Table 1 lists the experimental database for this study. Of a total of 125 specimens studied, only 69 specimens (25 for snug-tight, 4 for fully tensioned, and 40 for welded connectors) satisfying the following AISC requirements (AISC, 2005) for spacing between connectors are used in the data analysis,

$$\frac{a}{r_i} \le \frac{3}{4}\left(\frac{KL}{r}\right) \tag{15}$$

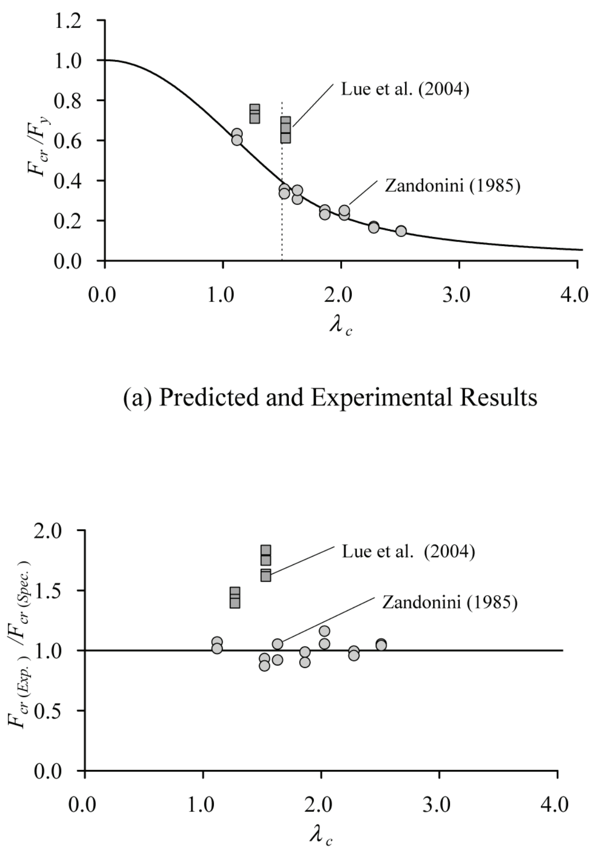
Specimens with Snug-Tight Bolted Connectors

$(KL/r)_m$ in Equation 9 for built-up compression members with snug-tight bolted connectors has not been changed since it was adopted in the first edition LRFD Specification. Although this study is focused on members with welded or pretensioned bolted connectors, it is worthwhile to verify Equation 9 based on the enhanced database.

Figures 2 and 3 compare the predicted and experimental compressive strengths for double-channel and double-angle members, respectively. In this data set, the end connections were welded or pretensioned bolted as required in the AISC Specification. F_y in part (a) of the figures is the measured yield strength of steel. Part (b) of these figures shows that Equation 9, in an average sense, gives an accurate prediction of the test results of Zandonini (1985) and Astaneh et al. (1982, 1985). This is not surprising because Equation 9 was indeed derived empirically from these two data sets. But Equation 9 appears to be conservative in predicting the compressive strength of two more recent data sets by Sherman and Yura (1998) and Lue, Yen, Liu, and Hsu (2004).

Specimens with Welded or Pretensioned Bolted Connectors

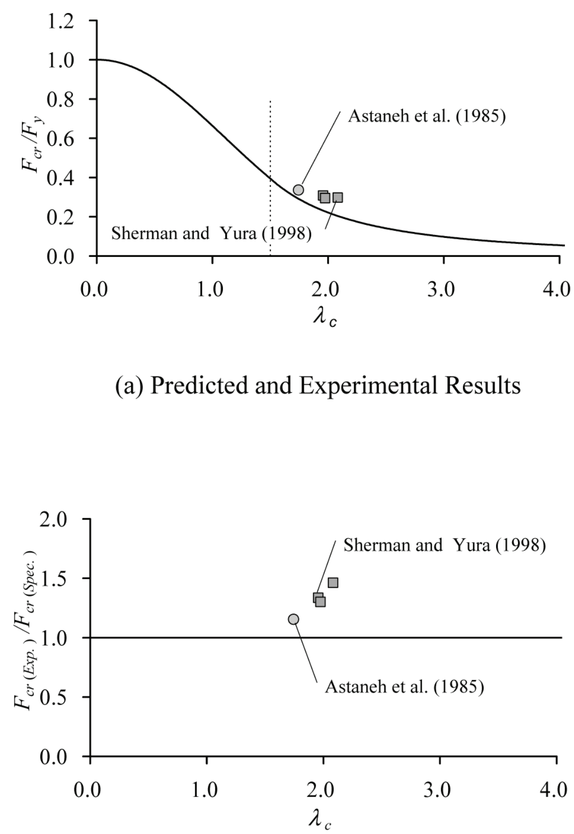
Presented in a similar style, Figures 4 to 8 show the correlation between the measured strength and the predicted strength based on both the first edition, 1986 *LRFD Specification for Structural Steel Buildings* (AISC, 1986) (Equations 10 and 11), hereafter referred to as the 1986 LRFD



(a) Predicted and Experimental Results

(b) Ratio of Compressive Strength (*Exp./Spec.*)

Fig. 2. Strength correlation of built-up members with snug-tight bolted connectors (Eq. 9) (double channels back-to-back).



(a) Predicted and Experimental Results

(b) Ratio of Compressive Strength (*Exp./Spec.*)

Fig. 3. Strength correlation of built-up members with snug-tight bolted connectors (double angles back-to-back).

Specification, and the 2005 *Specification for Structural Steel Buildings* (AISC, 2005) (Equation 13), hereafter referred to as the 2005 Specification. Note that the AISC procedures are unconservative for predicting the compressive strength of built-up members with rectangular bars (Figure 8); a different residual stress pattern than that assumed in the development of the AISC strength curve may be one contributing factor for such discrepancy. Since this particular shape is not used in practice, this data set will be excluded in the subsequent analysis.

Data Analysis

Combining all test data from Figures 4 to 7 for the cases of welded and pretensioned bolted connectors, a comparison is made in Figure 9 to the following models:

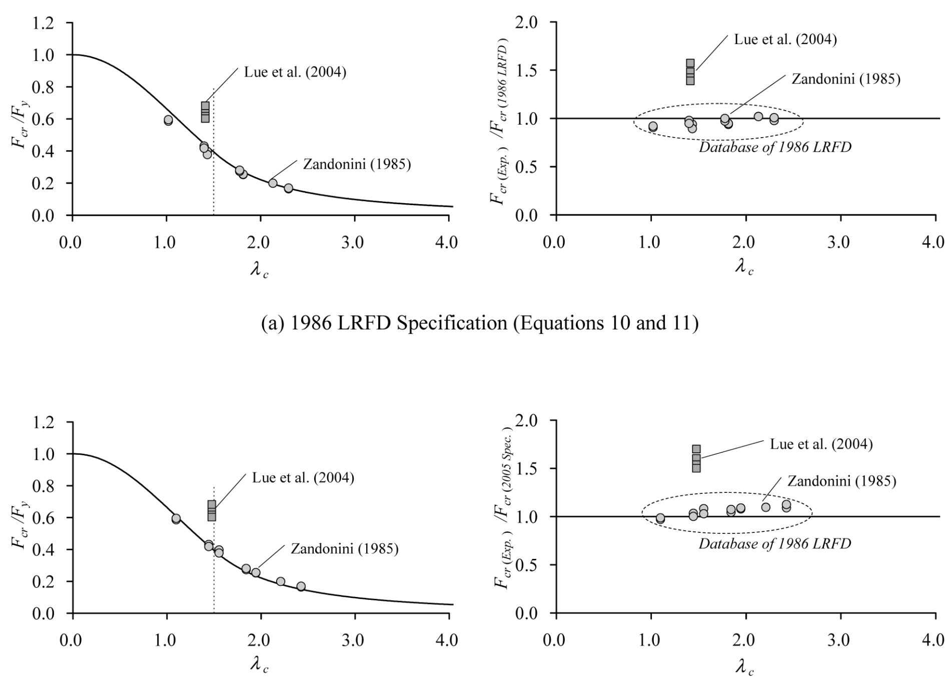
- 1986 LRFD Specification, in other words, Zahn and Haaijer's empirical formula (Equations 10 and 11)
- 2005 Specification, in other words, Aslani and Goel's, semi-theoretical formula (Equation 13)
- Bleich's exact solution (Equation 6)
- Bleich's simplified solution for widely spaced latticed members (Equation 8)
- Duan and Chen's empirical equation (Equation 14)

It is observed from Figure 9 that the correlation between the test data and each model is similar. But Bleich's exact solution gives the most conservative strength prediction. The more complicated formula in the 2005 Specification, which includes a separation factor, α, gives a predicted strength which is very similar to those provided by the first edition 1986 LRFD Specification approach, Bleich's simplified solution, and the formula proposed by Duan and Chen.

SIMPLIFICATION

The proposed model for built-up members with welded or pretensioned bolted connectors is patterned after that recommended by Duan and Chen (1988) for its simplicity, where $(KL/r)_m$ is defined as follows:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \tag{16}$$



(a) 1986 LRFD Specification (Equations 10 and 11)

(b) 2005 AISC Specification (Equation 13)

Fig. 4. Strength correlation of built-up members with welded or pretensioned bolted connectors (double channels back-to-back).

Table 2. Shearing Factor K_i				
Connectors	**Snug-Tight Bolted**	**Welded or Pretensioned Bolted**		
Shape	**Double Angles Back-to-Back** **Double Channels Toe-to-Toe and Back-to-Back**	**Double Angles Back-to-Back**	**Double Channels Back-to-Back**	**Others**
K_i	1.0	½	¾	6⁄7

Like Equation 10 used in the first edition of the LRFD Specification, Equation 16 requires only r_i to compute $(KL/r)_m$, while Equation 13 used in the 2005 AISC Specification requires both h and r_{ib}. The factor K_i in Equation 16 accounts for the shearing effect in the connectors. While Duan and Chen recommended a value of 0.65 for K_i, in this study the K_i value is a function of the built-up shapes.

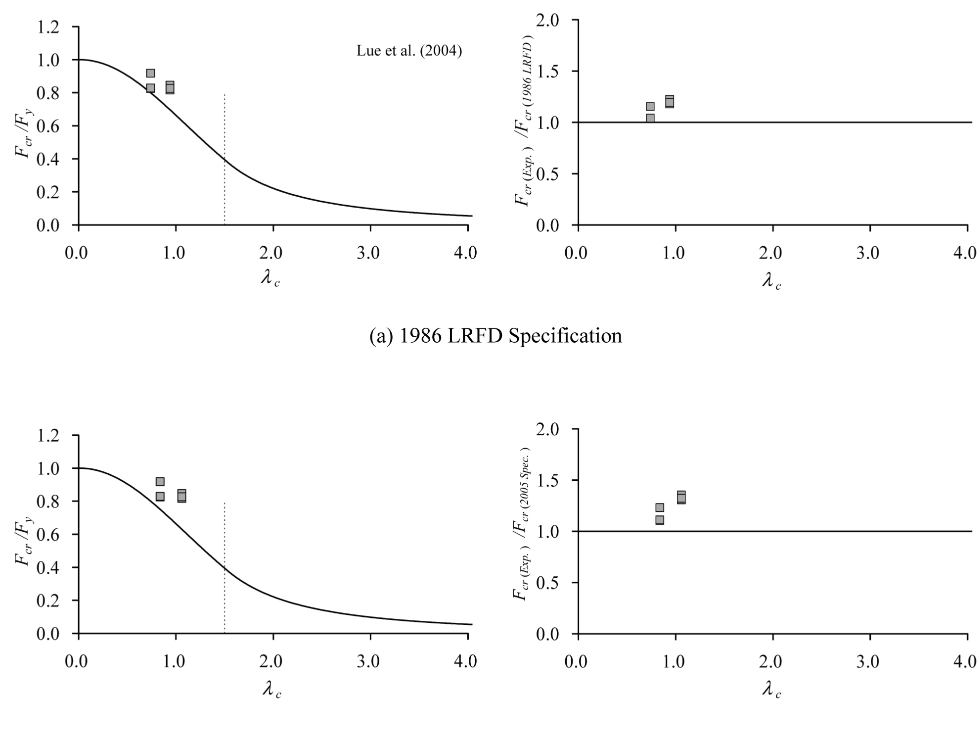
Rearranging Equation 13 in the form of Equation 16 gives the following expression for K_i,

$$K_i = \sqrt{0.82\left(\frac{\alpha^2}{1+\alpha^2}\right)\left(\frac{r_i}{r_{ib}}\right)} \tag{17}$$

The variations of the K_i value for the double angle and double channel sections listed in the AISC *Steel Construction Manual* (AISC, 2005b) are shown in Figure 10. Since the value of K_i varies in a narrow range for each configuration, the constant values in Table 2 are recommended.

A parametric study was conducted to demonstrate that the proposed model (Equation 16 and Table 2) gives very similar results as compared to the 2005 AISC Specification (Equation 13). In Figure 11, $P_{n(Pro.)}$ is based on the proposed model, and $P_{n(2005\ Spec.)}$ is based on the 2005 AISC Specification.

The reduction of compressive strength predicted by the proposed model as compared to P_{n0}, ignoring the shearing



(a) 1986 LRFD Specification

(b) 2005 AISC Specification

Fig. 5. Strength correlation of built-up members with welded or pretensioned bolted connectors (double channels toe-to-toe).

effect, can be observed in Figure 12. It is recommended that the shearing effect be ignored when $a/r_i \leq 40$.

CONCLUSIONS AND RECOMMENDATIONS

Based on a review of theory, background for the historical development of the AISC specifications, and an updated experimental database, a simple model which maintains the accuracy for the calculation of the modified slenderness ratio of built-up members with closely spaced individual components (for example, double angles and double channels) that are connected by welded or pretensioned bolted connectors is proposed:

a. with $a/r_i > 40$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2}$$

where the value of K_i is listed in Table 2.

b. with $a/r_i \leq 40$

$$\left(\frac{KL}{r}\right)_m = \left(\frac{KL}{r}\right)_o$$

NOTATION

I_0 = moment of inertia of the built-up section about the buckling axis, neglecting the moment of inertia of individual components about their own centroidal axis [$= 2A_{ib}\,(h/2)^2 = A_{ib}\,h^2/2$]

I_{ib} = moment of inertia of individual components about their own centroidal axis parallel to the axis of buckling ($= A_{ib}\,r_{ib}^2$)

I = moment of inertia of the built-up section, about the axis of buckling ($= I_0 + 2I_{ib}$)

A_{ib} = cross-sectional area of each individual component

A = cross-sectional area of the built-up member ($= 2A_{ib}$)

h = distance between centroids of individual components perpendicular to the member axis of buckling

r_{ib} = radius of gyration of individual component relative to its centroidal axis parallel to member axis of buckling

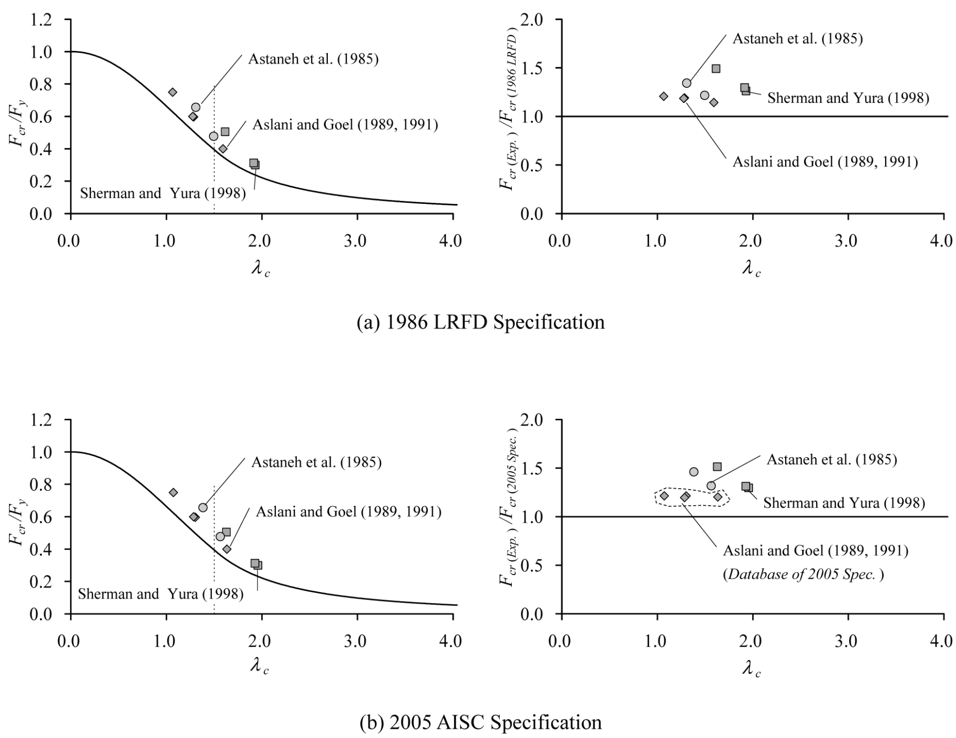


Fig. 6. Strength correlation of built-up members with welded or pretensioned bolted connectors (double angles back-to-back).

r_i = minimum radius of gyration of individual component

r = radius of gyration of the built-up section, about the axis of buckling

α = separation ratio ($= h/2r_{ib}$)

L = length of the member

a = distance between connectors

K = effective length factor for the overall member

K_m = local effective length factor

K_i = shearing factor

E = modulus of elasticity of steel

F_y = yield stress

F_{cr} = buckling stress

λ_c = column slenderness ratio $\left(= \left(KL/r\pi\right)\sqrt{F_y/E}\right)$

P_{cr} = buckling load

P_n = nominal compressive strength ($= AF_{cr}$)

P_{n0} = nominal compressive strength based on column slenderness ratio of built-up member acting as a unit

$P_{n(2005\ Spec.)}$ = nominal compressive strength based on 2005 AISC Specification modified column slenderness of built-up member

$P_{n(Pro.)}$ = nominal compressive strength based on proposed modified column slenderness of built-up member

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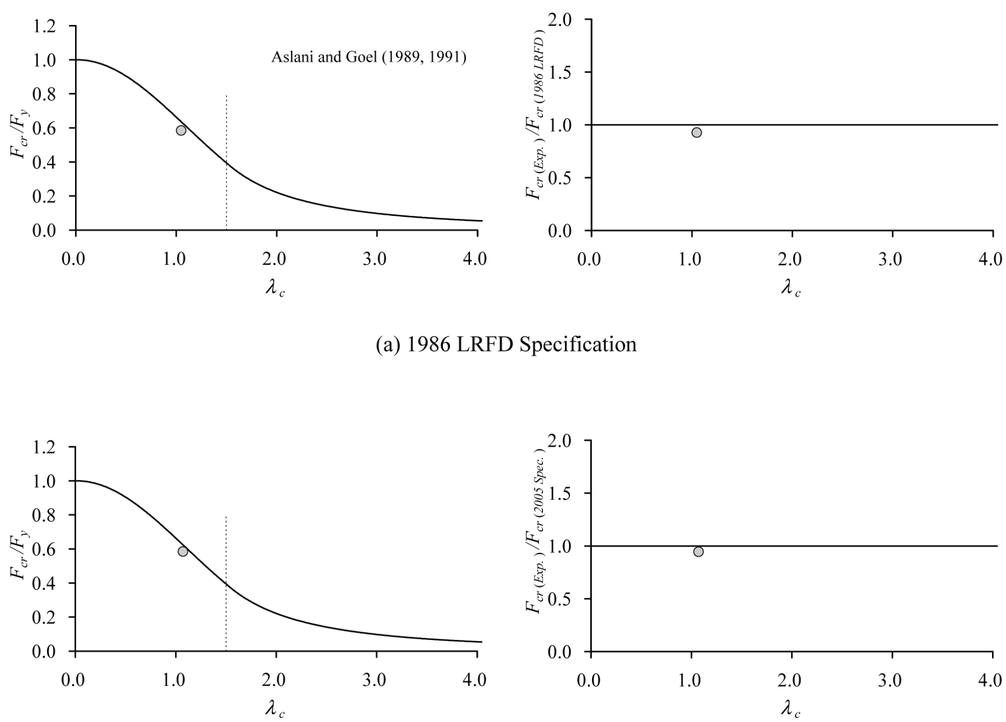

Fig. 7. Strength correlation of built-up members with welded or pretensioned bolted connectors (double angles toe-to-toe).

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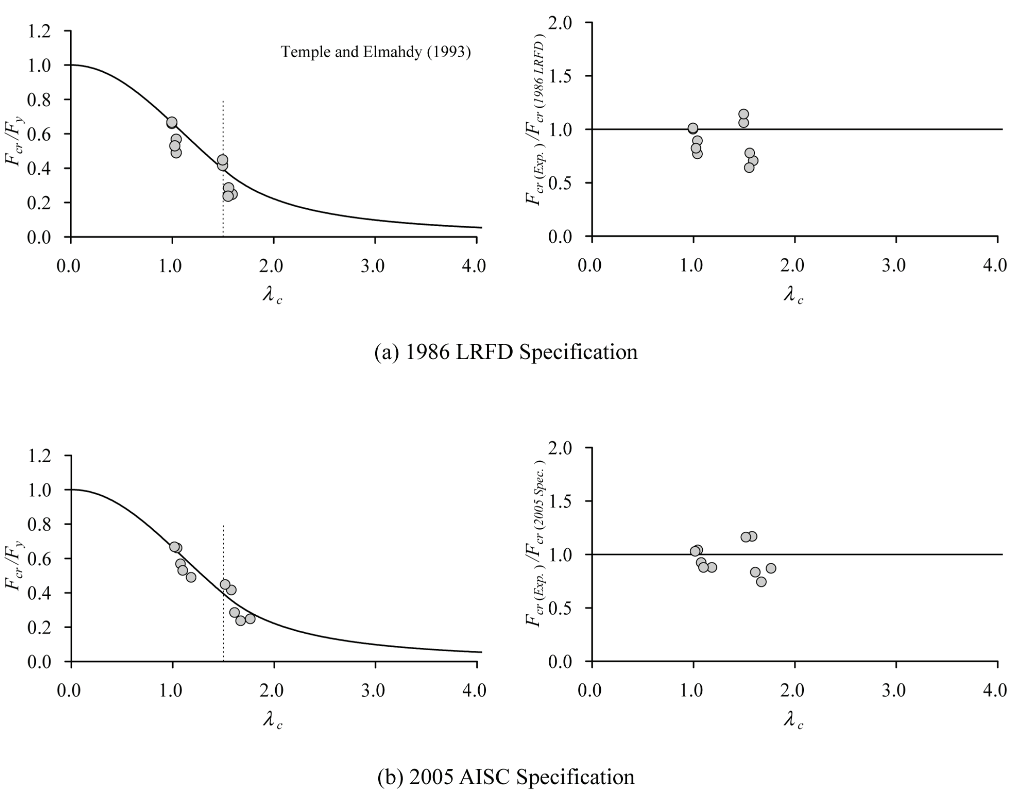

(a) 1986 LRFD Specification

(b) 2005 AISC Specification

Fig. 8. Strength correlation of built-up members with welded or pretensioned bolted connectors (rectangular bars).

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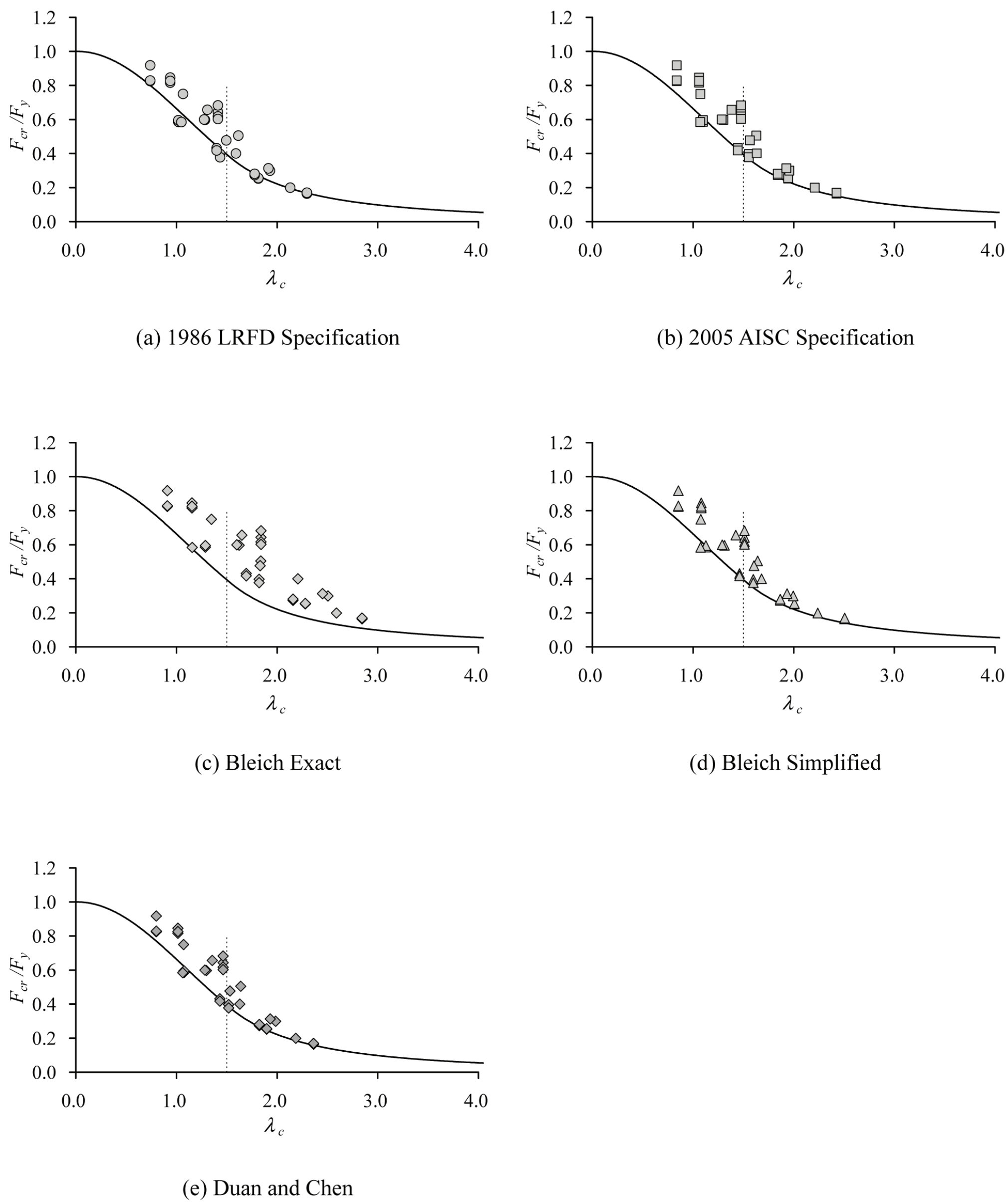
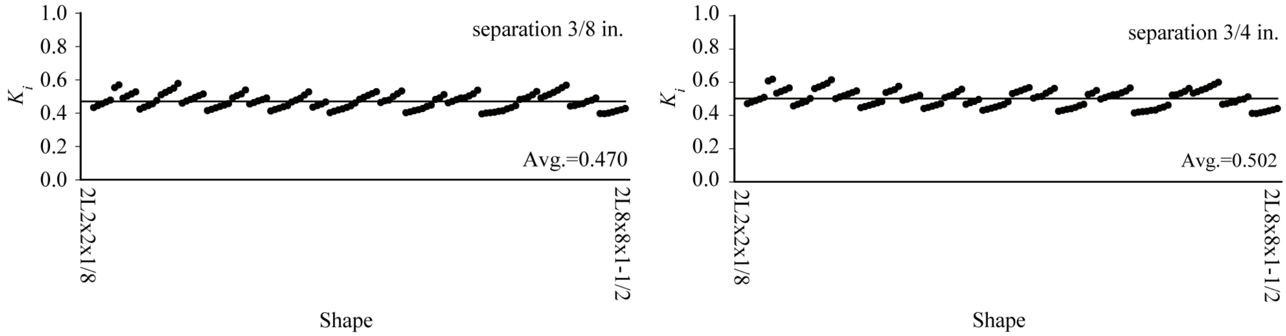
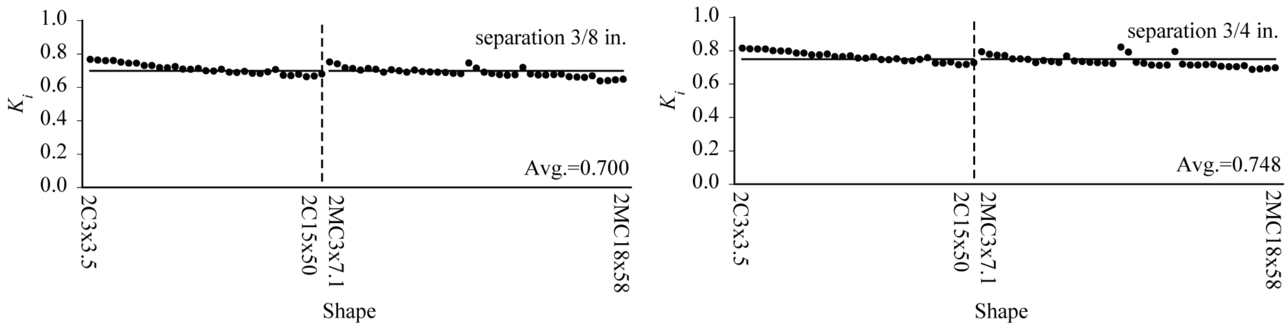


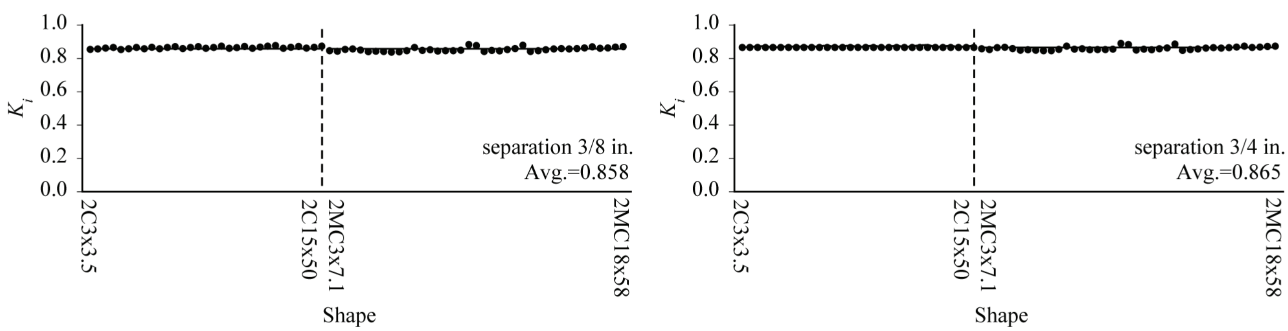
Fig. 9. Buckling strength of built-up members calculated from modified slenderness ratio; welded or pretensioned bolted connectors.



(a) Double Angles Long Legs Back-to-Back

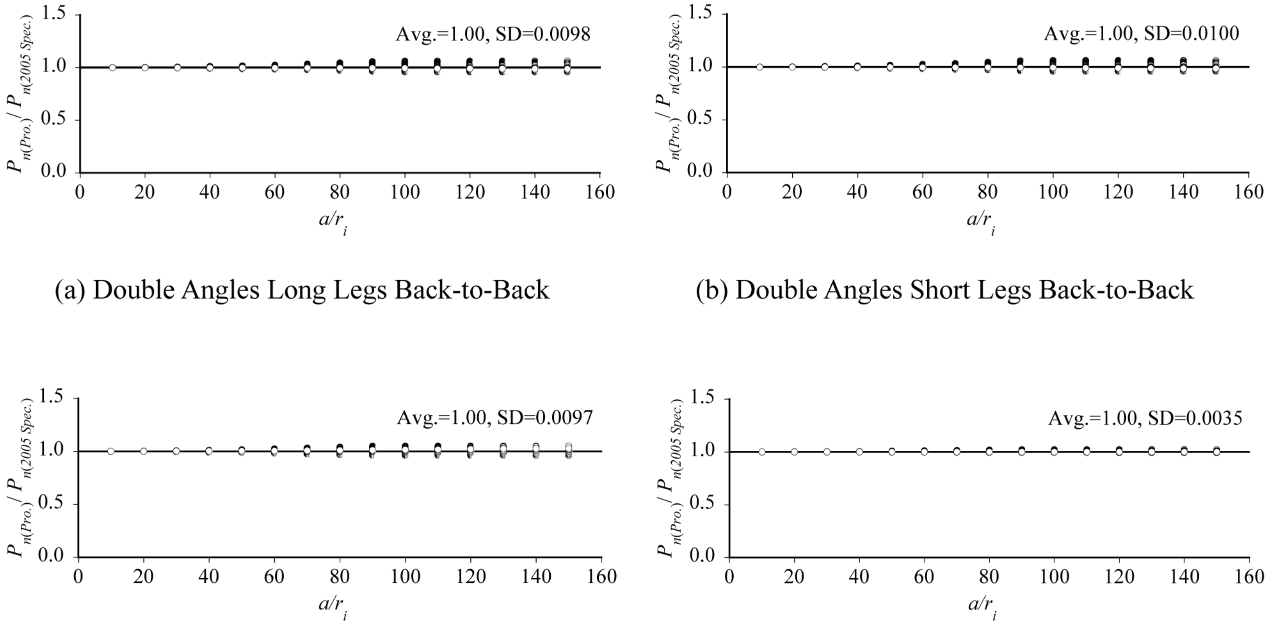


(b) Double Channels Back-to-Back



(c) Double Channels Toe-to-Toe

Fig. 10. Variations of factor K_i.



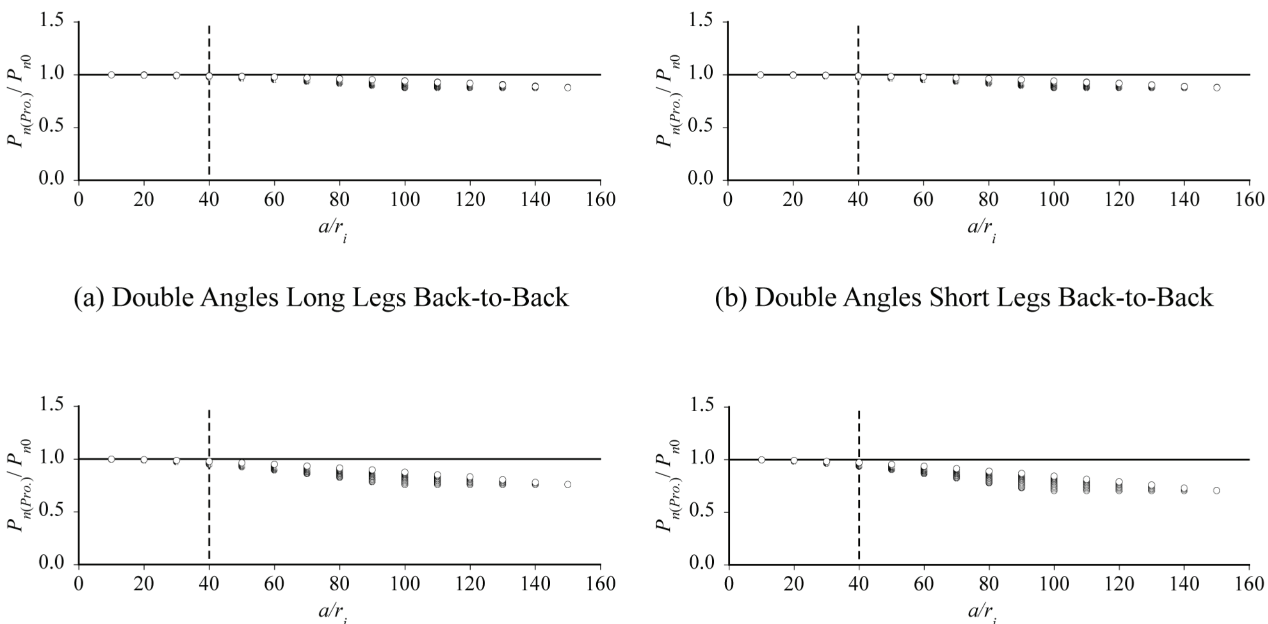
(a) Double Angles Long Legs Back-to-Back

(b) Double Angles Short Legs Back-to-Back

(c) Double Channels Back-to-Back

(d) Double Channels Toe-to-Toe

Fig. 11. Values of $P_{n(Pro.)}/P_{n(2005\ Spec.)}$ *(separation = ¾ in.).*



(a) Double Angles Long Legs Back-to-Back

(b) Double Angles Short Legs Back-to-Back

(c) Double Channels Back-to-Back

(d) Double Channels Toe-to-Toe

Fig. 12. Values of $P_{n(Pro.)}/P_{n0}$ *(separation = ¾ in.).*