

# Graphical Design Aid for Beam-Columns (LRFD)

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The AISC load and resistance factor design (LRFD) procedure for the design of a beam-column uses the following interaction equations.

$$\frac{P_u}{\phi_c P_n} + \left( \frac{8}{9} \right) \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \text{ for } \frac{P_u}{(\phi_c P_n)} \geq 0.20 \quad (1)$$

and

$$\frac{P_u}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \text{ for } \frac{P_u}{(\phi_c P_n)} < 0.20 \quad (2)$$

where

- $P_u$  = required axial strength  
 $M_u$  = required flexural strength  
 $\phi_c P_n$  = axial design strength  
 $\phi_b M_n$  = flexural design strength

These equations are taken from the *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 1999), hereafter referred to as the AISC LRFD Specification. Aminmansour (2004), has rewritten the interaction equations as

$$bP_u + m M_{ux} + n M_{uy} \leq 1.0 \text{ for } bP_u \geq 0.2 \quad (3)$$

$$0.50 bP_u + (9/8)\{m M_{ux} + n M_{uy}\} \leq 1.0 \text{ for } bP_u \geq 0.2 \quad (4)$$

with  $b$ ,  $m$ , and  $n$  replacing  $(1/\phi_c P_n)$ ,  $(1/\phi_b M_{nx})$ , and  $(1/\phi_b M_{ny})$ , respectively. He suggested that the smaller  $b$ ,  $m$ , and  $n$  values are more effective and therefore desirable. Also in the

case of relatively large axial load, a section with a smaller  $b$ -value may be more effective, although it may have a larger  $m$ -value. Similarly in the case of a member subjected to relatively large bending moment about the  $x$ -axis, a section with a smaller  $m$  value may be more effective and desirable though  $b$  may be larger.

Keil (2000) developed graphical design aids for beam-columns per the AISC LRFD Specification. The design aids are presented as interaction curves for W-sections with the conservative and simplifying assumption of  $C_b = 1$ , a factor accounting for moment gradient.

Zuraski (1992) elaborated upon the significance and application of  $C_b$  in beam design to account for moment gradient. The magnitude of compressive force within a beam can be determined by the inspection of moment diagram. Because the resistance to bending is composed of an internal compressive force,  $C$ , and tensile force,  $T$ , couple, the magnitude of  $C$  at any location along the span equals the bending moment divided by the internal moment arm. Thus the variation of the force within the compression flange has the same shape as the moment diagram.

In light of Zuraski's observations and the importance of the  $C_b$  factor, Keil's graphical design aids can be improved by considering the actual  $C_b$  value instead of a conservative value of  $C_b = 1$ , as the value of  $C_b$  varies from 1.0 to 2.3, which is quite significant.

## DERIVATION OF THE DESIGN PROCEDURE

The design procedure is developed taking into consideration the equivalent uniform moment factor,  $C_b$ , unbraced length,  $L_b$ , for the flexural design strength and effective length,  $KL$ , for the axial compressive design strength. Although the design aids are developed for beam-columns, the same curves can be used for beams and columns with  $P_u = 0$  and  $M_u = 0$ , respectively.

The interaction curves are derived with  $K = 1$  and  $C_b = 1.0$ . Therefore, the following conversion factors are defined to account for  $KL$  in axial load and  $L_b$  in bending moment.

$$\alpha_{(at \ KL)} = \frac{\phi_c \bar{P}_n}{\phi_c P_n} \geq 1.0 \quad (5)$$

$$\beta_{(at \ L_b)} = \frac{\phi_b \bar{M}_n}{\phi_b M_n \text{ (at } L_b \text{ and } C_b = 1\text{)}} \geq 1.0 \quad (6)$$

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where

- $\phi_c \bar{P}_n$  = maximum axial compressive design strength
- $\phi_b \bar{M}_n$  = maximum flexural design strength (plastic moment capacity) of the section

The axial load and bending moments are transformed as  $P'_u = \alpha P_u$  and  $M'_u = (\beta/C_b)M_u$  for use in the interaction equations, where  $P_u$  and  $M_u$  are the required axial compressive strength and required flexural strength, respectively. The interaction equations are rewritten for the convenience of preparing design aids as

$$\frac{P'_u}{\phi_c \bar{P}_n} + \left( \frac{8}{9} \right) \left( \frac{M'_u}{\phi_b \bar{M}_n} \right) \leq 1.0 \text{ for } \frac{P'_u}{(\phi_c \bar{P}_n)} \geq 0.20 \quad (7)$$

and

$$\frac{P'_u}{2\phi_c \bar{P}_n} + \left( \frac{M'_u}{\phi_b \bar{M}_n} \right) \leq 1.0 \text{ for } \frac{P'_u}{(\phi_c \bar{P}_n)} < 0.20 \quad (8)$$

The equivalence of  $P'_u/\phi_c \bar{P}_n$  and  $P_u/\phi_c P_n$  and also the equivalence of  $M'_u/\phi_b \bar{M}_n$  and  $M_u/\phi_b M_n$  are established here:

$$\begin{aligned} \frac{P'_u}{\phi_c \bar{P}_n} &= \frac{\alpha P_u}{\phi_c \bar{P}_n}, \text{ substituting for } \alpha \\ &= \frac{\left[ \frac{\phi_c \bar{P}_n}{\phi_c P_n} (\text{at } KL) \right] P_u}{\phi_c \bar{P}_n} \\ &= \frac{P_u}{\phi_c \bar{P}_n (\text{at } KL)} \\ \frac{M'_u}{\phi_b \bar{M}_n} &= \frac{\left( \frac{\beta}{C_b} \right) M_u}{\phi_b \bar{M}_n}, \text{ substituting for } \beta \\ &= \frac{\left[ \frac{\phi_b \bar{M}_n}{\phi_b M_n} (\text{at } L_b \text{ and } C_b = 1) \right] \left( \frac{1}{C_b} \right) M_u}{\phi_b \bar{M}_n} \\ &= \frac{M_u}{C_b \phi_b M_n (\text{at } L_b \text{ and } C_b = 1)} \end{aligned}$$

To ensure  $P_u \leq \phi_c P_n$  and  $M_u \leq \phi_b M_n$ , the conditions  $\alpha \geq 1.0$  and  $(\beta/C_b) \geq 1.0$  are to be imposed.

Based on the definitions of  $\alpha$  and  $\beta$  the design tables for  $\alpha$  and  $\beta$  (Appendix C) are prepared for different sections.

## THE USE OF DESIGN AIDS

The proposed design aids can be used for beam-columns, beams, and columns.

### Selection of a Beam-Column

1. For the member under consideration, effective length factor,  $K$ , unsupported length,  $L_b$ , factored moment,  $M_u$ , and factored axial load,  $P_u$ , are to be computed.
2. Compute  $C_b$  for the given moment distribution.
3. Compute the conversion factor,  $\alpha$ , for axial load and  $\beta$  for bending moment (or find  $\alpha$  and  $\beta$  from the table in Appendix C for the appropriate values of  $KL$  and  $L_b$ ).
4. In the tables for values of  $\alpha$  and  $\beta$  values shown in bold letters are for spans having slenderness ratio,  $KL/r > 200$ . So check for slenderness limits.
5. Find the ratio  $\beta/C_b$ . If this ratio is less than 1.0, the ratio is to be taken equal to 1.0, because  $\beta/C_b \geq 1.0$ .
6. Compute transformed axial load and bending moment as  $P'_u = \alpha P_u$  and  $M'_u = (\beta/C_b)M_u$ .
7. Enter the design curves with  $P'_u$  and  $M'_u$  to select the section.

An efficient section for the beam-column can be selected following Aminmansour's (2004) recommendations. Namely, for a beam-column with relatively large axial loads, a section having a smaller  $\alpha$  is to be selected. Similarly, for a beam-column with a relatively large bending moment, a section having a smaller  $\beta$  is to be selected.

### Use of Design Aid for a Beam

1. For the member with given unbraced length,  $L_b$ , and factored moment,  $M_u$ ,  $C_b$  is to be computed.
2. For a given  $L_b$ , compute  $\beta$  or determine the value from the table in Appendix C.
3. Find the ratio  $\beta/C_b$ . If this ratio is less than 1.0, the ratio is to be taken equal to 1.0, because  $\beta/C_b \geq 1.0$ .
4. Compute the transformed moment,  $M'_u = (\beta/C_b)M_u$ .
5. Enter the design curves with  $M'_u$  and  $P'_u = 0$ , to select the section. Check  $M'_u \leq \bar{M}_n$  (plastic moment capacity).
6. If condition 5 is satisfied, then the section selected is satisfactory. Otherwise Steps 2 to 5 are to be repeated. However, the iteration converges quickly.

### Use of Design Aid for a Column

1. For the member with given effective length,  $KL$ , and factored axial load,  $P_u$ , compute  $\alpha$  or determine the value from tables in Appendix C.

2. Compute the transformed axial load,  $P_u' = \alpha P_u$

3. Check  $P_u' \leq \bar{P}_n$  (squash load).

If condition 3 is satisfied, then the section selected is satisfactory, otherwise, steps 2 and 3 are to be repeated. However, the iteration converges quickly.

### ILLUSTRATIVE EXAMPLES

The following examples from the references as indicated are used to demonstrate the applicability of the proposed method.

#### Example 1 (Smith, 1996)

*Given*

The beam-column shown in Figure 1 is pinned at both ends and is subjected to the factored loads shown. Bending is about the strong axis. Determine whether this member satisfies the appropriate AISC LRFD Specification interaction equation.

*Solution*

From the AISC column load tables, for W8×58 with  $F_y = 50$  ksi and an effective length  $K_y L = 1.0 \times 17 = 17$  ft,  $\phi_c P_n = 365$  kips (AISC, 1998). (Note: This value was taken from the 2nd Ed. AISC LRFD Manual because the W8×58 was not tabulated in the 3rd Ed. AISC LRFD Manual.)

From the AISC beam design charts (AISC, 1999), for  $L_b = 17$  ft and  $C_b = 1.0$ ,  $\phi_b M_n = 202$  ft-kips.

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C} = 1.32 \leq 2.3$$

where

$$M_{max} = M_B = 93.5 \text{ ft-kips}, M_A = M_C = 46.75 \text{ ft-kips}$$

$$\phi_b M_n = 1.32 \times 202 = 267 \text{ ft-kips (for } C_b = 1.32\text{)}$$

But  $\phi_b M_p = 224$  ft-kips, therefore use  $\phi_b M_n = 224$  ft-kips

$$\begin{aligned} \frac{P_u}{\phi_c P_n} &= \frac{200}{365} = 0.5479 > 0.2 \\ \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) & \\ \frac{200}{365} + \frac{8}{9} \left( \frac{93.5}{224} + 0 \right) &= 0.919 < 1.0 \text{ o.k.} \end{aligned}$$

#### Calculation Using Proposed Method

For

$$M_u = 93.5 \text{ ft-kips}$$

$$P_u = 200 \text{ kips}$$

$$C_b = 1.32$$

$$KL = 17 \text{ ft} = L_b$$

$$\alpha = 2.01 \text{ and } \beta = 1.11 \text{ (interpolating from Appendix B, Table 1)}$$

$$\beta/C_b = 1.11/1.32 < 1.0, \text{ use } (\beta/C_b) = 1.0.$$

Therefore,  $P_u' = \alpha P_u = 2.01 \times 200 = 402$  kips and  $M_u' = (\beta/C_b) M_u = 1.0 \times 93.5 = 93.5$  ft-kips.

From Chart 1, W8×58 satisfies the requirements.

#### Example 2 (Smith, 1996)

*Given*

A W12×65, of ASTM A572 Grade 50 steel, 15 ft long, is to be investigated for use as a column in an unbraced frame. The axial load and end moments obtained from a first order analysis of the gravity loads (dead and live load) are shown in Figure 2(a). The frame is symmetrical and the gravity loads are symmetrically placed. Figure 2(b) shows the wind load moments obtained from a first-order analysis. All bending moments are about the strong axis. Effective length factors are  $K_x = 1.2$  for the sway case,  $K_x = 1.0$  for the nonsway case, and  $K_y = 1.0$ . Determine whether this member is in compliance with the AISC Specification. Check the following load combinations.

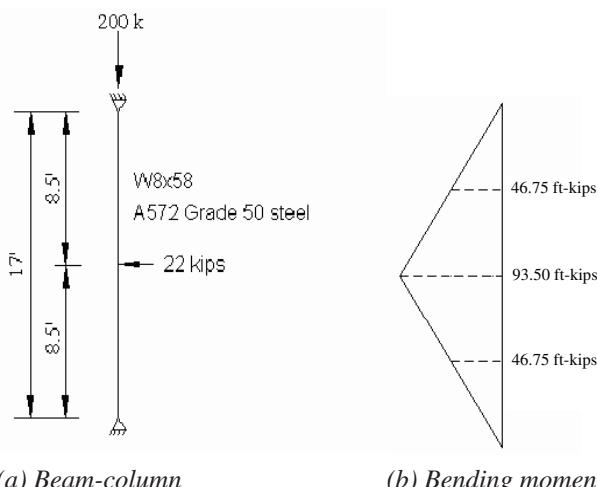


Fig. 1. Beam-column configuration and factored loading.

$$1.2D + 1.6L \quad (i)$$

$$1.2D + 0.5L + 1.3W \quad (ii)$$

*Solution*

Figures 2(c) and 2(d) show the axial loads and bending moments calculated for these two combinations. The critical axis for axial compressive strength is determined as follows.

$$K_y L = 15 \text{ ft}$$

$$\frac{K_x L}{r_x / r_y} = \frac{1.2 \times 15}{1.75} = 10.29 < 15, \text{ therefore use } KL = 15 \text{ ft}$$

**For Load Combination i:**

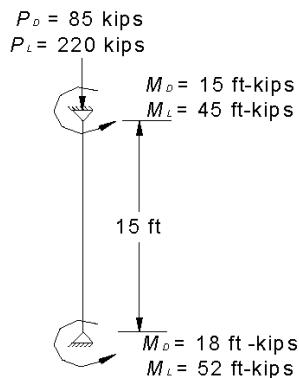
From the column load tables (AISC, 2001) for  $KL = 15 \text{ ft}$ ,  $\phi_c P_n = 626 \text{ kips}$ .

$$M_{lt} = 0 \text{ (no sidesway because of symmetry)}$$

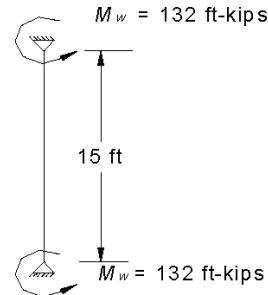
$$C_m = 0.6 - 0.4 (M_1/M_2)$$

$$= 0.6 - 0.4 (90/104.8)$$

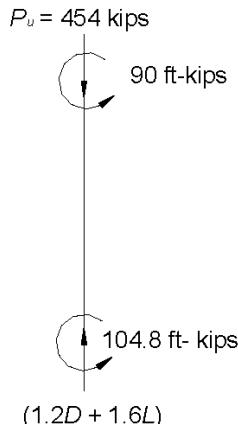
$$= 0.2565 \text{ [Eqn. C1-3 of AISC (1999)]}$$



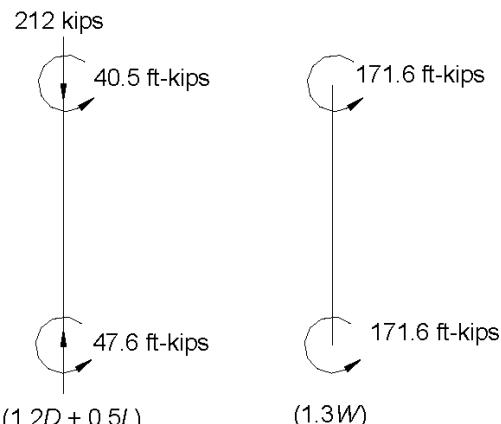
(a) Gravity loading



(b) Wind loading



(c) Load combination i



(d) Load combination ii ( $1.2D + 0.5L + 1.3W$ )

Fig. 2. Example 2 loading.

### For the axis of bending:

$KL/r = K_x L/r_x = 1 \times 15 \times 12/5.28 = 34.09$  ( $K_x = 1.0$  for no sidesway is used).

$$P_{e1} = \frac{\pi^2 EA_g}{(KL/r)^2} = \frac{\pi^2 \times 29,000 \times 19.1}{(34.09)^2} = 4,704 \text{ kips}$$

The amplification factor for nonsway moment is

$$\begin{aligned} B_1 &= \frac{C_m}{1 - (P_u/P_{e1})} && [\text{Eqn. C1-2 of AISC (1999)}] \\ &= \frac{0.2565}{1 - (454/4,704)} = 0.284 \\ &< 1.0, \text{ therefore use } B_1 = 1.0 \end{aligned}$$

$$M_u = B_1 M_{nt} + B_2 M_{lt} = (1.0 \times 104.8) + 0 = 104.8 \text{ kip-ft}$$

From the beam design charts, with  $L_b = 15 \text{ ft}$ ,

$$\phi_b M_n = 343 \text{ kip-ft (for } C_b = 1.0)$$

$$\phi_b M_p = 358 \text{ kip-ft}$$

$$\begin{aligned} C_b &= \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C} && [\text{Eqn. F1-3 of AISC (1999)}] \\ &= 2.24 \leq 2.3 \end{aligned}$$

where

$$M_{max} = 104.8 \text{ kip-ft}$$

$$M_A = 41.3 \text{ kip-ft}$$

$$M_B = 7.4 \text{ kip-ft}$$

$$M_c = 56.1 \text{ kip-ft}$$

For  $C_b = 2.24$ ,  $\phi_b M_n = 2.24 \times 343 > \phi_b M_p = 358 \text{ kip-ft}$ , therefore use  $\phi_b M_n = 358 \text{ kip-ft}$ .

$$\frac{P_u}{\phi_c P_n} = \frac{454}{626} = 0.725 > 0.2$$

$$\begin{aligned} \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) &= 0.725 + \frac{8}{9} \left( \frac{104.8}{358} + 0 \right) = 0.985 < 1.0 \text{ o.k.} \\ &\quad [\text{Eqn. H1-1a of AISC (1999)}] \end{aligned}$$

### Calculation Using Proposed Method (for load combination i)

For  $K = 1.0$ ,  $L = 15 \text{ ft}$ ,  $C_b = 2.24$ ,  $M_u = 104.8 \text{ kip-ft}$ , and  $P_u = 454 \text{ kips}$ .

For W12x65,  $KL = 15 \text{ ft}$ ,  $\alpha = 1.30$  and for  $L_b = 15 \text{ ft}$ ,  $\beta = 1.044$  (interpolating from Appendix B, Table 1).

$$P_u' = 454 \times 1.30 = 590.20 \text{ kips}$$

$$\beta/C_b = 1.044/2.24 < 1.0, \text{ therefore } \beta/C_b = 1.0 \text{ is used and } M_u' = 1 \times 104.8 = 104.8 \text{ kip-ft.}$$

From Chart 1, W12x65 satisfies the requirements as the coordinate (104.8, 590.2) lies below the curve for W12x65.

### For Load Combination ii:

$P_u = 212 \text{ kips}$ ,  $M_{nt} = 47.6 \text{ kip-ft (for } 1.2D + 0.5L)$ ,  $M_{lt} = 171.6 \text{ kip-ft (for } 1.3W)$

For the braced condition,

$$C_m = 0.6 - 0.4 (40.5 / 47.6) = 0.2597$$

$$P_{e1} = 4,704 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \left( \frac{P_u}{P_{e1}} \right)} = \frac{0.2597}{1 - \left( \frac{212}{4,704} \right)} = 0.272 < 1.0, \text{ use } B_1 = 1.0$$

Using  $K_x$  corresponding to unbraced condition for calculation of  $P_{e2}$ ,

$$P_{e2} = \frac{\pi^2 EA_g}{(KL/r)^2} = \frac{\pi^2 \times 29,000 \times 19.1}{(40.9)^2} = 3,268 \text{ kips}$$

$$\text{where } KL/r_x = 1.2 \times 15 \times 12/5.28 = 40.9$$

Assuming the ratio of  $P_u/P_e$  is the same for all the columns in the story, as for the column under consideration,

$$\begin{aligned} B_2 &= \frac{1}{1 - \left( \frac{\sum P_u}{\sum P_{e2}} \right)} && [\text{Eqn. C1-5 of AISC (1999)}] \\ &\approx \frac{1}{1 - \left( \frac{P_u}{P_{e2}} \right)} = \frac{1}{1 - \left( \frac{212}{3,268} \right)} = 1.069 \end{aligned}$$

The total amplified moment is

$$M_u = B_1 M_{nt} + B_2 M_{lt} = 1.0 \times 47.6 + 1.069 \times 171.6$$

$$= 231.0 \text{ kip-ft}$$

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C} = 2.2867 \leq 2.3$$

where

$$M_{max} = 219.2 \text{ kip-ft}$$

$$M_A = 107.87 \text{ kip-ft}$$

$$M_B = 3.45 \text{ kip-ft}$$

$$M_c = 104.27 \text{ kip-ft}$$

For  $C_b = 2.2867$ ,  $\phi_b M_n = 2.2867 \times 343 > \phi_b M_p = 358 \text{ kip-ft}$ , therefore use  $\phi_b M_n = 358 \text{ kip-ft}$

$$\frac{P_u}{\phi_c P_n} = \frac{212}{626} = 0.338 > 0.2$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right)$$

$$\frac{212}{626} + \frac{8}{9} \left( \frac{231.0}{358} + 0 \right) = 0.912 < 1.0 \quad \text{o.k.}$$

#### Calculation Using Proposed Method (for load combination ii)

For  $P_u = 212$  kips,  $M_u = 231.0$  kip-ft,  $KL = 15$  ft,  $L_b = 15$  ft and  $C_b = 2.2867$

For W12×65,  $KL = 15$  ft,  $\alpha = 1.30$  and for  $L_b = 15$  ft,  $\beta = 1.0645$  (interpolating from Appendix B, Table 1)

$$\beta/C_b = 1.0645/2.2867 < 1.0, \text{ therefore use 1.0}$$

$$M'_u = 1.0 \times 231 = 231 \text{ kip-ft}$$

$$P'_u = 1.30 \times 212 = 275.6 \text{ kips}$$

From Chart 1, W12×65 satisfies the requirements as the coordinates (231, 275.6) lie below the curve for W12×65.

#### **Example 3 (Aminmansour, 2004)**

*Given*

W24×131, ASTM A992 steel,  $L_b = 16$  ft,  $C_b = 1.67$

Determine  $\phi_b M_{nx}$ .

*Solution*

Referring to the AISC LRFD Manual of Steel Construction (Table 6-2) (AISC, 2001),  $m = 0.700 \times 10^{-3}$  (kip-ft) $^{-1}$ ;  $\phi_b M_p = 1,390$  kip-ft; and  $L_p = 10.5$  ft.

$$L_b = 16 \text{ ft} > L_p = 10.5 \text{ ft}$$

$$\begin{aligned} \phi_b M_{nx} &= 8/(9m) = 8/(9 \times 0.700 \times 10^{-3}) \\ &= 1,270 \text{ kip-ft for } C_b = 1.0 \end{aligned}$$

$$\begin{aligned} \text{For } C_b = 1.67, \phi_b M_{nx} &= (C_b)[\phi_b M_{nx} \text{ for } C_b = 1] \\ &= (1.67)(1,270 \text{ kip-ft}) \\ &= 2,120 \text{ kip-ft} > \phi_b M_p = 1,390 \text{ kip-ft} \end{aligned}$$

Therefore,  $\phi_b M_{nx} = \phi_b M_p = 1,390$  kip-ft.

#### Calculation Using Proposed Method

For  $L_b = 16$  ft,  $\beta = 1.093$  (from Appendix B, Table 1)

$$\beta/C_b = 1.093/1.67 < 1.0, \text{ therefore use 1.0}$$

$\phi_b M_p = 1,390$  (Chart 1), value of  $x$ -coordinate where curve for W24×131 meets the  $x$ -axis

$$\text{Therefore, } \phi_b M_{nx} = (\phi_b M_p)/\beta/C_b = 1,390 \text{ kip-ft.}$$

#### **Example 4 (Keil, 2000)**

*Given*

Given the following loading, select the lightest W14 using ASTM A992 steel.

*Solution*

$P_u = 3,400$  kips;  $M_u = 650$  kip-ft;  $K = 1.0$ ;  $L = 16$  ft;  $F_y = 50$  ksi

Try W14×398

$\phi_c P_n = 4,300$  kips from the column tables of AISC (2001), Part 3.

$\phi_b M_n = 2,997$  kip-ft from Chapter F of the AISC LRFD Specification.

$$\frac{P_u}{\phi_c P_n} = \frac{3,400}{4,300} = 0.7906 > 0.2, \text{ therefore,}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right)$$

$$\frac{3,400}{4,300} + \frac{8}{9} \left( \frac{650}{2,997} + 0 \right) = 0.983 < 1.0 \quad \text{o.k.}$$

#### Calculation Using Proposed Method

For  $P_u = 3,400$  kips,  $M_u = 650$  kip-ft,  $KL = 16$  ft,  $L_b = 16$  ft, and  $C_b = 1.0$ .

For W14×398, for  $KL = 15$  ft,  $\alpha = 1.157$  and for  $L_b = 15$  ft,  $\beta = 1.0019$  (interpolating from Appendix B, Table 1).

$$\beta/C_b = 1.0019/1 = 1.0019$$

$$M'_u = 1.0019 \times 650 = 651.235 \text{ kip-ft}$$

$$P'_u = 1.157 \times 3,400 = 3,933.8 \text{ kips}$$

From Chart 1, W14×398 satisfies the requirements as the coordinates (651.235, 3,933.8) lie below the curve for W14×398.

## **CONCLUSION**

The proposed graphical design aid and design procedure is an improvement over the design aid proposed by Keil (2000) as it includes the actual value of  $C_b$  in the computation of  $\phi_b M_n$  as opposed to the conservative value of  $C_b = 1$ . The same set of design charts can be used for computing the capacity of columns, beams and beam-columns, which is illustrated in Examples 1 to 4. This method reduces the number of iterations and simplifies the tedious computation of beam-column strength and provides ready-to-use charts for design engineers. Selection of an efficient section is possible through the use of Aminmansour's approach (Aminmansour, 2004).

## REFERENCES

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## APPENDIX A LIST OF NOTATIONS

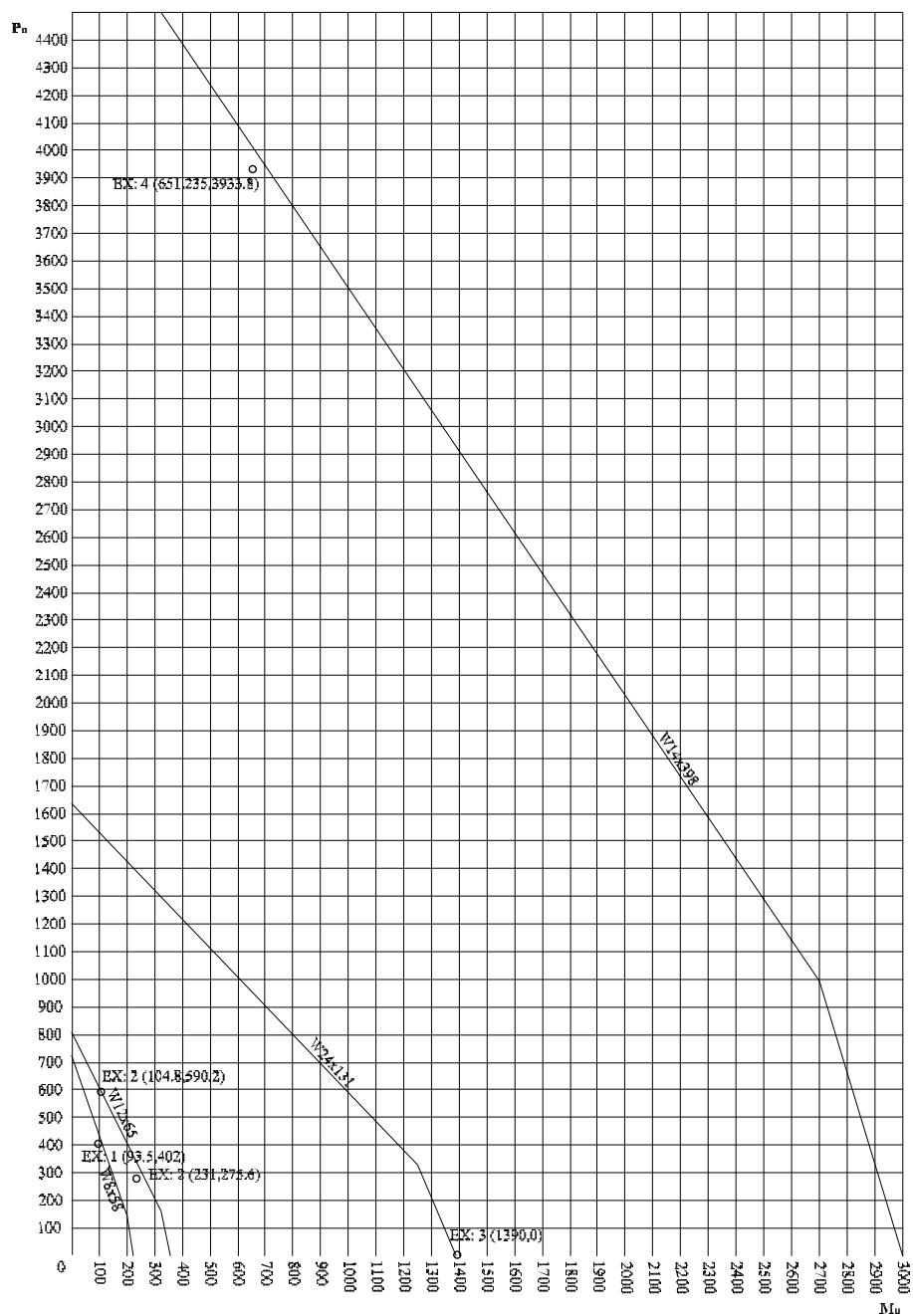
$B_1, B_2$	= Factors used in determining $M_u$ for combined bending and axial forces when first-order analysis is used.	$M_{max}$	= Absolute value of maximum moment in the unbraced beam segment.
$B_{KL}$ and $B_{LM}$	= Axial conversion and moment capacity conversion factors defined for Keil's graphical design aids (Keil, 2000).	$M_n$	= Nominal flexural strength.
$C_b$	= Modification factor for nonuniform moment.	$M_{nt}$	= Required flexural strength in member assuming there is no lateral translation of the frame.
$C_m$	= Coefficient applied to bending term in interaction formula for prismatic members and dependent on column curvature caused by applied moment.	$M_p$	= Plastic bending moment.
$D$	= Dead load due to weight of structural elements and permanent features on the structure.	$M_u$	= Required flexural strength.
$E$	= Modulus of elasticity of steel.	$M_1$	= Smaller moment at end of unbraced length of beam or beam-column.
$F_y$	= Specified minimum yield stress of steel.	$M_2$	= Larger moment at end of unbraced length of beam or beam-column.
$K$	= Effective length factor.	$P_{e1}, P_{e2}$	= Elastic Euler buckling load for braced and unbraced frame, respectively.
$L$	= Length of member.	$P_n$	= Nominal axial strength (tension or compression).
$L_p$	= Limiting laterally unbraced length for full plastic bending capacity, uniform moment case.	$P_u$	= Required axial strength (tension or compression).
$L_r$	= Limiting laterally unbraced length for inelastic lateral-torsional buckling.	$r_x, r_y$	= Radius of gyration about respective axis.
$M_A$	= Absolute value of moment at quarter point of the unbraced beam segment.	$\alpha$ and $\beta$	= Axial and moment capacity conversion factors for proposed method for AISC-LRFD.
$M_B$	= Absolute value of moment at center point of the unbraced beam segment	$\phi_c$	= Resistance factor for compression.
$M_C$	= Absolute value of moment at three-quarter point of the unbraced beam segment.	$\phi_b$	= Resistance factor for flexure.
$M_h$	= Required flexural strength in member due to lateral frame translation only.	$\phi_b \bar{M}_n$	= Maximum moment capacity (plastic capacity).
		$\phi_c \bar{P}_n$	= Maximum axial capacity of the section.

## APPENDIX B

KL (ft) or $L_b$ (ft)	$\alpha$ and $\beta$ Values for Sections Used in Illustrative Examples							
	W8×58		W12×65		W24×131		W14×398	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.010	1.000	1.005	1.000	1.005	1.000	1.002	1.000
4	1.039	1.000	1.019	1.000	1.019	1.000	1.009	1.000
6	1.090	1.000	1.042	1.000	1.044	1.000	1.021	1.000
8	1.165	1.006	1.077	1.000	1.079	1.000	1.037	1.000
10	1.270	1.028	1.123	1.000	1.126	1.000	1.058	1.000
12	1.411	1.050	1.181	1.003	1.186	1.024	1.085	1.000
14	1.597	1.073	1.254	1.030	1.262	1.057	1.118	1.000
16	1.844	1.098	1.344	1.059	1.355	1.093	1.157	1.002
18	2.169	1.123	1.454	1.089	1.469	1.131	1.202	1.007
20	2.604	1.150	1.588	1.121	1.608	1.172	1.255	1.012
22	3.151	1.178	1.749	1.155	1.776	1.216	1.317	1.017
24	3.750	1.208	1.946	1.192	1.981	1.264	1.387	1.022
26	4.401	1.239	2.184	1.230	2.231	1.315	1.468	1.028
28	5.104	1.271	2.474	1.272	2.536	1.371	1.561	1.033
30	5.860	1.306	2.833	1.316	2.910	1.336	1.668	1.038
32	6.667	1.342	3.224	1.239	3.311	1.471	1.789	1.044
34	7.527	1.380	3.639	1.341	3.738	1.606	1.929	1.049
36	<b>8.438</b>	<b>1.421</b>	4.080	1.442	4.190	1.742	2.089	1.055
38	<b>9.402</b>	<b>1.339</b>	4.546	1.544	4.669	1.879	2.272	1.060

Note: Values where  $KL/r > 200$  are shown in bold letters.

**Chart 1. Design Curves for Interaction Equation for Sections Used in Illustrative Examples  
( $P_u$  in kips and  $M_u$  in kip-ft)**



## APPENDIX C

KL (ft) or L <sub>b</sub> (ft)	Tables of $\alpha$ and $\beta$ Values									
	W6×20		W6×25		W8×21		W8×24		W8×28	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.019	1.000	1.018	1.000	1.027	1.000	1.016	1.000	1.016	1.000
4	1.078	1.000	1.076	1.000	1.112	1.000	1.067	1.000	1.066	1.000
6	1.184	1.016	1.178	1.011	1.270	1.051	1.158	1.007	1.156	1.007
8	1.350	1.065	1.339	1.051	1.529	1.127	1.297	1.059	1.293	1.053
10	1.597	1.120	1.578	1.094	1.942	1.216	1.502	1.116	1.494	1.103
12	1.963	1.180	1.929	1.141	2.604	1.320	1.796	1.180	1.783	1.159
14	2.505	1.246	2.445	1.192	3.545	1.347	2.219	1.252	2.197	1.220
16	3.267	1.321	3.181	1.248	4.630	1.603	2.836	1.333	2.801	1.289
18	4.135	1.278	4.027	1.310	5.860	1.857	3.589	1.328	3.545	1.366
20	5.104	1.448	4.971	1.377	7.234	2.110	4.431	1.518	4.376	1.353
22	6.176	1.618	6.015	1.338	<b>8.753</b>	<b>2.361</b>	5.361	1.707	5.295	1.515
24	7.350	1.786	7.158	1.472	<b>10.417</b>	<b>2.611</b>	6.380	1.895	6.302	1.677
26	<b>8.627</b>	<b>1.953</b>	<b>8.401</b>	<b>1.605</b>	<b>12.226</b>	<b>2.859</b>	7.488	2.081	7.396	1.837
28	<b>10.005</b>	<b>2.119</b>	<b>9.743</b>	<b>1.738</b>	<b>14.179</b>	<b>3.105</b>	<b>8.684</b>	<b>2.267</b>	<b>8.578</b>	<b>1.997</b>
30	<b>11.485</b>	<b>2.285</b>	<b>11.185</b>	<b>1.871</b>	<b>16.277</b>	<b>3.351</b>	<b>9.969</b>	<b>2.452</b>	<b>9.847</b>	<b>2.155</b>
32	<b>13.068</b>	<b>2.450</b>	<b>12.726</b>	<b>2.003</b>	<b>18.520</b>	<b>3.595</b>	<b>11.343</b>	<b>2.635</b>	<b>11.203</b>	<b>2.313</b>
34	<b>14.752</b>	<b>2.614</b>	<b>14.366</b>	<b>2.134</b>	<b>20.907</b>	<b>3.839</b>	<b>12.805</b>	<b>2.819</b>	<b>12.647</b>	<b>2.471</b>
36	<b>16.539</b>	<b>2.778</b>	<b>16.106</b>	<b>2.266</b>	<b>23.439</b>	<b>4.082</b>	<b>14.356</b>	<b>3.001</b>	<b>14.179</b>	<b>2.628</b>
38	<b>18.427</b>	<b>2.942</b>	<b>17.946</b>	<b>2.397</b>	<b>26.116</b>	<b>4.324</b>	<b>15.995</b>	<b>3.183</b>	<b>15.798</b>	<b>2.784</b>

Note: Values where  $KL/r > 200$  are shown in bold letters.

KL (ft) or L <sub>b</sub> (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W8×31		W8×35		W8×40		W8×48		W8×58	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.010	1.000	1.010	1.000	1.010	1.000	1.010	1.000	1.010	1.000
4	1.042	1.000	1.042	1.000	1.041	1.000	1.040	1.000	1.039	1.000
6	1.097	1.000	1.096	1.000	1.095	1.000	1.092	1.000	1.090	1.000
8	1.180	1.017	1.178	1.013	1.176	1.012	1.169	1.007	1.165	1.006
10	1.295	1.056	1.291	1.049	1.288	1.044	1.276	1.033	1.270	1.028
12	1.451	1.098	1.445	1.087	1.440	1.077	1.420	1.060	1.411	1.050
14	1.659	1.143	1.651	1.127	1.643	1.113	1.612	1.089	1.597	1.073
16	1.937	1.193	1.925	1.171	1.912	1.151	1.866	1.119	1.844	1.098
18	2.309	1.247	2.290	1.219	2.272	1.192	2.202	1.150	2.169	1.123
20	2.815	1.306	2.787	1.270	2.760	1.236	2.655	1.184	2.604	1.150
22	3.406	1.371	3.372	1.326	3.339	1.283	3.212	1.220	3.151	1.178
24	4.053	1.363	4.013	1.387	3.974	1.334	3.823	1.258	3.750	1.208
26	4.757	1.503	4.710	1.365	4.664	1.389	4.486	1.298	4.401	1.239
28	5.517	1.642	5.463	1.488	5.409	1.337	5.203	1.341	5.104	1.271
30	6.333	1.780	6.271	1.610	6.210	1.444	5.973	1.387	5.860	1.306
32	7.206	1.917	7.135	1.732	7.065	1.551	6.796	1.318	6.667	1.342
34	<b>8.134</b>	<b>2.054</b>	<b>8.055</b>	<b>1.853</b>	<b>7.976</b>	<b>1.658</b>	7.672	1.407	7.527	1.380
36	<b>9.120</b>	<b>2.191</b>	<b>9.030</b>	<b>1.974</b>	<b>8.942</b>	<b>1.764</b>	<b>8.601</b>	<b>1.495</b>	<b>8.438</b>	<b>1.421</b>
38	<b>10.161</b>	<b>2.327</b>	<b>10.061</b>	<b>2.094</b>	<b>9.963</b>	<b>1.870</b>	<b>9.583</b>	<b>1.583</b>	<b>9.402</b>	<b>1.339</b>

Note: Values where  $KL/r > 200$  are shown in bold letters.

## APPENDIX C (continued)

**Tables of  $\alpha$  and  $\beta$  Values (continued)**

KL (ft) or $L_b$ (ft)	W8x67		W10x22		W10x26		W10x30		W10x33	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.009	1.000	1.024	1.000	1.023	1.000	1.023	1.000	1.011	1.000
4	1.038	1.000	1.100	1.000	1.095	1.000	1.094	1.000	1.046	1.000
6	1.088	1.000	1.239	1.049	1.228	1.041	1.224	1.038	1.106	1.000
8	1.162	1.005	1.464	1.134	1.440	1.118	1.432	1.107	1.196	1.024
10	1.264	1.023	1.815	1.235	1.768	1.206	1.753	1.186	1.323	1.072
12	1.402	1.042	2.358	1.355	2.272	1.310	2.245	1.277	1.497	1.124
14	1.583	1.062	3.181	1.472	3.043	1.315	2.998	1.384	1.731	1.181
16	1.823	1.083	4.155	1.792	3.974	1.587	3.916	1.433	2.048	1.244
18	2.138	1.105	5.259	2.117	5.030	1.860	4.957	1.669	2.478	1.315
20	2.555	1.128	6.493	2.443	6.210	2.133	6.119	1.903	3.052	1.273
22	3.092	1.151	7.856	2.769	7.514	2.404	7.404	2.136	3.692	1.449
24	3.680	1.176	<b>9.350</b>	<b>3.095</b>	<b>8.942</b>	<b>2.675</b>	<b>8.812</b>	<b>2.368</b>	4.394	1.624
26	4.319	1.201	<b>10.973</b>	<b>3.420</b>	<b>10.494</b>	<b>2.944</b>	<b>10.341</b>	<b>2.599</b>	5.157	1.800
28	5.009	1.228	<b>12.726</b>	<b>3.743</b>	<b>12.171</b>	<b>3.211</b>	<b>11.994</b>	<b>2.828</b>	5.981	1.974
30	5.750	1.256	<b>14.609</b>	<b>4.064</b>	<b>13.971</b>	<b>3.477</b>	<b>13.768</b>	<b>3.055</b>	6.866	2.148
32	6.542	1.285	<b>16.622</b>	<b>4.385</b>	<b>15.896</b>	<b>3.742</b>	<b>15.665</b>	<b>3.282</b>	7.812	2.321
34	7.385	1.315	<b>18.764</b>	<b>4.703</b>	<b>17.946</b>	<b>4.006</b>	<b>17.684</b>	<b>3.508</b>	<b>8.819</b>	<b>2.493</b>
36	<b>8.280</b>	<b>1.348</b>	<b>21.037</b>	<b>5.021</b>	<b>20.119</b>	<b>4.268</b>	<b>19.826</b>	<b>3.733</b>	<b>9.887</b>	<b>2.665</b>
38	<b>9.225</b>	<b>1.381</b>	<b>23.439</b>	<b>5.337</b>	<b>22.416</b>	<b>4.530</b>	<b>22.090</b>	<b>3.958</b>	<b>11.016</b>	<b>2.836</b>

Note: Values where  $KL/r > 200$  are shown in bold letters.

**Tables of  $\alpha$  and  $\beta$  Values (continued)**

KL (ft) or $L_b$ (ft)	W10x39		W10x45		W10x49		W10x54		W10x60	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.011	1.000	1.010	1.000	1.007	1.000	1.006	1.000	1.006	1.000
4	1.044	1.000	1.043	1.000	1.026	1.000	1.026	1.000	1.026	1.000
6	1.102	1.000	1.098	1.000	1.061	1.000	1.060	1.000	1.059	1.000
8	1.188	1.019	1.182	1.015	1.110	1.000	1.108	1.000	1.108	1.000
10	1.308	1.060	1.298	1.051	1.177	1.015	1.174	1.012	1.173	1.011
12	1.473	1.104	1.456	1.089	1.265	1.045	1.261	1.040	1.258	1.036
14	1.694	1.152	1.667	1.131	1.377	1.077	1.371	1.069	1.367	1.063
16	1.990	1.204	1.950	1.175	1.519	1.111	1.509	1.100	1.505	1.091
18	2.389	1.262	2.329	1.223	1.698	1.148	1.684	1.133	1.677	1.120
20	2.930	1.325	2.843	1.276	1.922	1.187	1.903	1.168	1.893	1.152
22	3.545	1.267	3.440	1.333	2.205	1.229	2.178	1.205	2.165	1.184
24	4.219	1.413	4.094	1.395	2.562	1.273	2.525	1.245	2.507	1.219
26	4.951	1.559	4.804	1.381	3.009	1.322	2.962	1.287	2.939	1.256
28	5.742	1.703	5.572	1.505	3.489	1.374	3.435	1.332	3.408	1.296
30	6.592	1.847	6.396	1.628	4.005	1.337	3.943	1.381	3.912	1.337
32	7.500	1.991	7.278	1.751	4.557	1.446	4.486	1.346	4.452	1.382
34	<b>8.466</b>	<b>2.133</b>	<b>8.216</b>	<b>1.874</b>	5.145	1.555	5.065	1.445	5.025	1.324
36	<b>9.492</b>	<b>2.275</b>	<b>9.211</b>	<b>1.996</b>	5.768	1.663	5.678	1.544	5.634	1.413
38	<b>10.576</b>	<b>2.417</b>	<b>10.262</b>	<b>2.117</b>	6.427	1.771	6.326	1.642	6.277	1.501

Note: Values where  $KL/r > 200$  are shown in bold letters.

## APPENDIX C (continued)

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W10×68		W10×77		W10×88		W10×100		W10×112	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.006	1.000	1.006	1.000	1.006	1.000	1.006	1.000	1.006	1.000
4	1.025	1.000	1.025	1.000	1.025	1.000	1.024	1.000	1.024	1.000
6	1.058	1.000	1.058	1.000	1.056	1.000	1.056	1.000	1.054	1.000
8	1.106	1.000	1.105	1.000	1.102	1.000	1.101	1.000	1.098	1.000
10	1.170	1.009	1.169	1.008	1.165	1.006	1.162	1.005	1.158	1.003
12	1.254	1.031	1.252	1.028	1.245	1.023	1.241	1.020	1.235	1.017
14	1.361	1.055	1.357	1.049	1.348	1.041	1.342	1.036	1.333	1.031
16	1.495	1.079	1.491	1.070	1.477	1.060	1.468	1.052	1.456	1.045
18	1.664	1.105	1.657	1.093	1.638	1.079	1.626	1.069	1.609	1.060
20	1.875	1.132	1.866	1.116	1.840	1.099	1.823	1.086	1.799	1.075
22	2.139	1.161	2.127	1.141	2.091	1.120	2.068	1.104	2.034	1.091
24	2.472	1.191	2.455	1.167	2.405	1.142	2.374	1.123	2.329	1.107
26	2.893	1.222	2.871	1.193	2.806	1.164	2.764	1.142	2.702	1.124
28	3.356	1.255	3.330	1.221	3.254	1.187	3.206	1.162	3.134	1.141
30	3.852	1.290	3.823	1.251	3.736	1.212	3.680	1.183	3.598	1.159
32	4.383	1.327	4.349	1.282	4.251	1.237	4.187	1.204	4.094	1.177
34	4.948	1.366	4.910	1.314	4.799	1.264	4.727	1.226	4.621	1.196
36	5.547	1.269	5.505	1.349	5.380	1.291	5.299	1.249	5.181	1.215
38	6.181	1.347	6.133	1.385	5.994	1.320	5.904	1.273	5.773	1.235

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W12×22		W12×26		W12×30		W12×35		W12×40	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.061	1.000	1.019	1.000	1.018	1.000	1.018	1.000	1.011	1.000
4	1.265	1.060	1.077	1.000	1.076	1.000	1.074	1.000	1.046	1.000
6	1.697	1.205	1.181	1.024	1.178	1.019	1.173	1.018	1.107	1.000
8	2.561	1.396	1.344	1.098	1.339	1.089	1.329	1.083	1.199	1.028
10	4.002	1.701	1.588	1.185	1.578	1.169	1.560	1.157	1.327	1.077
12	5.763	2.219	1.946	1.285	1.929	1.262	1.896	1.242	1.503	1.131
14	7.844	2.742	2.474	1.301	2.445	1.371	2.389	1.340	1.741	1.191
16	<b>10.246</b>	<b>3.266</b>	3.224	1.615	3.181	1.495	3.099	1.360	2.063	1.258
18	<b>12.967</b>	<b>3.788</b>	4.080	1.939	4.027	1.782	3.923	1.608	2.501	1.333
20	<b>16.009</b>	<b>4.306</b>	5.037	2.271	4.971	2.073	4.843	1.857	3.083	1.314
22	<b>19.371</b>	<b>4.821</b>	6.095	2.607	6.015	2.366	5.860	2.107	3.731	1.498
24	<b>23.053</b>	<b>5.333</b>	7.253	2.946	7.158	2.660	6.974	2.357	4.440	1.683
26	<b>27.055</b>	<b>5.842</b>	<b>8.513</b>	<b>3.286</b>	<b>8.401</b>	<b>2.953</b>	<b>8.184</b>	<b>2.605</b>	5.211	1.868
28	<b>31.378</b>	<b>6.348</b>	<b>9.873</b>	<b>3.627</b>	<b>9.743</b>	<b>3.246</b>	<b>9.492</b>	<b>2.852</b>	6.043	2.052
30	<b>36.021</b>	<b>6.852</b>	<b>11.334</b>	<b>3.967</b>	<b>11.185</b>	<b>3.538</b>	<b>10.896</b>	<b>3.099</b>	6.938	2.236
32	<b>40.983</b>	<b>7.353</b>	<b>12.895</b>	<b>4.306</b>	<b>12.726</b>	<b>3.829</b>	<b>12.398</b>	<b>3.344</b>	7.893	2.418
34	<b>46.266</b>	<b>7.853</b>	<b>14.557</b>	<b>4.645</b>	<b>14.366</b>	<b>4.119</b>	<b>13.996</b>	<b>3.588</b>	<b>8.911</b>	<b>2.601</b>
36	<b>51.870</b>	<b>8.351</b>	<b>16.320</b>	<b>4.982</b>	<b>16.106</b>	<b>4.407</b>	<b>15.691</b>	<b>3.831</b>	<b>9.990</b>	<b>2.782</b>
38	<b>57.793</b>	<b>8.848</b>	<b>18.184</b>	<b>5.318</b>	<b>17.946</b>	<b>4.695</b>	<b>17.482</b>	<b>4.073</b>	<b>11.131</b>	<b>2.963</b>

Note: Values where  $KL/r > 200$  are shown in bold letters.

**APPENDIX C (continued)**

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W12×45		W12×50		W12×53		W12×58		W12×65	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.011	1.000	1.011	1.000	1.007	1.000	1.007	1.000	1.005	1.000
4	1.046	1.000	1.045	1.000	1.028	1.000	1.027	1.000	1.019	1.000
6	1.106	1.000	1.104	1.000	1.064	1.000	1.062	1.000	1.042	1.000
8	1.196	1.023	1.192	1.022	1.116	1.000	1.113	1.000	1.077	1.000
10	1.323	1.069	1.316	1.064	1.187	1.020	1.182	1.017	1.123	1.000
12	1.497	1.119	1.484	1.109	1.280	1.055	1.272	1.050	1.181	1.003
14	1.731	1.174	1.712	1.159	1.399	1.093	1.388	1.085	1.254	1.030
16	2.048	1.234	2.018	1.213	1.551	1.133	1.535	1.122	1.344	1.059
18	2.478	1.301	2.432	1.273	1.742	1.177	1.719	1.162	1.454	1.089
20	3.052	1.376	2.990	1.338	1.985	1.224	1.953	1.205	1.588	1.121
22	3.692	1.390	3.618	1.292	2.292	1.275	2.247	1.251	1.749	1.155
24	4.394	1.557	4.305	1.442	2.689	1.330	2.625	1.301	1.946	1.192
26	5.157	1.722	5.053	1.591	3.156	1.272	3.081	1.355	2.184	1.230
28	5.981	1.887	5.860	1.739	3.660	1.403	3.573	1.309	2.474	1.272
30	6.866	2.052	6.727	1.887	4.202	1.535	4.102	1.428	2.833	1.316
32	7.812	2.215	7.654	2.034	4.780	1.666	4.667	1.547	3.224	1.239
34	<b>8.819</b>	<b>2.378</b>	<b>8.640</b>	<b>2.181</b>	5.397	1.796	5.268	1.665	3.639	1.341
36	<b>9.887</b>	<b>2.540</b>	<b>9.687</b>	<b>2.327</b>	6.050	1.927	5.907	1.783	4.080	1.442
38	<b>11.016</b>	<b>2.702</b>	<b>10.793</b>	<b>2.472</b>	6.741	2.056	6.581	1.901	4.546	1.544

Note: Values where  $KL/r > 200$  are shown in bold letters.

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W12×72		W12×79		W12×87		W12×96		W12×106	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.005	1.000	1.005	1.000	1.004	1.000	1.004	1.000	1.004	1.000
4	1.018	1.000	1.018	1.000	1.018	1.000	1.018	1.000	1.018	1.000
6	1.042	1.000	1.042	1.000	1.041	1.000	1.041	1.000	1.040	1.000
8	1.076	1.000	1.075	1.000	1.074	1.000	1.073	1.000	1.072	1.000
10	1.121	1.000	1.120	1.000	1.118	1.000	1.117	1.000	1.115	1.000
12	1.178	1.016	1.177	1.014	1.175	1.012	1.172	1.010	1.170	1.009
14	1.250	1.042	1.249	1.037	1.245	1.033	1.242	1.030	1.238	1.027
16	1.339	1.069	1.336	1.062	1.331	1.056	1.327	1.051	1.322	1.045
18	1.447	1.098	1.444	1.088	1.437	1.079	1.430	1.072	1.423	1.064
20	1.578	1.128	1.573	1.116	1.564	1.104	1.555	1.094	1.546	1.084
22	1.737	1.159	1.730	1.145	1.718	1.130	1.706	1.117	1.695	1.105
24	1.929	1.193	1.921	1.175	1.904	1.157	1.889	1.141	1.873	1.127
26	2.162	1.229	2.151	1.207	2.130	1.185	2.109	1.166	2.089	1.149
28	2.445	1.267	2.431	1.241	2.403	1.215	2.376	1.192	2.350	1.172
30	2.796	1.307	2.778	1.276	2.742	1.246	2.706	1.220	2.672	1.197
32	3.181	1.350	3.161	1.314	3.120	1.279	3.079	1.249	3.040	1.222
34	3.592	1.265	3.568	1.354	3.522	1.314	3.476	1.279	3.432	1.248
36	4.027	1.359	4.000	1.263	3.948	1.351	3.897	1.310	3.847	1.276
38	4.486	1.452	4.457	1.348	4.399	1.389	4.342	1.344	4.287	1.305

**APPENDIX C (continued)**

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W12×120		W14×22		W14×26		W14×30		W14×34	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.004	1.000	1.040	1.000	1.037	1.000	1.019	1.000	1.018	1.000
4	1.017	1.000	1.169	1.015	1.156	1.009	1.079	1.000	1.075	1.000
6	1.039	1.000	1.420	1.130	1.384	1.113	1.186	1.025	1.176	1.020
8	1.071	1.000	1.866	1.274	1.783	1.240	1.355	1.102	1.334	1.091
10	1.114	1.000	2.655	1.373	2.468	1.400	1.608	1.192	1.569	1.173
12	1.168	1.007	3.823	1.868	3.545	1.667	1.981	1.298	1.912	1.269
14	1.235	1.023	5.203	2.395	4.825	2.115	2.536	1.310	2.417	1.381
16	1.317	1.039	6.796	2.942	6.302	2.574	3.311	1.632	3.140	1.510
18	1.417	1.056	<b>8.601</b>	<b>3.502</b>	<b>7.976</b>	<b>3.038</b>	4.190	1.969	3.974	1.810
20	1.538	1.073	<b>10.619</b>	<b>4.069</b>	<b>9.847</b>	<b>3.505</b>	5.173	2.314	4.906	2.115
22	1.683	1.091	<b>12.849</b>	<b>4.639</b>	<b>11.914</b>	<b>3.972</b>	6.260	2.666	5.937	2.424
24	1.858	1.110	<b>15.291</b>	<b>5.210</b>	<b>14.179</b>	<b>4.438</b>	7.449	3.021	7.065	2.735
26	2.069	1.129	<b>17.946</b>	<b>5.781</b>	<b>16.641</b>	<b>4.902</b>	<b>8.743</b>	<b>3.379</b>	<b>8.292</b>	<b>3.047</b>
28	2.324	1.149	<b>20.813</b>	<b>6.350</b>	<b>19.299</b>	<b>5.364</b>	<b>10.140</b>	<b>3.739</b>	<b>9.616</b>	<b>3.358</b>
30	2.638	1.170	<b>23.892</b>	<b>6.917</b>	<b>22.155</b>	<b>5.823</b>	<b>11.640</b>	<b>4.098</b>	<b>11.039</b>	<b>3.669</b>
32	3.001	1.191	<b>27.184</b>	<b>7.482</b>	<b>25.207</b>	<b>6.281</b>	<b>13.244</b>	<b>4.457</b>	<b>12.560</b>	<b>3.979</b>
34	3.388	1.213	<b>30.688</b>	<b>8.045</b>	<b>28.457</b>	<b>6.736</b>	<b>14.951</b>	<b>4.815</b>	<b>14.179</b>	<b>4.288</b>
36	3.798	1.236	<b>34.404</b>	<b>8.605</b>	<b>31.903</b>	<b>7.190</b>	<b>16.761</b>	<b>5.173</b>	<b>15.896</b>	<b>4.597</b>
38	4.232	1.260	<b>38.333</b>	<b>9.164</b>	<b>35.546</b>	<b>7.642</b>	<b>18.675</b>	<b>5.529</b>	<b>17.712</b>	<b>4.904</b>

Note: Values where  $KL/r > 200$  are shown in bold letters.

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W14×38		W14×43		W14×48		W14×53		W14×61	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.018	1.000	1.012	1.000	1.012	1.000	1.012	1.000	1.007	1.000
4	1.073	1.000	1.048	1.000	1.047	1.000	1.047	1.000	1.028	1.000
6	1.171	1.016	1.112	1.000	1.110	1.000	1.108	1.000	1.065	1.000
8	1.324	1.083	1.208	1.033	1.203	1.028	1.201	1.026	1.119	1.000
10	1.551	1.160	1.343	1.087	1.335	1.079	1.331	1.074	1.192	1.023
12	1.881	1.248	1.529	1.147	1.516	1.134	1.509	1.126	1.288	1.059
14	2.363	1.352	1.783	1.215	1.762	1.196	1.751	1.183	1.411	1.099
16	3.060	1.416	2.128	1.291	2.095	1.265	2.079	1.246	1.568	1.142
18	3.872	1.687	2.604	1.377	2.550	1.343	2.525	1.317	1.766	1.188
20	4.780	1.962	3.215	1.428	3.148	1.333	3.116	1.395	2.018	1.238
22	5.784	2.238	3.890	1.636	3.809	1.520	3.770	1.422	2.339	1.292
24	6.884	2.515	4.630	1.846	4.533	1.708	4.486	1.594	2.755	1.351
26	<b>8.079</b>	<b>2.792</b>	5.434	2.055	5.321	1.896	5.265	1.764	3.234	1.322
28	<b>9.370</b>	<b>3.068</b>	6.302	2.265	6.171	2.084	6.106	1.934	3.750	1.460
30	<b>10.756</b>	<b>3.343</b>	7.234	2.475	7.084	2.270	7.010	2.104	4.305	1.598
32	<b>12.238</b>	<b>3.617</b>	<b>8.231</b>	<b>2.684</b>	<b>8.060</b>	<b>2.456</b>	<b>7.976</b>	<b>2.272</b>	4.898	1.736
34	<b>13.816</b>	<b>3.890</b>	<b>9.292</b>	<b>2.892</b>	<b>9.098</b>	<b>2.642</b>	<b>9.004</b>	<b>2.440</b>	5.530	1.874
36	<b>15.489</b>	<b>4.162</b>	<b>10.417</b>	<b>3.099</b>	<b>10.200</b>	<b>2.826</b>	<b>10.094</b>	<b>2.607</b>	6.199	2.011
38	<b>17.258</b>	<b>4.433</b>	<b>11.607</b>	<b>3.306</b>	<b>11.365</b>	<b>3.010</b>	<b>11.247</b>	<b>2.774</b>	6.907	2.148

Note: Values where  $KL/r > 200$  are shown in bold letters.

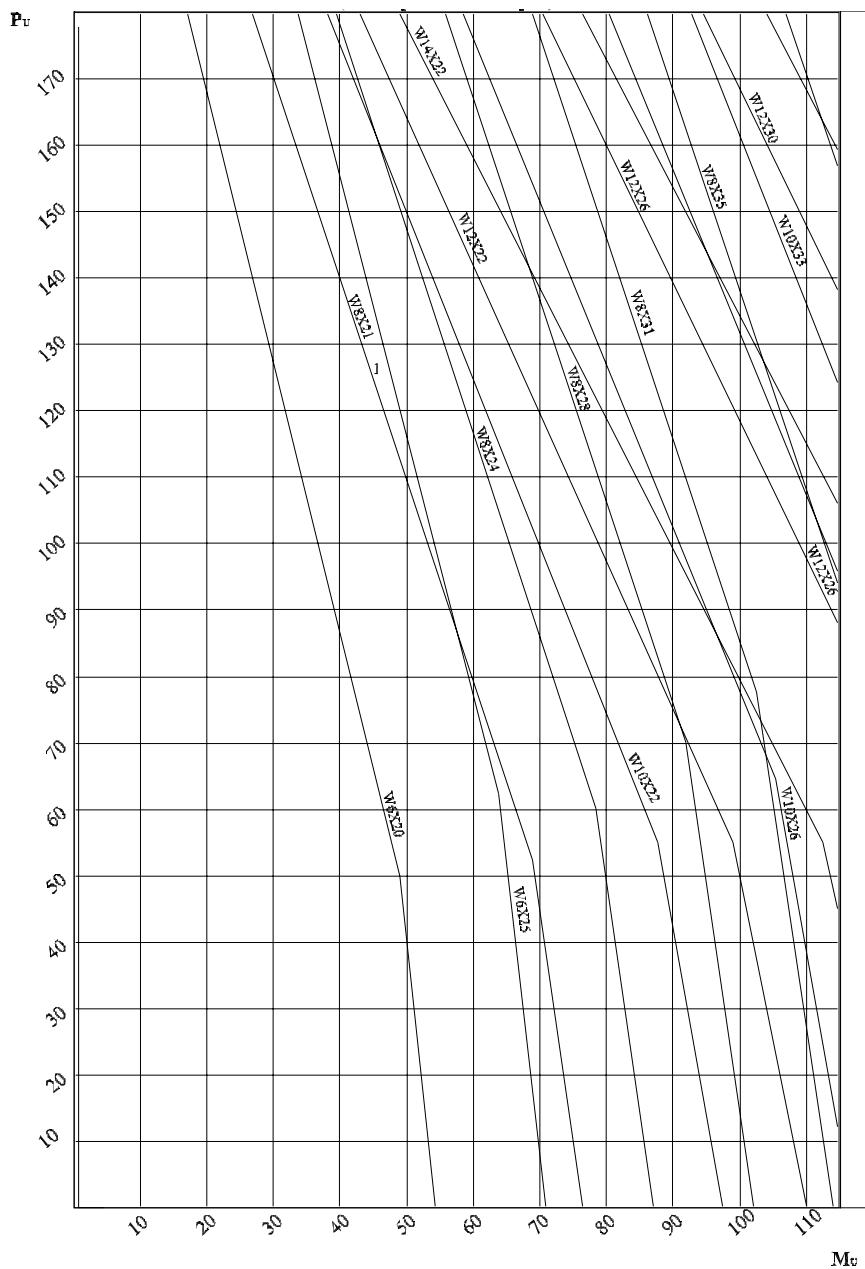
**APPENDIX C (continued)**

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)									
	W14x68		W14x74		W14x82		W14x90		W14x99	
	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.007	1.000	1.007	1.000	1.007	1.000	1.003	1.000	1.003	1.000
4	1.028	1.000	1.028	1.000	1.028	1.000	1.012	1.000	1.012	1.000
6	1.065	1.000	1.064	1.000	1.064	1.000	1.028	1.000	1.028	1.000
8	1.118	1.000	1.116	1.000	1.116	1.000	1.051	1.000	1.050	1.000
10	1.190	1.021	1.187	1.018	1.187	1.017	1.080	1.000	1.080	1.000
12	1.285	1.056	1.280	1.051	1.280	1.047	1.117	1.000	1.117	1.000
14	1.407	1.093	1.399	1.085	1.399	1.079	1.163	1.000	1.162	1.006
16	1.562	1.133	1.551	1.122	1.551	1.112	1.218	1.011	1.217	1.027
18	1.758	1.175	1.742	1.161	1.742	1.148	1.283	1.034	1.282	1.048
20	2.007	1.221	1.985	1.203	1.985	1.186	1.361	1.058	1.358	1.071
22	2.323	1.271	2.292	1.248	2.292	1.227	1.452	1.083	1.449	1.094
24	2.733	1.325	2.689	1.297	2.689	1.271	1.558	1.109	1.554	1.118
26	3.207	1.384	3.156	1.349	3.156	1.318	1.683	1.137	1.678	1.144
28	3.720	1.356	3.660	1.272	3.660	1.369	1.829	1.166	1.823	1.171
30	4.270	1.481	4.202	1.387	4.202	1.292	1.999	1.196	1.992	1.199
32	4.859	1.605	4.780	1.500	4.780	1.396	2.200	1.228	2.190	1.228
34	5.485	1.729	5.397	1.614	5.397	1.499	2.435	1.262	2.423	1.259
36	6.149	1.852	6.050	1.727	6.050	1.602	2.718	1.298	2.704	1.291
38	6.851	1.975	6.741	1.839	6.741	1.704	3.029	1.336	3.012	1.326

KL (ft) or $L_b$ (ft)	Tables of $\alpha$ and $\beta$ Values (continued)			
	W14x109		W14x120	
	$\alpha$	$\beta$	$\alpha$	$\beta$
0	1.000	1.000	1.000	1.000
2	1.003	1.000	1.003	1.000
4	1.012	1.000	1.012	1.000
6	1.028	1.000	1.027	1.000
8	1.050	1.000	1.049	1.000
10	1.079	1.000	1.078	1.000
12	1.115	1.000	1.115	1.000
14	1.160	1.008	1.159	1.007
16	1.214	1.027	1.213	1.025
18	1.278	1.047	1.277	1.043
20	1.354	1.068	1.352	1.062
22	1.443	1.089	1.440	1.082
24	1.547	1.112	1.543	1.102
26	1.669	1.135	1.664	1.123
28	1.811	1.160	1.805	1.145
30	1.977	1.185	1.970	1.168
32	2.172	1.212	2.163	1.192
34	2.401	1.240	2.389	1.217
36	2.675	1.269	2.660	1.243
38	2.980	1.300	2.964	1.270

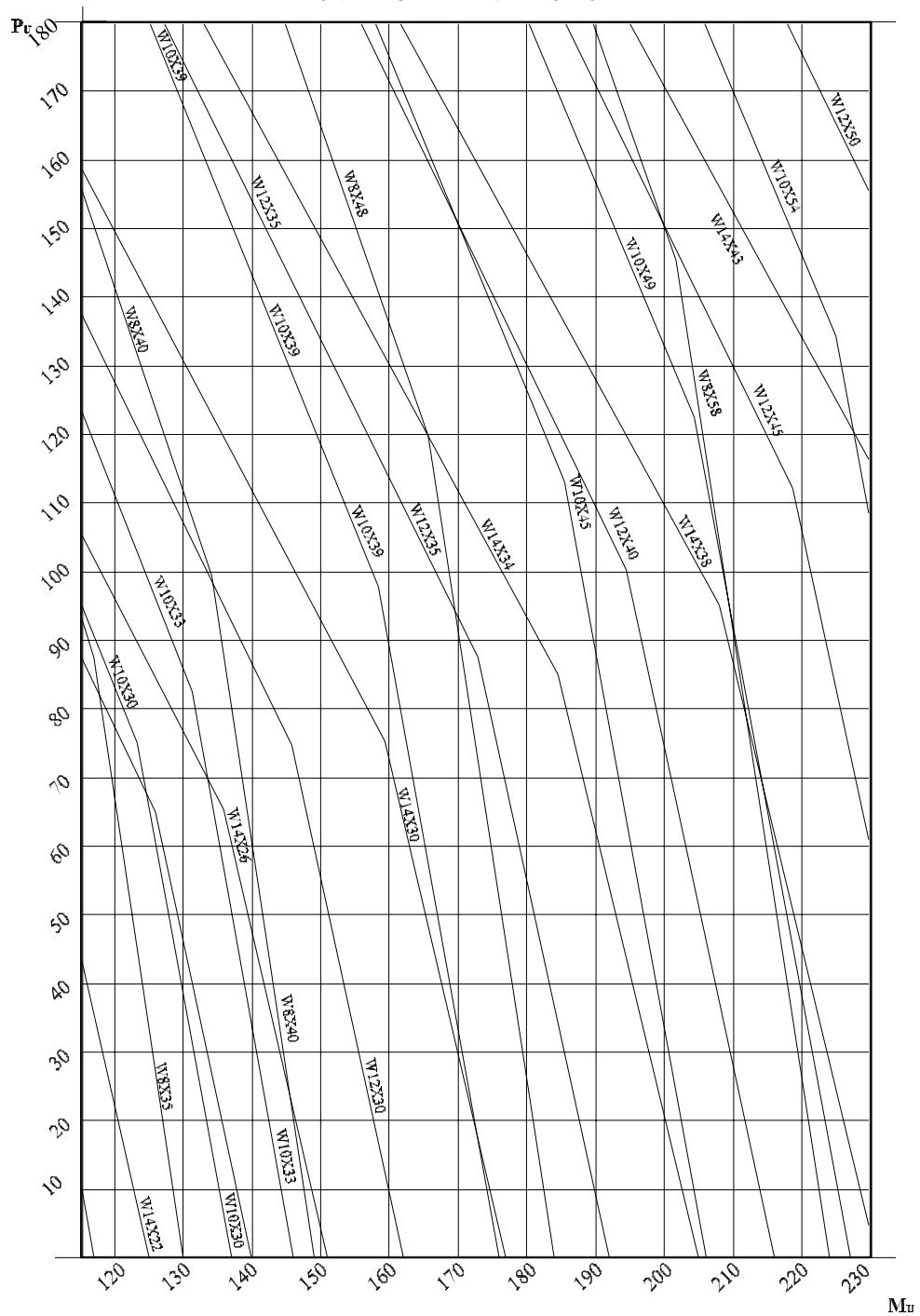
## APPENDIX D

Chart D1. Design Curves for Interaction Equation  
( $P_u$  in kips and  $M_u$  in kip-ft)



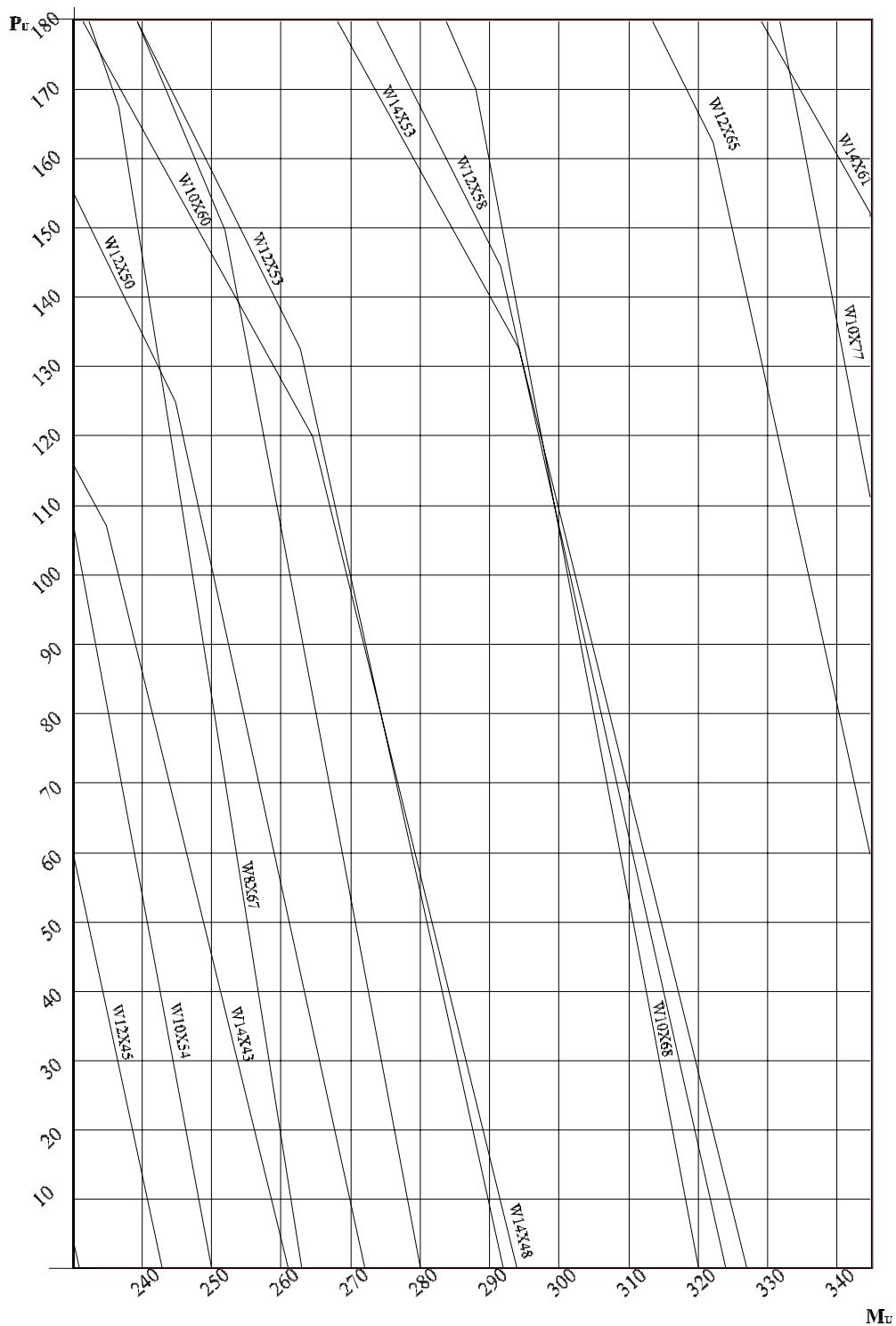
## APPENDIX D (continued)

**Chart D2. Design Curves for Interaction Equation (continued)**  
 $(P_u \text{ in kips and } M_u \text{ in kip-ft})$



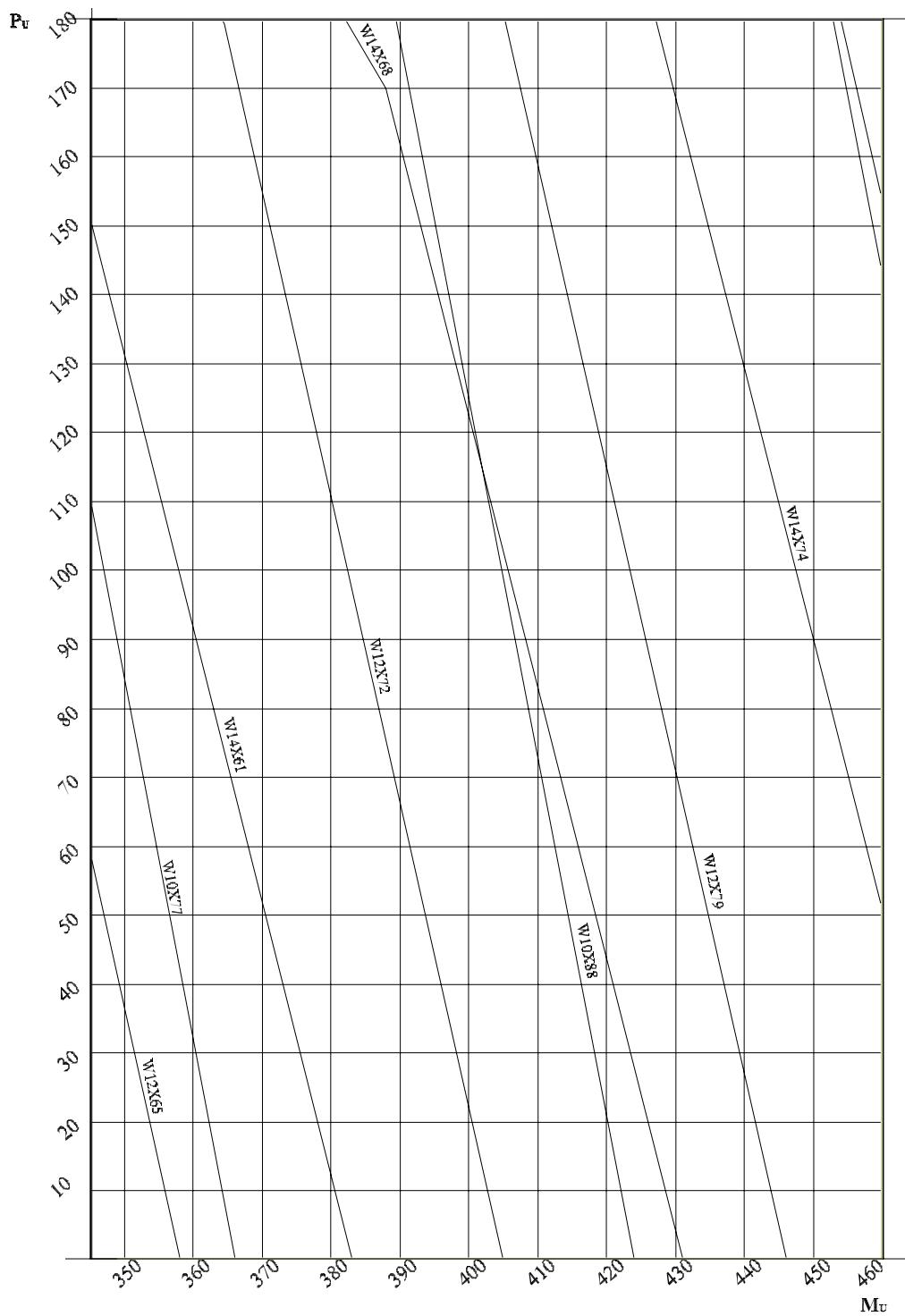
## APPENDIX D (continued)

**Chart D3. Design Curves for Interaction Equation (continued)**  
 $(P_u \text{ in kips and } M_u \text{ in kip-ft})$



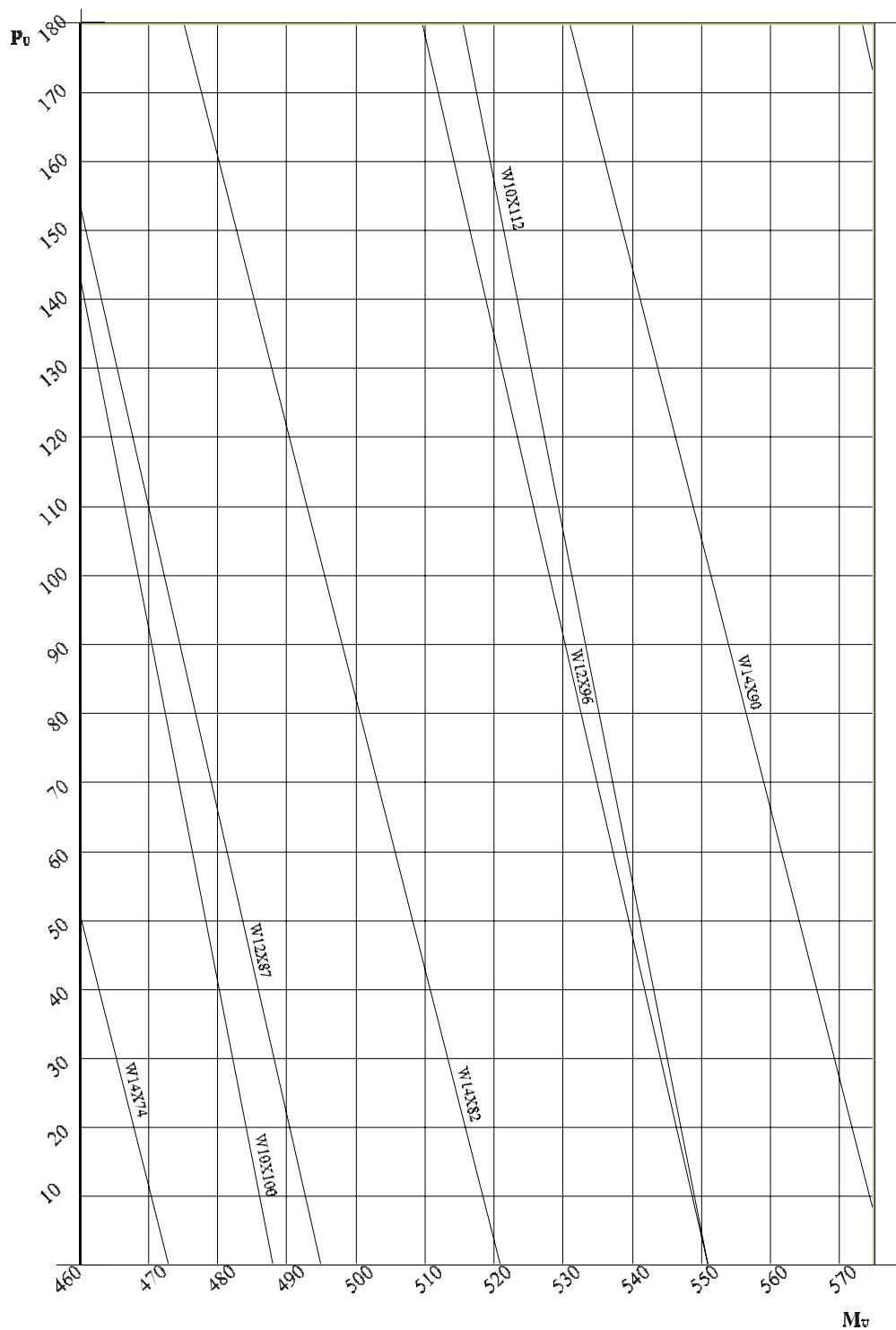
## APPENDIX D (continued)

**Chart D4. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



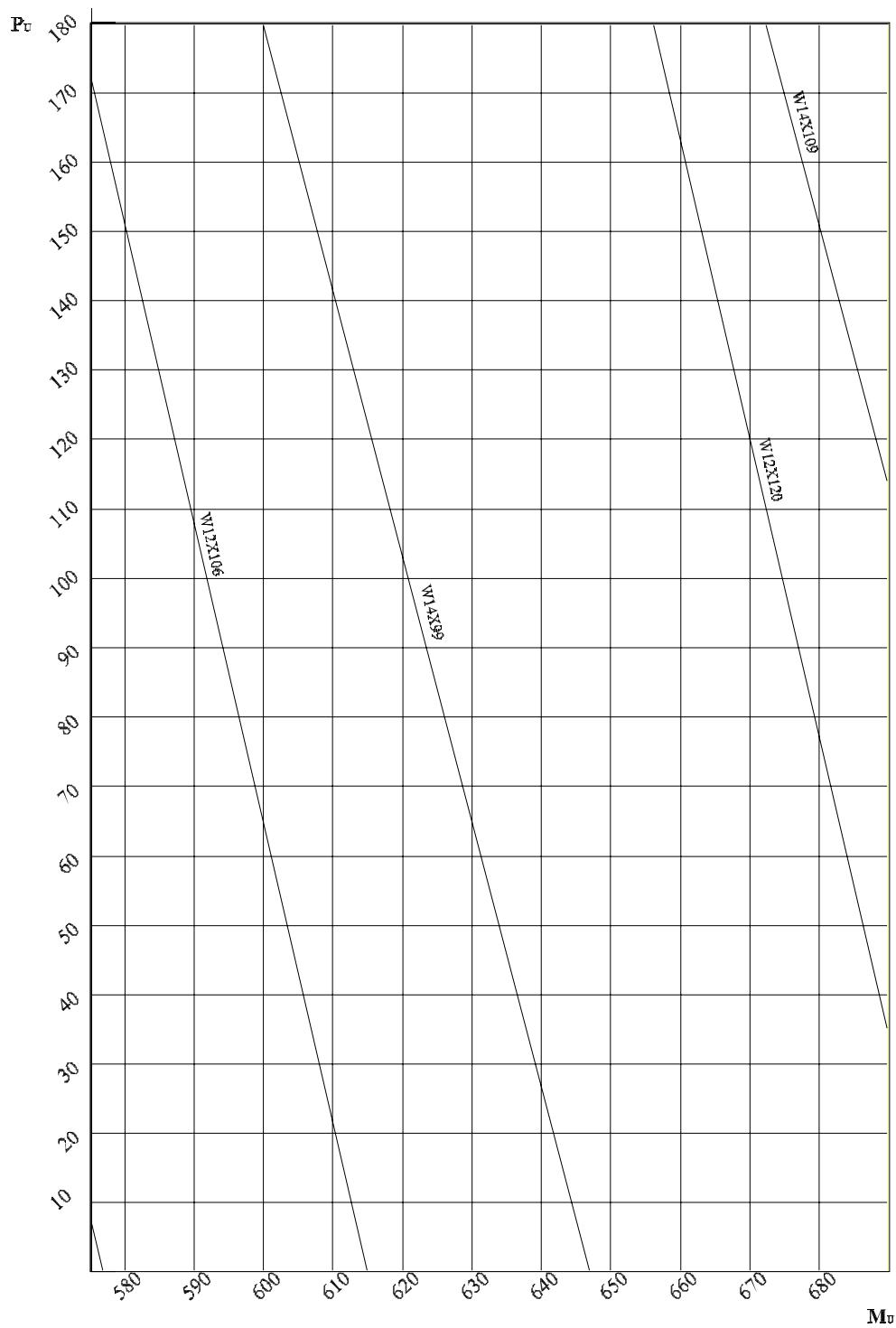
## APPENDIX D (continued)

**Chart D5. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



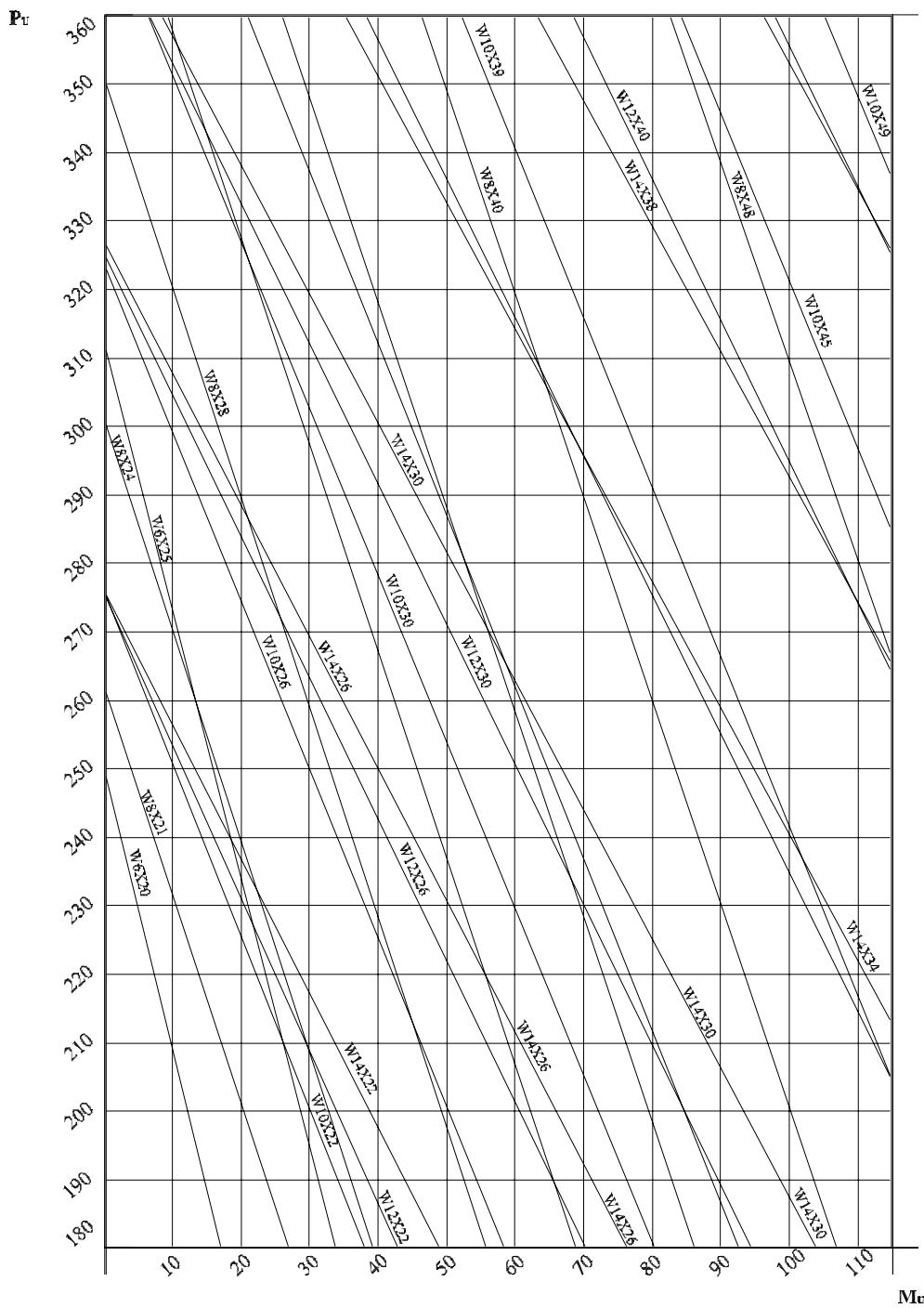
## APPENDIX D (continued)

**Chart D6. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



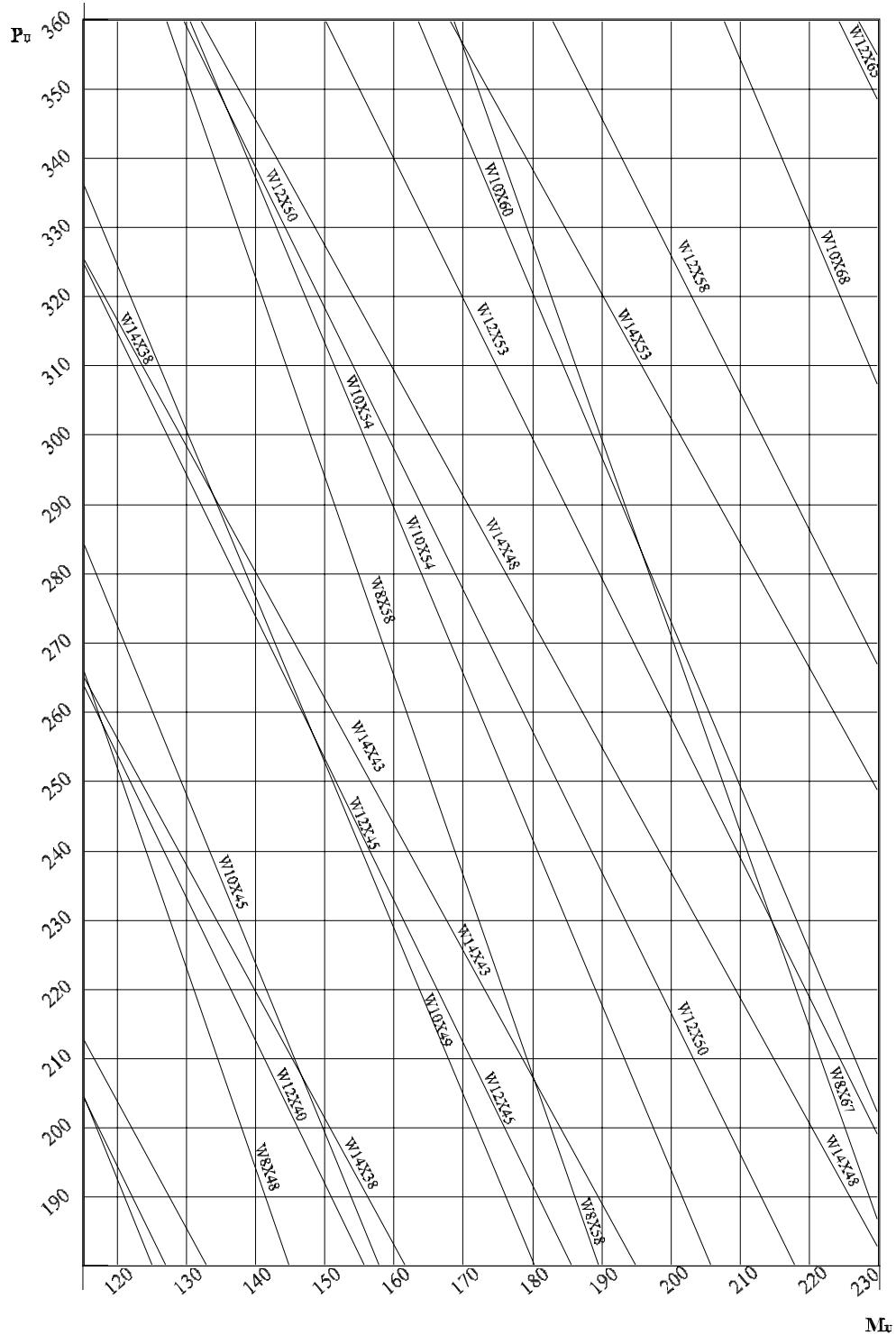
## APPENDIX D (continued)

**Chart D7. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



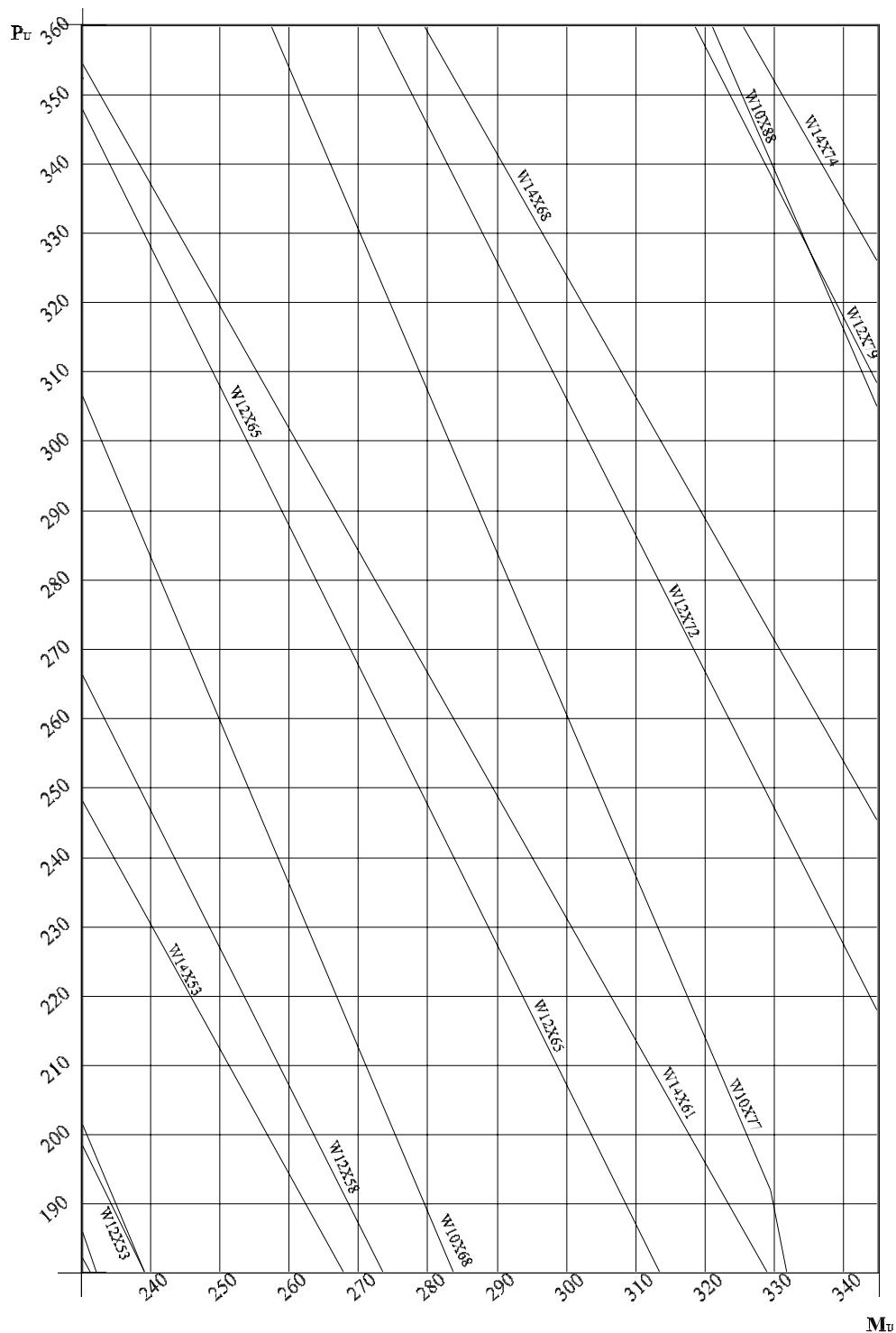
## APPENDIX D (continued)

**Chart D8. Design Curves for Interaction Equation (continued)**  
 $(P_u \text{ in kips and } M_u \text{ in kip-ft})$



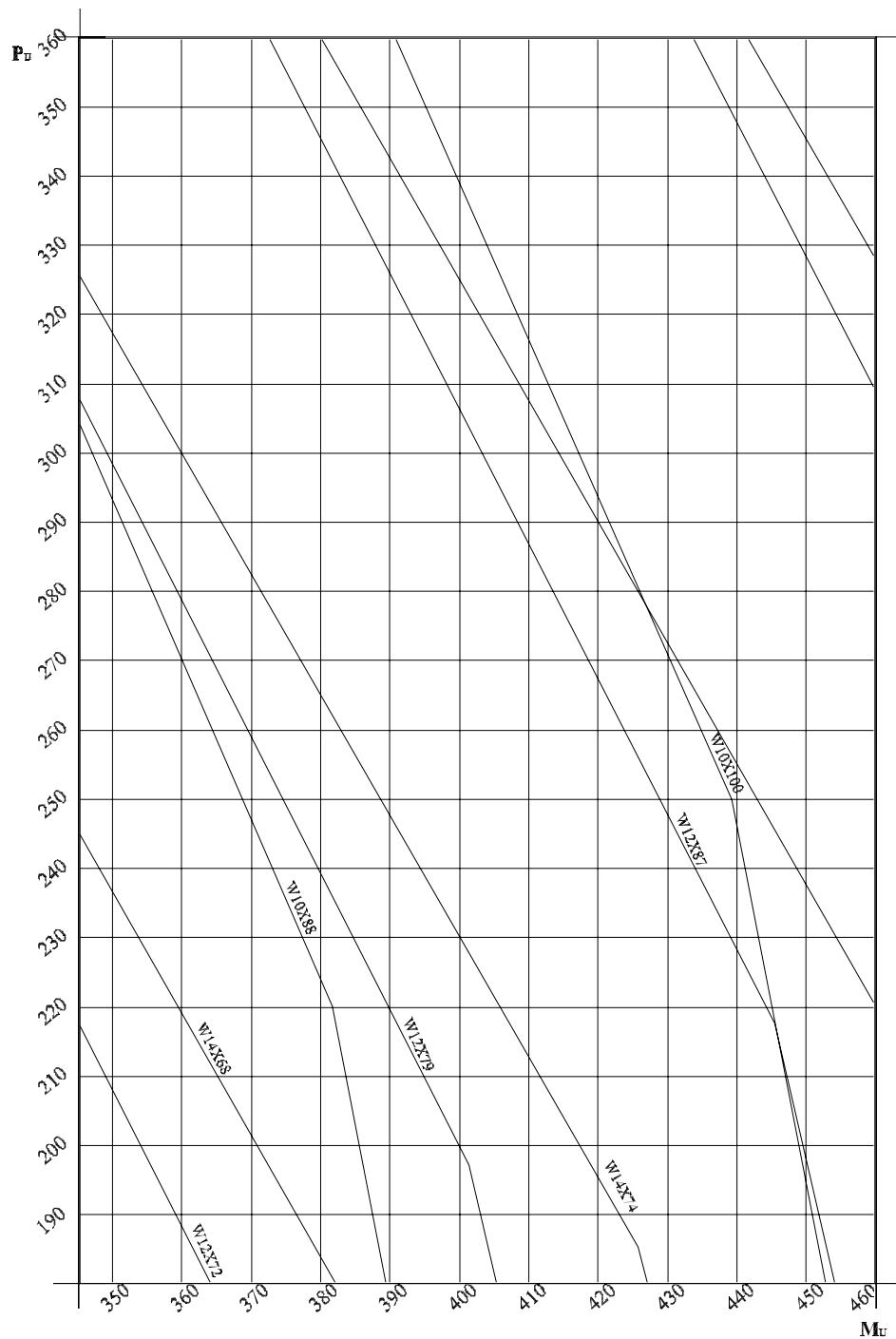
## APPENDIX D (continued)

**Chart D9. Design Curves for Interaction Equation (continued)**  
 $(P_u \text{ in kips and } M_u \text{ in kip-ft})$



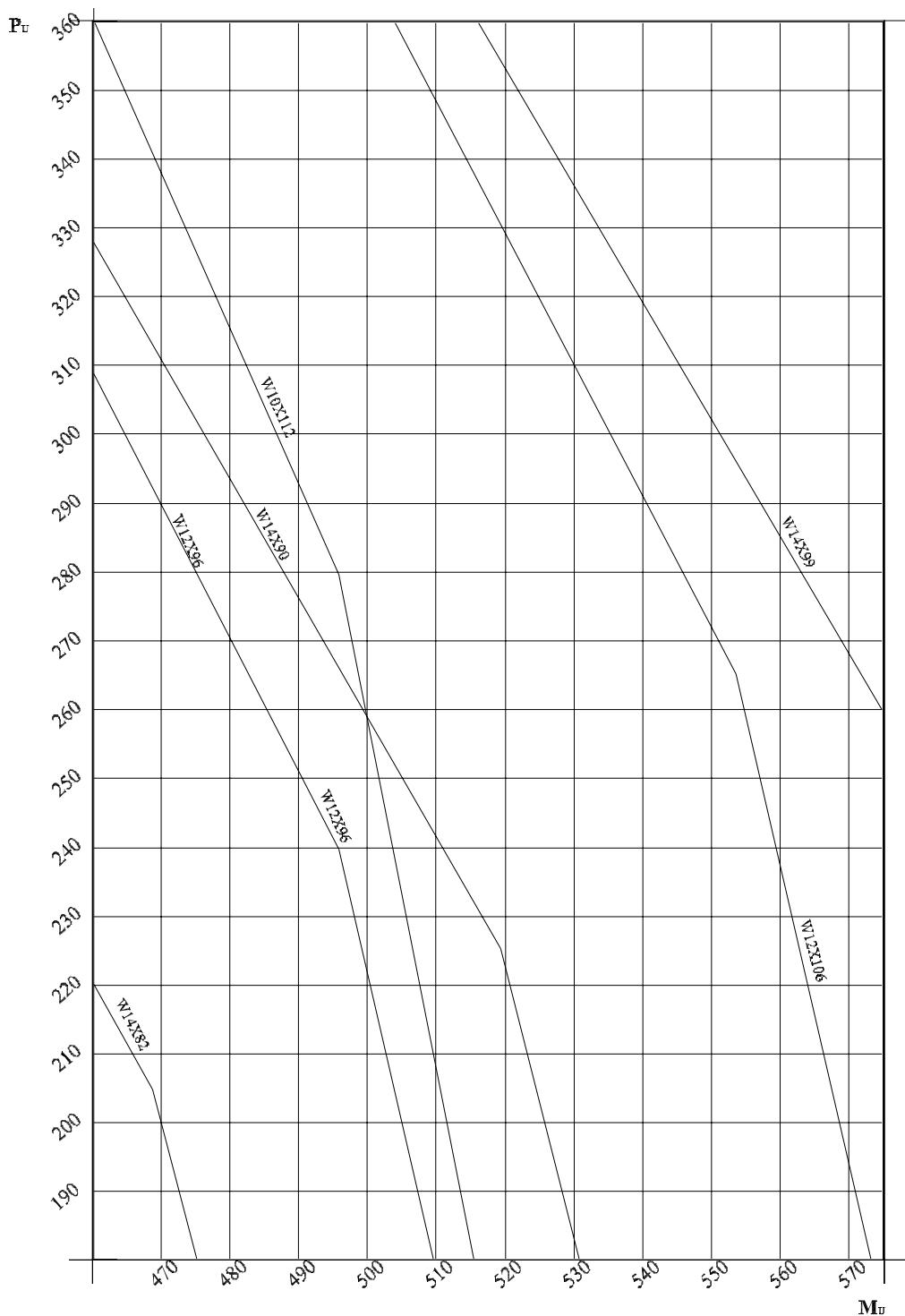
## APPENDIX D (continued)

**Chart D10. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



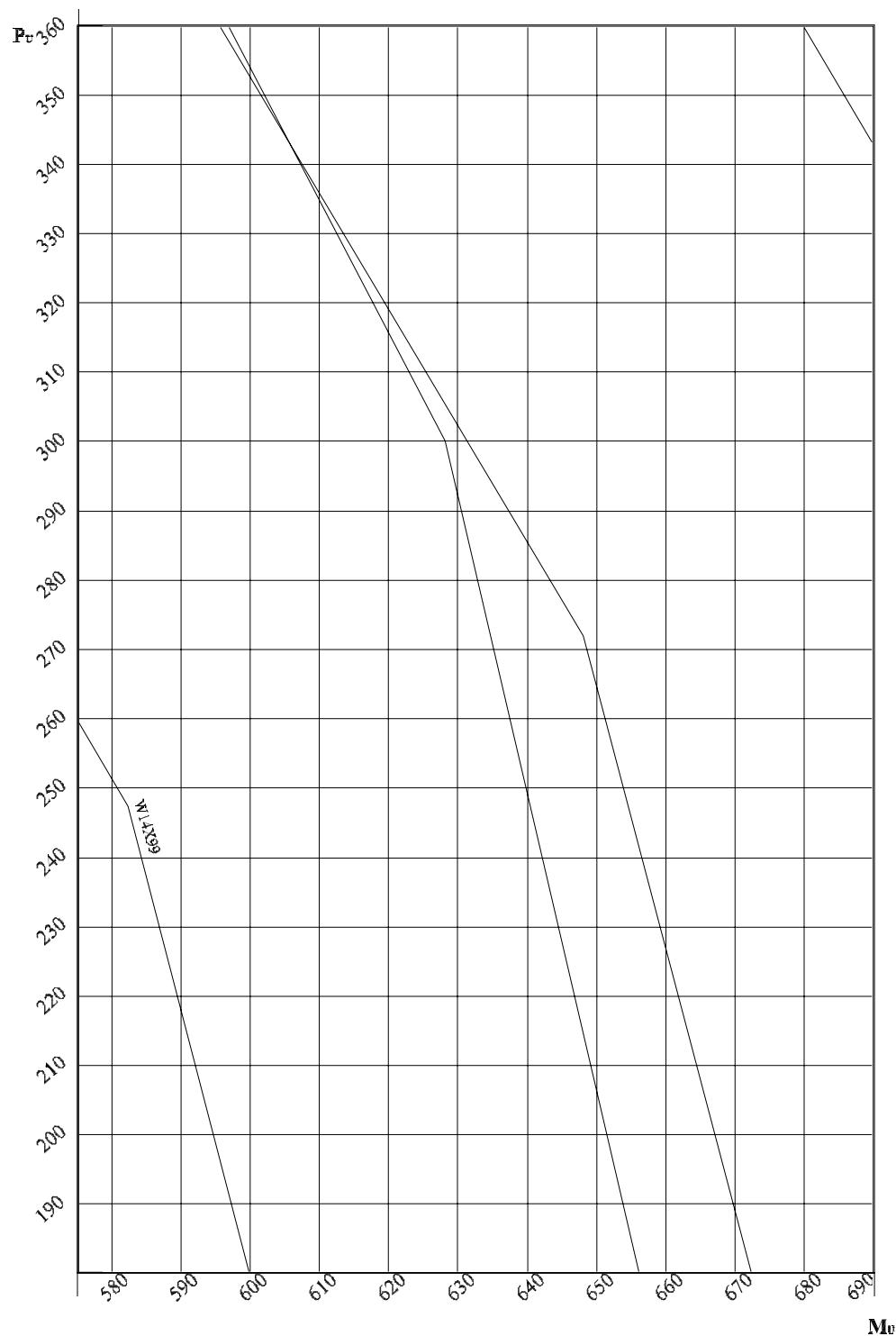
## APPENDIX D (continued)

**Chart D11. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



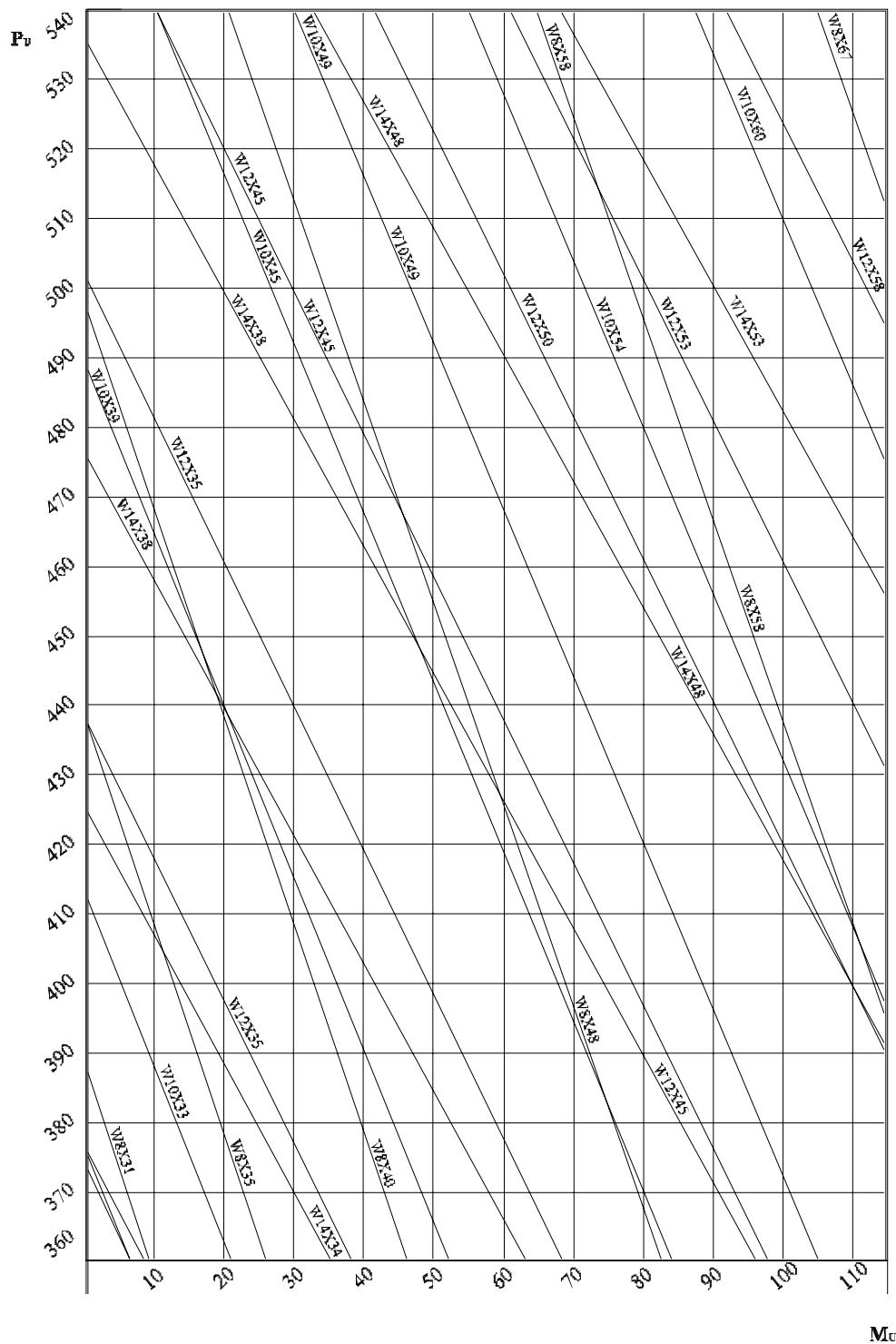
## APPENDIX D (continued)

**Chart D12. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



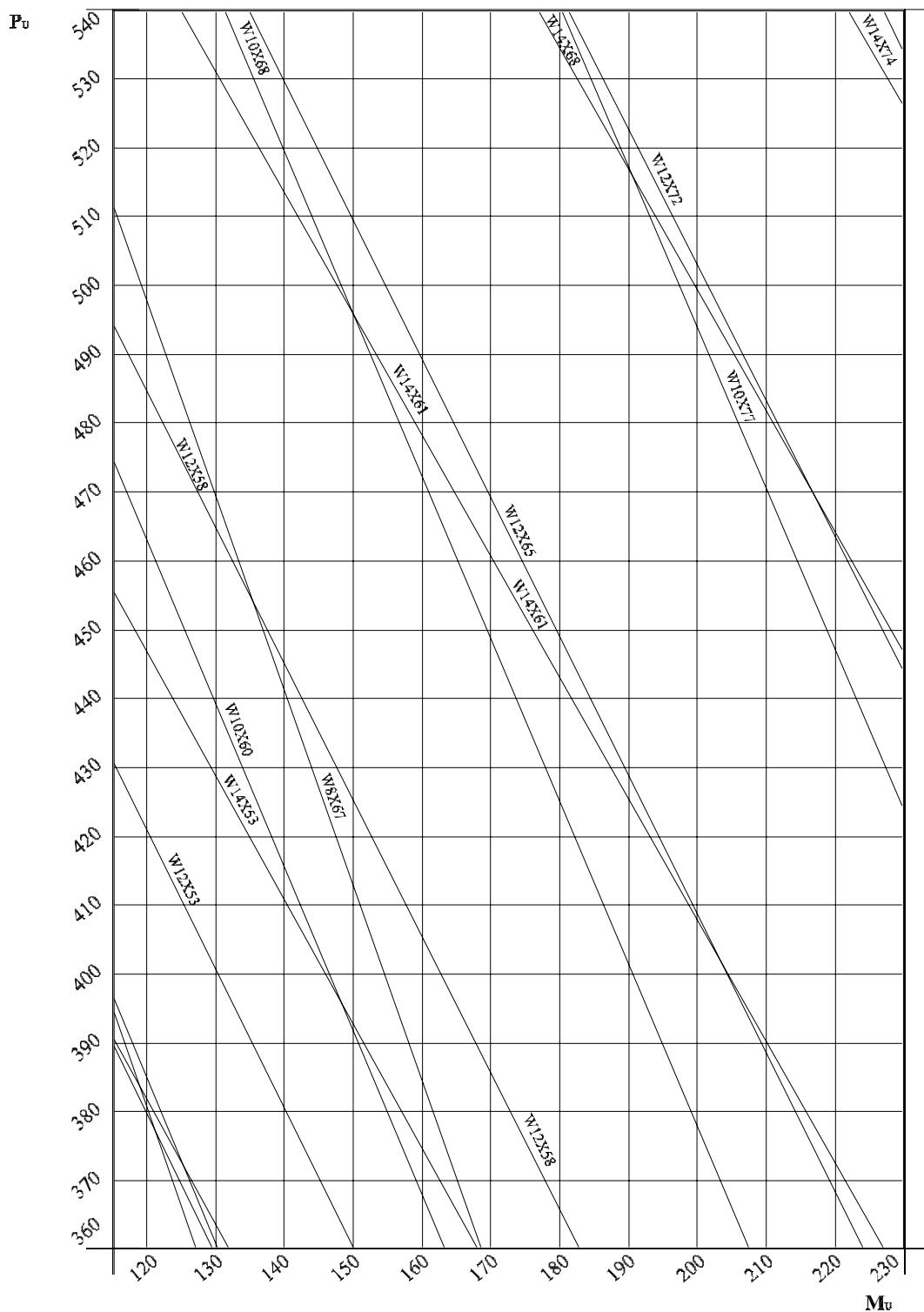
## APPENDIX D (continued)

**Chart D13. Design Curves for Interaction Equation (continued)**  
 $(P_u \text{ in kips and } M_u \text{ in kip-ft})$



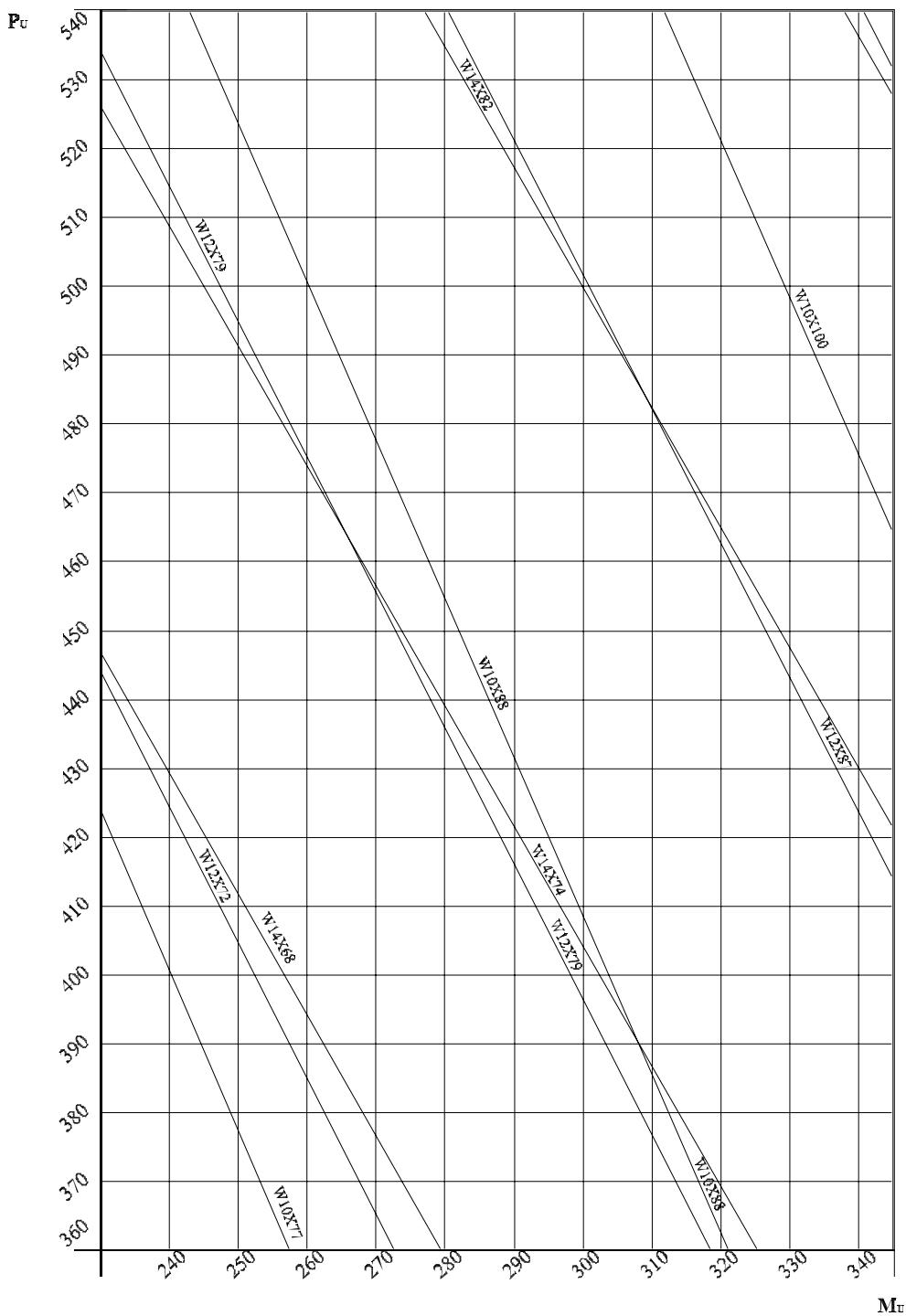
## APPENDIX D (continued)

**Chart D14. Design Curves for Interaction Equation (continued)**  
 $(P_u \text{ in kips and } M_u \text{ in kip-ft})$



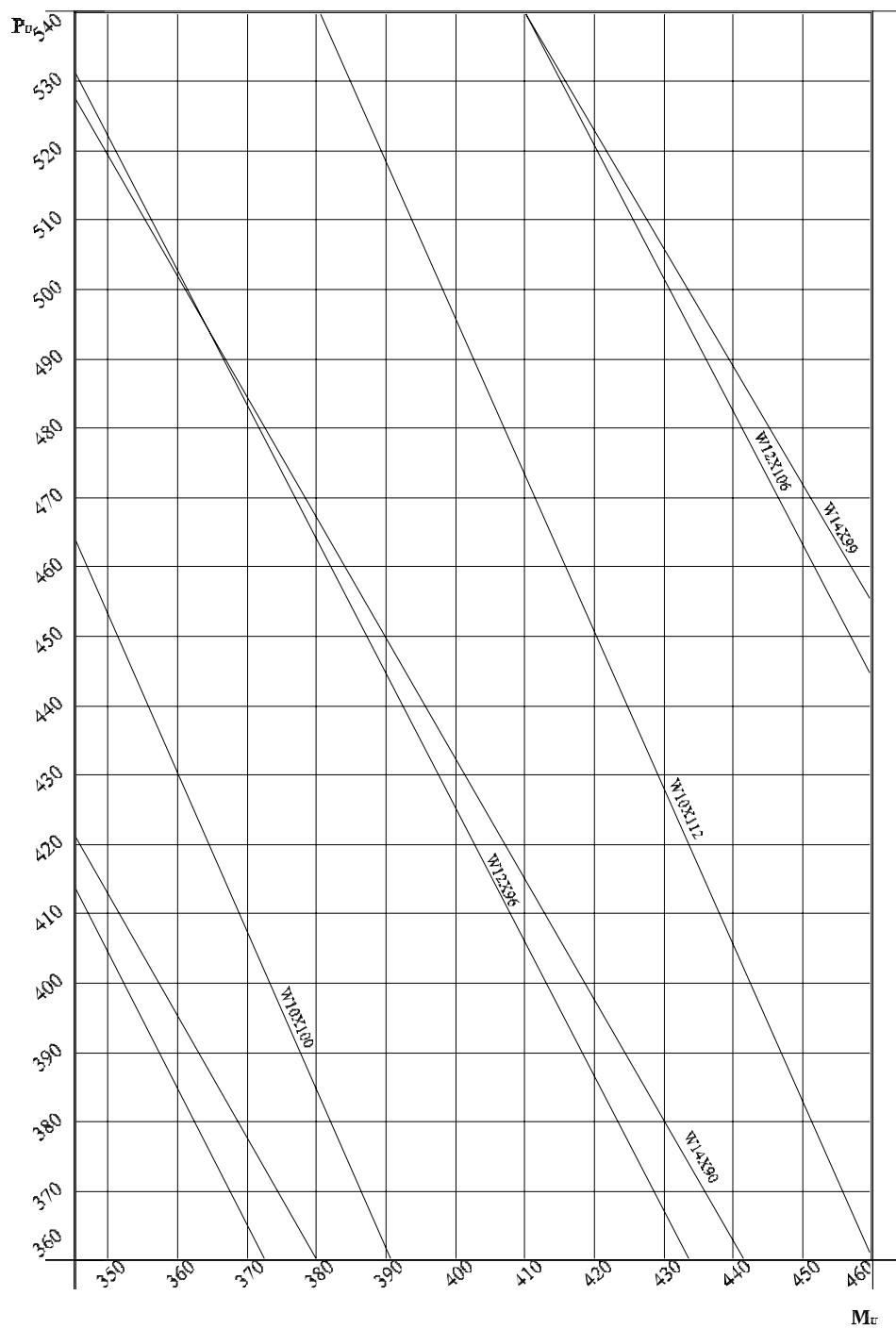
## APPENDIX D (continued)

**Chart D15. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



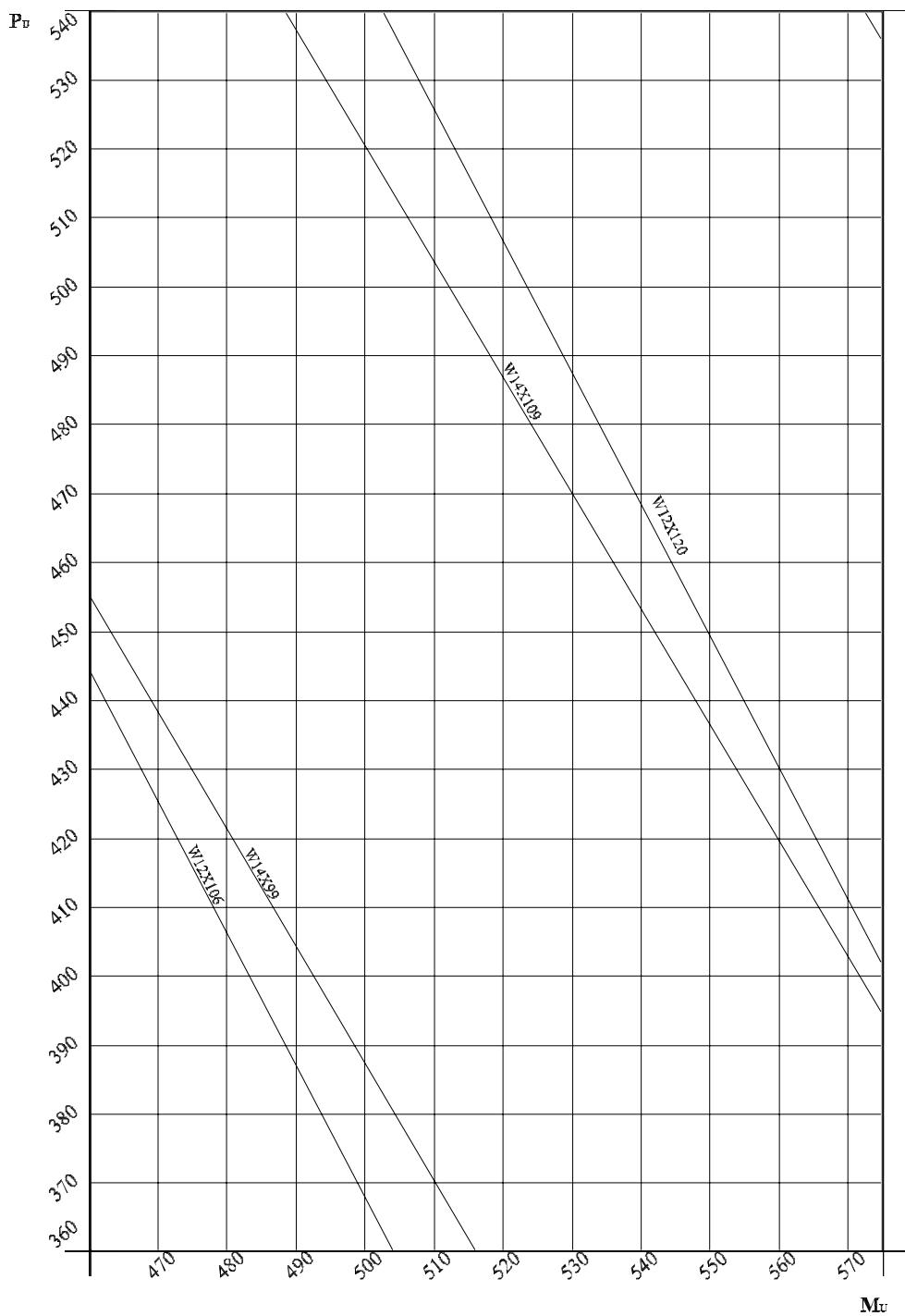
## APPENDIX D (continued)

**Chart D16. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



## APPENDIX D (continued)

**Chart D17. Design Curves for Interaction Equation (continued)**  
( $P_u$  in kips and  $M_u$  in kip-ft)



## APPENDIX D (continued)

**Chart D18. Design Curves for Interaction Equation (continued)**  
 $(P_u$  in kips and  $M_u$  in kip-ft)

