Cold Bending of Wide-Flange Shapes for Construction

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Horizontally and vertically curved members have seen much use in construction for years. Starting with cast iron bridges and bridge members as well as roof trusses for various exhibit halls, curved elements have attracted the interest of engineers and architects as expressions of structural function and artistic endeavor. But the elegance of the arch appears to be what always drew attention, covering large spaces with few or no interior obstructions.

As a structure, the action of an arch makes it possible to cover significant spans, since the load is carried largely in compression instead of through bending action. For certain arch geometries, support conditions, and load distributions, the effects of the imposed loads translate into uniform compression within the entire arch.

The early arch structures were made from cast iron, for which the compressive strength was paramount, since the material had little, if any tensile strength. These elements were cast at high temperatures, enabling the structure to be shaped as needed by the architect. The development of structural steel as it is known today took place in the second half of the 19th century, allowing for designs and structures that were capable of carrying tension as well as compression. The limitations imposed by the compression-only cast iron were overcome, and structures evolved that carried compression and tension equally well. The oldest example is the Eads Bridge of St. Louis, Missouri, generally regarded as the first structure to be built using what today is regarded as structural steel. As shown in Figure 1, the arches of the Eads Bridge (second bridge from the front of the picture) are excellent examples of such structures. Also shown in Figure 1 is the great Gateway Arch of St. Louis, a contemporary example of a parabolic arch.

The arches that are used in bridges and buildings impose relatively low deformation demands on the structure and its materials. The radius of curvature is generally very large. The shape is usually created by cutting the steel to form in smaller elements, connecting the curved elements to develop the arch, and then building the completed arch on the construction site. Details of the procedure may vary significantly, but the point is that the shaping process for large-span arches requires the steel only to undergo small plastic deformations during the fabrication. In-service conditions entail only elastic response demands for the arch, as is the case for most structures.

In addition to complete structures such as arches, curved structural members are also used extensively in construction, and Figures 2 through 4 give some examples. Some of the individual members are curved about the strong axis of the shapes (Figures 2 and 4); others are curved about the weak axis (Figure 3). Common to the examples shown in these figures and to most building applications is the fact that the curving of such members is mostly done at ambient temperature. Such cold bending places greater demands on the curving equipment that is used, to the effect that larger curving forces are needed. Higher temperature curving is certainly feasible for many applications, and a number of bending companies use it on a regular basis. It is almost always used when straightening repair of bridge girders is needed (Structural Damage Control, 1998).

Fig. 1. The Eads Bridge and other bridges across the Mississippi River at St. Louis, Missouri, along with the Gateway Arch (Princeton University Art Museum, 1974).

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For practical reasons the material that is presented focuses on applications of bent structural members for building-type structures. Bridge structures rarely use members that are curved as much as those described here, and fatigue and similar considerations therefore need not be addressed.

STRAIGHTENING, CAMBERING, AND CURVING

Concepts and Applications

Straightening, cambering, and curving of structural shapes are all representative of bending that involves local plastification of the steel to varying degrees. Although the principles and basic mechanics of these processes are the same, they are used for significantly different purposes and with very different magnitudes of bending deformations. The following presents brief descriptions of these forming operations. A detailed analysis of the large-deformation mechanics of shape curving is given later in the paper, as well as practical recommendations and limitations for cold curving of wide-flange shapes.

While this work focuses on the curving of wide-flange shapes, hollow structural sections (HSS) and other tubular forms, as well as angles and channels, are also used for bent construction. The bending mechanics theory applies equally to these types of cross-sections, although certain specialized conditions such as maintaining the geometry of a hollow section during the curving operation also have to be addressed. Changes in the depth through ovalization of a round tube, for example, will influence the stiffness of the element as a structural component. This and other subjects have been examined in Dutch research work on the performance requirements for buried steel pipelines (Gresnigt, 1986).

Cold bending of plates exhibits some of the characteristics of the processes that are addressed in this paper (Brockenbrough, 2006). However, the degree of bending and the associated strain level is usually much higher, with bending angles as large as 180°. The geometric parameters are also significantly different.

Straightening

ASTM Specification A6/A6M gives detailed requirements for bars, plates, shapes, and sheet piling used in construction (ASTM, 2006). Among these, the standard provides the permitted variations for straightness for the various forms of rolled shapes. For example, the maximum out-of-straightness for wide-flange shapes with a flange width larger than or equal to 6 in. is determined as

$$e = \tfrac{1}{8} \times [(\text{number of feet of total length})/10] \qquad (1)$$

which translates into approximately 1/1,000 of the length of the shape. For the maximum out-of-straightness this corresponds to a radius of curvature of 12.5 times the actual

Fig. 2. Trolley terminal (photo courtesy of Marks Metal Technology).

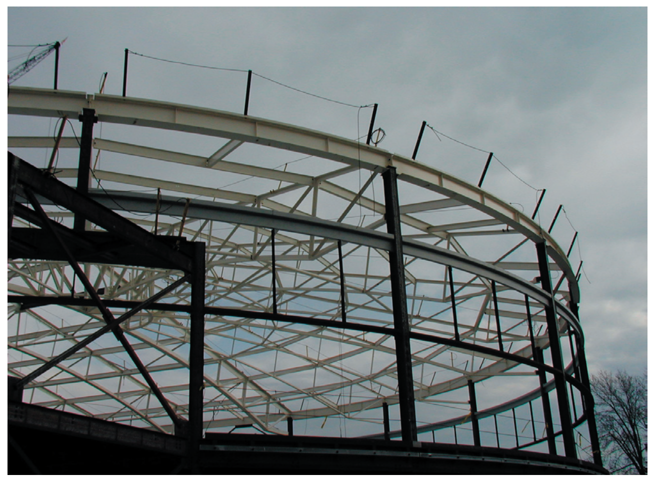

Fig. 3. Commercial structure (photo courtesy of Max Weiss Company, Inc.).

Fig. 4. Commercial mall structure (photo courtesy of Marks Metal Technology).

length of the member. As a purely theoretical example, if a member is 1,000 in. (approximately 83 ft) long, the maximum allowable value of e is 1 in. The radius of curvature is therefore approximately 1,042 ft. Such a large radius entails levels of bending strain that are extremely small.

The out-of-straightness requirement is the same for camber and sweep, which are the terms used for the out-of-straightness measured relative to the strong and weak axes, respectively. Figure 5 illustrates these two measurements (Geschwindner, Disque, and Bjorhovde, 1994). It is emphasized that the term camber as defined by ASTM A6/A6M is not the same as the camber that is sometimes used for beams and girders in buildings to arrive at near-level floors after the placement of concrete slabs.

Out-of-straightness is measured by the steel mill during the production of the shapes, and straightening is applied to make any nonconforming element meet the ASTM straightness requirements. Depending on the size of the shape, the straightening is either done in continuous fashion, or if the shape is heavy, it is done through point application of loads. The former procedure is referred to as roller or rotary straightening; the latter is known as gag straightening. Both involve local plastification of certain regions of the cross section.

In rotary straightening the yielding takes place continuously along the length of the shape, effectively altering the residual stress distribution in the shape in such a fashion that the strength of a wide-flange shape as a column is usually increased (Galambos, 1998). This is a side benefit of the rotary straightening process, although it is emphasized that strengthening does not apply to all types of shapes, usages, and straightener settings. Gag straightening causes local yielding in the shape only in short segments along the length of the member, surrounding each of the load application points. This procedure has no effect on the column strength of the shape. But common to both of these methods is the fact that the amount of curving and the accompanying strain demands within the cross section are very small, and the radius of curvature of the bent member is very large.

Straightening of shapes to meet delivery standards is used by all of the world's steel mills. The methods and results are the same, and the equipment that is used operates on the same principles and applications. Figure 6 shows rotary straightening equipment; Figure 7 pictures a gag straightening press.

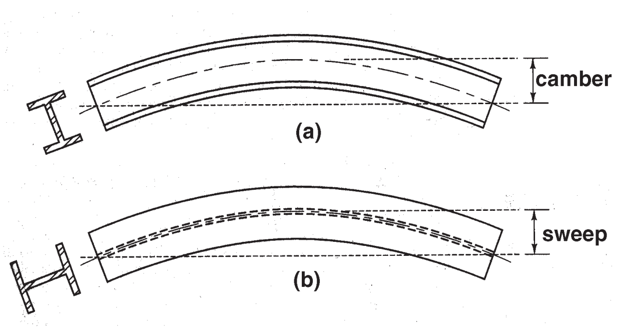

Fig. 5. Definitions of camber and sweep for wide-flange shapes.

Fig. 6. Rotary straightening equipment (photo courtesy of Nucor-Yamato Steel Co.).

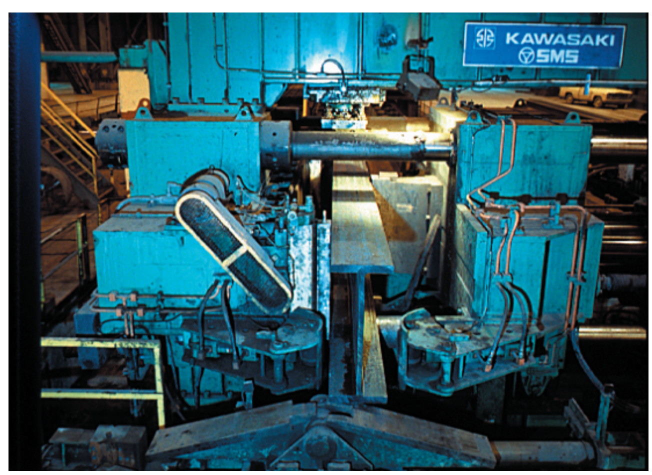

Fig. 7. Gag straightening equipment (photo courtesy of Nucor-Yamato Steel Co.).

Because of the way the rotary straightening rolls are applied to certain areas of a wide-flange shape, with the edges of the rolls resting on or near the fillet between the flange and the web, the region near the fillet that is known as the *k*-area develops higher strength and hardness as well as lower ductility and toughness than other regions of the shape. This is prompted by high shear yield strains in the through thickness direction at this location within the shape. The higher material strength and lower toughness have no effect for the overall strength of the member as a column or a beam. However, it has to be borne in mind when detailing certain types of welded connections, to the effect that welds should not be terminated in or near the *k*-area (Iwankiw, 1997; Bjorhovde, Goland, and Benac, 2000; Kaufmann and Fisher, 2001; Lee, Cotton, Dexter, Hajjar, Ye, and Ojard, 2002; Hajjar, Dexter, Ojard, Ye, and Cotton, 2003; Chi and Uang, 2004).

Cambering

For a structural engineer, cambering a beam means to prebend the member in the direction opposite to the deflection that will be developed by the anticipated gravity loads (Ricker, 1989; Larson and Huzzard, 1990). In this context the gravity loads usually mean the dead load. The aim is to have a structural component that is horizontal or nearly so following the application of the most closely known load component. Since the dead load generally is known more accurately than the live load, for example, cambering is almost always done to an extent that equals a fraction or even all of the dead load deflection.

The loads being considered are all at service levels, meaning that the deflections are expected to occur during the operation of the building. The most commonly used live load deflection limit equals the span divided by 360; dead load deflections may be on the same order of magnitude or somewhat larger. For a 30-ft-long simply supported beam, these values come out as deflections of approximately 1 in. It is therefore understood that the amount of curving that has to be done to counteract a dead load deflection of 1 in. for a 30 ft span is very small, and the corresponding radius of curvature is very large.

Structural cambering can be accomplished through selective heating of areas of the shape, or, as is most common, through gag pressing the member at ambient temperature while it is installed in a cambering frame (Ricker, 1989). Cold cambering does involve plastification of small areas of the cross section, similar to gag straightening, as discussed previously. But the accompanying deformation demands for the steel in the shape are very small, and the force(s) necessary to develop the camber curve tends to be fairly small.

Studies by Gergess and Sen (2005a, 2005b) have dealt with cold bending of symmetric and unsymmetric cross-section plate girders. Because of the depth of the girders, the lack of symmetry of some of the cross sections as well as the bending about the weak axis for some of the girders, the correlation to what is presented here for hot-rolled W-shapes is limited. The Gergess and Sen studies also made use of a single load application point. The associated radius of curvature is therefore much larger than what is applied to the W-shapes of the study presented here.

On the other hand, while the deformation and force demands associated with cambering are small, it is important to bear in mind the modifications of the cross-sectional area that will occur as a result of punching or drilling of holes and similar fabrication operations. Some fabricating shops are set up such that hole punching occurs at the beginning of the various operations, with the cambering done at a later stage. The cross-sectional area changes associated with hole punching or drilling can create a preferred plane for yielding, with the potential for localized failure (excessive deformations or even fracture in the net section) during the cold cambering process. This issue is further addressed later in this paper.

Curving

Curving or bending of steel members for structures such as those shown in Figures 2 through 4 involve substantial deformations and local plastification demands for the steel. It can be applied about the strong or weak axes of the shapes, depending on the structural requirements, to satisfy the architectural or engineering designs. The curving is typically performed to meet specific radii or other geometric configurations.

The strong axis stiffness of shapes is the largest, and the corresponding bending requirements are therefore the most demanding. The bending commonly entails large axial strains in the extreme fibers of the cross section. The procedure also involves local (point) application of forces from the bending equipment, with subsequent potential for local web or flange buckling of the shape or even overall lateral-torsional buckling of the member during the curving operation. In extreme cases, parts of the cross section may even undergo fracture. This has been observed in a number of actual curving cases, including one where the beams had been subjected to axial strains more than 19 times the yield strain of the steel. It has also occurred in members where local flange bending led to web buckling, which in turn caused a crack to develop in the flange close to the web-flange intersection. Similar failure scenarios have been observed in several cases.

The most common equipment that is used by bending companies focuses on passing the member through a set of rolls that gradually deform the shape into a circle (most commonly) with the required radius. The basic principle is illustrated in Figure 8, whereby forces are applied via the rolls on either side of the member. The shape is fed through the machine on repeated passes, with each one bending the member into successively smaller radii of curvature. Some machines also utilize rolls on the tension flange of the W-shape, to provide additional restraint against local flange bending as the curvature gets smaller. This is especially important for shapes with small web thicknesses (Steel Construction Institute, 2002). In Figure 8 the large rolls are the usual elements in the bending machine; the small (dashed outline) roll is actually a pair that is applied to the tension flange on both sides of the web.

The bending process indicated by the rolls in Figure 8 is the most common, but there are machines and bending companies that do not rely on such rolls. Certain patented processes utilize an articulated mechanism that distributes the load over a larger region than what is possible with the rolls. This is particularly the case for the smaller (tension flange) roll, which these processes do not use. A key feature of other curving operations is the use of restraining elements that prevent flange bending and web buckling, thereby allowing for smaller curving radii than what would otherwise be required. Some of the bending problems that are described here are therefore much less prevalent for the "no-rolls" and the "restraint" equipment operations, and the radius data that are described below can be expanded to include smaller radii.

Bending companies also provide curved elements with noncircular geometries. These may include elliptical forms, combinations of circles and ellipses, and members that are eventually made into S-type curves. However, such is usually achieved by joining elements that have been curved in opposite directions. An example of the latter is shown in Figure 9.

BENDING EQUIPMENT AND APPLICATIONS

There are a number of manufacturers of bending equipment, but the Swedish company Roundo is possibly the best-known supplier. Several bending companies use machinery of their own design, and certain firms have patented equipment and processes.

Figures 10 and 11 show two of the Roundo models and Figures 12 through 15 give additional examples of bending applications.

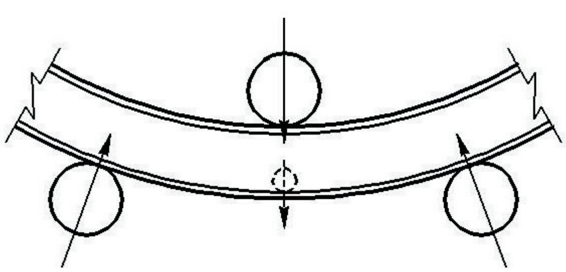

Fig. 8. Principle of bending process for shapes.

Fig. 9. Structure with members curved in opposite directions (photo courtesy of Marks Metal Technology).

Fig. 10. Bending machine, Roundo model R3 (photo courtesy of Comeq, Inc.).

Fig. 11. Bending machine, Roundo model R-15-S (photo courtesy of Comeq, Inc.).

CURVING MECHANICS AND ANALYSIS

Basic Concepts

As has been indicated for several applications of bending shapes into permanent curves, a certain amount of plastic deformation must take place in the cross section during the process in order for the curving operation to work. Elastic stress analysis and bending will not suffice, since any deformations taking place under such conditions will revert to zero once the applied force or moment is removed. Further, it has already been emphasized that bending about the strong axis of the member, also referred to as bending the hard way, is more demanding, because of the larger shape depth and distance to the extreme fibers in the cross section, as well as the larger bending stiffness. Finally, curving may be done to have a final product in the form of a circle or part of one; this can be arranged in many ways to arrive at combined geometric forms. However, the bending machinery is most commonly used to develop the circular form, for any number of reasons.

Using the principles of mechanics of materials, classical bending theory implies that to arrive at a circular form, a constant moment is developed along the full length of the member. Since the first derivative of the moment is the shear force and the moment is a constant, shear force, stresses, and strains will be zero. Only flexural stresses and strains will develop in the cross section under these conditions.

However, it is important to recall that transverse forces are developed by the rollers used in the bending machine (see Figure 8). These produce local compressive stresses in the web of the shape. Further, and especially with small radii, the curving operation has a tendency to promote local bending of the tension flange of the shape, which in turn may promote the possibility of local web buckling. Both of these phenomena can be somewhat controlled by careful alignment of the shape in the curving machine, but if the web of the shape is very thin, local buckling can be avoided by limiting the magnitude of the radius of curvature. The use of restraints, as mentioned previously, to prevent or at least delay the onset of flange bending and/or web buckling can also enhance the bending capacity significantly. The restraints facilitate the use of smaller bending radii.

Fig. 13. Curved W21×44 shapes (photo courtesy of Max Weiss Company).

Fig. 14. Wedding chapel (photo courtesy of Marks Metal Technology).

Fig. 12. Curved W12×53 shapes (photo courtesy of Max Weiss Company).

Fig. 15. Industrial installation (photo courtesy of Max Weiss Company).

Finally, if local web buckling occurs and it is not promptly discovered, chances are very good that the web or even the flange may fracture as a result of the combination of compressive stress in the web and the flange bending that accompanies the buckling action. Figures 16 and 17 demonstrate web buckling and flange bending of a W12×26 shape that had been curved to a tight radius. Figure 17 shows the flange actually broken away from the web. This occurred as a result of the web buckling and the subsequent flange bending.

Material Properties of Curved Members

The mechanical properties of steel are based on the common uniaxial tension test. This reflects a consensus approach to the properties of a wide range of materials and is very useful

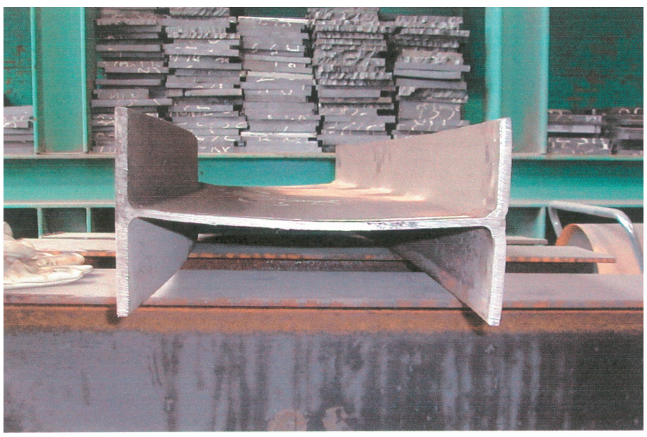

Fig. 16. Wide-flange shape with buckled web and flange bending (right side) (photo courtesy of Nucor-Yamato Steel Company).

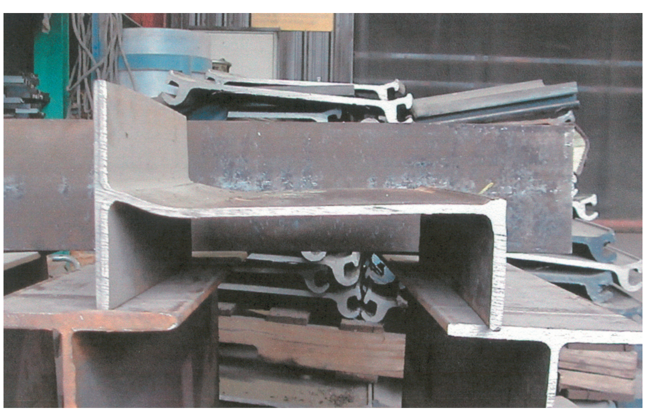

Fig. 17. Wide-flange shape with buckled web and one-half of flange broken off (photo courtesy of Nucor-Yamato Steel Company).

when it comes to correlation and comparison between steels from various producers. Of course, the tension test does not represent the response characteristics of the steel in actual structures, and was never intended to do so, but it is still a practical tool for designers, fabricators, and other users of the material.

As the beam is curved to the extent that plastic deformations occur in smaller or larger areas of the cross-section, the yielded regions may undergo material property changes as a result of the strains. As an example, Figure 18 shows the stress-strain curve for the proverbial "mild" steel (ASTM A36 in this case).

When the steel has been deformed plastically, it has undergone permanent deformations that are not removed upon unloading of the material (Bjorhovde, Engestrom, Griffis, Kloiber, and Malley, 2001). This is indicated by the dashed lines in Figure 18, to the right of the solid line initial stress-strain curves in the diagram.

Upon reloading, the steel responds in accordance with the dashed lines in the figures, seeming to indicate a material with a yield stress and elongation properties as defined by the "new" stress-strain curve. If there is only a small or even no yield plateau, which is typical of higher strength materials, the reloading response appears to be that of a steel material with a yield stress that is larger and an elongation at rupture that is smaller than the corresponding properties of the "original" material. If the steel has been strained into the strain hardening range, the change in the apparent mechanical properties can be substantial (Brockenbrough, 1992).

This elastic-inelastic behavior of the steel must be considered when planning the curving operations for structural shapes. It is taken into account in the curving criteria presented in the section entitled, "Practical Criteria for Structural Shape Curving." But specifically, since a smaller or larger part of the cross-section of the member must be deformed

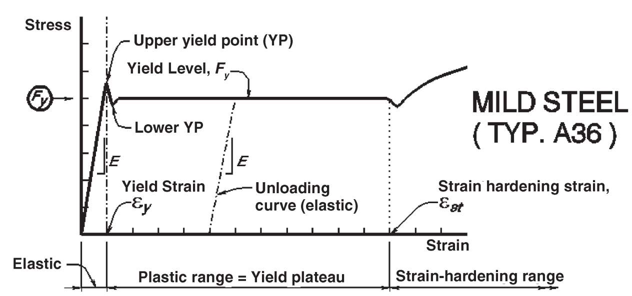

Fig. 18. Stress-strain curve for typical mild structural steel.

plastically in order for the curving to work, the extreme fibers in the cross section will be deformed well beyond the level of the initial yield strain, ε_y, of the steel. Depending on the curving radius, the maximum strain is likely to exceed the yield strain by a significant amount. If the curving radius is too small, or if there are stress concentrations such as holes in the flange(s) or coped areas of the beam, the magnitude of the strain that is imposed by the curving operation may reach the fracture level. This has been observed in a number of actual curving cases at bending facilities, but it has never been seen in the field. In brief, therefore, once the curving has been completed successfully, a failure due to the bending operation will not occur after the member has been placed into service. The flexural strains the member will experience during service are within the elastic range of behavior, and at least one order of magnitude smaller than those associated with the curving. In terms of service load stress levels, it is important to bear in mind that these stresses are well below the yield stress, typically no more than half of the yield level.

A study by Schlim (Schlim, 1987) used a range of curving radii for shapes in 36 ksi steel that were 220 mm (8.8 in.) deep. For a curving radius of 12.7 m (44 ft), which correlates very well with the curving criterion developed here, it was found that the yield and tensile strengths increased by about 10%; the elongation at rupture decreased by a small amount. There was no measurable change in the fracture toughness, as based on Charpy V-Notch tests. However, Schlim also noted that curving to much smaller radii would result in significant changes in the material properties.

Curving Mechanics

Curving Geometry

The curving is usually performed to meet a certain radius required for the member. Sometimes only the length of the span that is to be covered is given, and the offset of the curved member is specified. Figure 19 illustrates this for an element with a radius R, a span (in other words, circle chord length) of c, and an offset of b.

The mathematics of the circle gives the following relationships:

$$\text{Offset: } b = R - \tfrac{1}{2}\sqrt{\left(4R^2 - c^2\right)} \tag{2a}$$

$$\text{Radius: } R = \frac{4b^2 + c^2}{8b} \tag{2b}$$

The curving radius is the most important factor, since it determines the moment, curvature and levels of strain that will develop when the shape is bent to the specified configuration.

Moments and Strains Developed by Curving Operation

Given the radius of the member to be curved, R, the curvature, κ, is given by Equation 3.

$$\kappa = \frac{1}{R} \tag{3}$$

For a constant radius for the full length of the member, the moment-curvature relationship is defined by the expression

$$\kappa = \frac{1}{R} = \frac{M}{EI} \tag{4}$$

which gives the value of the constant moment along the length of the member as

$$M = M_{ce} = \frac{EI}{R} \tag{5}$$

where

M_{ce} = constant moment of the circular member when the material and the shape behave elastically

E = modulus of elasticity

I = moment of inertia for the axis of bending about which the shape is being curved

It is emphasized that the moment of Equation 5 reflects the elastic moment-curvature relationship. Since the curving operation necessitates a certain amount of plastification of the shape, Equation 5 does not apply. For example, the effective modulus of elasticity is zero for yielded material, and the moment of inertia of the shape is reduced by the loss of the stiffness contribution of the yielded areas of the cross section. Equation 5 can be replaced by an inelastic version, thus

$$M = M_{ie} = \frac{(EI)_{eff}}{R} \tag{5a}$$

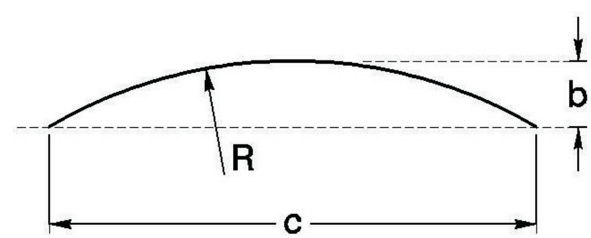

Fig. 19. Circle with radius, chord length, and offset.

where

M_{ie} = inelastic moment

$(EI)_{eff}$ = effective bending stiffness of the partially yielded cross section

However, this is a cumbersome and less than practical approach, especially since the effective stiffness changes as the radius changes.

For inelastic behavior, the moment-curvature expressions of Equations 4 and 5 do not apply. In this case it is necessary to go to the fundamental strain-curvature relationship, which dictates that plane sections remain plane. Although it is satisfied only approximately for curved members, it is sufficiently accurate for the applications addressed here.

For this case the following applies, as illustrated by the strain distribution of Figure 20, using a doubly symmetric cross section for the example (the linear strain distribution applies for all types of shapes):

$$\kappa = \frac{1}{R} = \frac{\varepsilon}{y} \tag{6a}$$

or

$$\varepsilon = \frac{y}{R} \tag{6b}$$

where

ε = strain at a distance y from the neutral axis

The linear strain distribution in Figure 20 applies for any level of applied moment.*

The traditional linear stress distribution applies only when all areas of the cross section are elastic, for which stress and strain are related by Hooke's Law, that is, $\sigma = E\,\varepsilon$.

For a doubly symmetric cross section, the maximum bending strains occur at the top and bottom extreme fibers, thus

$$\varepsilon_{\max} = \left[\frac{\left(\frac{d}{2}\right)}{R}\right] = \phi\left(\frac{d}{2}\right) \tag{7}$$

where

ϕ = rotation of the cross section, equal to the curvature, $\kappa = 1/R$

At the point when yielding first takes place, $\varepsilon_{\max} = \varepsilon_y = F_y/E$, which reflects the limit of elastic behavior. The yield strain, ε_y, and the corresponding yield rotation, ϕ_y, are:

$$\varepsilon_y = \frac{F_y}{E} \tag{8a}$$

$$\phi_y = \frac{\varepsilon_y}{\left(\frac{d}{2}\right)} \tag{8b}$$

which occur for a moment equal to the yield moment, M_y.

The yield and fully plastic moment capacities of the cross section are:

$$M_y = F_y S_x \tag{9a}$$

$$M_p = F_y Z_x \tag{9b}$$

where

S_x = elastic section moduli

Z_x = plastic section moduli

For a given radius, R, the maximum strain in the cross section is given by Equation 7, and the corresponding cross-sectional rotation is

$$\phi_{\max} = \phi_y\left(\frac{\varepsilon_{\max}}{\varepsilon_y}\right) = \alpha\phi_y \tag{10}$$

and $\varepsilon_{\max}$ is reformulated as

$$\varepsilon_{\max} = \frac{\left(\frac{d}{2}\right)}{R} = \alpha\varepsilon_y \tag{11}$$

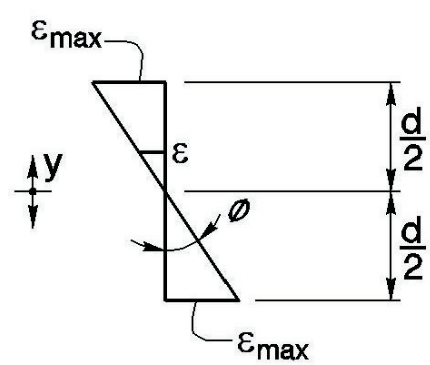

Fig. 20. Linear strain distribution over the depth of the cross section.

*The principle of "plane sections remain plane" applies for all *practical* structural engineering issues. If very large bending or shear deflections occur, it does not. However, such considerations need not be made here.

where

α = the maximum strain factor

Equation 11 indicates that the maximum strain in the cross section is a multiple, α, of the yield strain, ε_y. The maximum strain value can now be compared to the stress-strain curve for the material of the shape in question.

The strain and stress distributions in the cross section for the required amount of curving can now be determined. The stress distribution may then be used to calculate the magnitude of the curving moment, M_c, taking into account the fact that portions of the cross section have yielded.

Together with the yield and fully plastic moments and the corresponding yield and fully plastic rotations, the moment-rotation curve for the shape is then determined. The magnitudes of the curving moment and the corresponding rotation serve as an illustration of the degree to which the shape has been plastified.

Figure 21 shows a hypothetical strain and corresponding stress distribution for a doubly symmetric cross section, noting that the strain is larger than the yield strain ε_y over the two symmetrical portions of the depth, d_y. The remaining elastic portion of the cross section is the central area of depth equal to $2(d/2 - d_y)$.

Figure 22 shows the resulting moment-rotation diagram for the shape in question, including the data for the maximum rotation, ϕ_{max}, that is being imposed by the curving moment.

The size of the yielded portion of the cross section is determined from the relationship (see Figure 21)

$$\frac{d_y}{\left(\varepsilon_{max} - \varepsilon_y\right)} = \frac{\left(\dfrac{d}{2}\right)}{\varepsilon_{max}}$$

which gives

$$d_y = \left[\frac{\left(\varepsilon_{max} - \varepsilon_y\right)}{\varepsilon_{max}}\right]\left(\frac{d}{2}\right) \tag{12}$$

The actual magnitude of the curving moment, M_c, can now be found from the stress distribution shown in Figure 21b, along with the respective areas of the cross section and their moment arms.

Development of Curving Requirements

Based on the magnitudes of the maximum strains and rotations, an assessment can now be made as to whether it is feasible to curve the given shape and steel material to the specified radius. As will be seen in the following, the smaller the value of the maximum strain factor, the larger will be the portion of the shape that remains elastic when the maximum curving moment (in other words, the target radius) is applied. The value of α dictates the reserve moment capacity and hence the available ductility of the shape for the required radius of curvature.

Obviously, there are pros and cons in choosing small or large α values, but the key considerations are as follows:

1. A small value of α provides for larger reserve moment capacity and therefore a larger margin of safety against potential overstraining of the steel.

2. The smaller the α value, the larger the radius that should be used for a given shape.

3. A large value of α leaves a smaller portion of the cross section elastic by the time the target radius has been reached. This may be acceptable to the engineer and the curving company, but it is important to understand the

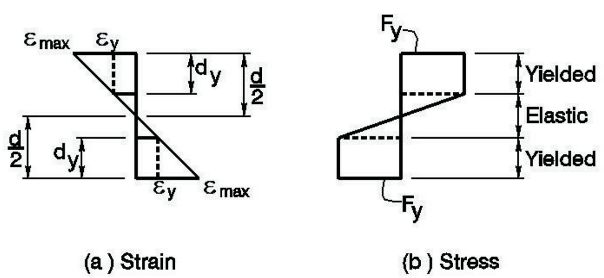

Fig. 21. Strain and stress distributions for the specified radius of curving.

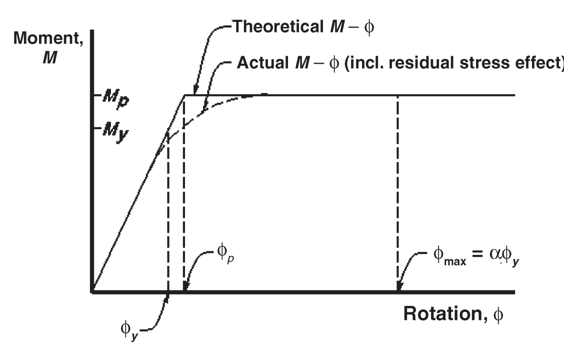

Fig. 22. Moment-rotation relationship for curved shape.

implications of the small reserve moment capacity. Accidental overloads or curving to a smaller radius than the required value in particular may result in steel fracture, local buckling, or other failure while the member is being curved at the bending facility.

4. Shapes with thin webs are particularly vulnerable to web buckling for small reserve moment capacities and, therefore, also to overstraining and fracture of the steel in the web-to-flange region.

5. Flange and/or web restraint elements are effective in preventing flange bending and web buckling. Such equipment will allow for the use of a larger α value, in other words, a smaller radius of curvature. The restraints allow the shape to be able to accommodate the kind of larger maximum strains that are associated with smaller radii.

6. Overstraining and consequent overloading of a shape through improper curving is more likely to occur for lower strength steels. This is because the yield strain is lower for such steels.

7. Pre-curving fabrication operations such as punching or drilling of holes in the flanges, coping of the flanges, and similar may provide areas of stress and consequently strain concentrations, with the possibility of fracture of the material at these locations. Such fractures have been observed in several cases.

For structural steels with yield stresses of 36 and 50 ksi and a modulus of elasticity of 29,000 ksi, maximum strain factor limitations of 4, 6, 8, 10, 16, 24, and 36 correspond to yield and maximum strain levels as illustrated below.

Yield Stress:	$F_y = 36$ ksi	$\varepsilon_y = 0.00124$	$= 0.124\%$
	$\alpha = 4$	$\varepsilon_{max} = 0.00496$	$= 0.496\%$
	$\alpha = 6$	$\varepsilon_{max} = 0.00744$	$= 0.744\%$
	$\alpha = 8$	$\varepsilon_{max} = 0.00992$	$= 0.992\%$
	$\alpha = 10$	$\varepsilon_{max} = 0.0124$	$= 1.24\%$
	$\alpha = 16$	$\varepsilon_{max} = 0.01984$	$= 1.98\%$
	$\alpha = 24$	$\varepsilon_{max} = 0.02976$	$= 2.98\%$
	$\alpha = 36$	$\varepsilon_{max} = 0.04464$	$= 4.46\%$
Yield Stress:	$F_y = 50$ ksi	$\varepsilon_y = 0.00172$	$= 0.172\%$
	$\alpha = 4$	$\varepsilon_{max} = 0.00688$	$= 0.688\%$
	$\alpha = 6$	$\varepsilon_{max} = 0.01032$	$= 1.03\%$
	$\alpha = 8$	$\varepsilon_{max} = 0.01376$	$= 1.38\%$
	$\alpha = 10$	$\varepsilon_{max} = 0.0172$	$= 1.72\%$
	$\alpha = 16$	$\varepsilon_{max} = 0.02752$	$= 2.75\%$
	$\alpha = 24$	$\varepsilon_{max} = 0.04128$	$= 4.13\%$
	$\alpha = 36$	$\varepsilon_{max} = 0.06192$	$= 6.19\%$

Expressing the equations in terms of the maximum level of strain, $\varepsilon_{max} = \alpha\,\varepsilon_y$, the yielded portions of the cross section become (see Figure 21),

$$d_y = \left[\frac{(\varepsilon_{max} - \varepsilon_y)}{\varepsilon_{max}}\right]\left(\frac{d}{2}\right) = \left[\frac{(\alpha\varepsilon_y - \varepsilon_y)}{(\alpha\varepsilon_y)}\right]\left(\frac{d}{2}\right) = \left[\frac{(\alpha - 1)}{\alpha}\right]\left(\frac{d}{2}\right) \tag{13}$$

and the central fraction of the web that remains elastic is

$$\text{Elastic portion} = EP = 1 - \left[\frac{(\alpha - 1)}{\alpha}\right] \tag{14}$$

Checking for Web Buckling and Similar Limit States

The installation and alignment of the shape in the curving machine are important considerations. These are very difficult to have done exactly, since the actual support conditions, the placement of the equipment rolls, etc., are rarely known. However, the following checks should be made for the shape by the structural engineer, as broad assessments of the potential for local failures:

1. Web compression buckling (AISC Equation J10-8)

2. Web sidesway buckling (AISC Equations J10-6 and J10-7)

3. Web crippling (AISC Equations J10-4 and J10-5a,b)

4. Web local yielding (AISC Equations J10-2 and J10-3)

The references to AISC give the equation numbers for the respective limit states, as addressed in the AISC *Specification for Structural Steel Buildings*, hereafter referred to as the AISC Specification (AISC, 2005).

Of the preceding four checks, the web compression buckling (No. 1) and web sidesway buckling (No. 2) are the ones most likely to indicate a potential local buckling problem associated with the curving of the shape. It is also noted that the application of No. 1 to the curving of a beam is conservative.

In some actual cases, during curving at a bending facility, web buckling has been found along with flange local bending. The flange bending was a result of the too-small radius and the phenomenon described earlier. However, the large axial strain demands imposed by the curving operation and the web buckling were the driving forces, eventually having caused the fracture of the web or the flange by through-thickness bending. Flange and web restraints might have eliminated these problems.

It has been noted that shapes with a larger web thickness tend to perform better during the bending operation. The increased web thickness provides a higher buckling strength and hence less opportunity for transverse flange bending to take place. As of 2005, there are no actual data or operational suggestions that can be used to arrive at enhanced radii when curving such shapes. An experienced bending company should be approached for advice in these cases.

A final comment is needed regarding gradually increasing the plastification of the shape during the curving operation. Most companies send a shape through the machine in several passes, each of which imposes additional yielding of the steel. This provides for curving cycles that demand smaller roller forces for each pass. However, the process does not limit the eventual maximum strains. In addition, since there is only a small time lag between the individual bending passes, issues such as strain aging most likely will not play a role. However, this is very difficult to confirm or reject without actual tests for material properties.

Some Other Considerations

The text has focused on the technical issues and potential difficulties associated with curving wide-flange shapes about the strong axis at ambient temperature. These are the most demanding conditions insofar as the material and the shapes are concerned. Some other considerations are outlined in the following.

Curving by Concentrated Load Application (Gag Pressing)

Depending on the available equipment in the bending shop, curving by concentrated load application is sometimes performed. The operation is similar to what is done for normal cambering of beams as described previously, except that the level of force and the amount of bending are significantly larger than the magnitudes associated with structural camber.

Using the model of a concentrated load applied centrally to a simply supported beam, the moment distribution varies linearly from zero at the supports to the maximum at the load application point. The shear force is constant for each half of the beam and equal to half of the applied load. The deflection is a maximum at mid-span.

The location of the maximum moment is also the location of the maximum strain demand within the length of the beam. Further, the deformation capacity of the steel is the smallest when the member is subjected to a uniform moment along the full length, or, in other words, when all cross-sections are subjected to the same maximum strain demand. The latter occurs for a beam that is bent to a constant curvature, when the moment is a constant for the full length of the member. It is clear that bending by cambering is at most as severe as curving by constant radius bending. The strain demand at the location of the maximum moment is always the critical consideration. The performance requirements as defined by the constant radius and the corresponding maximum strain factor therefore apply equally to gag pressing.

Curving about the Weak Axis of W-Shapes

The bending mechanics approach and the equations that have been developed here are equally applicable to the case of weak axis bending of W-shapes. However, the strain demands will be significantly different, due to the smaller dimension across the flange (as compared to the depth of the shape) and the smaller moment of inertia. In theory, a shape being bent about its weak axis will be able to accommodate smaller curving radii. However, it is important to bear in mind that the maximum strain will occur at the flange tips, and that the flange tips are laterally unsupported. That is, the flanges are unstiffened and will be subjected to flexural strains and stresses whose values are zero at the level of the web. For smaller flange thicknesses and/or larger flange widths the potential problem of flange buckling (in a direction transverse to the centroidal axis of the shape) must be addressed. This is a situation similar to having the web (stem) of a tee shape subjected to compression.

Section B4 (Table B4.1) of the *AISC Specification for Structural Steel Buildings* (AISC, 2005) details the criteria for compactness of shapes. A compact shape by definition is capable of reaching the fully plastic moment and to sustain significant plastic rotation before local buckling or strain hardening occur. However, compactness does not guarantee that the shape can be curved to any radius. The compactness concept and criteria were not developed with shape curving in mind. Further stability considerations are presented in the "Guide" of the Structural Stability Research Council (Galambos, 1998).

Curving Other Types of Shapes

Channels, tees, angles, and various hollow structural sections (HSS) are sometimes used in curved applications. The basic mechanics criteria also apply to these types of shapes, although some special considerations need to be made for local buckling of outstanding legs and flanges, unstiffened flanges, stems and angle legs, and the shape of circular and square/rectangular HSS. The AISC Specification provides width-to-thickness criteria for the plate elements of these shapes (AISC, 2005); however, other local buckling and stability assessments can be made for the individual types of shapes.

Considerations for Pre- and Post-Curving Operations

The curving model determines the requirements to deform a shape to the desired curvature. The forming operations can create areas of higher strain in the shape than predicted by the model. Local areas of higher plastic strains (for example, when small amounts of web or flange buckling occur) can influence the ability to perform post-curving operations such as coping or galvanizing. Any post-curving operations need to be considered when selecting a value for the maximum strain factor and may necessitate the use of a more conservative (in other words, smaller) value of α.

Bracing members or purlins are sometimes attached to the outside flange of a curved member, most commonly by bolts. It is recommended that bolt holes in the flange of a curved shape be made after the curving has been completed, even if this may be somewhat inconvenient. The stress concentrations associated with the holes may create additional strain limitations for the material in the flanges, and fractures have been known to occur at the exact locations of the bolt holes. However, if the curving is completed first, no difficulties are anticipated. It is known that this approach has been used successfully.

Galvanizing is sometimes used to provide corrosion protection for curved members that will be exposed to the environment during regular service. This is effectively a form of heat treatment, and although difficulties have been reported in some cases, there are no hard data that can be used to analyze the influence of the galvanization. Most applications have been successful, as reported by the industry, but careful inspection of post-galvanizing is very important.

PRACTICAL CRITERIA FOR STRUCTURAL SHAPE CURVING

Development of Criteria

The following parameters are paramount in the initial assessment of the curving needs for wide-flange structural members:

1. Radius of curvature, R
2. Depth of shape, d
3. Yield stress of steel, F_y

In addition, to prevent excessive straining of the steel and to ensure a minimum of reserve bending capacity of the shape:

4. Maximum strain factor, α
5. Maximum bending strain, $\varepsilon_{max} = \alpha\varepsilon_y$

With the maximum strain factor of α, this gives:

$$\varepsilon_{\max} = \alpha\varepsilon_y = \frac{\left(\frac{d}{2}\right)}{R}$$

which means that the minimum curving radius must satisfy the following criterion:

$$R > \frac{\left(\frac{d}{2}\right)}{\alpha\varepsilon_y} = \frac{d}{\left(2\alpha\varepsilon_y\right)}$$

and substituting for $\varepsilon_y = F_y/E$, this gives

$$R > \frac{d}{\left[2\alpha\left(\frac{F_y}{E}\right)\right]} = \frac{(dE)}{\left(2\alpha F_y\right)} \tag{15}$$

With the value of E as 29,000 ksi, this gives the basic curving criterion, thus

$$R > 14,500\left[\frac{d}{\left(\alpha F_y\right)}\right] \tag{16}$$

with R and d expressed in inches and F_y in ksi. The maximum strain factor α is dimensionless.

The limiting radius is used here for the curving criterion, not the least because of the physical illustration that is provided by the radius. It is possible to express the curving criterion in terms of the maximum longitudinal strain, which is given directly by the magnitude of maximum strain factor. An α value of 8, for example, implies a maximum total strain of $8\varepsilon_y$. Another definition might be to use the total plastic strain that is developed during curving, especially when it is done to repair damaged girders (Brockenbrough and Barsom, 1992). However, it is felt that the radius approach is preferable.

Sample Limits of Curving Using the Basic Curving Criterion

The following data are given only as examples of the radii that are implied for two grades of steel and two values of the maximum strain factor. Again, it is noted that the radii that are used for the higher strength steel are smaller than those that can be used with the lower strength material (in other words, higher strength implies higher curving capacity for otherwise identical shapes).

1. **50 ksi yield stress steel and α = 8:**

   R* > 36.25*d

d = 12 in.	R > 435 in.	= 36 ft
d = 18 in.	R > 653 in.	= 54 ft
d = 24 in.	R > 870 in.	= 73 ft
d = 30 in.	R > 1,088 in.	= 91 ft
d = 40 in.	R > 1,450 in.	= 121 ft

2. **36 ksi yield stress steel and α = 8:**

   R* > 50.35*d

d = 12 in.	R > 604 in.	= 50 ft
d = 18 in.	R > 906 in.	= 76 ft
d = 24 in.	R > 1,208 in.	= 101 ft
d = 30 in.	R > 1,511 in.	= 126 ft
d = 40 in.	R > 2,014 in.	= 168 ft

3. **50 ksi yield stress steel and α = 12:**

$R > 24.2d$

d = 12 in.	R > 290 in.	= 24 ft
d = 18 in.	R > 426 in.	= 36 ft
d = 24 in.	R > 581 in.	= 48 ft
d = 30 in.	R > 726 in.	= 61 ft
d = 40 in.	R > 968 in.	= 81 ft

4. **36 ksi yield stress steel and α = 12:**

$R > 33.6d$

d = 12 in.	R > 403 in.	= 34 ft
d = 18 in.	R > 605 in.	= 50 ft
d = 24 in.	R > 806 in.	= 67 ft
d = 30 in.	R > 1,008 in.	= 84 ft
d = 40 in.	R > 1,344 in.	= 112 ft

These data reflect the first and most important check for curving capacity, taking into account the properties of the steel as well as the key dimension of the shape. It is emphasized that these data are based on unrestrained steel shapes, without the use of restraint equipment or tensile force application, as is done in some processes.

It is recommended that if an α value larger than 12 is being considered, advice should be sought from an experienced bending company.

The basic curving citerion demonstrates that the larger the depth of the shape, the larger the curving radius must be in order to avoid overstrain. Similarly, for two otherwise identical shapes, the one with the lower yield stress steel requires a larger curving radius to avoid overstrain, simply because yielding will occur at a smaller moment than it will for the higher strength material. In other words, overstraining is more likely to occur in lower strength steel, as would be expected.

The second stage of checking by the structural engineer requires an evaluation of the potential for web compression buckling and web sidesway buckling, as outlined earlier.

SUMMARY

These guidelines present a discussion of current cold bending practices for hot-rolled steel wide-flange shapes for application to building-type structures, with illustrations using a number of current projects. The criteria do not apply to stainless steel.

A theoretical formulation has been developed to emphasize that the key consideration for shape bending is the strain demand that will be imposed on the material. Following fundamental moment-curvature relationships and aiming for a certain maximum strain in the extreme fibers of the cross section, the basic curving criterion has been developed. The expression is a simple relationship between the radius of curvature, the depth of the shape, the yield stress of the steel and the maximum strain factor. It is also noted that other limit states need to be evaluated. Practical applications of the basic curving criterion may focus on a range of suitable curving radii. The radius depends on the preferences and experience of the bending contractor, as well as the equipment that is used.

When a shape has been curved successfully, with no buckling or localized cracking in the steel, the strains the member will experience under actual service conditions will be much smaller than those associated with the curving operation. Once the curving is done, the member can be expected to perform as intended.

Although the focus of the guidelines has been on the use of wide-flange shapes bent about their strong axis at room temperature, the basic curving criterion applies equally to other cross-sections. However, for such other shapes additional attention must be given to the potential limit states that may influence the response of the cross-section during bending.

Developments in bending equipment and techniques continue to improve and expand the capacities of companies to curve shapes to increasingly tighter radii. Maintaining sectional integrity with minimal distortion and avoidance of localized material failures will aid in providing bent members that will perform effectively in service. Additional research studies may be warranted to continue to expand the offerings of bending companies.

The analyses and the criteria that have been developed are not intended for application with structures that are subjected to high-cycle loads, such as bridges.

ACKNOWLEDGMENTS

Sincere appreciation for detailed and very useful comments on the study presented in this paper are extended to the members of the AISC Rollers and Benders Committee, in particular George Wendt, committee chair, Allan Flamholz, David Marks, Joe Rogers, and Ray Weiss. Additional thanks are due the Technical Committee on Structural Shapes, in particular Mike Engestrom, and to numerous Nucor-Yamato Steel Company staff members.

NOMENCLATURE

E = modulus of elasticity

$(EI)_{eff}$ = effective bending stiffness of a partially yielded cross section

EP = remaining elastic portion of cross section

F_y = yield stress of steel

I = moment of inertia

M = bending moment (general)

M_c = curving moment

M_{ce} = elastic constant moment of a circular beam segment

M_{ie} = inelastic moment

M_p = fully plastic moment of a cross section

M_y = yield moment

R = radius

S_x = elastic section modulus of cross section about the x-axis

Z_x = plastic section modulus of cross section about the x-axis

b = offset of a circle segment

c = chord length of a circle segment

d = depth (height) of beam

d_y = yielded portion of cross section

e = initial out-of-straightness

t_w = web thickness

y = coordinate within beam cross section

α = maximum strain factor

ε = strain

ε_{max} = maximum strain within cross section

ε_y = yield strain

ϕ = rotation of cross section

ϕ_{max} = maximum rotation of cross section

ϕ_p = rotation of cross section upon reaching fully plastic moment

ϕ_y = rotation of cross section upon reaching yield moment

κ = curvature

σ = stress

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