Performance of the AISC LRFD Specification in Predicting the Capacity of Eccentrically Loaded Single-Angle Struts

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Over the past few decades, the AISC design rules for structural steel single-angle members have evolved from simple assumptions to the present ultimate-strength approach. The revisions to the AISC specification have always reflected the state of knowledge of the behavior of these structural members resulting in better-performing and more reliable structures.

Prior to 1993, the load and resistance factor design of single-angle struts was governed by the general provisions of the AISC *LRFD Specification for Structural Steel Buildings* (AISC, 1986). The first separate specification addressing single-angle members was the *Specification for Load and Resistance Factor Design of Single-Angle Members* (AISC, 1993). This 1993 specification was then superseded by the *Load and Resistance Factor Design Specification for Single-Angle Members* (AISC, 2000), hereafter referred to as the *Single-Angle Member Specification*, which represents the state-of-the-art in single angle design.

The *Single-Angle Member Specification* considers three different limit states when evaluating the flexural strength of a single-angle: (1) limit state of local buckling when the tip of an angle is in compression; (2) limit state of yielding when the tip of an angle is in tension; and (3) limit state of lateral-torsional buckling. A bilinear interaction equation is then used to estimate the load-carrying capacity of the single-angle strut. This interaction equation was mainly derived from research on doubly symmetric W-sections, which may be improper for use with eccentrically loaded single-angle struts that are either monosymmetric or asymmetric (Adluri and Madugula, 1992; Trahair, 2001).

Based on contemporary research work by Earls and Galambos (1997), the *Single-Angle Member Specification* limits the flexural strength at full yielding to a shape factor of 1.5 applied to the yield moment as this value represents a better lower bound. The current shape factor of 1.5 replaced the 1.25 value used in the previous single-angle specification (AISC, 1993) which was known to be a conservative estimate. This increase in the shape factor affected the flexural strength of single-angles evaluated using the aforementioned three limit states. The leg width-to-thickness limits of local buckling, b/t, when the tip of an angle leg is in compression have been modified in the *Single-Angle Member Specification* to be more representative of flexural limits rather than using those for single angles under uniform compression.

Prior to the appearance of the first separate specification addressing single-angle members, Adluri and Madugula (1992) compared results of an experimental investigation on eccentrically loaded single-angle struts with the load-carrying capacity calculated according to the two AISC specifications governing the design of single-angle struts at that time, the *Specification for Structural Steel Buildings — Allowable Stress Design and Plastic Design* (AISC, 1989) and the first AISC *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 1986), hereafter referred to as the LRFD Specification. A total of 71 test results from three different sources were used for comparison purposes. Their study, which focused primarily on equal-leg single-angle struts, was originally undertaken in order to verify a growing feeling among some practicing engineers that the AISC specifications were conservative in predicting the load carrying capacity of eccentrically loaded single-angle struts. In the 1986 edition of the AISC LRFD Specification, the flexural strength of single angles was taken equal to the yield moment of the cross section, M_y. Adluri and Madugula concluded that the AISC LRFD Specification (AISC, 1986) was highly conservative for predicting the load-carrying capacity of eccentrically loaded single-angle struts. Two suggestions were proposed to improve the performance of the AISC LRFD Specification: (1) interaction equations should be applied at all critical points of the angle cross section separately with due consideration of the sign of the stress; and (2) reducing the moment interaction factor from $8/9$ to $2/3$ for the range of $P_u/\phi P_n$ between 0.5 and 1.0. As a result of their study, the first suggestion was incorporated in the superseding editions of the AISC LRFD Specification leading to a significant increase in the load-carrying capacity (Lutz, 1996). The commentary of the *Single-Angle Member Specification*

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(AISC, 2000) refers to the second suggestion as an alternative special interaction equation for single-angle struts.

In addition to considering the sense of stresses at a certain location, the first separate AISC specification addressing single-angle members (AISC, 1993) established an upper limit of flexural strength at 1.25 of the yield moment which was changed to 1.5 of the yield moment in the *Single-Angle Member Specification*. Also, the resistance factor for compression was changed from 0.85 in the 1986 edition of the AISC LRFD Specification to 0.9 in first separate single-angle member AISC specification (AISC, 1993) and has remained unchanged in the *Single-Angle Member Specification*.

With all the aforementioned amendments in mind, it is necessary to evaluate the performance of the *Single-Angle Member Specification* in predicting the load-carrying capacity of eccentrically loaded single-angle struts and their safety index. A literature review revealed that the performance of that specification in predicting the load-carrying capacity of eccentrically loaded unequal-leg single-angle struts has never been assessed. In this paper, experimental results of 91 tests conducted on equal-leg angles and 42 tests conducted on unequal-leg angles are examined and compared with the prediction of AISC (2000) and previous AISC LRFD specifications. The reliability indices inherent in the *Single-Angle Member Specification* will also be reviewed. The common practice of detailing the end connection of single-angle members in such a manner that the load point, located along the connected leg, is as close as possible to the projection of the angle centroid on the connected leg will be investigated in accordance with the provisions of the *Single-Angle Member Specification*.

AISC BEAM-COLUMN DESIGN MODEL

The *Single-Angle Member Specification* and previous editions of the AISC LRFD specification addressing single-angle members have required that an eccentrically loaded single angle be designed as a beam-column subjected to biaxial bending moments. There are two steps involved in applying the beam-column model. Firstly, the load point is determined and the bending moments about both principal axes due to the eccentricity of the applied load are calculated as shown in Figure 1. Secondly, an interaction equation is then used to estimate the load-carrying capacity of the single-angle strut. The following beam-column interaction criteria have been given by all editions of the AISC specification to check the adequacy of an eccentrically loaded single-angle strut.

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}}\right) \le 1.0 \qquad \text{for } \frac{P_u}{\phi P_n} \ge 0.2 \qquad (1)$$

$$\frac{P_u}{2\phi P_n} + \left(\frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}}\right) \le 1.0 \qquad \text{for } \frac{P_u}{\phi P_n} \le 0.2 \qquad (2)$$

where

P_u = required (factored) design axial load
P_n = nominal compressive strength for concentric axial loading
M_u = required (factored) bending moment
M_n = nominal flexural strength for tension or compression as appropriate
ϕ = resistance factor for compression = 0.90
ϕ_b = resistance factor for flexure = 0.90
w = subscript symbol relating to major-axis bending
z = subscript symbol relating to minor-axis bending

Adluri and Madugula (1992) recommended reducing the moment interaction factor in Equation 1 from 8⁄9 to 2⁄3 in order to improve the prediction of the load-carrying capacity of single angle struts. The following set of interaction equations was suggested:

$$\frac{P_u}{\phi P_n} + \frac{2}{3}\left(\frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}}\right) \le 1.0 \qquad \text{for } \frac{P_u}{\phi P_n} \ge 0.5 \qquad (3)$$

$$\frac{P_u}{2\phi P_n} + \left(\frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}}\right) \le 1.0 \qquad \text{for } \frac{P_u}{\phi P_n} \le 0.5 \qquad (4)$$

This reduction in the moment interaction term resulted in higher load-carrying capacities for angles with axial term, $P_u/\phi P_n$ greater than 0.5. The commentary of the *Single-Angle Member Specification* refers to the set of interaction equations given by Equations 3 and 4 as an alternative special interaction equation for single-angle struts.

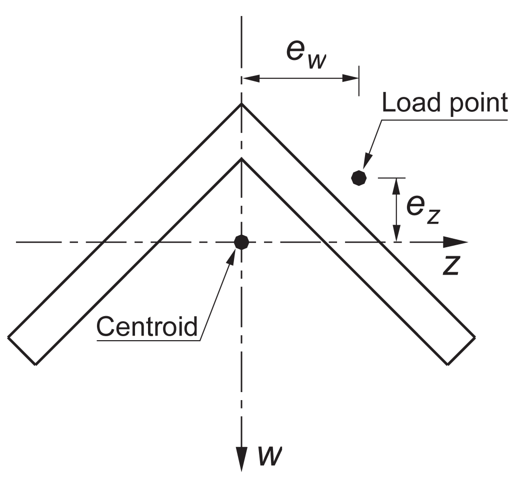

Fig. 1. Eccentrically-loaded single angle.

Table 1. A Brief Description of Experimental Studies Used for Evaluating Equal-Leg Single-Angle Struts							
Source Reference	No. of Tests	Slenderness Ratios (Approx.)	Angle Sizes	F_y (ksi)	Load Eccentricity		
						e_z/b	e_w/b
Bathon et al. (1993)	31	60, 90, 120, and 180	$1^3/_4 \times 1^3/_4 \times {}^1/_4$ in. $2 \times 2 \times {}^1/_8$ in. $2^1/_2 \times 2^1/_2 \times {}^3/_{16}$ in. $3 \times 3 \times {}^1/_4$ in. $3^1/_2 \times 3^1/_2 \times {}^1/_4$ in. $4 \times 4 \times {}^1/_4$ in. $5 \times 5 \times {}^5/_{16}$ in. $5 \times 5 \times {}^3/_8$ in. $6 \times 6 \times {}^3/_8$ in.	47.2 ~ 58.4	The angle was loaded at the outer face of one leg at a distance, g, measured from the heel of the angle. The distance g ranged from 0.36 to 0.56 times the width of the loaded leg.		
Wakabayashi and Nonaka (1965)	40	20, 40, 60, 70, 80, 90, 100, 110, 130, and 150	3.54 × 3.54 × 0.28 in.	42.7 ~ 46.9	Four cases of load eccentricity		
					(I)	0.20	0.00
					(II)	–0.20	0.00
					(III)	0.00	0.19
					(IV)	0.20	0.19
Mueller and Erzurumlu (1983)	14	60, 110, and 192	3 × 3 × ¼ in.	50.6 ~ 61.3		–0.07 ~ 0.07	–0.37 ~ 0.46
Ishida (1968)	7	20, 40, 60, 80, and 100	2.95 × 2.95 × 0.24 in.	58.8 ~ 63.9		0.20	0.00

Table 2. A Brief Description of Experimental Studies Used for Evaluating Unequal-Leg Single-Angle Struts						
Source Reference	No. of Tests	Slenderness Ratios (approx.)	Angle Sizes	F_y (ksi)	Load Eccentricity	
					e_z/b	e_w/b
Bathon et al. (1993)	42	60, 90, 120, 150, 180, and 210	$2^1/_2 \times 2 \times {}^3/_{16}$ in. $3 \times 2 \times {}^3/_{16}$ in. $3 \times 2 \times {}^1/_4$ in. $3 \times 2^1/_2 \times {}^1/_4$ in. $3^1/_2 \times 2^1/_2 \times {}^1/_4$ in. $3^1/_2 \times 3 \times {}^1/_4$ in. $4 \times 3^1/_2 \times {}^1/_4$ in. $5 \times 3^1/_2 \times {}^5/_{16}$ in.	47.1 ~ 56.4	The angle was loaded at the outer face of the long leg at a distance, g, measured from the heel of the angle. The distance g ranged from 0.38 to 0.49 times the width of the loaded leg.	

EXPERIMENTAL DATABASE

Published test reports on the behavior of eccentrically loaded single-angle struts have been collected and reviewed and a database containing the experimental test results along with the associated design variables has been established. For the purpose of evaluating the performance of the AISC design procedure, the database was reduced to consider only single-angle struts that satisfy the following two conditions: (i) the strut is unrestrained against rotation and twist in all directions, in other words, clear end restraints and effective lengths about both principal axes could be defined; and (ii) the load point at the ends of the strut can be clearly located. This resulted in a reduced database of 133 test results comprising 91 tests on equal-leg single-angles and 42 tests on unequal-leg single-angles. The records of the database were taken from the experimental studies reported by (1) Bathon, Mueller and Kempner (1993); (2) Wakabayashi and Nonaka (1965); (3) Mueller and Erzurumlu (1983); and (4) Ishida (1968). Results reported in the last three sources were used in the aforementioned study by Adluri and Madugula (1992). A brief description of the properties of the angles used in each of the four experimental studies is given in Tables 1 and 2.

As could be shown from Tables 1 and 2, the experimental results reported by Bathon et al. (1993) were the most comprehensive for evaluating the performance of the

Table 3. Experimental-to-Prediction Ratio Statistics for Single-Angle Struts

		No. of Tests	Method of Calculation					
			AISC LRFD 1993		AISC LRFD 2000		AISC LRFD 2000 using an interaction term of ⅔	
			Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
Equal-leg single-angle struts	Wakabayashi and Nonaka (1965)	40	1.308	0.130	1.140	0.148	1.034	0.151
	Mueller and Erzurumlu (1983)	14	1.220	0.157	1.112	0.142	1.032	0.143
	Ishida (1968)	7	1.240	0.109	0.999	0.060	0.895	0.044
	Bathon et al. (1983)	30	1.252	0.201	1.165	0.182	1.077	0.169
	All	91	1.271	0.161	1.133	0.159	1.037	0.156
Unequal-leg single-angle struts	Bathon et al. (1983)	42	1.263	0.178	1.187	0.155	1.101	0.132

Single-Angle Member Specification. Three factors contribute to the importance of this experimental study: (1) it covered equal- and unequal-leg angles; (2) nine different sizes of equal-leg angles and eight different sizes of unequal-leg angles were used for making the specimens; and (3) the angle was loaded through one leg only at the outer face of the loaded leg, which simulates practical applications of single angles in towers, trusses and buildings.

COMPARISON OF THE AISC DESIGN MODEL WITH EXPERIMENTAL DATA

The AISC design model was used to predict the load-carrying capacities of the aforementioned experimental tests. The resistance factors for compression, ϕ, and for flexure, ϕ_b, were both taken as 1.0. Figures 2 and 3 show a comparison between the AISC set of interaction equations and the flexure and axial terms calculated using experimental load-carrying capacities for equal- and unequal-leg single-angle struts, respectively. In these figures, the axial load term ($P_u/\phi P_n$) is shown on the y-axis while the corresponding flexure term is shown on the x-axis. The flexure terms in Figures 2(a) and 3(a) are calculated using the provisions of the previous AISC LRFD single-angle specification (AISC, 1993) while the flexure terms shown in Figures 2b and 3b are calculated in accordance with the provisions of the *Single-Angle Member Specification*. As could be noted from Figures 2 and 3, the *Single-Angle Member Specification* provides a much better estimate of the load-carrying capacity of eccentrically loaded single-angle struts when compared with the previous specification as indicated by closeness of the experimental results to the bilinear interaction curve representing Equations 1 and 2. This improved performance in predicting the load-carrying capacity is a result of increasing the flexural strength of single-angles in the *Single-Angle Member Specification*. The performance of this AISC specification in predicting the load carrying capacity of equal-leg single angle struts is better than that of unequal-leg single-angle struts. Figures 2(a) and 3(a) indicate that, except for one equal-leg test result, the earlier version of the specification (AISC, 1993) provided a lower bound for all equal- and unequal-leg single-angle struts as all experimental data points are located outside the bilinear interaction curve of the 1993 AISC Specification. In spite of the improved performance of the *Single-Angle Member Specification*, Figure 3(b) indicates that the Specification still provides a lower bound for all unequal-leg single angle struts. The *Single-Angle Member Specification* overestimated the load-carrying capacity of some 15 equal-leg angles by as much as 18% in one case and by less than 10% for the remaining 14 cases. See Figure 2(b).

Table 3 shows the experimental-to-predicted ratio statistics of load-carrying capacity for the equal- and unequal-leg single angle struts reviewed in this study. Using the provisions of AISC (2000) and previous specifications, the average experimental-to-prediction ratios for equal-leg angles were found to be 1.133 and 1.271, respectively, indicating an average increase in the calculated load-carrying capacity of about 12.2%. For unequal-leg single-angle struts, the average experimental-to-prediction ratios were 1.187 and 1.263 as per the provisions of AISC (2000) and previous specifications, respectively, reflecting an average increase in the calculated load carrying capacity of about 6.4%. The standard deviation of the experimental-to-prediction ratio remained basically the same for equal-leg angles (around 0.160) and slightly improved for unequal-leg angles (changed from 0.178 to 0.155).

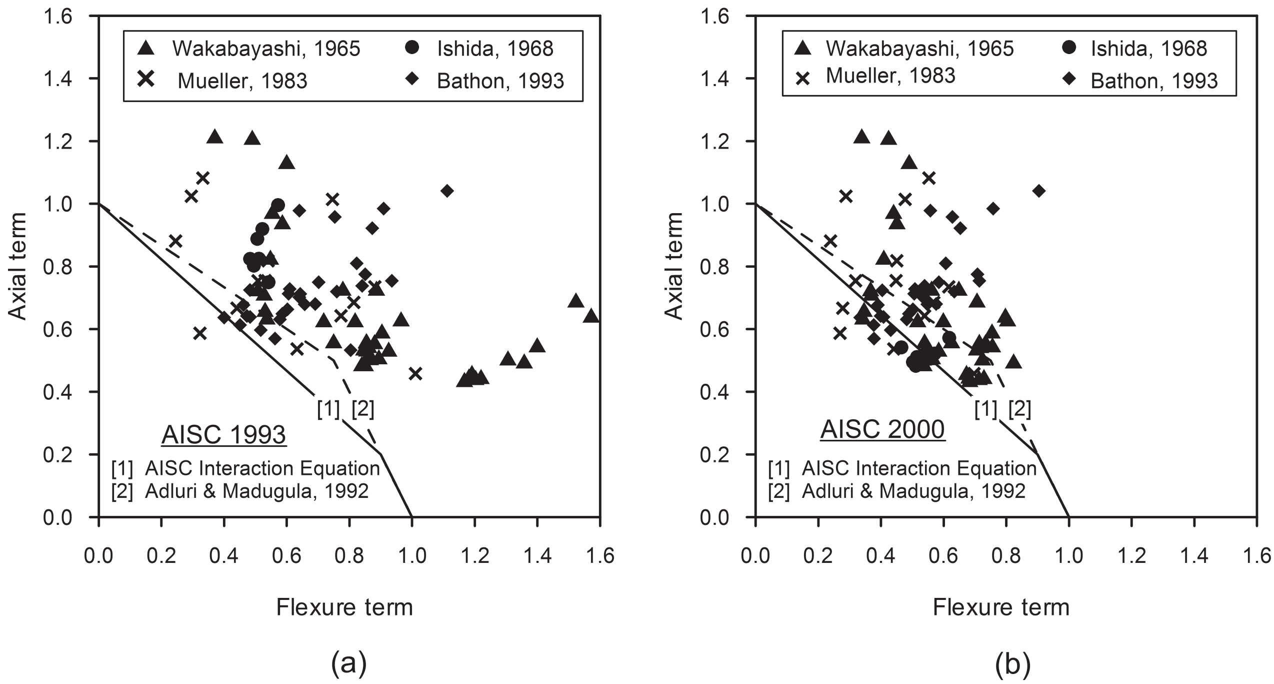

Fig. 2. A comparison between experimental load-carrying capacities of equal-leg single-angle struts and: (a) AISC (1993); (b) AISC (2000).

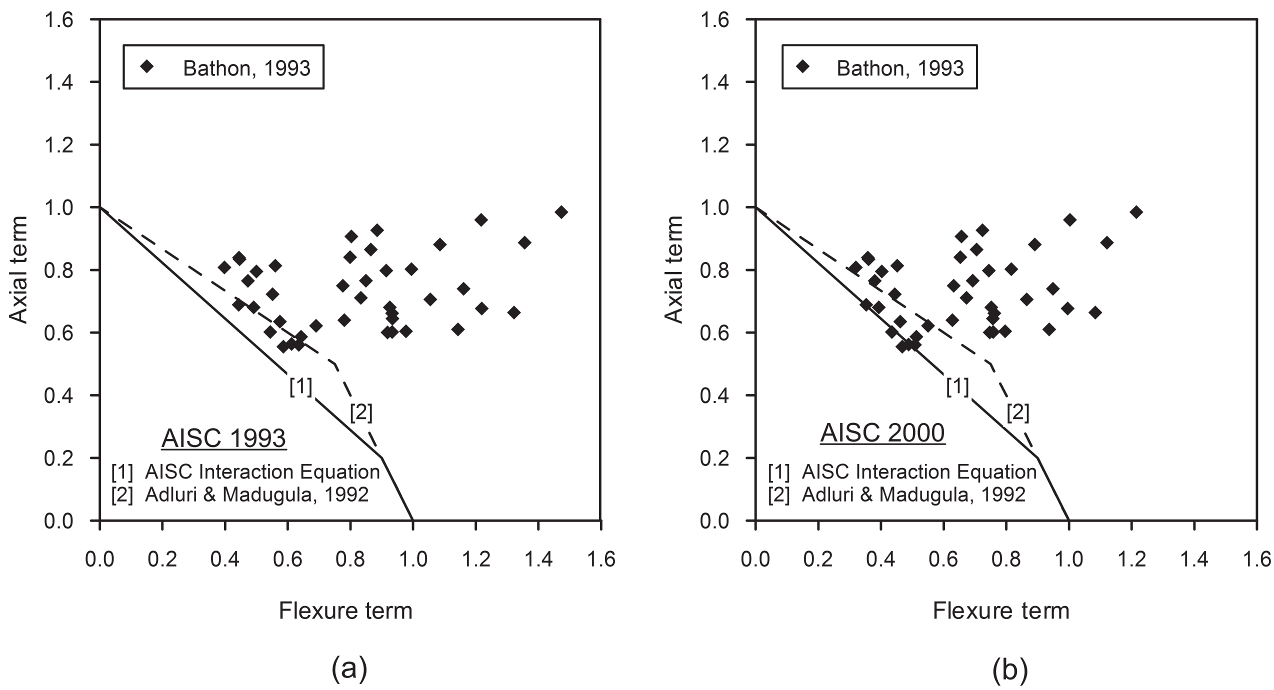

Fig. 3. A comparison between experimental load-carrying capacities of unequal-leg single-angle struts and: (a) AISC (1993); (b) AISC (2000).

In order to assess the safety margin for LRFD of single-angle struts, the reliability index β was examined using Monte Carlo simulation. Based on the statistics of the random variables and the AISC prediction model, the mean and the standard deviation of the checking function *g* were determined by simulation and the reliability index β were calculated from the following equation:

$$\beta = \frac{g_m}{\sigma_g} \tag{5}$$

where

g_m = mean value of the checking function
σ_g = standard deviation

The reliability index, β, was calculated using the results shown in Table 3 and the statistical properties given by Galambos, Ellingwood, MacGregor and Cornell (1982) as these particular values were used to determine the load factors and resistance factors in the AISC specification (Ellingwood, MacGregor, Galambos and Cornell, 1982).

The calculated reliability indices were found to be in the range of 2.65 to 2.90 for equal-leg angles and in the range of 2.95 to 3.20 for unequal-leg angles. These calculated reliability indices are consistent with the values associated with other types of structural members as given in the AISC *LRFD Specification for Structural Steel Buildings* (AISC, 1999). According to the Commentary in AISC (1999), this typical reliability index is equal to 2.6.

Using Equations 3 and 4 to predict the load-carrying capacities of experimental tests resulted in an average experimental-to-prediction ratio of 1.037 and 1.101 for equal- and unequal-leg single-angle struts, respectively, as could be noted from Table 3. In spite of this improvement in the prediction of the load carrying capacity, the standard deviation of the experimental-to-prediction ratio remained basically the same for equal-leg angles (around 0.160) and slightly improved for unequal-leg angles (changed from 0.155 to 0.132) when compared with the predictions of the *Single-Angle Member Specification*. However, the computed reliability indices were in the range of 2.20 to 2.40 and 2.75 to 3.00 for equal- and unequal-leg angles, respectively. This indicates that Equations 3 and 4 have fallen short of achieving the target reliability index of 2.6 when used to predict the load-carrying capacity of equal-leg single-angle struts.

LOCATION OF LOAD POINT FOR ANGLES CONNECTED BY ONE LEG

In the majority of practical applications, single-angles are typically attached to other structural members by one leg only. It is a common practice to detail the end connection of single-angle members in such a manner that the load point, located along the connected leg, is as close as possible to the projection of the angle centroid on the connected leg.

Consider an L3×3×¼ in. with a yield stress of 50 ksi loaded through one leg. Figure 4 shows the effect of changing the location of the applied load along the connected leg on the load carrying capacity as per the *Single-Angle Member Specification* for three different slenderness ratios. The resistance factors for compression, ϕ, and for flexure, ϕ_b, were both taken as 1.0. The location of the applied load along the connected leg, *g*, measured from the heel of the angle was varied from 0.25 to 0.60 times the width of the connected leg width. As could be noted from Figure 4, the calculated load carrying capacity of an angle is sensitive to the location of the applied load along the connected leg. The effect of load eccentricity becomes more predominant as the slenderness ratio of the angle strut decreases. As per the *Single-Angle Member Specification*, the highest possible calculated load-carrying capacity is achieved by locating the applied load along the angle leg at a distance of 0.46, 0.47 and 0.47 times the width of the connected angle measured from the heel of the angle for slenderness ratios of 60, 120 and 180, respectively. The maximum load carrying capacity is higher than that computed when the load is located at the projection of the centroid on the connected leg by 44%, 31%, and 20% for the slenderness ratios of 60, 120, and 180, respectively. This implies that there is a discrepancy between the *Single-Angle Member Specification* and the common practice regarding the optimum detailing of the end connection of single-angle struts attached to other structural members. While it is a common practice to locate the centroid of the bolt pattern as

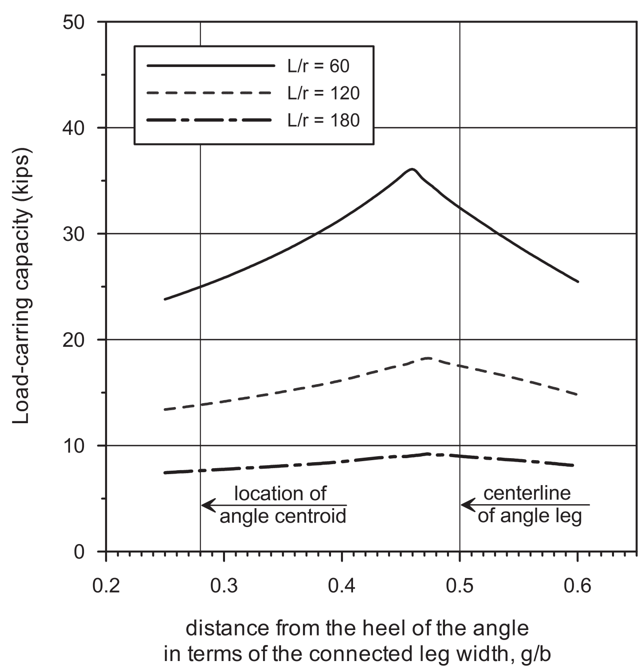

Fig. 4. Effect of changing the location of the applied load along the connected leg on the load carrying capacity.

close to the centroid as practicable, it is evident that, as per the AISC provisions, the optimum location of the centroid of the bolt pattern is near the center of the connected leg.

CONCLUSIONS

Using experimental data available in the literature, the performance of the *Single-Angle Member Specification* and previous AISC LRFD single-angle member specifications (AISC, 1993) in predicting the load-carrying capacity of eccentrically loaded single-angle struts was examined. Comparisons of available test results with the design rules of the *Single-Angle Member Specification* show that

1. The performance of the *Single-Angle Member Specification* (AISC, 2000) addressing single-angle members in predicting the load-carrying capacity of eccentrically loaded single-angle struts has significantly improved when compared with the previous versions of the specification. As per the *Single-Angle Member Specification,* the average test-to-prediction ratio of the load carrying capacity was found to be 1.133 and 1.187 for equal- and unequal-leg single-angle struts, respectively.

2. The *Single-Angle Member Specification* provides a lower bound for all unequal-leg test results examined in this study and overestimates the load-carrying capacity of some 15 equal-leg test results.

3. Both equal- and unequal-leg angles designed using Equations 1 and 2 and unequal-leg angles designed using Equations 3 and 4 met the target reliability index of 2.6 specified in the Commentary of the *Single-Angle Member Specification.* However, equal-leg angles designed using Equations 3 and 4 have fallen short of this target reliability index.

It was also noted that a discrepancy exists between the *Single-Angle Member Specification* and the common practice of detailing the end connection of single-angle members in such a manner that the centroid of the bolt pattern is located as close to the centroid as practicable. As per the *Single-Angle Member Specification*, locating the load point at the projection of the centroid on the connected leg does not result in the largest achievable load-carrying capacity.

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