A Historical and Technical Overview of the C_b Coefficient in the AISC Specifications

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This paper was written as a refresher for experienced engineers as well as to document the technical basis for the C_b coefficient (bending coefficient) for entry-level engineers and designers. It also discusses the historical changes to the C_b coefficient from the 1989 *Specification for Structural Steel Buildings—Allowable Stress Design and Plastic Design* (AISC, 1989), hereafter referred to as the 1989 *ASD Specification*, through the 1999 *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 2000), and hereafter referred to as the 1999 *LRFD Specification*. Incorporated into this paper are answers to the most frequently asked questions concerning the use and interpretation of the C_b expressions found in the AISC Specifications.

TECHNICAL BASIS

The C_b coefficient is used in flexural expressions to account for the variation of bending moment along the length of a member. End supported beams or braced segments, either due to their loading arrangement and/or support restraint condition, can have nonlinear bending moment diagrams. The AISC flexural expressions in Chapter F of the 1989 *ASD Specification* and the 1999 *LRFD Specification* were developed with the conservative assumption of a constant bending moment along the member. The C_b coefficient was created to account for departures from this assumption. It is important to note that C_b is only of consequence in cases where lateral-torsional buckling may become an issue, which typically occurs for bending about the strong axis for unbraced span lengths greater than L_c (ASD) and L_p (LRFD).

Flexural members should always be designed based on the applied maximum bending moment. Applying a C_b value greater than unity results in an increase in the section capacity, in other words, an increase in the allowable flexural stress (ASD) and the nominal flexural strength (LRFD). This allows the designer to select a lighter beam, or alternatively, it permits a larger service load if the initially selected member is used. Section F1.3 of the 1989 *ASD Specification* and Section F1.2 of the 1999 *LRFD Specification* address the C_b coefficient, which is used directly in flexural expressions as a multiplier.

When a vertical load is applied to a beam, the top flange experiences compression while the bottom flange experiences tension. Lateral-torsional buckling occurs when the compression flange buckles about its strong axis. Buckling will not occur about the weak axis of the compression flange since it is braced by the web. The bottom flange will not buckle, as tension elements are not susceptible to buckling.

In composite design, the top flange is braced by the floor system and lateral-torsional buckling is typically not an issue (except in rigid frame and continuous beam design). However, in noncomposite design, the top flange may span long distances without lateral bracing. These unbraced spans are subject to lateral-torsional buckling if they are longer than L_p. Part 5 of the AISC 3rd Edition *LRFD Manual of Steel Construction* (AISC, 2001) gives values for M_n, the nominal flexural strength, for a multitude of beam sizes with various unbraced segments.

The nominal flexural strength is conservative for beams with nonlinear moment diagrams (varying moment values along their length.) The C_b coefficient is used to more accurately model the actual strength of the beams and can be explained by an analogy to columns, considering the compression flange as a column under eccentric axial load. As the bending moment increases, the axial force in the column increases. Unlike most columns though, the axial force in the compression flange varies along the length due to the moment variation. M_n corresponds to P_{cr}, the critical buckling load of a column. A column with no compressive axial load at its top and an increasing axial compressive load that results in P_{cr} at the bottom would not buckle. Similarly, a beam where the bending moment is equal to the nominal flexural strength M_n at one point along the beam will not buckle. This results in an average bending moment along the length of the beam that is less than the nominal flexural strength, M_n. The

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coefficient C_b increases the average permitted bending moment to that of the nominal flexural strength, M_n.

USE OF C_b IN AISC SPECIFICATIONS

The upper bound for C_b is stipulated in the 1989 *ASD Specification* Section F1.3 as:

$$C_b = 1.75 + 1.05\frac{M_1}{M_2} + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3 \tag{1}$$

In this expression, M_1 and M_2 are the moments at the ends of the unbraced length (in other words, segment) and M_1 is always smaller than M_2. The ratio of (M_1/M_2) can be positive or negative, depending on the orientation of the moment couples. Note that Equation 1 should only be used for loading arrangements and/or support restraint conditions that result in straight-line moment diagrams. An upper bound of 2.3 was established to limit the equation, as C_b values for straight-line moment diagrams are typically at or below this value. When using Equation 1, if the internal moment anywhere along a beam exceeds the magnitude of its maximum end moment, a C_b value of 1.0 should be used.

The 1999 *LRFD Specification* Section F1.2a contains an entirely different expression for C_b, as shown below:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \tag{2}$$

It is important to realize that M_{max}, M_A, M_B and M_C are absolute values (always positive) of the maximum, quarter-, mid- and three-quarter point moments along an unbraced segment, respectively.

One advantage of Equation 2 is that it accounts for the maximum bending moment as well as the quarter-point interval values along the moment diagram. Hence, this particular equation can be used with any moment diagram, regardless if it is straight-line or nonlinear. The second advantage is that Equation 2 does not require an associated upper bound since it naturally limits itself to an upper bound value of 2.27 for straight-line moment diagrams. Although there are an infinite number of possible nonlinear moment diagrams, the upper bound of this expression for nonlinear moment diagrams, in most cases, does not significantly exceed the value of 3.0. One example is the moment diagram for a fixed-fixed end-supported beam with a uniformly distributed load, which yields a C_b value of 2.38 from Equation 2.

The *LRFD Specification* Commentary Figure C-F1.3 (Figure 1) compares the ASD and LRFD C_b expressions for straight-line moment diagrams. It can be seen from this figure that Equation 2 changes the trend and becomes nearly straight from $\pm 0.33 \le M_1/M_2 \le \pm 1.0$. This sudden change in trend occurs when either M_A or M_C is zero at the point $M_1/M_2 = 0.33$. The use of absolute values creates this near linearity in Equation 2.

Under certain circumstances and for some structural steel shapes, the C_b coefficient is not significant because the member reaches its ultimate plastic bending capacity before it reaches its nominal flexural strength. The following must always be checked:

$$C_b M_n \le M_p \tag{3}$$

If C_b is already incorporated within the M_n expression, as is the case for most flexural expressions in the AISC *LRFD Specification*, then M_n must be less than or equal to M_p. The lower bound for C_b is unity, which does not result in any increase in the nominal flexural strength of the member. This is the case where the end moments of the member are equal (straight-line moment diagrams) or when the designer wants to assume $C_b = 1.0$ for additional conservatism, even though the moment diagram may be nonlinear.

It should be noted that there is nothing wrong with assuming $C_b = 1.0$ since doing so simplifies the calculation and also assumes the worst-case scenario for the bending moment diagram. It is conservative, but as a result, can lead to less economical designs. In addition, because Equations 1 and 2 are completely independent of design methodology, either expression can be used interchangeably in ASD and LRFD flexural calculations. However, Equation 1 is only applicable to straight-line moment diagrams while Equation 2 can be applied to any moment diagram.

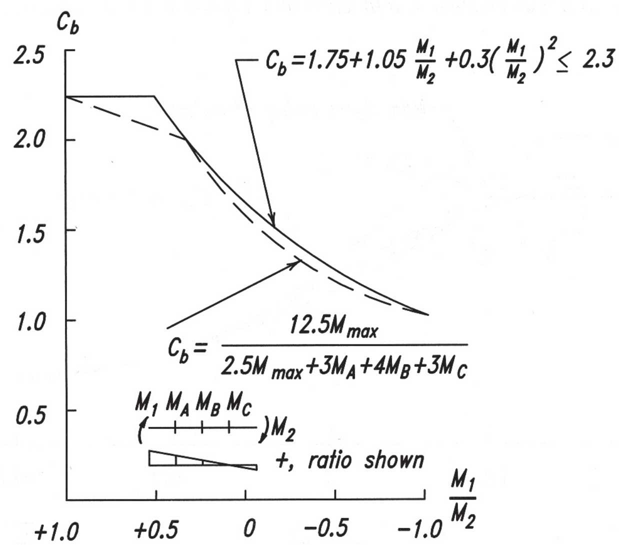


Fig. 1. Various C_b *equations.*

CANTILEVERED BEAMS

If the internal moment anywhere along a beam exceeds the magnitude of its maximum end moment, a C_b value of 1.0 can be used according to Section F1.3 of the 1989 *ASD Specification*. In Section F1.2a of the 1999 *LRFD Specification*, the coefficient C_b is taken as 1.0 for cantilevers and overhangs where the free end is unbraced. Expressions for C_b when evaluated for a cantilevered beam can lead to a value of approximately 2.0, depending on loading conditions, and one may be inclined to increase the moment capacity of the member by an equal amount. This is not only unconservative, but incorrect. Analogous to a flagpole under axial compressive load where $K = 2.0$, the effective unbraced length is twice the actual length. These two factors cancel each other since C_b would increase the moment capacity and K would decrease it. A conservative approach for the nominal flexural strength for a cantilever uses the actual length and a C_b coefficient of unity. For cases of restraint to the compression and/or tension flanges at the free end of the cantilever, refer to the SSRC *Guide to Stability Design Criteria for Metal Structures* (Galambos, 1998).

FRAME MEMBERS

The concept of C_b is also incorporated into the design of frames. Chapter C of the 1999 *LRFD Specification* discusses second-order analysis of frames. The required flexural strength given by Equation C1-1 is $M_u = B_1M_{nt} + B_2M_{lt}$. M_{nt} is the required flexural strength in the member assuming no lateral translation of the frame while M_{lt} is the required flexural strength resulting from the lateral translation alone. The load on the beam or column is partly due to the applied load and partly due to the load from lateral translation. Although these maximums do not always occur at the same point along the beam or column, Equation C1-1 is a valid approximation of the actual load on the member. When designing a frame, one needs to calculate the coefficient C_b based on the combined moment diagram from both factors. Once the required flexural strength and the actual flexural strength of the beam or column are determined, one can use the beam-column interaction equations from Chapter H of the 1999 *LRFD Specification*.

HISTORICAL DEVELOPMENT OF C_b

For more than fifty years, engineers have recognized that beam end conditions can affect nominal flexural strength. In 1956, Mario Salvadori developed one of the first relationships between beam moment diagrams and nominal flexural strength. Salvadori discovered that the value of the coefficient C_b also depends on the warping properties of individual sections as shown in Figure 2. For deeper beams with short spans, the value of C_b will be toward the top end of the area between the two curves which represents cases where warping is significant. However, where warping is not a problem (in other words, shallow beams with long spans), values for C_b will reside closer to the bottom curve.

Over the years, engineers have tried to derive mathematical equations to fit the curves developed by Salvadori. To be as accurate as possible without being unduly unconservative, most of these equations have an associated upper bound of 2.3 or 2.5. Various methods have been used to match the results of Salvadori's work; some use inverse functions, some use quadratics, and others take less traditional forms. In 1979, Kirby and Nethercot (Kirby and Nethercot, 1979) published an equation similar to Equation 2 that was self-limiting, thus eliminating the need to define an upper bound limit. This equation was the basis of Equation 2 and the first to apply to nonlinear moment diagrams.

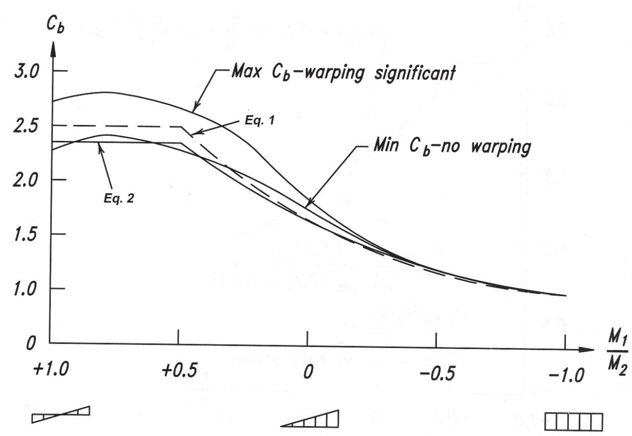


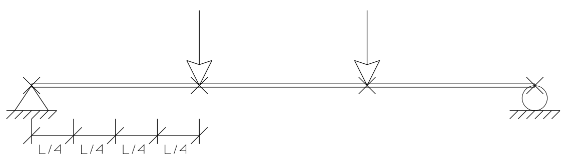
Fig. 2. Effect of warping properties on C_b.

DESIGN EXAMPLES

Example 1:

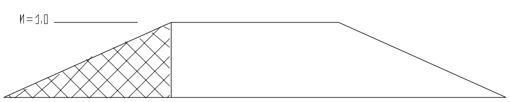
Determine C_b for each of the beam spans shown when lateral bracing is located at third-points.

Left segment value using Equation 1:



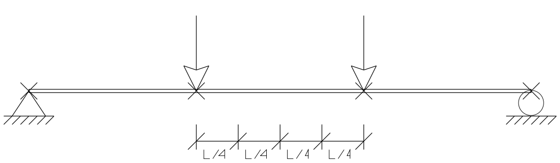
$$C_b = 1.75 + 1.05\frac{M_1}{M_2} + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3$$
$$= 1.75 + 1.05(0) + 0.3(0)^2$$
$$= 1.75$$

Left segment value using Equation 2:



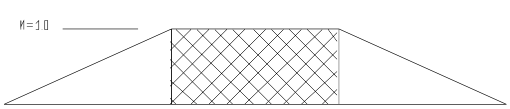
$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$
$$= \frac{12.5(1)}{2.5(1) + 3(0.25) + 4(0.5) + 3(0.75)}$$
$$= \frac{12.5}{7.5} = 1.67$$

Center segment value using Equation 1:



$$C_b = 1.75 + 1.05\frac{M_1}{M_2} + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3$$
$$= 1.75 + 1.05(-1) + 0.3(-1)^2 = 1.0$$

Center segment value using Equation 2:



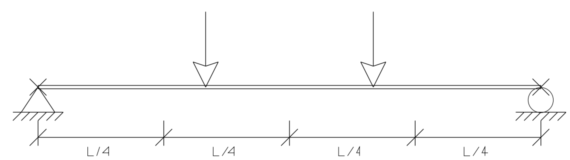
$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$
$$= \frac{12.5(1)}{2.5(1) + 3(1) + 4(1) + 3(1)} = 1.0$$

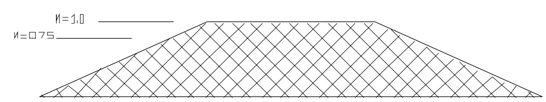
In this example, the strength of the beam is limited by the center segment where C_b is equal to unity. Note that either Equation 1 or 2 can be used since the moment diagram consists of three straight-line segments.

Example 2:

Determine the C_b value for the unbraced beam shown below.

Considering that the entire unbraced span contains a nonlinear moment diagram, use Equation 2:





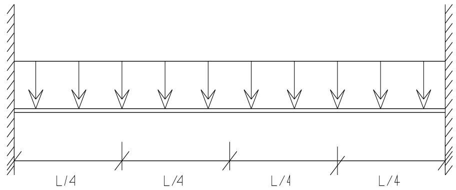
$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$
$$= \frac{12.5(1)}{2.5(1) + 3(0.75) + 4(1) + 3(0.75)} = 1.14$$

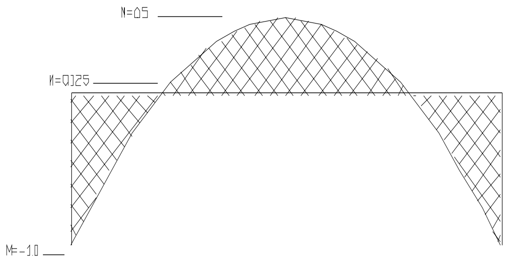
It is important to note that even though the unbraced case in Example 2 has a larger C_b value than that in Example 1, the braced case in Example 1 will still have a larger nominal flexural capacity. The greater capacity for Example 1 results from the fact that the unbraced length in Example 1 is only one-third of the unbraced length of Example 2. The longer unbraced length in Example 2 greatly increases the detrimental effects of lateral-torsional buckling and therefore reduces the nominal flexural strength of the member. In other words, even though $C_{b1} < C_{b2}$, the determination of nominal flexural strength for Examples 1 and 2 results in $C_{b1}M_{n1} > C_{b2}M_{n2}$ because $M_{n1} >> M_{n2}$.

Example 3:

Determine C_b for the unbraced beam span below.

The moment diagram is nonlinear, therefore use Equation 2

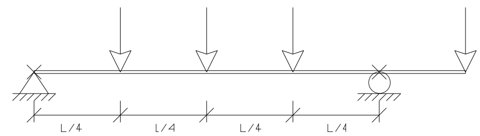


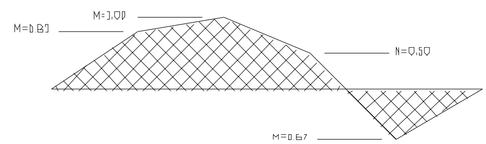


$$
\begin{aligned}
C_b &= \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \\
&= \frac{12.5(1)}{2.5(1) + 3(0.125) + 4(0.5) + 3(0.125)} \\
&= 2.38
\end{aligned}
$$

Example 4:

This example shows a girder with floor beams framing into it. The girder contains a cantilevered span, which supports a floor beam at the end. Assuming the girder is braced at both supports, there are two unbraced spans. The C_b coefficient for the cantilevered span should be taken as 1.0. The C_b coefficient for the main span is calculated using Equation 2:





$$
\begin{aligned}
C_b &= \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \\
&= \frac{12.5(1)}{2.5(1) + 3(0.83) + 4(1) + 3(0.5)} \\
&= 1.19
\end{aligned}
$$

CONCLUSIONS

This paper discussed some fundamental, yet not well-understood aspects of the C_b coefficient. Engineers should be aware of these concepts and limitations. It is often advantageous to consider the C_b coefficient since it allows for more economical and efficient sizing of a flexural member. It is particularly applicable to long unbraced spans in flexural members that cannot reach their plastic moment strength, M_p. Another important fact is that Equations 1 and 2 may be used with either LRFD or ASD design, as they are independent of design methodology. Equation 2 is applicable to all moment diagrams whereas Equation 1 is applicable only to spans having straight-line moment diagrams. In LRFD, it is always necessary to check that the calculated nominal flexural strength of the member, after applying C_b, is no greater than the member's plastic moment strength, M_p. The same general concept applies to ASD.

ACKNOWLEDGMENTS

The authors wish to thank Donald Sherman and Charles Carter for their thoughts and input. Their contributions and comments led the authors to publish this paper.

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