

Fabrication Aids for Cold Straightening I-Girders

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Fabricated steel girders sometimes fail to meet permissible tolerances (AISC, 2001), and it is necessary to rectify the out-of-straightness (Figure 1). Two methods are used: (1) heat straightening and (2) cold straightening. In heat straightening, localized heat is applied in accordance with established procedures (Avent and Mukai, 2001). In cold straightening, rectification is achieved by applying loads. No guidance is available and success is dependent on the skill and knowledge of the fabricator. This guesswork may be minimized if the relationship between residual deformation and load were available.

This paper presents closed-form solutions for cold straightening for the general case where a single concentrated load is used to rectify permanent out-of-straightness in a symmetrical or unsymmetrical I-girder (Figure 2). Maximum limits on loads, deformations and load frame set-up are then established using prevailing LRFD criteria.

To facilitate the determination of loads required for straightening purposes, fabrication aids in the form of charts were developed for steel grades commonly used, in other words, Grade 250 ($F_y = 36$ ksi) and Grade 345 ($F_y = 50$ ksi). A step-by-step procedure is outlined and its application illustrated by three numerical examples.

COLD STRAIGHTENING SET-UP

The analysis presented is based on an actual system developed by a fabricator shown in Figure 2. In the set-up, the corrective load is applied to a portion of the girder using a mobile, self-straining frame that can be positioned anywhere along the girder so that it coincides with the location of the maximum out-of-straightness (Figure 3). The spacing of the two longitudinal arms of the loading frame, S , (usually smaller than the bent segment length L), varies between

1.5 m and 3.6 m (5 ft to 12 ft) depending on the girder size and out-of-straightness. Larger spacing is used for larger sized girders or more severe out-of-straightness. The hydraulic jacks (Figure 2) are unequal to allow straightening unsymmetrical (smaller size top flange compared to the bottom flange) or hybrid (smaller steel grade top flange compared to bottom flange) girders. Their capacity should be within practical limits, for example, 450 metric tons (500 tons) in the set-up shown in Figure 2.

During straightening, the girder may be placed in a vertical (Figure 4a—strong axis bending) or horizontal (Figure 4b—weak axis bending) position. The latter configuration necessitates consideration of selfweight (w) stresses within span S and the overhanging portion ($L-S$) (Figure 3a). While the vertical strong axis position is preferred by steel fabricators, both cases are considered in the analysis.

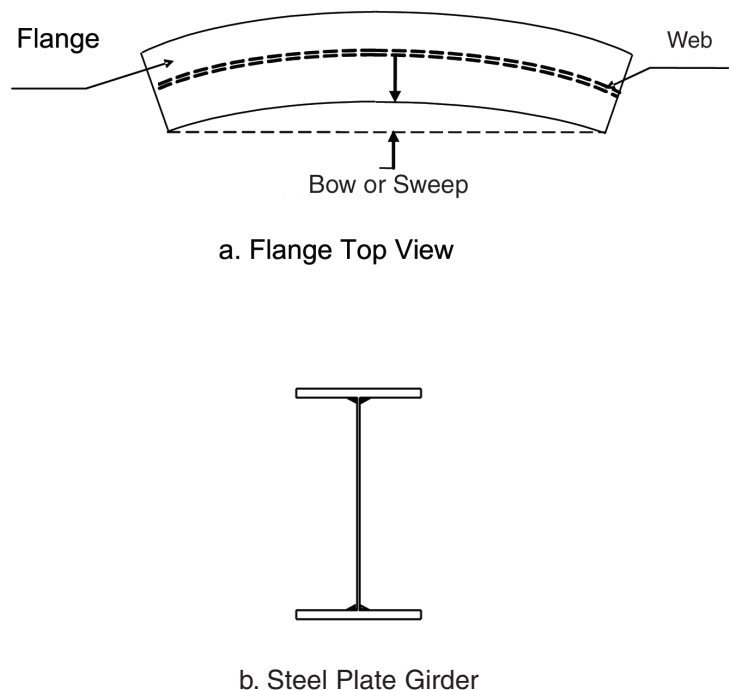


Fig. 1. Fabrication Out-of-Straightness in a Steel Girder.

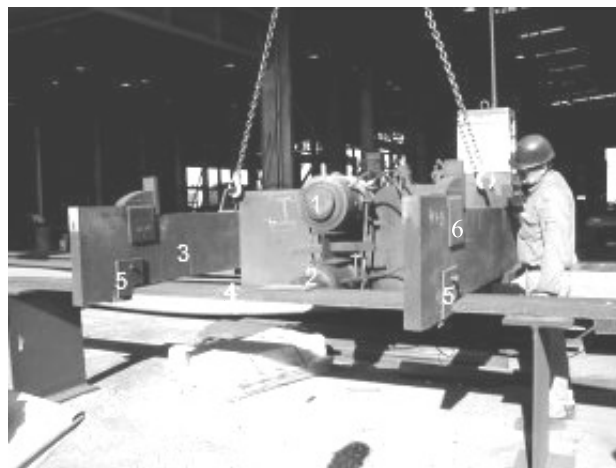
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ASSUMPTIONS

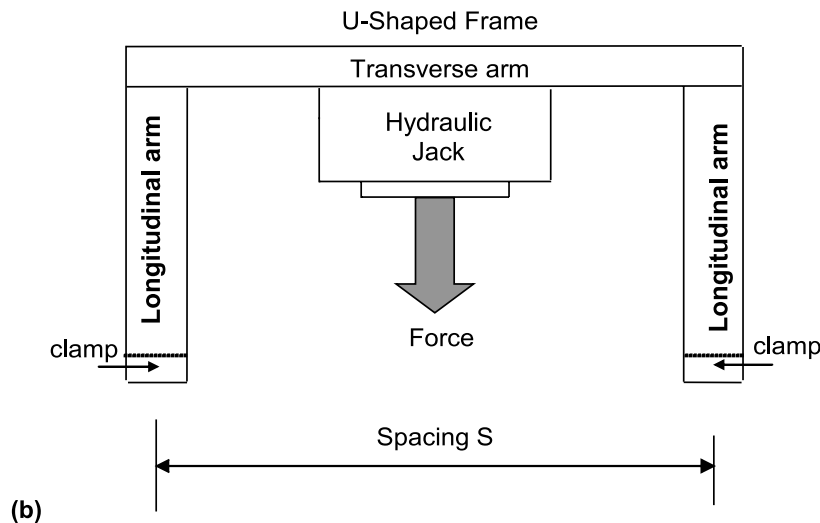
The analysis presented is based on the following assumptions:

1. Steel is elastic-perfectly plastic. The cumulative strain due to multiple load applications is kept below the strain-hardening limit so that Bauschinger effects can be neglected.
2. The girder is unstiffened and the proportions meet requirements for compact sections (AISC, 2001; AASHTO, 1996).
3. The point of maximum bow or sweep subjected to inelastic concentrated loads is idealized as a plastic hinge (Figure 3b). This allows free rotation of the bent segment at the load frame arms and idealization of the deformed shape as straight lines.
4. The resultant of the applied loads lies in the plane of symmetry of the girder.
5. Deformations are small compared to the length of the girder.
6. Loads are applied in incremental steps but the effect of residual stresses that build-up after each step is disregarded. Finite element investigations showed that the effect of residual stresses decreases deformations by about 15 percent (Gergess, 2001).



1. Hydraulic jack (bot. flange)
2. Hydraulic jack (top flange)
3. Longitudinal arms
4. Steel plate
5. Clamps (top flange)
6. Clamps (bot. flange)

(a)



(b)

Fig. 2. Cold Bending Fixture: (a) Equipment; (b) Schematic Plan View (Photo: Courtesy Tampa Steel Erecting Company).

7. For weak axis bending, consideration of selfweight in the closed form solution necessitates that the overhangs, $a = (L-S)/2$, (Figure 3a) be equal. While this is restrictive for asymmetrically located out-of-straightness (refer to Example 3), it can be by-passed by positioning the girder vertically (Figure 4a), for example, ignore the self-weight.

FORMULATION

The I-section to be straightened is of length (L), flange width ($2c$), flange thickness (t_f), web depth (d) and web thickness (t_w) (Figure 5a). During straightening, the girder is simply supported by the U-shaped steel frame's longitudinal arms (Figure 3a). Stability is ensured by anchoring the bottom flanges at the girder's ends to a rigid platform that

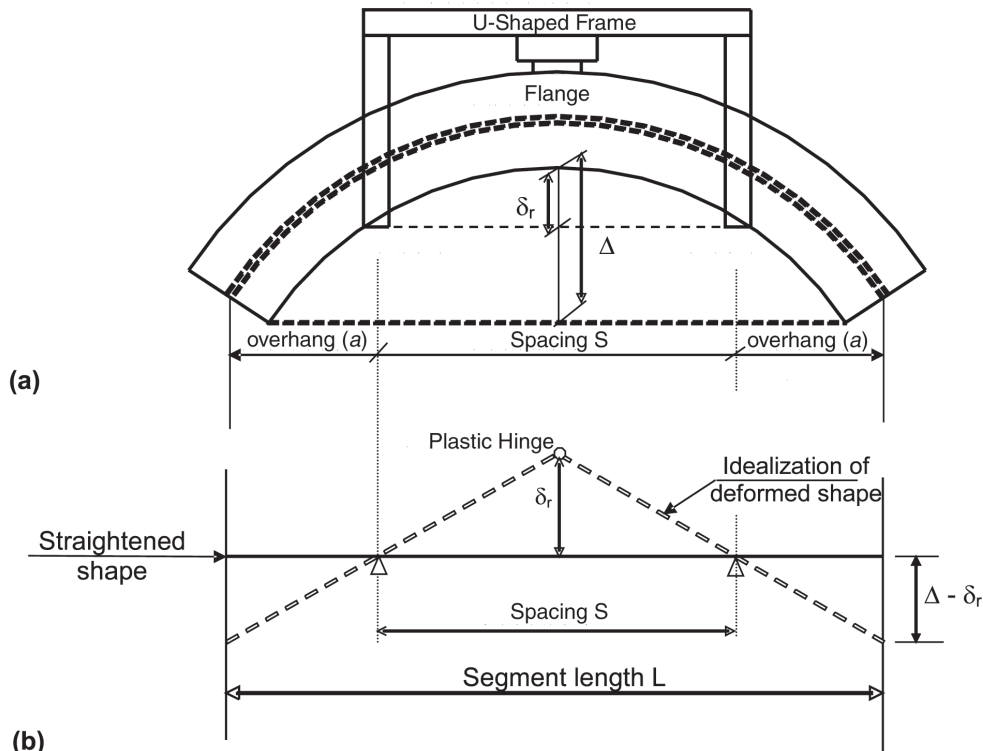


Fig. 3. Cold Bending Load Set-up: (a) Configuration; (b) Idealization.

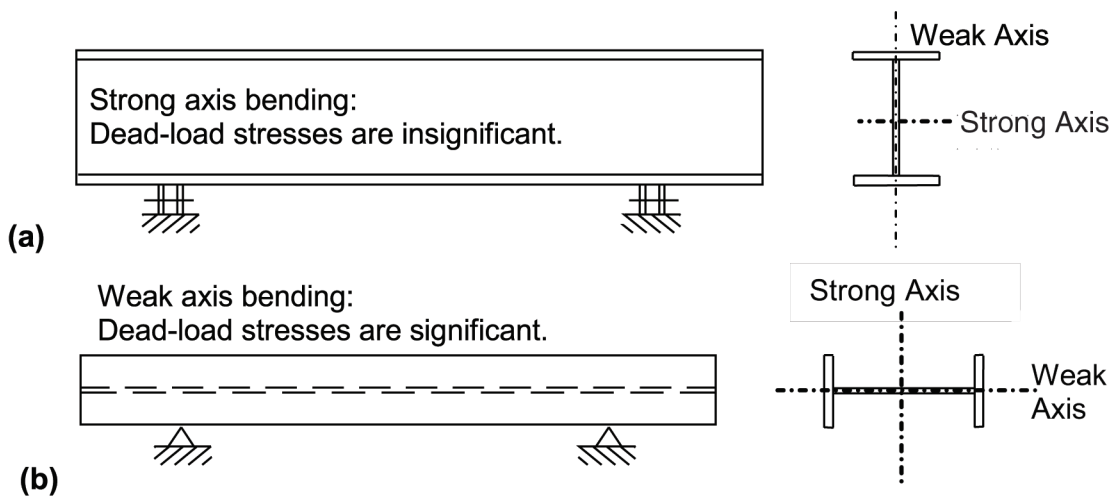


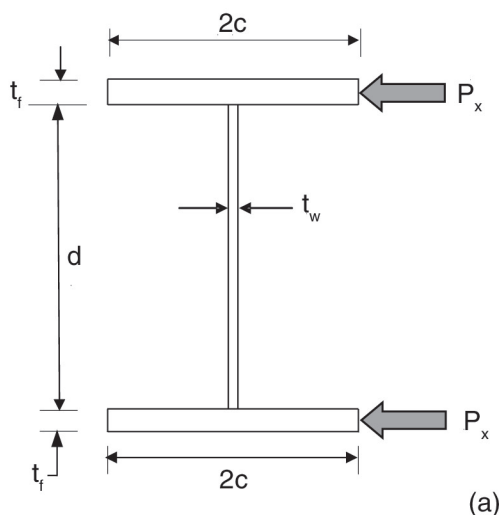
Fig. 4. Girder Position during Bending: (a) Vertical; (b) Horizontal (Brockenbrough, 1972).

only prevents vertical movement (Figure 5b). The load frame span S is selected based on lateral buckling requirements for compact sections (refer to Application section), therefore $S < L$ (Figure 3a).

The load frame is positioned so that its centerline coincides with the maximum out-of-straightness (Δ) location, idealized as a plastic hinge (Figure 3b). The corresponding offset intersected by the arms of the loading frame is designated δ_r . The bending load P_x applied through the loading frame ensures nullification of δ_r . At the same time, free rotation of the overhanging portions $a = (L-S)/2$ idealized as straight lines (Figure 3b), ensures full straightening of the bent segment of length L . Thus, the main focus of the analysis is to determine the load frame configuration required to rectify the offset δ_r .

In symmetrical girders, two equal loads of magnitude (P_x) are applied to the top and bottom flanges. In unsymmetrical girders, the loads applied to the top (P_{xtf}) and bottom (P_{xbf}) flanges are different. For this case, the analysis is performed twice, based on (1) top flange size and/or steel grade, (2) bottom flange size and/or steel grade to determine the respective straightening loads that are separately applied to the flanges.

External moments M_x (refer to Equation in Appendix 1) develop due to the effect of applied load ($2P_x$ /girder) and the uniformly distributed selfweight (w). A plastic zone initiates over portions of the flange (Figure 6a) in which the bending stress (σ_x) reaches yield stress (F_y). This zone is identified by dimension $2(c-y_y)$ in Figure 6a. When the loads are removed, moments reduce to zero and in an elastic-perfectly plastic material, the stress and strain reduction is represented by a straight line (Brockenbrough and Meritt, 1999). Residual stresses (Figure 6c) and deformations are then determined.



RESIDUAL DEFORMATION

The application of post-yield load results in a plastic deformation δ at $S/2$. This is calculated by successive integration of curvatures (Popov, 1976). In the elastic zone (Figure 6a), M_x/EI_x approximates the curvature while in the inelastic zone it is determined from strain variation. Detailed derivations may be found in Gergess (2001).

Equations for plastic deformation δ and residual deformation δ_r (determined by subtracting the elastic rebound due to removal of the load), written in terms of coefficient I_x , x_2 , m , r , s , p , n , and u (listed in Appendix 1) are as follows:

$$\delta = r \left[m - x_2 + \frac{s}{2} - p + \frac{s}{2} \ln \left[\frac{m}{n - \frac{P_x}{w}} \right] \right] + u \left[-\frac{2}{3} \frac{s}{S} + \frac{x_2}{2S} \right] x_2^3 + \frac{a^2 x_2^2 u}{S} \quad (1)$$

$$\delta_r = \delta + \frac{wS^2}{48EI_x} \left[\frac{5S^2}{8} - 3a^2 \right] + \frac{P_x S^3}{24EI_x} \quad (2)$$

Note that in Equations 1 and 2, section properties (for example, I_x and w) include the top and bottom flange properties, while P_x is the load per flange.

CRACK PREVENTION

Concerns regarding “cracking” and “fracture” have limited the application of cold bending. These adverse effects are of primary concern when steel becomes strain-hardened, in



Fig. 5. Structural Steel I-Girder: (a) Cross-Section; (b) End Supports.

other words, strains are 15 to 20 times the maximum elastic strain (Salmon and Johnson, 1996) as ductility is reduced. In the derivations, steel was assumed to be elastic perfectly plastic, in other words, strain hardening was ignored. The load and resistance factor design (LRFD) method also limits strains to the plastic range, in other words, the yield plateau where the average of strain limits for the plastic range is set at $10\epsilon_y$ (McCormac, 1994). Consequently, limits can be established for curvatures and offsets that are consistent with prevailing LRFD standards.

Curvature Limits

For the two grades of steel considered, $(10\epsilon_y)$ is 0.0124 for Grade 250, and 0.0172 for Grade 345 steel. Strain recovery following load removal in a fully-plastic material is $1.5\epsilon_y$ [Gergess (2001), the fully plastic moment per flange is $M_p = F_y t_f c^2$ and the corresponding flexural stress at the tip of the flange is $M_p \times c / (2t_f c^3/3) = 1.5F_y$], resulting in a residual strain of $10\epsilon_y - 1.5\epsilon_y = 8.5\epsilon_y$. Residual strains are then used to calculate the maximum curvature ($1/R$) given by Equation 3 as:

$$\frac{1}{R} = \frac{\epsilon}{y} \quad (3)$$

For Grade 250, $\epsilon = 8.5 \times 0.00124 = 0.0106$, then $1/R$ at the flange tip ($y = c$) is:

$$\frac{1}{R} = \frac{0.0106}{c} \quad (3a)$$

For Grade 345, $\epsilon = 8.5 \times 0.00172 = 0.0147$, then $1/R$ at the flange tip ($y = c$) is:

$$\frac{1}{R} = \frac{0.0147}{c} \quad (3b)$$

Offset Limits

Numerical limits on offsets (Figure 3) may be determined for the special case where the bending fixture spacing (S) is set equal to the braced length for compact sections L_p (Equation 4) to prevent lateral buckling of the flange (AISC, 2001). The effect of the web is neglected since its stiffness is small compared to that of the flange:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \quad (4)$$

where

$$r_y = \sqrt{\frac{I_{cg}}{A}}$$

$$I_{cg} = \frac{2t_f c^3}{3}$$

$$A = 2ct_f$$

$$\text{Then } r_y = \sqrt{\frac{\frac{2t_f c^3}{3}}{2ct_f}} = 0.58c \text{ and}$$

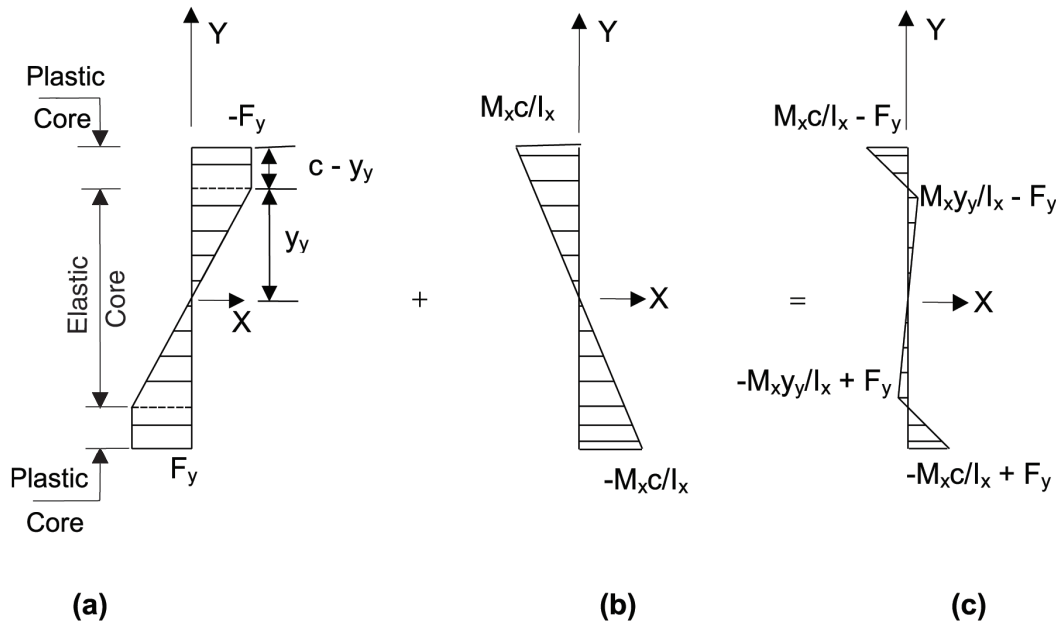


Fig. 6. Stress Distributions: (a) Loading; (b) Unloading; (c) Residual.

Table 1. Limiting Minimum Radii and Maximum Out-of-Straightness

Item	Grade 250 ($F_y = 36$ ksi)	Grade 345 ($F_y = 50$ ksi)
Residual Strain $\epsilon_r \approx 8.5\epsilon_y$	0.0106	0.0147
Minimum Radius $\frac{1}{R} = \frac{\epsilon_r}{y}$ Eq. 3	$\frac{1}{R} = \frac{0.0106}{c}$ Eq. 3a	$\frac{1}{R} = \frac{0.0147}{c}$ Eq. 3b
Maximum Frame Spacing, S $L_p = 1.76r_y \sqrt{\frac{E}{F_y}}$ Eq. 4	$L_p = 28.8c$ Eq. 4a	$L_p = 24.5c$ Eq. 4b
Maximum out-of-straightness δ_r	$\delta_r = \frac{(28.8c)^2}{8\left(\frac{c}{0.0106}\right)} = 1.11c$ Eq. 5a	$\delta_r = \frac{(24.5c)^2}{8\left(\frac{c}{0.0147}\right)} = 1.11c$ Eq. 5b

$$L_p = 1.76 \times 0.58c \sqrt{\frac{200000 \text{ MPa}}{250 \text{ MPa}}} = 28.8c \text{ for Grade 250} \quad (4a)$$

$$L_p = 1.76 \times 0.58c \sqrt{\frac{200000 \text{ MPa}}{345 \text{ MPa}}} = 24.5c \text{ for Grade 345} \quad (4b)$$

Approximating the offset δ_r as $L_p^2/8R$ (Brockenbrough, 1970), the maximum permissible offsets are obtained by substituting $1/R$ and L_p values from Equations 3a, 3b and 4a, 4b respectively in this expression leading to:

$$\delta_r = \frac{(28.8c)^2}{8\left(\frac{c}{0.0106}\right)} = 1.11c \text{ for Grade 250} \quad (5a)$$

$$\delta_r = \frac{(24.5c)^2}{8\left(\frac{c}{0.0147}\right)} = 1.11c \text{ for Grade 345} \quad (5b)$$

Note that δ_r is the same for Grade 250 and 345 steel. The calculated limits apply for the special case of symmetric, homogeneous steel girders that are prevalent in building but not in bridge construction. For unsymmetrical, non-homogenous girders, the same process may be adopted by analyzing the top and bottom flanges separately (see Example 2). Should out-of-straightness be asymmetric (for example, not located at midspan), straightening should be carried out

with the girder placed in a vertical position (Figure 4a) so that selfweight can be neglected thereby avoiding complicated analyses due to the unsymmetrical overhang configuration (Example 3).

Equations 3 through 5, give $L_p = 6.6$ m (21.6 ft) for the smallest W100×19 (W4×13) section and $L_p = 5.6$ m (18.4ft) for the largest W920×1188 (W36×798) section [data on rolled shapes is based on AISC (2001)]. Table 1 summarizes the recommended limits given by Equations 3 through 5. Note that they are applicable for US and SI units.

DESIGN CHARTS

Equations 1 and 2 determine a load P_x /flange that induces a deformation equal and opposite to offset δ_r . However, its application requires time-consuming trial and error procedures because of dependence of the residual deformation on the unknown load. Analysis showed that the ratio of δ_r to (S^2/c) and P_x to $(t_f c^2/S)$ were interrelated. This relationship was used to construct design charts to reduce computational effort.

Using defined coefficients $\alpha = (a/S)$, $\beta = (P_x/wS)$, and $\gamma = [P_x/(t_f c^2/S)]$, generalized Equations 1 and 2 are recast as (refer to Appendix 1 for coefficients $x'_2, m', n', p', r', s',$ and u'):

$$\frac{\delta}{S^2} = r' \left[m' - x'_2 + \frac{s'}{2} \ln \left[\frac{m'}{n' - \beta} \right] \right] + u' \left[-\frac{2}{3} s' + \frac{x'_2}{2} \right] x'_2{}^3 + \alpha^2 x'_2{}^2 u' \quad (6)$$

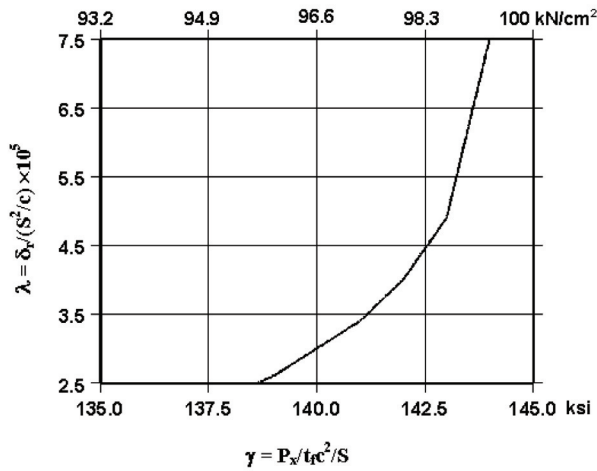


CHART 1: Grade 250 Steel - $\alpha = 0.5$

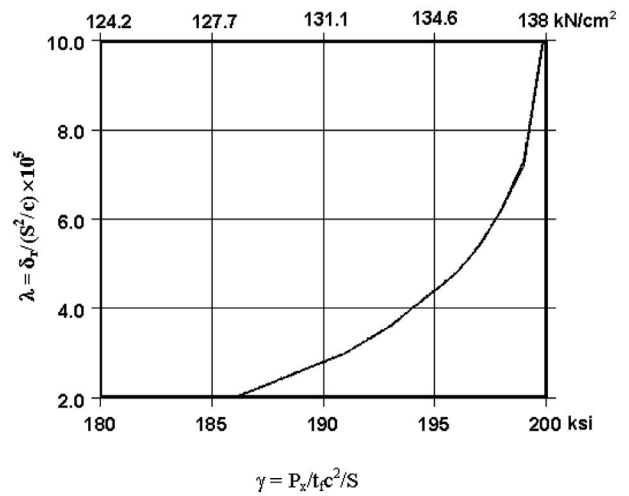


CHART 2: Grade 345 Steel - $\alpha = 0.5$

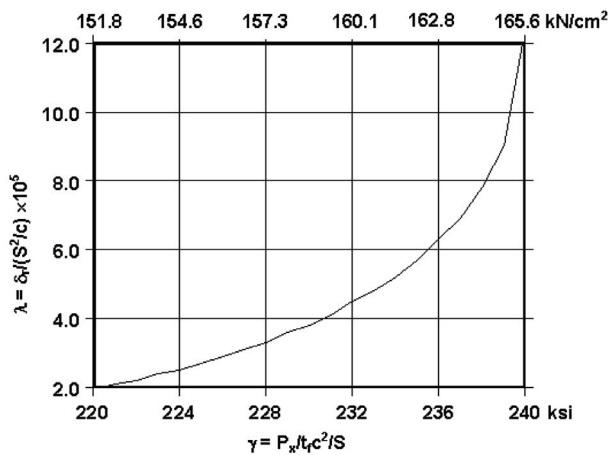


CHART 3: $F_y = 414 \text{ MPa} - \alpha = 0.5$

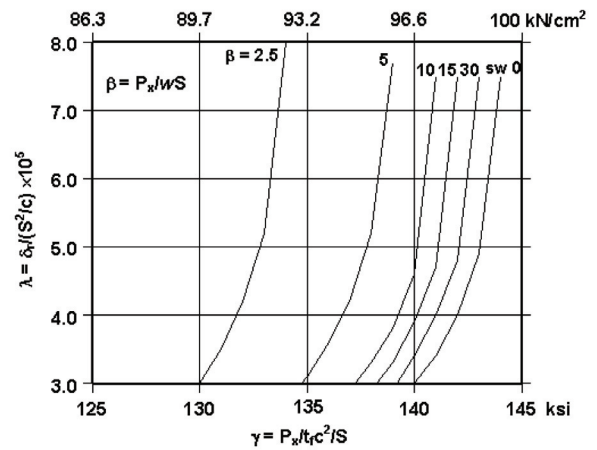


CHART 4: Grade 250 - $\alpha = 0.25$

$$\frac{\delta_r}{S^2/c} = \frac{\delta}{S^2/c} + \frac{\gamma}{32E} + \frac{\gamma}{64E\beta} \left(\frac{5}{8} - 3\alpha^2 \right) \quad (7)$$

Design charts relating $\lambda = [\delta_r / (S^2/c)] \times 10^5$ to $\gamma = [P_x / (t_f c^2 / S)]$ were developed from Equation 7, as functions of F_y and coefficients β and α .

When self weight stresses are excluded (β and α not considered, Figure 4a), Charts 1 and 2 are generated for Grade 250 and 345 steel. As the actual yield is usually higher than the specified value (Brockenbrough, 2001), an additional chart is provided for $F_y = 414 \text{ MPa}$ (60 ksi) (Chart 3) to allow interpolation.

When selfweight stresses are included (Figure 4b), Charts 4 through 9 (Appendix 2) are generated. These

incorporate α and β . α values selected were 0 (no overhangs, $S = L$), 0.25 ($S = 2/3 L$) and 0.5 ($S = 1/2 L$). For $\alpha = 0.5$, Charts 1 through 3 may also be used, in other words, one curve fits various β values. This makes sense since the self-weight effects within span $S = L/2$ are nullified by the overhanging portions ($2a = L/2$). For $\alpha = 0.25$, Charts 4 through 6 were developed for Grade 250, 345 and 414 steel respectively. For $\alpha = 0$, Charts 7 through 9 were developed for Grade 250, 345 and 414 steel respectively. This leads to three charts for each steel grade (Charts 1, 4, 7 for Grade 250, Charts 2, 5, 8 for Grade 345 and Charts 3, 6, 9 for Grade 414), in other words, a total of nine charts. Note that Charts 4 through 9 contain six curves each, corresponding to specific ratios of the applied load to selfweight $\beta = (P_x / w S)$. A curve (*sw 0*) that corresponds to strong axis bending (Figure 4a) is also presented where it is used for

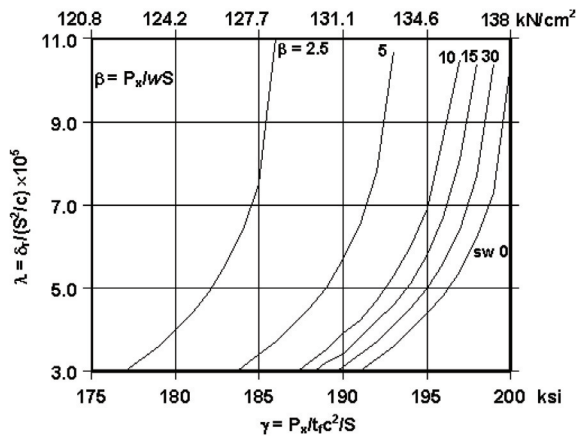


CHART 5: Grade 345 Steel - $\alpha = 0.25$

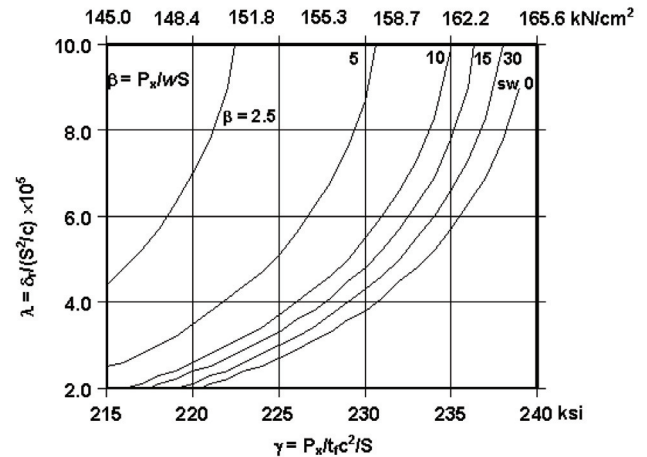


CHART 6: $F_y = 414 \text{ MPa} - \alpha = .25$

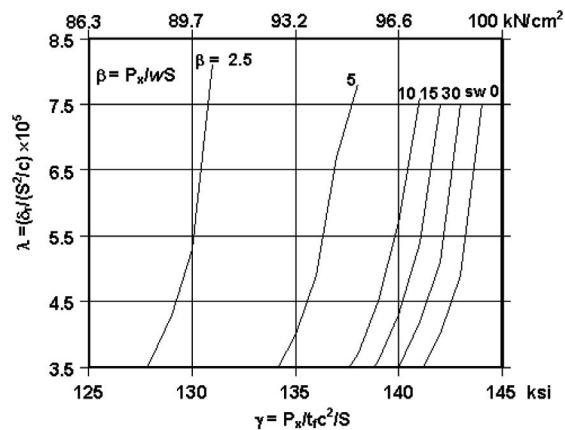


CHART 7: Grade 250 - $\alpha = 0$

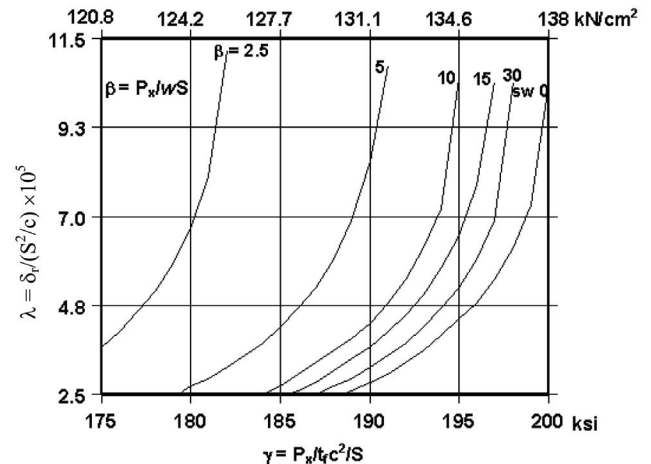


CHART 8: Grade 345 Steel - $\alpha = 0$

$\beta > 30$ (this is the same curve from Charts 1 through 3). Other curves correspond to β varying from 2.5 ($\beta \leq 2.5$ were impractical since they require very large loads) to 30. The curves are bunched together so only intermediate ($\beta = 5, 10, \text{ and } 15$) values are shown.

Derived equations are bounded by the full-plastic zone $M_p = F_y t_f c^2$ leading to a maximum load (P_{max}) per flange, determined as follows (Gergess and Sen, 2001):

$$P_{max} = \frac{4c^2 t_f F_y}{S} + \frac{wa^2}{S} - \frac{wS}{4} \quad (8)$$

In terms of F_y , λ , γ , β and α coefficients, Equation 8 is recast as:

$$\gamma_{max} = \frac{4F_y}{\left(1 - \frac{\alpha^2}{\beta} + \frac{1}{4\beta}\right)} \quad (9)$$

γ_{max} values are obtained from Equation 9 for various steel grades and replaced in Equation 7 to obtain the maximum values (λ_{max}) on the ordinate axis of the charts. Charts 4 through 9 show that for $\beta > 10$, selfweight effects reduce significantly and the corresponding curves are closer.

Results obtained using the design charts showed excellent agreement with those using the complete equations (Gergess and Sen, 2001). Good agreement was also obtained with solutions from a three-dimensional finite element analysis incorporating non-linearity and initial residual stresses. The design charts results were within 10 percent of the finite element solution.

APPLICATION

The concentrated straightening load per flange P_x is calculated using design charts. In symmetrical girders, the load is the same for both flanges. In unsymmetrical girders, the

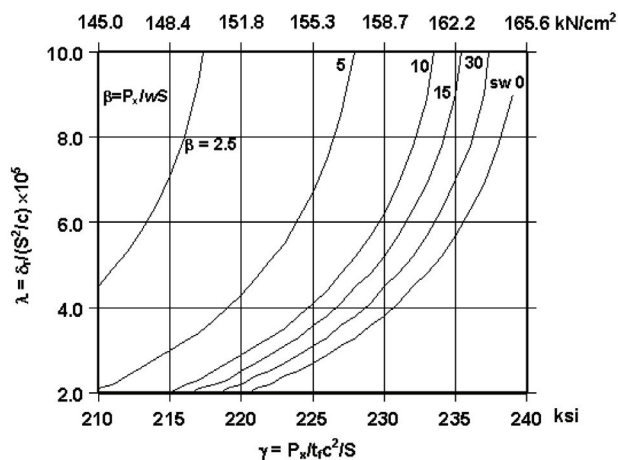


CHART 9: $F_y = 414 \text{ MPa} - \alpha = 0$

load for the top flange, $P_{x_{tf}}$, differs from that for the bottom flange, $P_{x_{bf}}$.

The load frame is positioned so that P_x coincides with Δ (Figure 3). Δ is measured with respect to the bent segment (span L), while the offset δ_r , nullified by P_x is the chord intersected by the loading frame. δ_r is obtained from similar triangles as follows:

$$\delta_r = \frac{\Delta}{\left(\frac{2a}{S} + 1\right)} = \frac{\Delta}{\left(\frac{L}{S}\right)} \quad (10)$$

The girder should preferably be positioned vertically (Figure 4a) so that selfweight is ignored. However, design charts are also included for the case where the girder is placed horizontally (Figure 4b) that necessitates consideration of selfweight (Appendix 2).

The steps required to determine the rectification load are described as follows:

Step 1: Set Parameters and frame spacing, S

c : half the flange width, t_f : flange thickness

L : length of bent segment

Δ : out-of-straightness relative to the bent segment of length L (Figure 3a)

Based on lateral buckling limits for compact sections, select S (Equation 4):

$S \leq 28.8c$ (Grade 250 Steel)

$S \leq 24.5c$ (Grade 345 Steel)

For unsymmetrical girders, use the smaller width (usually top flange c_{tf}).

Step 2: Check that calculated offset δ_r is within limits

From Equation 10: $\delta_r = \Delta / (L / S)$

For crack prevention, check that $\delta_r \leq 1.11c$

Use smaller c for unsymmetrical girders.

For $\delta_r > 1.11c$, the proposed method cannot be used for repair.

Step 3: Select chart

Use Chart 1 for Grade 250, Chart 2 for Grade 345 and Chart 3 for Grade 414 steel. Interpolate for intermediate yield stress values.

If the girder is placed horizontally, refer to Appendix 2.

Step 4: Calculate λ and number of passes η

$$\lambda = \left[\frac{\delta_r}{\frac{S^2}{c}} \right] \times 10^5$$

If $\lambda < \lambda_{max}$ (λ_{max} is the maximum value on the chart ordinate-axis), go to Step 5. Otherwise, multiple load applications or passes (η) are required in which the same load is applied and removed to satisfy limitations of elasto-plastic analysis (Chen and Han, 1988).

$\eta = \lambda / \lambda_{max}$, round-up to the nearest higher integer. Analysis is performed for one pass λ / η .

Step 5: Determine rectification load P_x

For λ (or λ / η) from Step 4, read $\gamma = [P_x / t_f c^2 / S]$ from the appropriate chart [units kN/cm² (ksi)].

Calculate $P_x = \gamma (t_f c^2 / S)$, [units kN (kips)].

In unsymmetrical girders, different loads $P_{x_{tf}}$ and $P_{x_{bf}}$ are calculated using the respective top flange (c_{tf} , t_{tf}) or bottom flange (c_{bf} , t_{bf}) dimensions.

NUMERICAL EXAMPLES

Three numerical examples illustrate the application of the charts. Example 1 compares the effect of higher yield strength on the rectification load for symmetric out-of-straightness in a wide flange section. Example 2 evaluates straightening of unsymmetrical girders. Example 3 illustrates asymmetric rectification in a symmetrical girder.

Example 1

Two 4.27 m (14 ft) long W610×153 (W24×103) girders (Grade 250 and Grade 345) did not meet fabrication tolerances with an out-of-straightness sweep of 80 mm ($3\frac{1}{8}$ in.) measured at the middle. Determine the load required to rectify this defect.

Solution

Several applications of the same load are usually required to correct the out-of straightness. For the load frame configuration selected (S , Step 1), offset δ_r and the maximum

allowable rectification are calculated in Step 2. If the offset δ_r is within maximum limits, the appropriate chart based on steel grade (Step 3) is selected. Otherwise, the proposed method cannot be used for repair. In step 4, chart parameter λ and number of passes η (if required), given by the ratio λ/λ_{max} (the maximum limit in a chart) are calculated. The load per pass P_x is then calculated in Step 5.

Step 1: Set parameters and frame spacing, S

$c = 114.3$ mm (4.5 in.), $t_f = 25.4$ mm (1 in.)
 $L = 4.27$ m (14 ft)
 $\Delta = 80$ mm ($3\frac{1}{8}$ in.)
 Select L based on lateral buckling requirements for compact sections:
 $S \leq 28.8c = 28.8 \times 114.3/1000 = 3.3$ m (10.8 ft) for Grade 250 steel.
 $S \leq 24.5c = 24.5 \times 114.3/1000 = 2.8$ m (9.2 ft) for Grade 345 steel.
 Use $S = 2.74$ m (9 ft) for straightening both girders. A 3 m (10 ft) spacing that leads to a smaller straightening load may be used for Grade 250.

Step 2: Check that calculated offset δ_r is within limits

$$\delta_r = \frac{\Delta}{\left(\frac{L}{S}\right)} = \frac{80}{\left(\frac{427}{274}\right)} = 51.33 \text{ mm (2 in.)}$$

Plastic limit: $\delta_r = 51.33 \text{ mm} \leq 1.11c = 1.11 \times 114.3 \text{ mm} = 127 \text{ mm (5 in.) OK}$

Step 3: Select chart

Use Chart 1 for Grade 250 and Chart 2 for Grade 345 steel.

Step 4: Calculate λ and number of passes η

$$\lambda = \left[\frac{\delta_r}{\frac{S^2}{c}} \right] \times 10^5 = \left[\frac{51.3 \text{ mm}}{\frac{(2740 \text{ mm})^2}{114.3 \text{ mm}}} \right] \times 10^5 = 78.1$$

Steps 4 and 5 are continued separately for Grade 250 and 345 steels.

Step 5: Determine rectification load P_x

Grade 250 $\lambda_{max} = 7.5$ in Chart 1. Since λ is $78.1 \gg 7.5$, multiple passes are needed. $\eta = \lambda/\lambda_{max} = 78.1/7.5 = 10.4$, round-up to the nearest higher integer, $\eta = 11$, each cycle corresponding to $\lambda/\eta = 78.1/11 = 7.1$.

For $\lambda/\eta = 7.1$ (Step 4), read $\gamma = 99.3 \text{ kN/cm}^2$ (144 ksi) from Chart 1.

For $t_f = 25.4$ cm, $c = 11.43$ cm, $S = 274$ cm, calculate:

$$P_x = \gamma \left(\frac{t_f c^2}{S} \right) = 99.3 \left(\frac{2.54 \times 11.43^2}{274} \right) = 120.3 \text{ kN (27 kips)}.$$

This load ($P_x = 120.3 \text{ kN (27 kips)}$) should be applied and removed eleven times to each flange to rectify the 80 mm ($3\frac{1}{8}$ in.) sweep.

Grade 345 $\lambda_{max} = 10$ in Chart 2. Since λ is $78.1 \gg 10$, multiple passes are needed.

$\eta = \lambda/\lambda_{max} = 78.1/10 = 7.81$, round-up to the nearest higher integer, $\eta = 8$, each cycle corresponding to $\lambda/\eta = 78.1/8 = 9.8$.

For $\lambda/\eta = 9.8$ read $\gamma = 137 \text{ kN/cm}^2$ (199 ksi) from Chart 2.

$$P_x = \gamma \left(\frac{t_f c^2}{S} \right) = 137 \left(\frac{2.54 \times 11.43^2}{274} \right) = 165.9 \text{ kN (37.3 kips)}.$$

This load/flange (39 percent larger than Grade 250 load) should be applied and removed eight times to rectify the 80 mm ($3\frac{1}{8}$ in.) sweep.

In summary, a bending fixture of spacing $S = 2.74$ m (9 ft) is used to rectify the out-of-straightness $\Delta = 80$ mm ($3\frac{1}{8}$ in.) in the Grade 250 and Grade 345 steel girders (both 4.27 m (14 ft) long). Although a larger S could be used for the Grade 250 girder, the loading frame set-up was kept the same for convenience. A load $P_x = 120.3 \text{ kN (27 kips)}$ must be applied eleven times to the top and bottom flanges, i.e. 22 load applications, for the Grade 250 girder. For the Grade 345 girder, a load $P_x = 167.1 \text{ kN (37.6 kips)}$ should be applied eight times to each flange, in other words, 16 load applications in total. Note that since the plastic load limit for Grade 250 ($\lambda_{max} = 7.5$) is smaller than that for Grade 345 ($\lambda_{max} = 10$), a larger number of passes is required ($\eta = 11$ compared to $\eta = 8$ for Grade 345).

Example 2

A 12.2 m (40 ft) long, Grade 345 ($F_y = 50$ ksi), unsymmetrical steel plate girder with a 30.5 cm $\times$ 2.54 cm (12 in. $\times$ 1 in.) top flange, a 61 cm $\times$ 5.08 cm (24 in. $\times$ 2 in.) bottom flange, and a 116.8 cm $\times$ 1.27 cm (46 in. $\times$ $\frac{1}{2}$ in.) web did not meet fabrication tolerances with an out-of-straightness of 300 mm ($11\frac{13}{16}$ in.) at mid-length. Determine the load configuration required for rectification.

Solution

For an unsymmetrical girder, the procedure is identical to that of a symmetrical girder excepting that separate analyses are required for the top and bottom flanges.

Step 1: Set parameters and frame spacing, S

$$c_{bf} = 305 \text{ mm (12 in.)}, t_{bf} = 50.8 \text{ mm (2 in.)}$$

$$c_{tf} = 152.5 \text{ mm (6 in.)}, t_{tf} = 25.4 \text{ mm (1 in.)}$$

$$L = 12.2 \text{ m (40 ft)}, \Delta = 300 \text{ mm (11}^{13}/_{16} \text{ in.)}$$

Based on lateral buckling requirements for compact sections select S :

$$S \leq 24.5c \text{ where smallest } c \text{ is } c_{tf} = 152.5 \text{ mm (6 in.)},$$

$$S \leq 24.5 \times 152.5 / 1000 = 3.74 \text{ m (12.3 ft) for Grade 345 steel.}$$

Select $S = 3.66 \text{ m (12 ft)}$ (maximum limit recommended by fabricators).

Step 2: Check that calculated offset δ_r is within limits

$$\delta_r = \frac{\Delta}{\left(\frac{L}{S}\right)} = \frac{300}{\left(\frac{1220}{366}\right)} = 90 \text{ mm (3}^{9}/_{16} \text{ in.)}$$

Check plastic limit requirement (based on smaller top flange width):

$$\delta_r = 90 \text{ mm} \leq 1.11c = 1.11 \times 152.5 \text{ mm} = 169 \text{ mm (7 in.) OK.}$$

Step 3: Select chart

For Grade 345 steel, use Chart 2.

Step 4: Determine chart parameter λ and number of passes η (if required)

$$\text{Calculate: } \lambda = \left[\frac{\delta_r}{\frac{S^2}{c}} \right] \times 10^5$$

$$\text{Bottom Flange: } \lambda = \left[\frac{90 \text{ mm}}{\frac{(3660 \text{ mm})^2}{305 \text{ mm}}} \right] \times 10^5 = 205$$

$$\text{Top Flange: } \lambda = \left[\frac{90 \text{ mm}}{\frac{(3660 \text{ mm})^2}{152.5 \text{ mm}}} \right] \times 10^5 = 102.5$$

$\lambda_{max} = 10$ in Chart 2. Since λ (205 bottom flange and 102.5 top flange) $\gg 10$, multiple passes are needed.

Bottom Flange: $\eta = \lambda / \lambda_{max} = 205 / 10 = 20.5$, round-up to the nearest higher integer, $\eta = 21$, each cycle corresponds to $\lambda / \eta = 205 / 21 = 9.8$

Top Flange: $\eta = \lambda / \lambda_{max} = 102.5 / 10 = 10.25$, round-up to the nearest higher integer, $\eta = 11$, each cycle corresponds to $\lambda / \eta = 102.5 / 11 = 9.3$

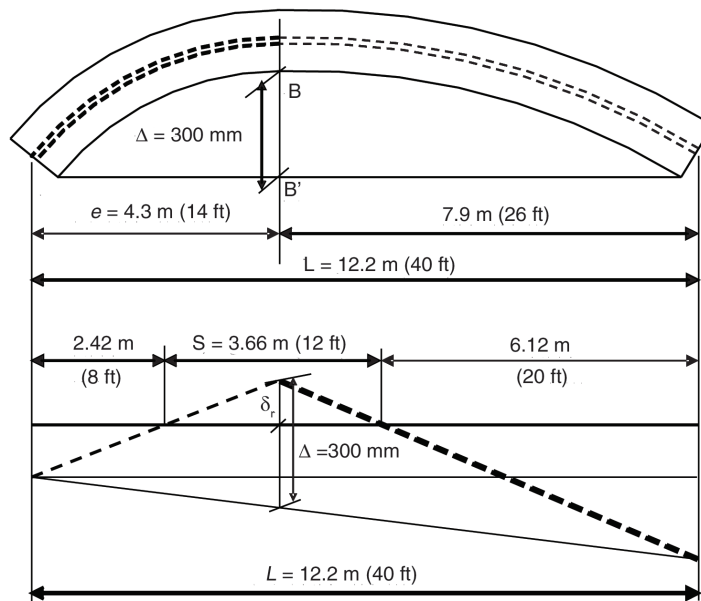


Fig. 7. Example 3: Asymmetric Fabrication Out-of-Straightness.

Step 5: Determine rectification load P_x

Bottom Flange: for $\lambda/\eta = 9.8$ (Step 4), read $\gamma = 137$ kN/cm² (199 ksi) from Chart 2. For $t_{bf} = 5.08$ cm, $c_{bf} = 30.5$ cm, $S = 366$ cm, calculate:

$$P_{xbf} = \gamma \left(\frac{t_{bf} c_{bf}^2}{S} \right) = 137 \left(\frac{5.08 \times 30.5^2}{366} \right) = 1770 \text{ kN (398 kips)}$$

Top Flange: for $\lambda/\eta = 9.3$ (Step 4), read $\gamma = 136.5$ kN/cm² (198 ksi) from Chart 2. For $t_{tf} = 2.54$ cm, $c_{tf} = 15.25$ cm, $S = 366$ cm, calculate:

$$P_{xtf} = \gamma \left(\frac{t_{tf} c_{tf}^2}{S} \right) = 136.5 \left(\frac{2.54 \times 15.25^2}{366} \right) = 221 \text{ kN (50 kips)}$$

In summary, a bending fixture ($S = 3.66$ m (12 ft)) is used to rectify $\Delta = 300$ mm ($11^{13}/16$ in.) at mid-length of the 12.2 m (40 ft) long, Grade 345 girder. Loads $P_{xtf} = 221$ kN (50 kips) should be applied eleven times to the top flange and $P_{xbf} = 1770$ kN (398 kips) should be applied twenty one times to the bottom flange (a total of 32 load applications).

Example 3

A 12.2 m (40 ft) long, Grade 345 ($F_y = 50$ ksi) symmetrical girder with 61 cm $\times$ 5.08 cm (24 in. $\times$ 2 in.) flanges and 116.8 cm $\times$ 1.27 cm (46 in. $\times$ 1/2 in.) web has an asymmetric out-of-straightness sweep of 300 mm ($11^{13}/16$ in.) measured at $e = 4.3$ m (14 ft) from its left end (Figure 7). Determine the set up and loads required to correct it.

Solution

Since load P_x is concentric with the asymmetric out-of-straightness Δ , an asymmetric load frame configuration (Figure 7) is used. While the proposed analysis is tailored for symmetric defects (located at $L/2$), δ_r from Equation 10 can be modified to incorporate asymmetry. From geometry (Figure 7),

$$\delta_r = \frac{\Delta}{\frac{4e}{S} \left(1 - \frac{e}{L} \right)}$$

The correctness of the modified equation can be verified by substituting $e = L/2$ for symmetrical defects to obtain

$$\delta_r = \frac{\Delta}{\frac{4L}{2S} \left(1 - \frac{L}{2L} \right)} = \frac{\Delta}{\frac{L}{S}} \text{ (same as Equation 10)}$$

Step 1: Set parameters and frame spacing, S

$c = 305$ mm (12 in.), $t_f = 50.8$ mm (2 in.)

Based on lateral buckling requirements for compact sections select S :

$S \leq 24.5c = 24.5 \times 305/1000 = 7.5$ m (24.6 ft) for Grade 345 steel.

Select $S = 3.66$ m (12 ft) (maximum limit recommended by fabricators). The bending fixture is placed such that the hydraulic jack is located at the point of out-of-straightness (Figure 7, point B, $e = 4.26$ m (14 ft) from the left girder's end), the 1st clamp at $(e - S/2) = (4.26 \text{ m} - 3.66 \text{ m}/2) = 2.44$ m (8 ft), and the 2nd clamp at $(e + S/2) = (4.26 \text{ m} + 3.66 \text{ m}/2) = 6.1$ m (20 ft).

Step 2: Check that calculated offset δ_r is within limits

$$\delta_r = \frac{\Delta}{\frac{4e}{S} \left(1 - \frac{e}{L} \right)} = \frac{300}{\frac{4 \times 4.3}{3.66} \left(1 - \frac{4.3}{12.2} \right)} = 98.6 \text{ mm (3 } \frac{7}{8} \text{ in.)}$$

Check plastic limit requirement:

$\delta_r = 98.6 \text{ mm} \leq 1.11c = 1.11 \times 305 \text{ mm} = 338.6 \text{ mm (13.3 in.) OK}$

Step 3: Select chart

For Grade 345 steel, use Chart 2.

Step 4: Determine chart parameter λ and number of passes η (if required)

$$\begin{aligned} \text{Calculate } \lambda &= \left[\frac{\delta_r}{\frac{S^2}{c}} \right] \times 10^5 \\ &= \left[\frac{98.6 \text{ mm}}{\left(\frac{3660 \text{ mm}}{305 \text{ mm}} \right)^2} \right] \times 10^5 = 224.5 \end{aligned}$$

$\lambda_{max} = 10$ in Chart 2. Since λ is 224.5 $\gg$ 10, multiple passes are needed. $\eta = \lambda/\lambda_{max} = 224.5/10 = 22.5$, round-up to the nearest integer, $\eta = 23$, each cycle corresponding to $\lambda/\eta = 224.5/23 = 9.8$

Step 5: Determine rectification load P_x

For $\lambda/\eta = 9.8$ (Step 4), read $\gamma = 137$ kN/cm² (199 ksi) from Chart 2.

For $t_f = 5.08$ cm, $c = 30.5$ cm, $S = 366$ cm, calculate:

$$\begin{aligned} P_x &= \gamma \left(\frac{t_f c^2}{S} \right) = 137 \times \left(\frac{5.08 \times 30.5^2}{366} \right) \\ &= 1780.6 \text{ kN (400.2 kips)}. \end{aligned}$$

Table 2. Numerical Examples ($\gamma = P_x/t_f c^2/S$, $\lambda = (\delta_r/S^2/c)\times 10^5$)

#	Section Steel Grade	Step 1 Parameters	Step 2 δ_r	Step 3 Chart	Step 4 λ, η	Step 5 P_x
1	Grade 250 W610×153 (W24×103)	$\Delta=80\text{mm}$ ($3\frac{1}{8}"$) $S=2.74\text{m}$ (9 ft) $L=4.27\text{m}$ (14ft)	51.3mm ($2"$) < 127mm ($5"$)	Chart 1 $\lambda_{\max}=7.5$	$\lambda=78.1$ $\eta=11$ $\frac{\lambda}{\eta}=7.1$	$\gamma=99.3\text{ kN/cm}^2$ (143.9 ksi) $P_x = 120.3\text{ kN}$ (27 kips)
1	Grade 345 W610×153 (W24×103)	$\Delta=80\text{ mm}$ ($3\frac{1}{8}"$) $S=2.74\text{m}$ (9 ft) $L=4.27\text{m}$ (14ft)	33mm ($1\frac{5}{16}"$) < 127mm ($5"$)	Chart 2 $\lambda_{\max}=10$	$\lambda=78.1$ $\eta=8$ $\frac{\lambda}{\eta}=9.8$	$\gamma=199\text{ kN/cm}^2$ (137 ksi) $P_x = 167.1\text{ kN}$ (37.6 kips)
2	Un-symmetrical Grade 345 Bot.FI: 61cm×5.08cm Top 30.5cm×2.54cm Web: 116.8cm×1.27cm	$\Delta=300\text{mm}$ ($11\frac{13}{16}"$) $S=3.66\text{ m}$ (12 ft) $L=12.2\text{ m}$ (40 ft)	90mm ($3\frac{9}{16}"$) < 169 mm ($7"$)	Chart 2 $\lambda_{\max}=10$	Bot. $\lambda=205$ $\eta=21$ $\frac{\lambda}{\eta}=9.8$ Top $\lambda=102.5$ $\eta=11$ $\frac{\lambda}{\eta}=9.3$	Bot.: $\gamma=137\text{kN/cm}^2$ (199 ksi) $P_{\text{xbf}} = 1770\text{ kN}$ (398 kips) Top: $\gamma=136.5\text{kN/cm}^2$ (198 ksi) $P_{\text{xtr}} = 221\text{ kN}$ (50 kips)
3	Asymmetric sweep FI: 61cm×5.08cm Web: 116.8cm×1.27cm	$\Delta=300\text{mm}$ ($11\frac{13}{16}"$) $S=3.66\text{ m}$ (12 ft)	98.6mm ($3\frac{7}{8}"$) < 338.6mm (13.3")	Chart 2 $\lambda_{\max}=10$	$\lambda=224.5$ $\eta=23$ $\frac{\lambda}{\eta}=9.8$	$\gamma=137\text{ kN/cm}^2$ (199 ksi) $P_x = 1780.6\text{ kN}$ (400.2 kips)

In summary, a bending fixture of spacing $S = 3.66\text{ m}$ (12 ft) is used to rectify the unsymmetrical defect $\Delta = 300\text{ mm}$ ($11\frac{13}{16}\text{ in.}$) located at $e = 4.3\text{ m}$ (14 ft) from the left end of the 12.2 m (40 ft) long, Grade 345 steel girder. The repair load $P_x = 1780.6\text{ kN}$ (400.2 kips) should be applied twenty three times per flange; for example, a total of 46 load applications.

SUMMARY

The numerical examples illustrate the applicability of the charts for straightening girders bent during fabrication. Example 1 confirms that larger loads are required to straighten identical defects for higher strength steels. Example 2 shows how unequal loads have to be applied to the top and bottom flanges in unsymmetrical sections. Example 3 shows modifications to the proposed procedure to determine the bending fixture configuration required to rectify asymmetrical out-of-straightness in a symmetrical steel plate girder. Results are summarized in Table 2.

CONCLUSIONS

Cold straightening is a simple operation that offers significant economies to steel fabricators. This paper presents an idealized analysis for a set-up developed by a fabricator in which concentrated loads are used to rectify the out-of-straightness. Plastic analysis was used to derive generalized equations relating applied loads to residual deformation for the special case of unstiffened steel girders. Maximum limits were then established to optimize the solution. Such limits allow determination of optimal spacing and load required to straighten steel girders with a known bow or sweep (Figure 1). This makes the operation safer, more efficient and less costly since it simplifies calculations and eliminates trial and error.

Design aids in the form of charts that relate the straightening load to the out-of-straightness were developed for conventional Grade 250 and Grade 345 steel. A step-by-step procedure is described and three numerical examples presented to illustrate its application.

The proposed procedure provides fabricators with information on loads required to rectify excessive bow or sweep along the length of the girder. However, because of the many uncontrolled variables and assumptions involved, it may still be necessary to apply a final corrective straightening load. Steel fabricators should rely on their judgment in selecting the load.

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NOTATIONS

α	length of overhang
c	half flange width
d	web depth
E	modulus of elasticity
F_y	yield stress
I_x	moment of inertia about weak axis

S	bending fixture spacing
L	length of bent segment
LRFD	Load and Resistance Factor Design
L_p	limiting laterally unbraced length for full plastic flexural strength
M_p	plastic moment
M_x	bending moment due to selfweight plus concentrated applied loads
M_y	yield moment
P_{max}	maximum load based on fully-plastic analysis
P_x	magnitude of applied concentrated load
P_{xtf}	magnitude of applied concentrated load based on top flange data
P_{xbf}	magnitude of applied concentrated load based on bottom flange data
r_y	radius of gyration about weak axis
t_f	flange thickness
t_w	web thickness
w	uniformly distributed self-weight of girder
x_2	length of elastic portion
y_y	half depth of elastic core
δ	maximum deflection during loading
δ_r	maximum residual deflection after unloading
Δ	maximum bow or sweep within bent segment L

APPENDIX 1. COEFFICIENTS

$$I_x = \frac{4t_f c^3}{3}, w = \gamma_{\text{steel}} \times A_{\text{girder}}$$

$$M_x = P_x \frac{x}{2} + \frac{wSx}{2} - \frac{wx^2}{2} - \frac{wa^2}{2}, x \text{ measured from support}$$

$$y_y = \sqrt{3c^2 - \frac{3M_x}{2t_f F_y}}$$

Equations (1-2):

$$a = \left(\frac{L - S}{2} \right)$$

$$m = \left(\sqrt{x_2^2 - sx_2 + t} + x_2 - \frac{s}{2} \right)$$

$$n = \left(\sqrt{\frac{S^2}{4} - s\frac{S}{2} + t} \right)$$

$$p = \sqrt{-\frac{S^2}{4} + t - v}$$

$$r = \sqrt{\frac{4t_f F_y^3}{3wE^2}}$$

$$s = \left(S + \frac{2P_x}{w} \right)$$

$$u = \frac{3wS}{16Et_f c^3} = \frac{wS}{4EI_x}$$

$$x_2 = \frac{\left(S + \frac{2P_x}{w} \right) - \sqrt{\left(S + \frac{2P_x}{w} \right)^2 - \frac{8F_y I_x}{wc} - 4a^2}}{2}$$

Charts (Equations 6-7):

$$\alpha = \left(\frac{a}{S} \right)$$

$$m' = \left(\sqrt{x_2'^2 - s'x_2' + t'} + x_2' - \frac{s'}{2} \right)$$

$$n' = \left(\sqrt{\frac{1}{4} - \frac{s'}{2} + t'} \right)$$

$$p' = \sqrt{-\frac{1}{4} + \alpha^2 - \beta + \frac{4F_y \beta}{\gamma}}$$

$$r' = \frac{1}{E} \sqrt{\frac{4\beta F_y^3}{3\gamma}}$$

$$s' = (1 + 2\beta)$$

$$u' = \frac{3\gamma}{16E\beta}$$

$$x_2' = \frac{(1 + 2\beta) - \sqrt{(1 + 2\beta)^2 - \frac{32F_y \beta}{3\gamma} - 4\alpha^2}}{2}$$

APPENDIX 2. SELFWEIGHT EFFECTS

The steps required to determine the rectification load considering selfweight are listed as follows:

- Steps 1–2:** Same as for selfweight excluded (refer to Application Section)
- Step 3:** Calculate $\alpha = a/S$ where $\alpha = 1/2 (L - S)$. Based on α , select appropriate chart:
 $\alpha = 0.5$: Chart 1 for Grade 250, Chart 2 for Grade 345, Chart 3 for Grade 414.
 $\alpha = 0.25$: Chart 4 for Grade 250, Chart 5 for Grade 345, Chart 6 for Grade 414.
 $\alpha = 0$: Chart 7 for Grade 250, Chart 8 for Grade 345, Chart 9 for Grade 414.
 Interpolate for other α values
- Step 4:** Same as for selfweight excluded (refer to Application)
- Step 5:** Assume β (varies from 2.5 to 30). Based on β , read $\gamma = P_x/(t_f c^2/S)$ from appropriate chart for λ (or λ/η) from Step 4. Verify validity of β by calculating $\beta = P_x/wS$, where $w = \gamma_{steel} \times A_{girder}$. If β is different from the assumed value, repeat Step 5 until they are in agreement. Calculate P_x by multiplying (γ) by $t_f c^2/S$.

Application:

Example 1 is repeated for the Grade 250 girder placed horizontally. Steps 1–2 are as in Example 1.

- Step 3:** $a = 1/2 (4.27 \text{ m} - 2.74 \text{ m}) = 0.77 \text{ m}$, $\alpha = 0.77 \text{ m}/2.74 \text{ m} = 0.28$. Interpolate between Chart 1 ($\alpha = 0.5$) and Chart 4 ($\alpha = 0.25$).
- Step 4:** As in example 1 ($\lambda = 78.1$): $\lambda_{max} = 7.5$ in Chart 1, 8 in Chart 4. $\lambda = 78.1 \gg 7.5$ or 8, multiple passes are needed. Since β is function of the not yet known P_x , use trial and error.
- Step 5:** Iteration #1: Assume $\beta = 5$, $\lambda_{max} = 7.5$, $\lambda/\lambda_{max} = 78.1/7.5 = 10.4$, round-up to $\eta = 11$. $\lambda = 78.1/11 = 7.1$. For $\lambda = 7.1$ & $\beta = 5$, read $\gamma = 99.3 \text{ kN/cm}^2$ (chart 1), 95.6 kN/cm^2 (chart 4). $\gamma = 96 \text{ kN/cm}^2$ by interpolation. $P_x = 96 \times 2.54 \times 11.43^2/274 = 116.3 \text{ kN}$ (26.1 kips). For $w = 1.5 \text{ kN/m}$, $\beta = 16.3 \text{ kN}/(1.5 \text{ kN/m} \times 2.74 \text{ m}) = 28.3 > 5$, a second iteration is required, use $\beta = 30$ (closest to 28.3). $P_x = 119.7 \text{ kN}$ (26.9 kips). $\beta = 119.7/(1.5 \times 2.74) = 29.1 \approx 30$, OK
- A repair load $P_x = 119.7 \text{ kN}$ (26.9 kips) should be applied eleven times per flange to the Grade 250 girder (compared to 120.3 kN (27 kips) for $sw = 0$) Note that the repair loads are almost identical since selfweight stresses are inconsiderable.

REFERENCES

- AASHTO (1996), *Standard Specifications for Highway Bridges*, 16th Edition, Washington, DC.
- Avent, R.R. and Mukai, D. (2001), "What You Should Know about Heat-Straightening Repair of Damaged Steel," *Engineering Journal*, AISC, Vol. 38, No. 1, pp. 27-49.
- Brockenbrough, R.L. (1970), "Theoretical Stresses and Strains from Heat-Curving", ASCE, *Journal of Structural Division*, July, vol. 96, no. ST7, pp. 1421-1444.
- Brockenbrough, R.L. (1972), "Fabrication Aids for Continuously Heat-Curved Girders," United States Steel Corporation, Pittsburgh, PA, April.
- Brockenbrough, R. (2003), "MTR Survey of Plate Material Used in Structural Fabrication," *Engineering Journal*, Volume 40, Number 1, pp. 42-49.
- Brockenbrough, R.L. and Merrit, F.S. (1999), *Structural Steel Designer's Handbook*, 3rd Edition, McGraw-Hill, NY, NY (page 1.18).
- Chen, W.F. and Han, D.J. (1988), *Plasticity for Structural Engineers*, Springer-Verlag, NY.
- Gergess, A. (2001), "Cold Bending and Heat Curving of Structural Steel I-Girders", PhD Dissertation, Department of Civil and Environmental Engineering, University of South Florida, Tampa, FL, August.
- Gergess, A. and Sen, R. (2001), "Inelastic Response of Simply Supported I-Girders subjected to Weak Axis Bending," *Proceedings of the International Conference on Structural Engineering, Mechanics and Computation*, Cape Town, South Africa, Vol. I, pp.243-250.
- AISC (2001), *Manual of Steel Construction Load and Resistance Factor Design*, 3rd Edition, American Institute of Steel Construction, Chicago, IL.
- McCormac, J. (1994), *Structural Steel Design LRFD Method*, Harper-Collins, NY, NY, Second Edition, 1994 (p. 14).
- MSC/NASTRAN for Windows, The MacNeal-Schwendler Corporation (2000), "Finite Element Modeling and Post-processing System," Los Angeles, CA.
- Popov E.V. (1976), "Mechanics of Materials", Prentice-Hall Inc., Englewood Cliffs, New Jersey, 2nd Edition (Page 135 - 142, 377 - 393).
- Salmon, C.G. and Johnson, J.E. (1996), "Steel Structures: Design and Behavior", Harper-Collins, NY, NY, Fourth Edition.
- Sen, R., Gergess, A. and Issa, C. (2003), "Finite Element Modeling of Heat Curved I-Girders", ASCE, *Journal of Bridge Engineering*, Vol. 8, No. 3, May/June, pp. 153-161.