Lateral-Torsional Buckling of Wide Flange Cantilever Beams

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Lateral-torsional buckling is an important consideration in the design of steel beams. Most beams that are used in steel construction have a much greater stiffness about the axis which resists the bending moment than in the perpendicular direction. This results in the possibility of lateral-torsional buckling.

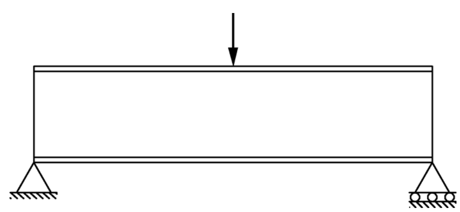
Lateral-torsional buckling is the result of lateral deflection and twisting as shown in Figure 1b. The resistance to this type of buckling is dependent on the lateral bending stiffness of the cross section, the torsional stiffness of the cross section, the type of bracing (lateral or torsional), the position of the bracing along the length of the beam, the position of the bracing on the cross section, the stiffness of the bracing, the bending moment distribution along the length of the beam, the position of the load on the cross section, the material properties, the magnitude and distribution of residual stresses, the initial twist of the cross section, and the initial bow along the length of the cross section.

To prevent lateral-torsional buckling, the beam can be braced against twisting of the cross section. Two types of bracing are commonly used: (1) torsional bracing, which resists twisting of the cross section directly; (2) lateral bracing, which resists twisting of the cross section by limiting the lateral deflection at a point away from the level of virtual rotation. In practice, it is common for beams to be braced laterally at the top flange by the floor or roof system. Previous studies have shown that, for cantilever beams, torsional bracing is more effective than lateral bracing, assuming that both types have infinite stiffness (Nethercot and Al-Shankyty, 1979; Kitipornchai and Richter, 1983).

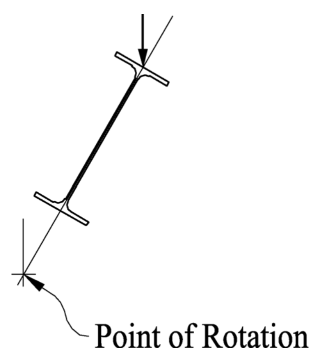
Problem Statement

The American Institute of Steel Construction, *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 1999), hereafter referred to as the AISC *LRFD Specification*, includes provisions for the design of beams to prevent lateral-torsional buckling. The design method uses an equation that was derived for a simply supported beam with equal end moments. The equation is not accurate for cantilever beams because the mechanics of buckling are different from simply supported beams. When a simply supported beam buckles, the compression flange moves farther than the tension flange, whereas the opposite is true for cantilever beams. The buckled shape of an unbraced cantilever beam is shown in Figure 2.

Previous studies have shown that lateral bracing for cantilever beams is most efficient when it is located at the tension flange and that compression flange bracing is only slightly beneficial (Nethercot and Al-Shankyty, 1979; Kitipornchai and Richter, 1983). Figure 2 shows that, for cantilever beams, the tension flange is the point on the cross section that is farther from the point of rotation. According



a. Simply supported beam.



b. Section at midspan.

Fig. 1. Buckled shape of a simply supported beam.

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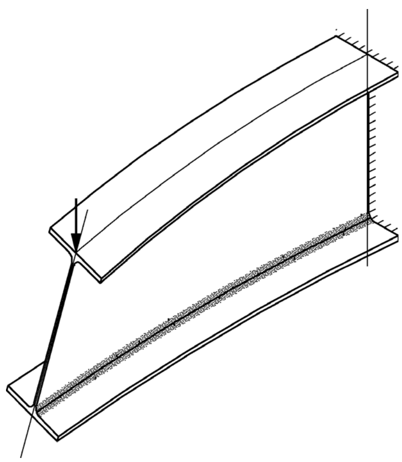
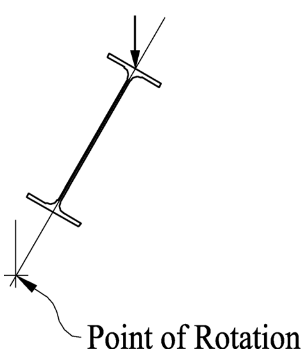
to the AISC *LRFD Specification* (AISC, 1999), a beam is braced against lateral-torsional buckling at locations along the length of the beam where it is "braced against lateral displacement of the compression flange," or "braced to prevent twist of the cross section." The AISC *LRFD Specification* will produce unsafe results in cases where the compression flange of a cantilever beam is laterally braced to prevent lateral-torsional buckling. The beneficial effect of tension flange lateral bracing is not always realized in practice because some designers are not aware that this type of bracing will prevent twist of the cross section. The buckled shape of a cantilever beam with a discrete lateral brace at the free end is shown in Figure 3.

Objectives

A simple and unified method to calculate lateral-torsional buckling capacity of cantilever beams is needed to provide safe and economical steel structures.The purpose of this research was to develop a method to design cantilever beams with various bracing configurations and load conditions. Equations to predict the buckling capacity of cantilever beams were developed by curve fitting data from a finite element buckling analysis. The equations explicitly account for the effects of bracing, load height, and moment distribution along the length of the beam. The finite element program Buckling Analysis of Stiffened Plates (BASP) was used. BASP was developed at the University of Texas at Austin.



a. Unbraced cantilever beam.



b. Section at free end.

Fig. 2. Buckled shape of an unbraced cantilever beam.

BACKGROUND

Timoshenko and Gere Equation for Simply Supported Beams

Timoshenko and Gere (1961) derived the theoretical equation for the lateral-torsional buckling capacity of a simply supported beam with equal end moments. A doubly symmetric, prismatic, linear elastic beam was assumed. At the ends, the beam was free to warp, but torsional rotation and lateral deflection were prevented. The critical buckling moment is

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \quad (1)$$

where L is the beam length, E is the modulus of elasticity, G is the shear modulus, I_y is the weak axis moment of inertia, J is the St. Venant torsion constant, and C_w is the warping constant.

AISC Procedure

The AISC *LRFD Specification* (AISC, 1999) uses the Timoshenko and Gere equation modified by a factor, C_b, which accounts for non-uniform moments along the length of the beam. For inelastic buckling, the capacity is interpolated linearly between the elastic buckling moment and the plastic capacity of the cross section. Residual stresses are accounted for explicitly by limiting the elastic stress to the yield stress minus the residual stress. The equation for the elastic buckling moment is

$$M_{cr} = C_b \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \quad (2)$$

To obtain the design capacity, M_{cr} is multiplied by a reduction factor, ϕ_b, to account for inaccuracies in the design. For unbraced cantilevers, the AISC *LRFD Specification* recommends using C_b = 1.0.

Timoshenko and Gere Equation For Cantilever Beams

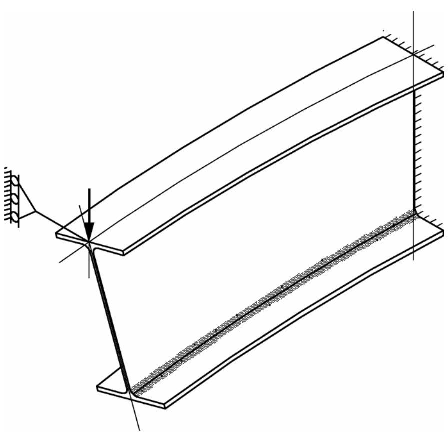
For an unbraced cantilever I section beam, Timoshenko and Gere (1961) expressed the differential equations of equilibrium using infinite series. They assumed full fixity at the root and a point load at the tip applied at the level of the centroid. From the boundary conditions, a transcendental equation for calculating the critical load was obtained. The critical load is

$$P_{cr} = \gamma_2 \frac{\sqrt{EI_y GJ}}{L^2} \tag{3}$$

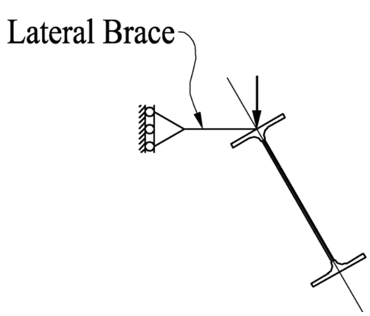
where γ_2 is provided in Table 6-3 of Timoshenko and Gere (1961).

Trahair Procedure

Trahair (1983, 1993) proposed solutions for doubly symmetric, unbraced, cantilever beams with varying load height. He used an elastic finite element program to obtain equations for the critical moment. When the load is at the level of the centroid, the equations reduce to:



a. Braced cantilever beam.



b. Section at free end.

Fig. 3. Buckled shape of a cantilever beam with a discrete lateral brace at the free end.

Point load at tip,

$$M_{cr} = \left(3.95 + 3.52\,X\right)\frac{\sqrt{EI_y GJ}}{L} \tag{4}$$

Uniformly distributed load,

$$M_{cr} = \left(5.83 + 8.71\,X\right)\frac{\sqrt{EI_y GJ}}{L} \tag{5}$$

where

$$X = \frac{\pi}{L}\sqrt{\frac{EC_w}{GJ}} \tag{6}$$

SOLUTION PROCEDURE

Because closed form solutions exist only for the simplest cases of lateral-torsional buckling, numerical methods were used for this study. The finite element program BASP was used to find the critical buckling moments of cantilever beams with various loading and restraint conditions. Three different wide flange sections were used, W12×53, W16×26, and W8×15, and six different lengths, 90, 120, 150, 180, 210, and 240 in. A total of 216 finite element models were generated.

Equations were developed by curve fitting the finite element data. Only lateral bracing located at the top flange was considered. The equations will be conservative where they are used with combined lateral and torsional bracing, such as metal decking or a concrete slab. Continuous bracing was considered, as well as discrete bracing at the free end of the cantilever. Assadi and Roeder (1985) showed that only a small bracing stiffness is required for lateral bracing to be fully effective; therefore, only infinitely stiff braces were considered. Two loading conditions were considered, uniformly distributed load and point load at the free end. The load was located at the top flange or the centroid of the cross section. The effects of initial twist were not considered. Because most design methods use the elastic buckling load as a starting point, the effects of inelastic action were not considered. All of the beams were prismatic and doubly symmetric. Warping, twist, and deflection in all directions were prevented at the fixed end of the beam.The modulus of elasticity was 29,000 ksi.

Finite Element Program

The finite element program that was used for the buckling analysis is BASP. BASP was developed at the University of Texas at Austin. The program uses a two-dimensional idealization. The analysis is performed in two steps. First, the in-plane analysis is performed to calculate the stresses arising from the applied loading. Using these stresses, an out-of-plane analysis is performed to solve for the buckling load. Cross section distortion and local buckling are accounted for. The program provides an elastic solution and does not take initial twist into account.The program is described in more detail in Akay, Johnson, and Will (1977).

The beam webs were composed of 4 elements vertically and 10 elements along the length. The beam flanges were composed of 10 elements along the length. The fixed end of the beam was modeled with pinned supports at each node for the in-plane analysis. For the out-of-plane analysis each node at the fixed end was restrained against translation and rotation. The lateral bracing was modeled with a pinned support for the out-of-plane analysis. Transverse beam web stiffeners were used at locations of concentrated loads to prevent excessive local web distortion.

The accuracy of the program was verified by comparing the critical loads from the program to the critical loads from the Timoshenko and Gere (1961) equation for cantilever beams. The ratio of the BASP load to the calculated load varied from 0.76 to 1.07. The main source of this variation is believed to be the effect of web distortion. The effect of web distortion was accounted for in the finite element models, but was neglected in the derivation of the Timoshenko and Gere equation.

RESULTS

The proposed design equation for the critical buckling moment is

$$M_{cr} = C_L C_H C_B \frac{\sqrt{EI_y GJ}}{L} \tag{7}$$

where C_L is a coefficient to account for the moment distribution along the length of the beam, C_H is a coefficient to account for the effect of load height, and C_B is a coefficient to account for the effect of bracing.

Moment Distribution Coefficient

The buckling moments from BASP are compared with the equations proposed by Trahair (1983, 1993) in Figure 4. The BASP results agree well with Trahair's equations; therefore, his equations are proposed as the moment distribution coefficient:

Point load at the free end,

$$C_L = 3.95 + 3.52X \tag{7}$$

Uniformly distributed load,

$$C_L = 5.83 + 8.71X \tag{8}$$

Load Height Coefficient

The data for the load height coefficient is presented as a ratio of the critical moment of a beam loaded at the top flange to the critical moment of an identical beam loaded at the shear center. The ratios are plotted versus X in Figure 5

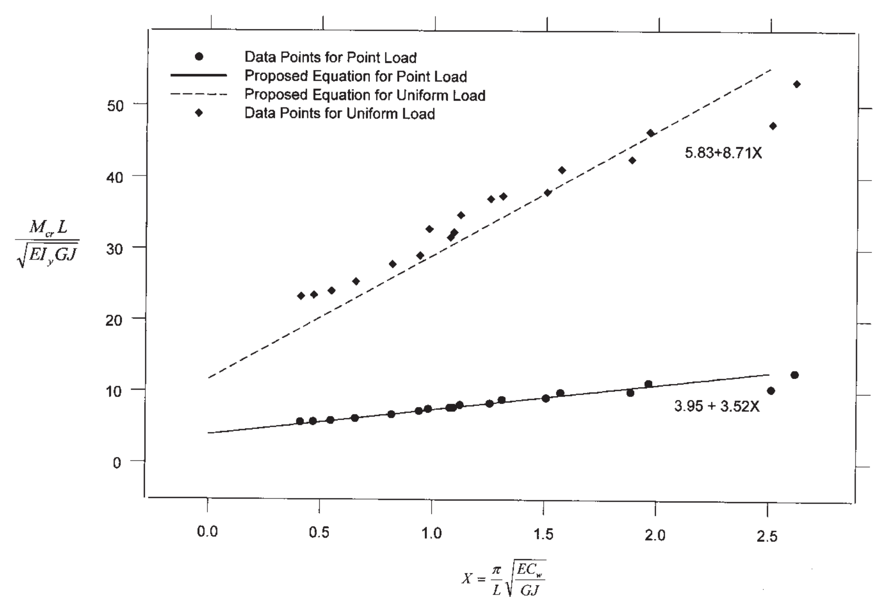


Fig. 4. Effect of loading condition on the buckling capacity of unbraced cantilever beams loaded at the shear center.

for a beam with a concentrated load at the free end and continuous bracing. The following equations were obtained by curve fitting the data:

concentrated load at the free end, continuous bracing,

$$C_H = 0.76 - 0.51X + 0.13X^2 \tag{10}$$

concentrated load at the free end, no bracing,

$$C_H = 0.97 - 0.59X + 0.14X^2 \tag{11}$$

concentrated load at the free end, discrete bracing at the free end,

$$C_H = 0.87 - 0.59X + 0.15X^2 \tag{12}$$

uniform load, continuous bracing,

$$C_H = 0.49 - 0.27X + 0.06X^2 \tag{13}$$

uniform load, no bracing,

$$C_H = 0.83 - 0.54X + 0.12X^2 \tag{14}$$

uniform load, discrete bracing at the free end,

$$C_H = 0.64 - 0.43X + 0.10X^2 \tag{15}$$

Where the load is at the level of the shear center, $C_H = 1.0$. Where the load is below the level of the shear center, it is conservative to use $C_H = 1.0$.

Bracing Coefficient

The data for the bracing coefficient is presented as a ratio of the critical moment of a beam braced at the top flange to the critical moment of an identical unbraced beam. The ratios are plotted versus X in Figure 6 for a beam with continuous bracing and a point load at the free end, at the level of the shear center. Figure 7 shows a beam with discrete bracing and a point load at the free end, at the level of the shear center. The following equations were obtained by curve fitting the data:

concentrated load at the free end, at the level of the shear center, continuous bracing,

$$C_B = 2.38 + 0.26 \cdot \ln X + 0.08(\ln X)^2 - 0.60(\ln X)^3 \tag{16}$$

concentrated load at the free end, at the level of the top flange, continuous bracing,

$$C_B = 1.75 + 0.13 \cdot \ln X + 0.27(\ln X)^2 - 0.23(\ln X)^3 \tag{17}$$

concentrated load at the free end, at the level of the shear center, discrete bracing at the free end,

$$C_B = 1.42 + 0.88X - 0.26X^2 \tag{18}$$

concentrated load at the free end, at the level of the top flange, discrete bracing at the free end,

$$C_B = 1.48 + 0.16X \tag{19}$$

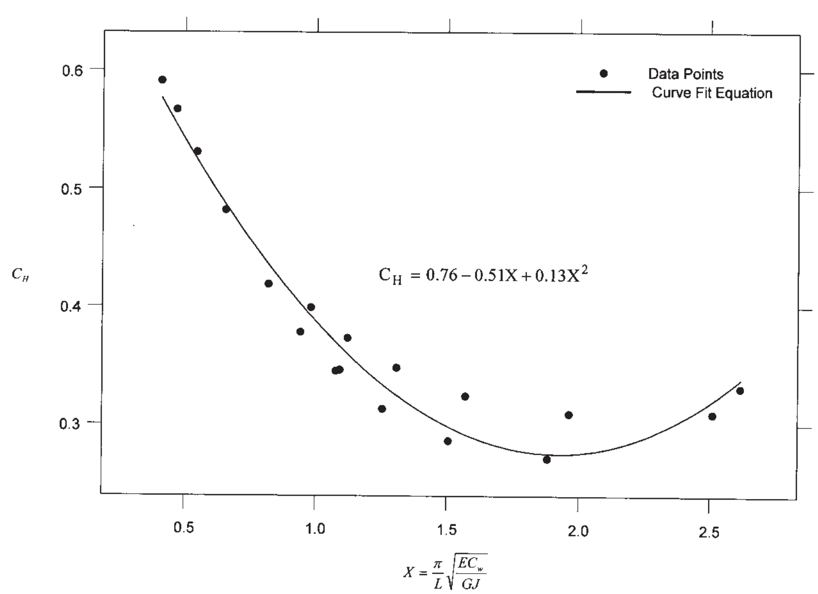


Fig. 5. Effect of top flange loading on the buckling capacity of cantilever beams with a concentrated load at the free end and continuous bracing.

uniform load at the level of the shear center, continuous bracing,

$$C_B = 2.62 + 0.08 \ln X + 0.24 (\ln X)^2 - 0.38 (\ln X)^3 \quad (20)$$

uniform load at the level of the top flange, continuous bracing,

$$C_B = 1.77 + 0.27 \ln X + 0.11 (\ln X)^2 - 0.27 (\ln X)^3 \quad (21)$$

uniform load at the level of the shear center, discrete bracing,

$$C_B = 1.92 + 0.53 X - 0.11 X^2 \quad (22)$$

uniform load at the level of the top flange, discrete bracing,

$$C_B = 1.63 - 0.12 X + 0.31 X^2 - 0.09 X^3 \quad (23)$$

Where no bracing is present, $C_B = 1.0$.

Simplified Equations

In order to simplify the design process, the equations in this section are proposed. The following equations produce conservative results for C_H:

concentrated load at the free end,

$$C_H = 0.76 - 0.51 X + 0.13 X^2 \quad (24)$$

uniform load,

$$C_H = 0.49 - 0.27 X + 0.06 X^2 \quad (25)$$

The following equations produce conservative results for C_B:

load at the level of the shear center,

$$C_B = 1.42 + 0.88 X - 0.26 X^2 \quad (26)$$

load at the top flange,

$$C_B = 1.48 + 0.16 X \quad (27)$$

Modified AISC Procedure

The proposed equations can be used with the AISC procedure by solving for an equivalent C_b factor. Setting Equation 2 equal to Equation 7 and solving for C_b,

$$C_b = \frac{C_L C_H C_B}{\pi} \sqrt{\frac{GJ}{GJ + \left(\frac{\pi}{L}\right)^2 EC_w}} \quad (28)$$

If E = 29,000 ksi and G = 11,200 ksi are substituted into the equation, it reduces to

$$C_b = \frac{C_L C_H C_B}{\sqrt{9.87 + 252 \frac{C_w}{JL^2}}} \quad (29)$$

The accuracy of the proposed equations has not been proven when the beam is in the inelastic zone of the AISC buckling curve. The designer should use caution when using these equations when inelastic buckling is predicted.

Comparison of Equations to Finite Element Results

A total of 216 finite element models were used in this study in order to obtain accurate curve fit equations. Tables 1

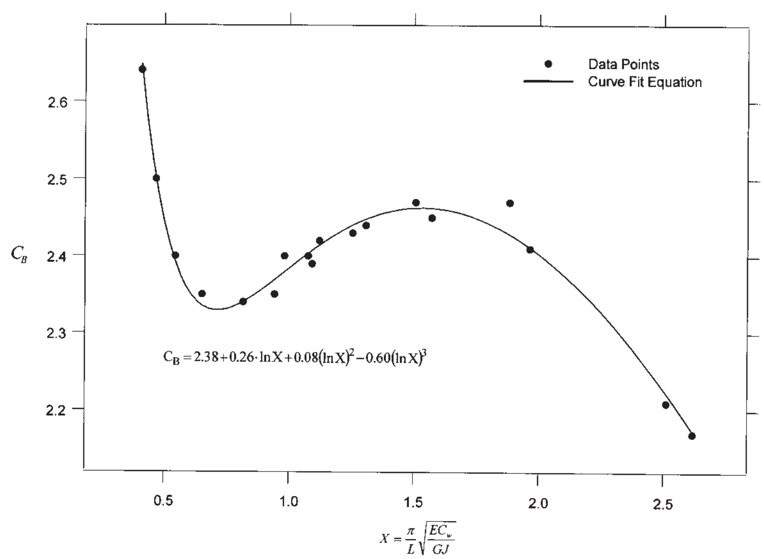


Fig. 6. Effect of continuous bracing on the buckling capacity of cantilever beams with a concentrated load at the free end, at the level of the shear center.

through 12 compare the critical load from the finite element (BASP) models to the calculated critical loads. The AISC load was calculated using the AISC *LRFD Specification* (AISC, 1999) with ϕ_b and $C_b = 1.0$.

CONCLUSIONS

It was shown that the AISC (1999) procedure produces conservative results for cantilever beams that are braced laterally at the tension flange, and unsafe results for cantilever beams that are braced laterally at the compression flange.

Equations to predict the lateral-torsional buckling capacity of cantilever beams were developed by curve fitting the finite element data. Simplified equations were proposed that were generally conservative compared with the finite element data. A design method was proposed that uses a modified C_b factor to be used with the AISC (1999) procedure.

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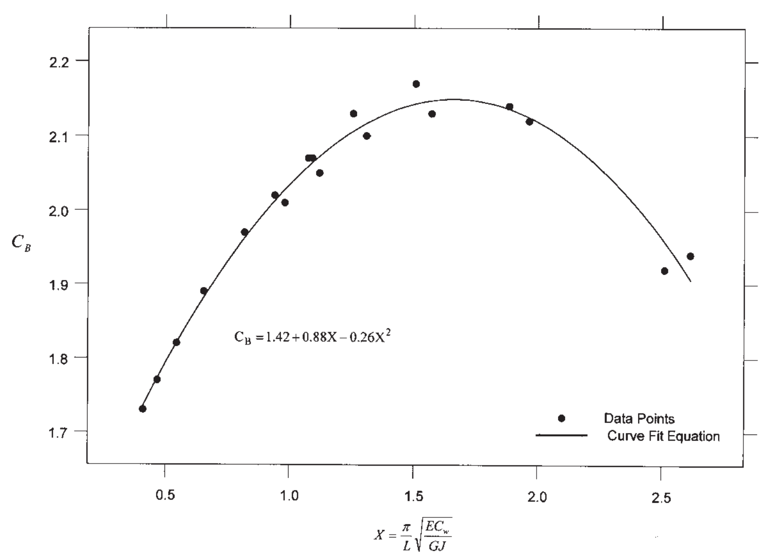

Fig. 7. Effect of discrete bracing at the free end on the buckling capacity of cantilever beams with a concentrated load at the free end, at the level of the shear center.