# Dynamic Amplitude Prediction for Ballroom Floors

LINDA M. HANAGAN


## ABSTRACT

To avoid excessive vibration in ballroom-type floors, engineers can evaluate a floor system design before construction using the procedures outlined in the AISC Design Guide 11. These procedures are most appropriately applied to simply supported beam systems where dancing is expected to occur over the entire span. This paper presents a modification to assess beam/girder systems subjected to dance type loads over only a portion of the bay. This modification is especially useful for long-span systems where meeting the serviceability requirements is difficult.


## INTRODUCTION

Rhythmic activities, such as dancing, have been reported to cause excessive vibration levels of steel-framed floor systems. To combat this problem, a criterion has been developed to assess a floor system design before it is constructed. This criterion has been most recently presented in the AISC *Design Guide 11* (Murray, Allen, and Ungar, 1997). The material presented in this paper is intended to add to the usefulness of the "Design for Rhythmic Excitation" criteria presented in the Design Guide.

The Rhythmic Excitation Criteria for dancing was derived for a simply supported beam-like floor construction with the dynamic excitation applied over its entire span (Allen, Rainer, and Pernica, 1985). Perhaps more commonly found than this case would be a simply supported beam/girder floor system subjected to a dance excitation over only a portion of the bay. Such a case is not addressed by the current criterion. A modification to address this common condition is the main subject of this paper. Structural engineers may find this modification especially helpful when designing long span ballroom floors, where dancing activities are likely to take place in only a limited area of the bay. In the long span case, satisfying the existing criterion, with the assumption of loading over the entire floor area, results in a design so massive that the cost is prohibitive.

Linda M. Hanagan is an assistant professor, architectural engineering department, Pennsylvania State University, University Park, PA.

## THE CURRENT RHYTHMIC EXCITATION CRITERION FOR DANCING

To understand the derivation of the modifications proposed herein, a review of the current criterion derivation is presented. The peak dynamic amplitude, in this case acceleration, of a simply supported beam-like floor system subjected to dance-type loads can be closely approximated as the steady-state acceleration peak at the mid-span for the dynamic beam model shown in Figure 1 where *f(t)* is the uniformly distributed sinusoidal load specified in *Design Guide 11* (Murray et al.,1997), $\bar{m}$ is the uniformly distributed mass, and $\ddot{y}_{ss}$ is the steady state acceleration response at mid-span. The dynamic behavior of this beam is governed by the partial differential equation (Meirovitch, 1997)

$$Ly(x,t) + C\dot{y}(x,t) + M\ddot{y}(x,t) = f(x,t) \quad (1)$$

where

- $L$ = homogeneous differential stiffness operator
- $C$ = homogeneous differential damping operator
- $M$ = homogeneous differential mass operator
- $y(x, t)$ = displacement of point $x$
- $x$ = a coordinate location along the span of the beam
- $f(x, t)$ = force density

Assuming a solution to Equation 1 of the form

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\eta_i(t) \quad (2)$$

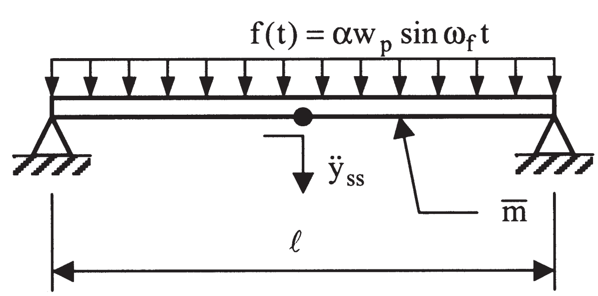


*Fig. 1. Dynamic beam model.*

where

$\phi_i(x)$ = eigenfunctions of the undamped system

$\eta_i(t)$ = time-dependent generalized coordinates often referred to as normal or modal coordinates

The eigenfunction describes the shape of the structure while vibrating in that mode. Substituting Equation 2 into Equation 1, multiplying through by $\phi_i(x)$, assuming proportional damping, and considering the orthonormality relations, we obtain the independent set of modal equations

$$\ddot{\eta}_i(t) + 2\beta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i(t) = N_i(t), \mathrm{i} = 1, 2, ... \tag{3}$$

where

$\omega_i$ = the natural frequencies, the square roots of the eigenvalues, of the system

$\beta_i$ = the viscous damping factors

$N_i(t)$ = the modal force for the $i^{th}$ mode

where

$$N_i(t) = \int \phi_i(x) \cdot f(x,t) \cdot dx, \ \mathrm{i} = 1, 2, ... \tag{4}$$

For a simply supported beam with a uniformly distributed mass, as represented in Figure 1, the eigenfuctions and natural frequencies are

$$\phi_i(x) = A_i \sin\frac{i\pi x}{\ell} \tag{5}$$

$$\omega_i = C_i\sqrt{\frac{EI}{\bar{m}\ell^4}} \tag{6}$$

where $C_1 = \pi^2$ for the first mode of vibration, $\bar{m} = w_t / g$, and $g$ is the acceleration of gravity. The amplitude of the mode shape, $A_i$, is found from setting the modal mass for the $i^{th}$ mode shape equal to unity, or

$$m_i = \int \bar{m}\phi_i^2(x)dx = 1 \tag{7}$$

This corresponds to the assumption of unity on the acceleration term in the expressions represented by Equation 3.

For dance step frequencies less than the first natural frequency of the beam, as is usually the case, the peak acceleration will be primarily the result of vibration of the first mode.

For the first mode

$$m_1 = \int_0^\ell \bar{m}\left(A_1 \sin\frac{\pi x}{\ell}\right)^2 dx = 1 \tag{8}$$

therefore

$$A_1 = \sqrt{\frac{2}{\bar{m}\ell}} = \sqrt{\frac{2g}{w_t\ell}} \tag{9}$$

From Equation 4, the force for the first mode, $N_1(t)$, due to the dynamic load shown in Figure 1 becomes

$$\begin{aligned} N_1(t) &= \int \phi_1(x) \cdot f(x,t) \cdot dx \\ &= \int_0^\ell \left[\sqrt{\frac{2g}{w_t\ell}} \sin\frac{\pi x}{\ell}\right]\left[\alpha w_p \sin\omega_f t\right] \cdot dx \\ &= 2\alpha w_p \frac{\ell}{\pi}\sqrt{\frac{2g}{w_t\ell}} \sin\omega_f t \end{aligned} \tag{10}$$

Substituting Equation 10 into Equation 3 for the first mode yields an equivalent single-degree-of-freedom subjected to a sinusoidal load, $N_1(t)$.

The maximum acceleration from the classical steady-state solution for

$$\ddot{\eta}_1(t) + 2\beta_1\omega_1\dot{\eta}_1 + \omega_1^2\eta_1(t) = 2\alpha w_p \frac{\ell}{\pi}\sqrt{\frac{2g}{w_t\ell}} \sin\omega_f t \tag{11}$$

is

$$\ddot{\eta}_{1\max} = \frac{2\alpha w_p \frac{\ell}{\pi}\sqrt{\frac{2g}{w_t\ell}}}{\sqrt{\left(\frac{\omega_1^2}{\omega_f^2} - 1\right)^2 + \left(\frac{2\beta_1\omega_1}{\omega_f}\right)^2}} \tag{12}$$

Substituting Equation 12 into Equation 2 yields the peak steady-state acceleration response at the center of the beam, $a_p$, as follows

$$\begin{aligned} a_p &= \ddot{y}_{\max} = \phi_{1\max}\ddot{\eta}_{1\max} = \sqrt{\frac{2g}{w_t\ell}}\ddot{\eta}_{1\max} \\ &= \frac{\alpha \frac{w_p}{w_t}\frac{4}{\pi} g}{\sqrt{\left(\frac{\omega_1^2}{\omega_f^2} - 1\right)^2 + \left(\frac{2\beta_1\omega_1}{\omega_f}\right)^2}} \end{aligned} \tag{13}$$

The expression for peak acceleration of the floor due to a harmonic rhythmic force, Equation 2.4 in Design Guide 11 (Murray et al., 1997), is given as

$$\frac{a_p}{g} = \frac{1.3\alpha \frac{w_p}{w_t}}{\sqrt{\left(\frac{f_n^2}{f^2} - 1\right)^2 + \left(\frac{2\beta f_n}{f}\right)^2}} \tag{14}$$

where $f_n = \omega_1 / 2\pi$ and $f = \omega_f / 2\pi$. When comparing Equations 13 and 14, one can see that the $4/\pi$ is replaced by a constant, 1.3, rounded to one decimal place. It is important to note that this constant would be different if either the mode shape expression, Equation 5, or the load distribution changed.

It is convenient to refer to the constant noted above as $k$ and understand that $k$ can be found from the more general expression

$$k = \phi_{1\max} N_{1\max} \frac{w_t / g}{\alpha w_p} \tag{15}$$

## DETERMINATION OF THE MODIFIED CONSTANT

This section, and the thrust of this paper, will derive a method for determining the constant, to replace 1.3 in Equation 14, when the floor has a two dimensional mode shape, as in a beam/girder floor system, and only a partial loading of the bay.

The first step is to define an expression that reasonably characterizes the mode shape of a beam/girder bay for the fundamental mode. This expression is shown below and the parameters are defined in Figure 2.

$$\phi_1(x, y) = A_j \sin\frac{\pi x}{\ell_j} + A_g \sin\frac{\pi y}{\ell_g} \tag{16}$$

Equation 16 can be compared to Equation 5 to understand the parallel with the simple beam case. Note that the expression of the mode shape necessarily becomes two-dimensional.

The next step is to determine the amplitudes, $A_j$ and $A_g$, in the mode shape such that the modal mass, $m_1$, is unity. Paralleling Equations 7 through 9 in the previous section,

$$m_i = \iint \bar{m}\phi_i^2(x, y)dxdy = 1 \tag{17}$$

$$m_1 = \int_0^{\ell_j}\int_0^{\ell_g} \bar{m}\left(A_j \sin\frac{\pi x}{\ell_j} + A_g \sin\frac{\pi y}{\ell_g}\right)^2 dxdy = 1 \tag{18}$$

From inspection of Equation 18, it is obvious that the two unknowns, $A_j$ and $A_g$, cannot be determined from this single expression. It is therefore reasonable and necessary to assume that

$$\frac{A_g}{A_j} = \frac{\Delta_g}{\Delta_j} \tag{19}$$

where $\Delta_g$ and $\Delta_j$ are as defined in the unmodified criterion of *Design Guide 11*. After some manipulation and substituting $w_t/g$ for $\bar{m}$, $A_j$ and $A_g$ are determined to be

$$A_j = \sqrt{\frac{2g\pi^2\Delta_j^2}{w_t\ell_j\ell_g(\pi^2\Delta_j^2 + 16\Delta_j\Delta_g + \pi^2\Delta_g^2)}} \tag{20}$$

$$A_g = \sqrt{\frac{2g\pi^2\Delta_g^2}{w_t\ell_j\ell_g(\pi^2\Delta_j^2 + 16\Delta_j\Delta_g + \pi^2\Delta_g^2)}} \tag{21}$$

Therefore, $\phi_{1\max}$, to be substituted in Equation 15, becomes

$$\phi_{1\max} = A_j + A_g \tag{22}$$

The final step in determining the modified constant, as expressed by Equation 15, is to determine $N_{1\max}$. The expression for determining $N_1$ is shown below along with the derivation of $N_{1\max}$ for partial loading.

$$N_1(x, y) = \iint \phi_1(x, y) f(x, y, t)dxdy \tag{23}$$

For the limits of partial loading shown in Figure 3, a sinusoidal force ( $f(t) = \alpha w_p \sin\omega_f t$), and the mode shape expressed by Equation 16,

$$N_1(t) = \int_{\ell_{j1}}^{\ell_{j2}}\int_{\ell_{g1}}^{\ell_{g2}} \left(A_j \sin\frac{\pi x}{\ell_j} + A_g \sin\frac{\pi y}{\ell_g}\right)\left(\alpha w_p \sin\omega_f t\right)dxdy \tag{24}$$

where $A_j$ and $A_g$ are defined in Equations 20 and 21. Therefore,

$$N_{1\max} = \int_{\ell_{j1}}^{\ell_{j2}}\int_{\ell_{g1}}^{\ell_{g2}} \left(A_j \sin\frac{\pi x}{\ell_j} + A_g \sin\frac{\pi y}{\ell_g}\right)\left(\alpha w_p\right)dxdy \tag{25}$$

After integrating and simplifying,

$$N_{1\max} = \frac{\alpha w_p}{\pi}\left[A_j\ell_j\left(\ell_{g2} - \ell_{g1}\right)\left(\cos\frac{\pi\ell_{j1}}{\ell_j} - \cos\frac{\pi\ell_{j2}}{\ell_j}\right) + A_g\ell_g\left(\ell_{j2} - \ell_{j1}\right)\left(\cos\frac{\pi\ell_{g1}}{\ell_g} - \cos\frac{\pi\ell_{g2}}{\ell_g}\right)\right] \tag{26}$$

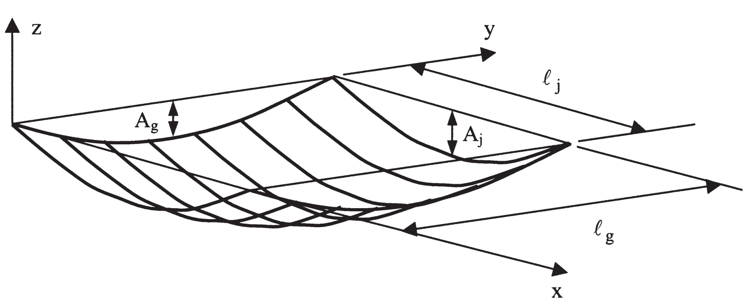


*Fig. 2. Fundamental mode shape for a beam/girder bay.*

Substituting Equations 20, 21, and 26 into Equation 15, the constant, $k$, becomes

$$k = \frac{2\pi}{\ell_g \ell_j}\left(\sqrt{c_j}+\sqrt{c_g}\right)\left[\ell_j\sqrt{c_j}\left(\ell_{g2}-\ell_{g1}\right)\left(\cos\frac{\pi\ell_{j1}}{\ell_j}-\cos\frac{\pi\ell_{j2}}{\ell_j}\right)\right.$$
$$\left.+\ell_g\sqrt{c_g}\left(\ell_{j2}-\ell_{j1}\right)\left(\cos\frac{\pi\ell_{g1}}{\ell_g}-\cos\frac{\pi\ell_{g2}}{\ell_g}\right)\right] \quad (27)$$

where

$$c_j = \frac{\Delta_j^2}{\pi^2\Delta_j^2+16\Delta_j\Delta_g+\pi^2\Delta_g^2} \quad (28)$$

$$c_g = \frac{\Delta_g^2}{\pi^2\Delta_j^2+16\Delta_j\Delta_g+\pi^2\Delta_g^2} \quad (29)$$

The expression in Equation 27 can be simplified for the case of a fully loaded bay with the following substitutions: $\ell_{j1} = 0$, $\ell_{j2} = \ell_j$, $\ell_{g1} = 0$, $\ell_{g2} = \ell_g$. Therefore, for a fully loaded beam/girder bay

$$k = 4\pi\left(\sqrt{c_j}+\sqrt{c_g}\right)^2 \quad (30)$$

To facilitate the determination of $k$ for the fully loaded case, $k$ is plotted in Figure 4 for a range of deflection ratios, $\Delta_g / \Delta_j$. From this plot, it can be noted that the maximum possible $k$ value is approximately 1.41. Therefore, the assumption of $k = 1.3$, implied by the Rhythmic Excitation Criterion for dancing (Murray et al., 1997), is not always conservative when assessing beam/girder systems. It should also be noted that Equation 30 reduces to the fully loaded, simple beam case, $k = 4/\pi$ derived in the previous section, by setting $\Delta_g = 0$.

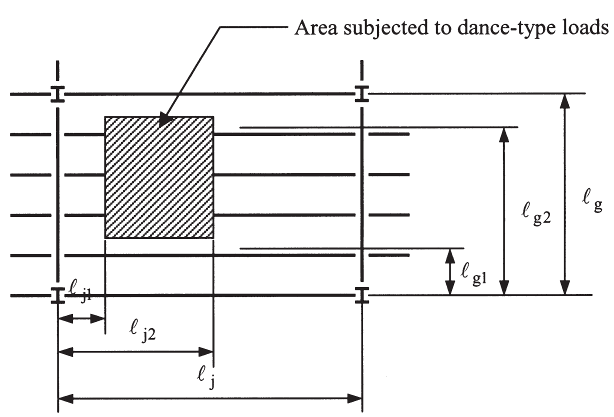


*Fig. 3. Framing plan of bay subjected to partial loading.*

## EXAMPLE USING THE MODIFIED *k* FACTOR

The beam girder system shown in Figure 5 represents a typical bay in a large ballroom that has been designed for strength assuming a 100-psf live load and meets a static live load deflection limit of $L/360$. This example illustrates the use of the modified $k$ factor in determining whether this system will perform acceptably when subjected to dancing over the 20 ft × 20 ft area noted in Figure 5.

### Deck Properties

Concrete: $w_c$ = 110 pcf
$f'_c$ = 3,000 psi

### Beam Properties

W36 × 135
$A$ = 39.7 in.$^2$
$I_x$ = 7,800 in.$^4$
$d$ = 35.55 in.

### Girder Properties

W44 × 262
$A$ = 77.2 in.$^2$
$I_x$ = 24,200 in.$^4$
$d$ = 43.3 in.

### Beam Natural Frequency

With an effective concrete slab width of 120 in. < $0.4L_j$ = 0.4 × 60 × 12 = 288 in., considering only the concrete above the steel deck, and using a dynamic concrete modulus of

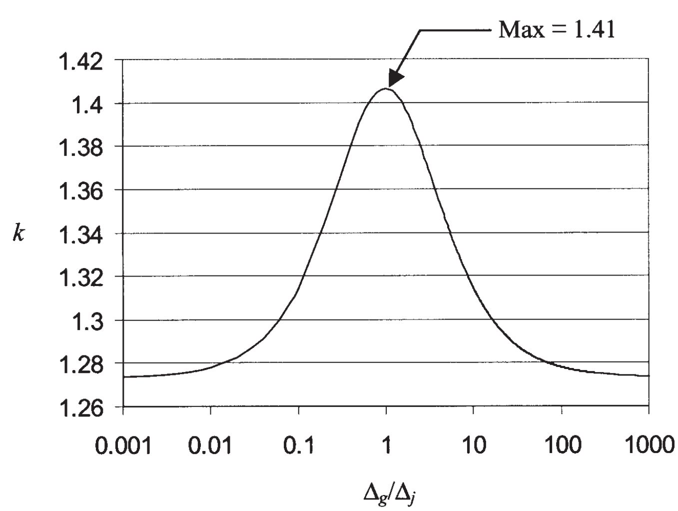


*Fig. 4. Graph of k vs. $\Delta_g/\Delta_j$ for fully loaded bay.*

elasticity of $1.35E_c$, the transformed moment of inertia is calculated as follows

$$E_c = w^{1.5}\sqrt{f'_c} = 110^{1.5}\sqrt{3.0} = 2,000 \text{ ksi}$$

$$n = \text{modular ratio}$$

$$= E_s/1.35E_c = 29,000/(1.35\times 2,000) = 10.75$$

The transformed moment of inertia, assuming composite behavior, is

$$\bar{y} = \frac{(120/10.75)(3.25)(3.25/2) + 39.7(6.25 + 35.55/2)}{(120/10.75)(3.25) + 39.7}$$

$$= 13.33 \text{ in. below the top of slab}$$

$$I_j = 7,800 + (120/10.75)(3.25)^3/12 + 39.7(6.25 + 35.55/2 - 13.33)^2 + (120/10.75)(3.25)(13.33 - 3.25/2)^2 = 17,340 \text{ in.}^4$$

For each beam, the uniformly distributed load is

$$w_j = (46 + 4 + 12.5)(10) + 135 = 760 \text{ plf}$$

where 46 psf is the weight of the slab + deck, 4 psf is an estimate of the actual superimposed dead load, and 12.5 psf is the estimated weight of the participants as recommended in Table 5.3 in *Design Guide 11* (Murray et al., 1997). The corresponding deflection is

$$\Delta_j = \frac{5w_jL_j^4}{384E_sI_j} = \frac{5(0.76)(60)^4(1,728)}{384(29,000)(17,340)} = 0.441 \text{ in.}$$

The beam mode fundamental frequency is

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} = 0.18\sqrt{\frac{386}{0.441}} = 5.32 \text{ Hz}$$

**Girder Natural Frequency**

With an effective slab width of $0.4L_g = (0.4)(40)(12) = 192$ in. $< L_j = (60)(12) = 720$ in. and considering the concrete in the deck ribs, the transformed moment of inertia is calculated as follows

$$\bar{y} = \frac{\left[\begin{array}{l}(192/10.75)(3.25)(3.25/2)\\ +(192/2/10.75)(3.0)(3.25+3.0/2)\\ +77.2(6.25+43.3/2)\end{array}\right]}{\left[\begin{array}{l}(192/10.75)(3.25)\\ +(192/2/10.75)(3.0)+77.2\end{array}\right]}$$

$$= 14.66 \text{ in. below the top of slab}$$

$$I_g = 24,200 + (192/10.75)(3.25)^3/12 + (192/2/10.75)(3.0)^3/12 + 77.2(6.25 + 43.3/2 - 14.66)^2 + (192/10.75)(3.25)(14.66 - 3.25/2)^2 + (192/2/10.75)(3.0)(14.66 - 3.25 - 3.0/2)^2 = 50,300 \text{ in.}^4$$

For each girder, the uniformly distributed load is

$$w_g = (760/10)(60) + 262 = 4,822 \text{ plf}$$

The corresponding deflection is

$$\Delta_g = \frac{5w_gL_g^4}{384E_sI_g} = \frac{5(4.822)(40)^4(1,728)}{384(29,000)(50,300)} = 0.190 \text{ in.}$$

The girder mode fundamental frequency is

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} = 0.18\sqrt{\frac{386}{0.190}} = 8.11 \text{ Hz}$$

**System Natural Frequency**

The system fundamental natural frequency is

$$f_n = 0.18\sqrt{\frac{386}{\Delta_j + \Delta_g}} = 0.18\sqrt{\frac{386}{0.441 + 0.190}} = 4.45 \text{ Hz}$$

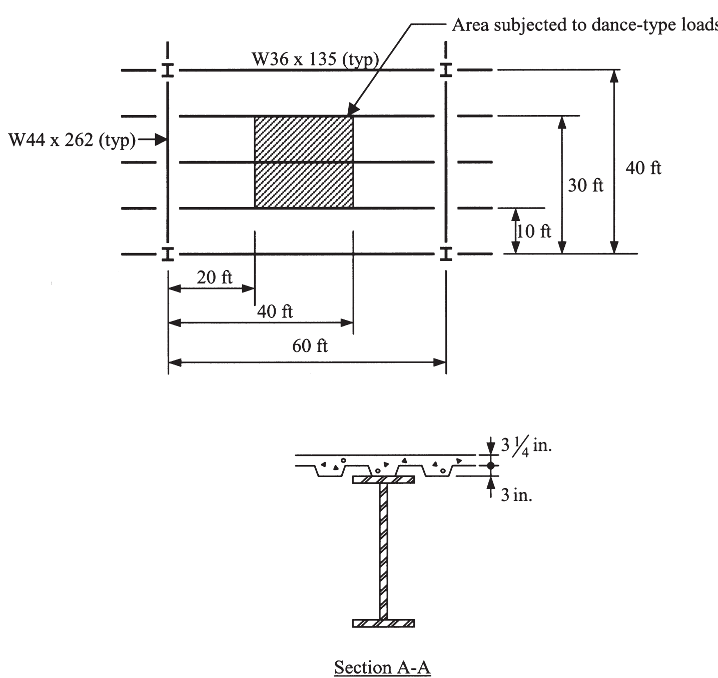


*Fig. 5. Floor plan and section for example problem.*

**Modified $k$ Value for Partial Loading**

The modified $k$ value is computed from Equations 27 through 29 as follows

$$c_j = \frac{\Delta_j^2}{\pi^2\Delta_j^2 + 16\Delta_j\Delta_g + \pi^2\Delta_g^2} = \frac{(0.441)^2}{\pi^2(0.441)^2 + 16(0.441)(0.190) + \pi^2(0.190)^2} = 0.0537$$

$$c_g = \frac{\Delta_g^2}{\pi^2\Delta_j^2 + 16\Delta_j\Delta_g + \pi^2\Delta_g^2} = \frac{(0.190)^2}{\pi^2(0.441)^2 + 16(0.441)(0.190) + \pi^2(0.190)^2} = 0.0100$$

$$k = \frac{2\pi}{\ell_g\ell_j}\left(\sqrt{c_j} + \sqrt{c_g}\right) \left[\ell_j\sqrt{c_j}\left(\ell_{g2} - \ell_{g1}\right)\left(\cos\frac{\pi\ell_{j1}}{\ell_j} - \cos\frac{\pi\ell_{j2}}{\ell_j}\right) + \ell_g\sqrt{c_g}\left(\ell_{j2} - \ell_{j1}\right)\left(\cos\frac{\pi\ell_{g1}}{\ell_g} - \cos\frac{\pi\ell_{g2}}{\ell_g}\right)\right]$$

$$= \frac{2\pi}{(40)(60)}\left(\sqrt{0.0537} + \sqrt{0.0100}\right)\left[60\sqrt{0.0537}\,(30-10)\left(\cos\frac{20\pi}{60} - \cos\frac{40\pi}{60}\right) + 40\sqrt{0.0100}\,(40-20)\left(\cos\frac{10\pi}{40} - \cos\frac{30\pi}{40}\right)\right]$$

$$= 0.340$$

**Peak Steady State Acceleration Ratio**

The peak steady state acceleration can be computed from Equation 14 by substituting the modified constant, $k$, above for 1.3 as follows

$$\frac{a_p}{g} = \frac{k\alpha\frac{w_p}{w_t}}{\sqrt{\left(\frac{f_n^2}{f^2} - 1\right)^2 + \left(\frac{2\beta f_n}{f}\right)^2}} = \frac{(0.340)(0.5)(12.5/76)}{\sqrt{\left(\frac{4.45^2}{2.8^2} - 1\right)^2 + \left(\frac{2(0.03)(4.45)}{2.8}\right)^2}}$$

$$= 0.0183$$

$$a_p = 1.8\%g$$

where the dynamic coefficient, $\alpha = 0.5$, is recommended in Table 5.2 of *Design Guide 11* (Murray et al., 1997), the damping coefficient, $b$, is estimated at 3 percent of critical, and the forcing frequency, $f = 2.8$ Hz, is the upper limit for the first harmonic of group dancing given in Table 2.1 of *Design Guide 11*.

An appropriate limit for dining and dancing is 2%$g$ (Murray et al., 1997); therefore, the floor represented in Figure 5 is considered acceptable.

## CONCLUDING REMARKS

A method to consider partial dance-type loading on a bay of a steel framed, beam/girder system has been presented. The essence of this method is a modification to the 1.3 constant in the "Rhythmic Excitation" Criterion for dancing in AISC *Design Guide 11*. The modification uses Equations 27 through 29 to compute a new constant based on the area of the dance floor. This modification is particularly useful in assessing long span ballroom floors where dancing activities would only take place over a portion of the bay. The derivation is presented as a justification of the proposed modification and is not particularly significant to its application. A detailed example is presented to illustrate the use of the "modified $k$ factor."

## ACKNOWLEDGMENT

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