Effect of Compound Buckling on Compression Strength of Built-up Members

LIAN DUAN, MARK RENO and CHIA-MING UANG

INTRODUCTION

Design engineers frequently use built-up members for steel building and bridge construction for economical reasons. The riveted or bolted laced member and battened member shown in Figures 1(a) and 1(b) have been used in some historical bridges and currently used in heavy industrial buildings to carry high axial load. The smaller, bolted or welded built-up members, like the ones shown in Figures 1(c) and 1(d) are common when axial load is not high. Unlike single members with a solid web, additional issues related to the behavior and design of built-up compression members need to be considered.

The first issue is the shearing effect. For laced or battened members, the shear deformation produced by laces or battens would reduce the buckling capacity. The phenomenon is well understood (Bleich, 1952; Timoshenko and Gere, 1961), and the shearing effect can be considered in design by including a factor α_v, in the computation of effective length, $(KL)_{eff} = \alpha_v (KL)$:

$$\alpha_v = \sqrt{1 + \frac{P_e}{S_v}} \tag{1}$$

where

P_e = elastic buckling load

S_v = shear stiffness of the member (= shear force required to produce a unit shear deformation)

Bleich (1952) suggested the following approximation for α_v:

$$\alpha_v = \sqrt{1 + \frac{300}{(KL/r)^2}} \quad \text{when } KL/r > 40 \tag{2a}$$

$$= 1.1 \quad \text{when } KL/r \le 40 \tag{2b}$$

where

K = effective length factor of a built-up compression member as a whole unit

L = laterally unsupported length of a built-up member in buckling plane

r = radius of gyration of built-up section about axis of buckling acting as a whole unit

For laced member design, the shearing effect has been considered in several major codes (Beedle, 1991), but not in the AISC LRFD Specification (AISC, 1999).

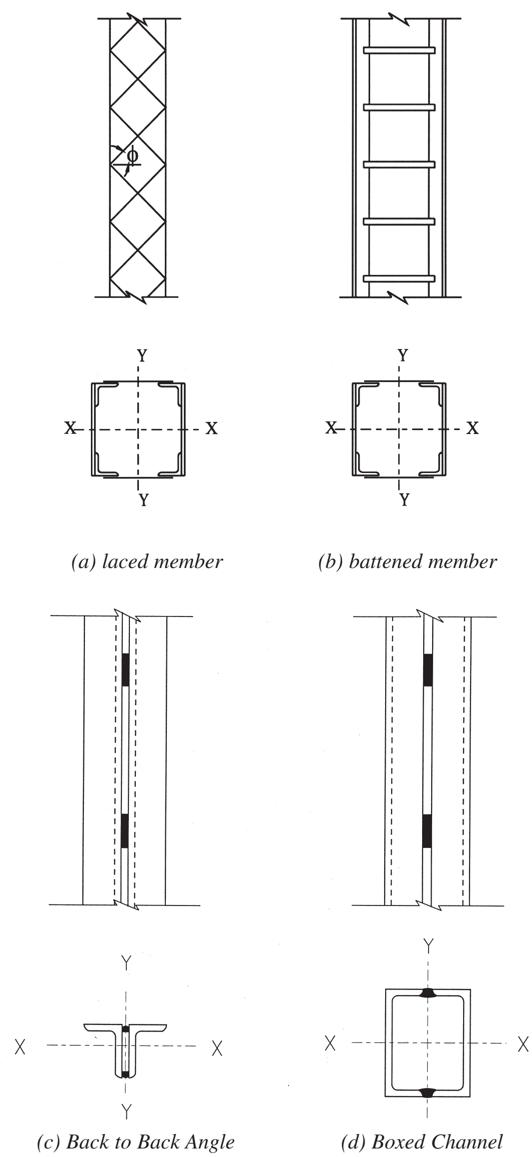

Fig. 1. Typical latticed members.

Lian Duan, senior bridge engineer, Engineering Service Center, California Department of Transportation, Sacramento, CA; professor of structural engineering, Taiyuan University of Technology, China.

Mark Reno, senior bridge engineer, Engineering Service Center, California Department of Transportation, Sacramento, CA.

Chia-Ming Uang, professor, department of structural engineering, University of California, San Diego, La Jolla, CA.

For built-up members like those shown in Figures 1(c) and 1(d) with components interconnected at intervals, a design procedure that considered the shearing effect of the connectors (either high-strength bolts or welds) on the member compression strength was first introduced in the first edition of the AISC LRFD Specification (AISC, 1986; Zahn and Haaijer, 1988). The design provisions were developed based on experimental results of back-to-back channel members interconnected by intermediate welded or bolted filler plates (Zandonini, 1985) and double-angle struts (Astaneh, Goel and Hanson, 1985). This design procedure was subsequently modified based on the work of Aslani and Goel (1991 and 1992) and appeared as Equations E4-1 and E4-2 in the second and third editions (AISC, 1993 and 1999). Note that these equations were derived for stitched members with slip characteristics in the connectors; they are not related to the shearing effect (Equation 1 or Equation 2) of lacing in laced members. The buckling of built-up members in the plane of the connectors was reported by Temple and Elmahdy (1992, 1993, 1995 and 1996).

Aside from the shearing effect, the second issue that would also reduce the compression strength of a built-up member is compound buckling, that is, the interaction between the global buckling mode and the localized buckling mode of flange components between connectors (see Figure 2). The effect of compound buckling on the laced member behavior has been investigated by Koiter and Kuilken (1971), Thompson and Hunt (1973), and Bazant and Cedolin (1991). This issue, which is the subject of this paper, has been largely ignored in major design codes.

Before analytical derivations are presented, it is worthwhile to examine briefly the physical meaning of compound buckling. Geometric imperfections always exist in steel members. Consider the laced member in Figure 2a with an assumed sinusoidal geometric imperfection profile. The imperfection is amplified under an axial load due to the P-δ effect, which results in an axial deformation in addition to axial shortening of the flange components without initial imperfection. The axial stiffness is, by definition, equal to the axial force required for a unit axial deformation. Therefore, the effective axial stiffness is smaller than the elastic axial stiffness of each flange component when the effect of initial geometric imperfection is considered. For a laced member with widely spaced flange components (i.e., large separation between flange components), its moment of inertia is directly proportional to the effective axial stiffness of the flange components. Therefore, the overall buckling capacity of the laced member is also reduced.

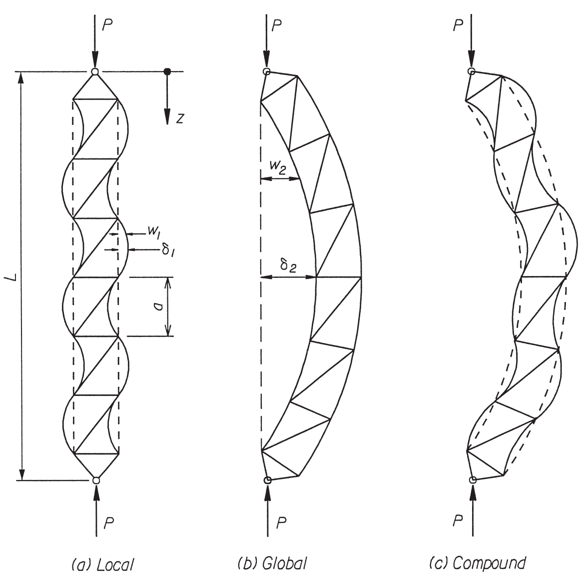

Fig. 2. Buckling models of built-up members.

COMPOUND BUCKLING: ANALYTICAL DERIVATIONS

Analytical derivations of the effect of compound buckling on the elastic buckling load of a laced member have been made by previous researchers (e.g., Bazant and Cedolin, 1991). The derivations assume that the local moment of inertia of flange components can be ignored for the computation of moment of inertia for the whole section, an assumption that is reasonable for laced members with widely spaced flange components. Such an assumption is removed in the following derivations. In this paper, a procedure is developed that uses a factor β to consider the effect of compound buckling in a format which is easy to implement for design purposes.

Consider a pin-ended laced member shown in Figure 2. The area and moment of inertia of each flange component on either side of the axis of buckling (Y-axis) are defined as A_f and I_f (see Figure 3). In the local mode (Figure 2a), the flange component is assumed to buckle as an infinite continuous beams of spans a, in a sinusoidal curve with zero bending moments at the joints (Bazant and Cedolin, 1991). Assume that the member geometric imperfection takes the following form (see Figure 3):

$$w_o = \delta_o \sin\frac{\pi z}{a} \tag{3}$$

where

δ_o = out-of-straightness (see Figure 3)

a = length of each laced panel (see Figure 3)

When a compressive load, P, is applied, the deflection is increased from δ_o to δ_1 due to the P-δ effect:

$$\delta_1 = \delta_o\left(\frac{1}{1-P/P_L}\right) \tag{4}$$

where P_L is the elastic buckling load taken about the axis parallel to the member axis of buckling of the two flange components between the laced panel:

$$P_L = \frac{2\pi^2 EI_f}{a^2} \tag{5}$$

An increase of the out-of-straightness from δ_o to δ_1 results in an additional axial deformation, u, of each laced panel:

$$u = \frac{1}{2}\int_o^a \left[\left(w_o' + w_1'\right)^2 - \left(w_o'\right)^2 \right] \; dz = \frac{\pi^2 \delta_o^2 \left(2P_L - P\right)}{4a\left(P_L - P\right)^2} \tag{6}$$

where $w_1 = \delta_1 \sin(\pi z/a)$. Considering lateral deflection w_1 and the above additional axial deformation, it can be shown that the effective axial stiffness, EA_f^*, is (Bazant and Cedolin, 1991):

$$\frac{1}{EA_f^*} = \frac{1}{EA_f} + \frac{\pi^2 \left(\delta_o / a\right)^2 P_L{}^2}{\left(P_L - P\right)^3} \tag{7}$$

Next, consider the overall buckling of the laced member [see Figure 2b]. For a perfectly straight member, the global buckling load, P_G, is

$$P_G = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EA_f h^2}{2L^2}\left(\frac{1+\alpha^2}{\alpha^2}\right)$$

where

a = separation factor = $h/2r_f$

h = distance between centroids of individual components perpendicular to the member axis of buckling

r_f = radius of gyration of individual flange component relative to its centroidal axis parallel to member axis of buckling = $\sqrt{I_f / A_f}$

A_f = cross-sectional area of individual flange component

I_f = moment of inertia of individual flange component relative to its centroidal axis parallel to the member axis of buckling

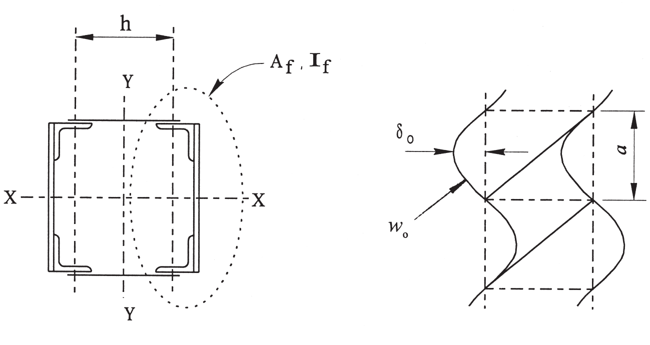

Fig. 3. Typical cross-section and individual flange components.

To include the effect of flange component buckling on the global buckling capacity, replace EA_f and α in Equation 8 with EA_f^* and α_o, respectively, where

$$\alpha_o = \frac{h}{2}\sqrt{\frac{A_f^*}{I_f}} = \frac{L}{a}\sqrt{\frac{\dfrac{\alpha^2}{1+\alpha^2}}{\dfrac{P_L}{P_G} + \dfrac{\left(L/r\right)^2 \left(\delta_o / a\right)^2}{2\left(1 - P/P_L\right)^3}}} \tag{9}$$

The resulting equation that can be used to solve the buckling load is

$$\frac{P_L}{P}\left(\frac{\dfrac{1+\alpha_o^2}{\alpha_o^2}}{\dfrac{1+\alpha^2}{\alpha^2}}\right) - \frac{P_L}{P_G} = \frac{\left(\dfrac{L}{r}\right)^2 \left(\dfrac{\delta_o}{a}\right)^2}{2\left(1 - \dfrac{P}{P_L}\right)^3} \tag{10}$$

For a laced member with widely spaced flange components, the local moment of inertia (I_f) can be ignored for computing the moment of inertia of the built-up section. Such an approximation leads to $\alpha \to \infty$ and $\alpha_o \to \infty$. Thus, Equation 10 can be simplified to that derived by Bazant and Cedolin (1991):

$$\frac{P_L}{P} - \frac{P_L}{P_G} = \frac{\left(\dfrac{L}{r}\right)^2 \left(\dfrac{\delta_o}{a}\right)^2}{2\left(1 - \dfrac{P}{P_L}\right)^3} \tag{11}$$

For design purposes, it is convenient to include the effect of compound buckling on buckling load in the following form:

$$P = \frac{P_G}{\beta^2} = \frac{\pi^2 EI}{\left(\beta L\right)^2} \tag{12}$$

From Equations 10 and 12, β can be solved from the following equation:

$$\beta^2 = \frac{1 + \alpha^2}{1 + \dfrac{\alpha^2}{1 + \dfrac{\left(\delta_o / a\right)^2 \left(a / r_f\right)^2}{2\left(1 - \dfrac{\left(a/r_f\right)^2}{\left(\beta L / r\right)^2}\right)^3}}} \tag{13}$$

Ignoring the local moment of inertia (I_f) for a laced member with widely spaced flange components, the above equation can be reduced to the following:

$$\beta^2 = 1 + \frac{\left(\delta_o / a\right)^2 \left(a / r_f\right)^2}{2\left(1 - \dfrac{\left(a / r_f\right)^2}{\left(\beta L / r\right)^2}\right)^3} \tag{14}$$

The factor β is a function of four variables: the separation factor (α), out-of-straightness ratio (δ_o/a), global slenderness ratio (L/r), and local slenderness ratio (a/r_f). An iterative numerical procedure is needed to solve Equation 13 or 14 for β.

PARAMETER STUDY

Solutions for β based on Equation 13 are presented in graphic forms. Figure 4 shows the variations of β for different α values. The six curves correspond to the increasing α values given in the graph, with the bottom curve corresponding to $\alpha = 0.5$ and the top curve corresponding to $\alpha = \infty$. For $\alpha > 2$, it is observed that variations of α have little effect on the β value, indicating that Equation 14 can be used under this circumstance.

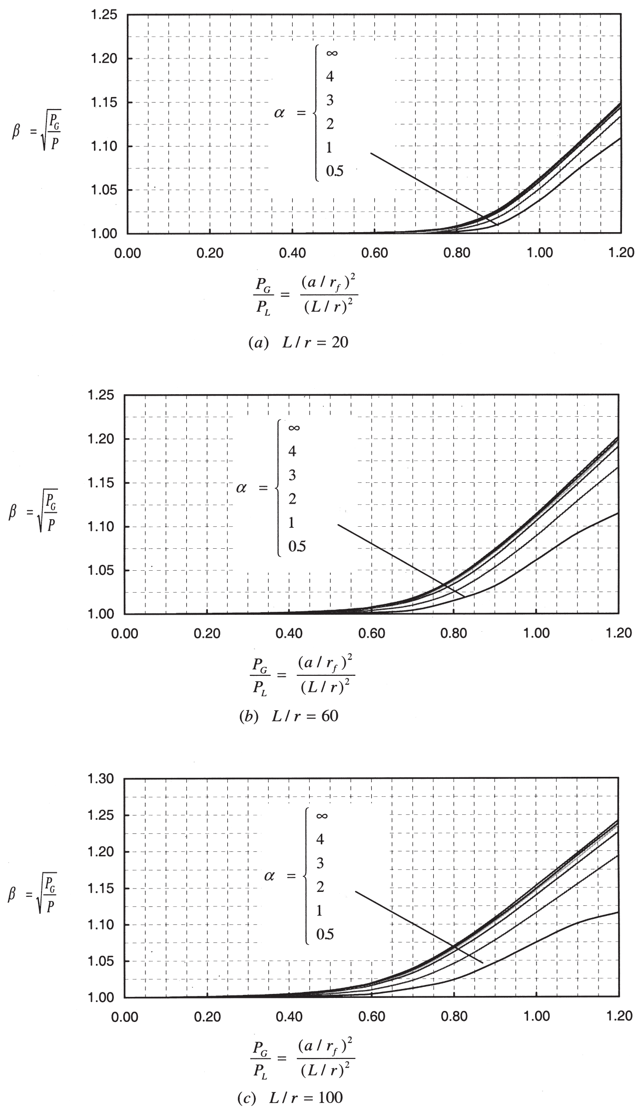

Fig. 4. Effects of Separation Ratio on β *Factor* (δ_o/a = *1/1000).*

The effect of out-of-straightness on the value of β is shown in Figure 5. The three curves given in Figure 5 correspond with the δ_o/a values given in the graph, with the bottom curve corresponding with $\delta_o/a = 1/1500$ and the top curve corresponding with $\delta_o/a = 1/500$. For the particular slenderness ratio ($L/r = 100$) considered, the figure shows that compound buckling is insignificant when P_G/P_L is less than approximately 0.4. The effect of global slenderness ratio on the value of β is shown in Figure 6; the effect of compound buckling is more significant for slender members. The five curves given in Figure 6 correspond consecutively with the L/r values given in the graph, with $L/r = 20$ corresponding to the bottom curve, etc.

DESIGN IMPLICATIONS

Equations 13 and 14 were derived for pin-ended cases. These equations can be generalized for other boundary conditions by replacing L/r by KL/r, where K is the effective length factor for the laced member as a whole unit.

Although the β factor is derived from an elastic theory, the effect of material nonlinearity, residual stresses, and geometrical imperfections on the actual compression strength of the member is included in the F_{cr} formulae in Chapter E of the LRFD Specification (AISC, 1999).

Previous researchers (Bazant and Cedolin, 1991) have pointed out the danger of "naive optimization" in design, i.e., designing a laced member such that the slenderness

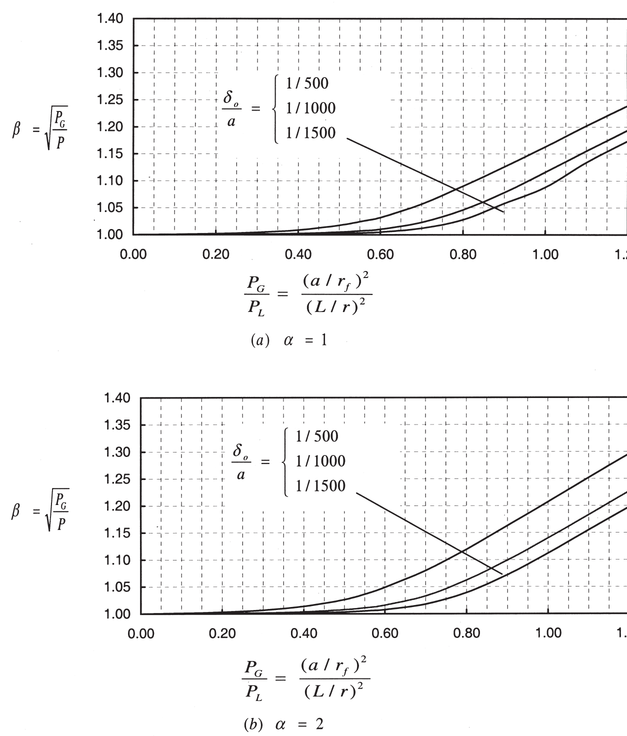

Fig. 5. Effects of Out-of-Straightness Ratio on β *Factor* (L/r = *100).*

ratio of the flange included between lacing connections is close or equal to the overall slenderness ratio of the compression member. For built-up members like those shown in Figures 1c and 1d, LRFD stipulates that $K_a a/r_f$ be no larger than three-fourths of KL/r. It implies that $P_G/P_L = (0.75)^2 = 0.56$. It can be seen from Figure 4 that the effect of compound buckling can be safely ignored for $P_G/P_L \leq 0.56$.

For the design of laced members like those shown in Figures 1a and 1b, however, the AISC LRFD Specification (1999) states that:

> Lacing, including flat bars, angles, channels, or other shapes employed as lacing, shall be so spaced that l/r of the flange included between their connections shall not exceed the governing slenderness ratio for the member as a whole.

Therefore, "naive optimization" may result. Consider a scenario that is not unlikely: a laced member of $KL/r = 100$ with a sinusoidal shape of initial geometric imperfection. Assuming that $\delta_o/a = 1/1500$, which is the basis for establishing the LRFD compressive strength formulae, Figure 5 shows that KL/r should be increased by 12 percent for compound buckling. This increase is significantly higher than the 1.5 percent increase in KL/r for the shearing effect (see Equation 2a).

The next example would demonstrate that the upper limit of $\frac{3}{4}(KL/r)$ for a/r_f effectively mitigates the compound buckling problem. Consider a pin-ended built-up member composed of two channels (see Figure 7) that are interconnected by welded stitches at every one-eighth location. The values of KL/r and a/r_f are 100 and 69, respectively. According to Equation E4-2 of the LRFD Specification (AISC, 1999), KL/r needs to be increased by 17 percent for the shearing effect of welded connectors. Assuming $\delta_o/a = 1/1500$, the β value in Figure 8c for $P_G/P_L = 0.48$ is very close to one, indicating that the effect of compound buckling can be safely ignored. (Note that the curves in Figure 8 correspond consecutively with the KL/r values given in the graph, i.e. the bottom curve represents $KL/r = 20$ and the top curve $KL/r = 200$.) Therefore, it is recommended that the existing AISC LRFD (1999) requirement be revised as follows:

> Lacing, including flat bars, angles, channels, or other shapes employed as lacing, or batten plates, shall be so spaced that l/r of the flange included between their connections shall not exceed three-fourths times the governing slenderness ratio for the laced member as a whole.

Information on the out-of-straightness (or crookedness) at the member level is available. For wide-flange members, the maximum permissible crookedness is about 1/1000th of the member length (AISC, 1999), and actual measurements showed a value of 1/1470th (Bjorhovde, 1972). For a channel like the flange component in Figure 7, the maximum permissible crookedness for the X-X axis buckling is about 1/500th of the member length; for weak-axis buckling of a single channel, the 3rd Edition LRFD Manual (AISC, 2001) states that "Due to the extreme variations in flexibility of these shapes, straightness tolerances for sweep are subject to negotiations between manufacturer and purchaser for individual sections involved." Unfortunately, little data is available. The crookedness mentioned above refers to the geometric imperfection at the member level, not δ_o that occurs within a laced panel (see Figure 3). Since data of δ_o from actual measurements is not available, it is necessary to exercise engineering judgement to determine δ_o/a, say,

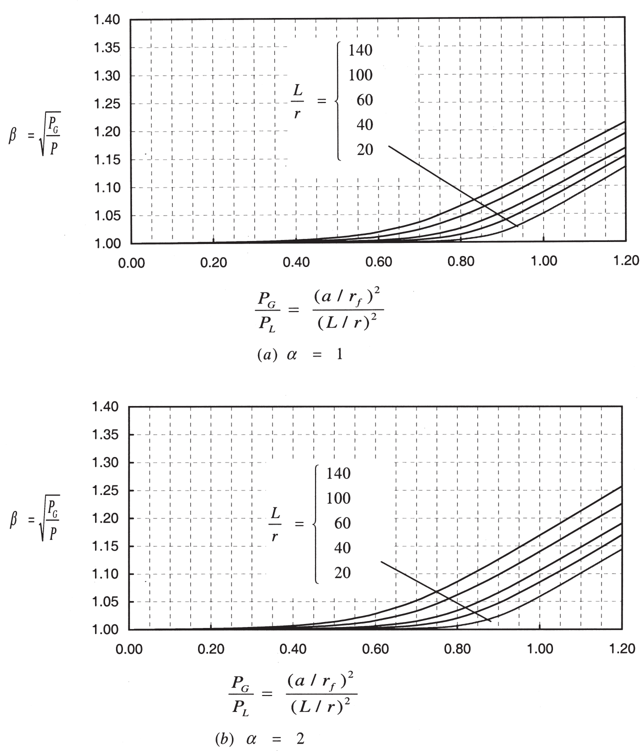

Fig. 6. Effects of global slenderness ratio on β factor (δ_o/a = 1/1000) (a) a = 1, (b) a = 2.

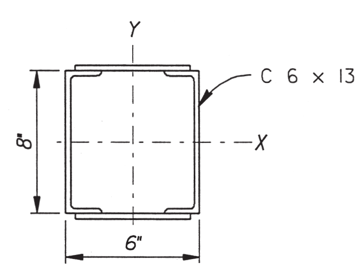

Fig. 7. Built-up member composed of two channels.

1/1000, before β can be determined.

For the purpose of evaluating an existing structure, Figure 8 provides engineers alternative graphical solutions for widely separated built-up members with $\alpha > 2$. In these figures, out-of-straightness ratios (δ_o/a) are 1/500, 1/1000 and 1/1500, and effective slenderness ratios (KL/r) are 20, 40, 60, 100, 140 and 200. In all these figures, the top line represents $KL/r = 200$, and the bottom line represents $KL/r = 20$.

DESIGN EXAMPLE

Given:

A built-up member of 2- C8×18.75 as shown in Figure 9 is subjected to a factored axial load, P_u = 320 kips. Unsupported length L = 26′-10″ and effective length factor K_x = 0.5 and K_y = 1.0; F_y = 50 ksi; E = 29,000 ksi.

Use AISC LRFD Specification (1999) and the proposed procedure to check axial load-carrying capacity and determine the batten plates spacing a.

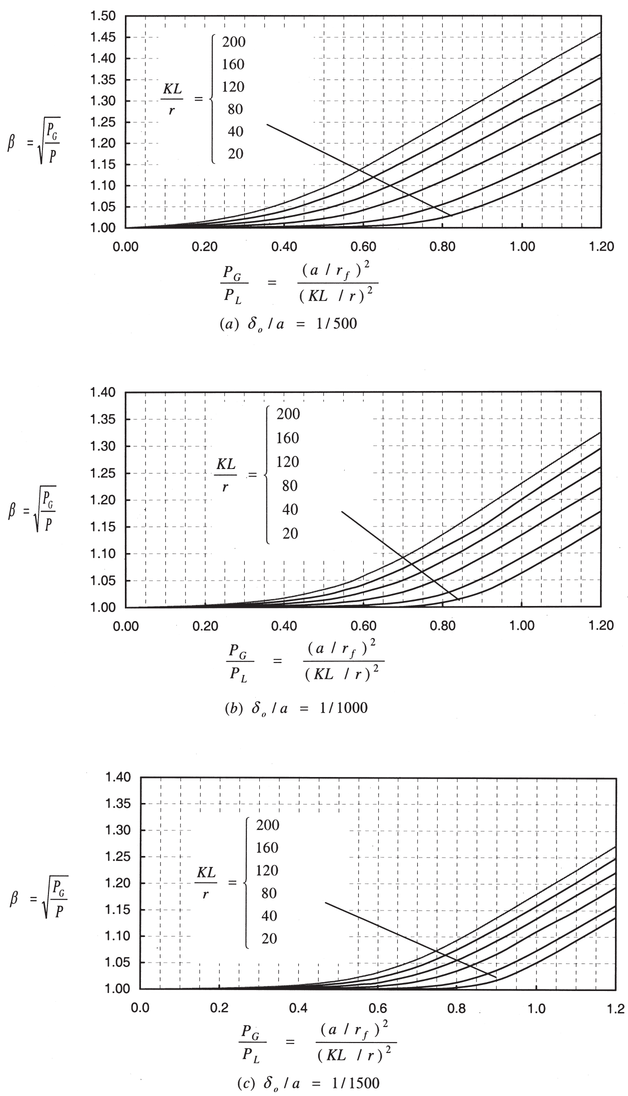

Fig. 8. Buckling mode interaction factor β for $\alpha \geq 2$ (a) δ_o/a = 1/500 , (b) δ_o/a = 1/1000, (c) δ_o/a = 1/1500

Solution:

1. Calculate section properties:

   For a channel section C8×18.75:

   A_g = 5.51 in.^{2}

   I_x = 43.9 in.^{4} $\quad$ r_x = 2.82 in.

   I_y = 1.97 in.^{4} $\quad$ r_y = 0.598 in.

   For built-up section Properties:

   $A_g = 2 \times 5.51 = 11.02$ in.^{2}

   $I_x = 2 \times 43.9 = 88$ in.^{4}

   r_x = 2.82 in.

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{233.59}{11.02}} = 4.6 \text{ in.}$$

$$I_y = 2(1.97) + 2(5.51)(4+0.565)^2 = 233.59 \text{ in.}^4$$

$$\left(\frac{KL}{r}\right)_y = \frac{1.0(322)}{4.6} = 70 > \left(\frac{KL}{r}\right)_x = \frac{0.5\ (322)}{2.82} = 57$$

2. Determine governing buckling plane

   ∴ $(KL/r)_y$ governs and buckling will be in the lacing plane.

3. AISC LRFD Procedure—check axial load-carrying capacity and determine lacing spacing a

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} = \frac{70}{\pi}\sqrt{\frac{50}{29{,}000}} = 0.925 \qquad \text{(AISC E2-4)}$$

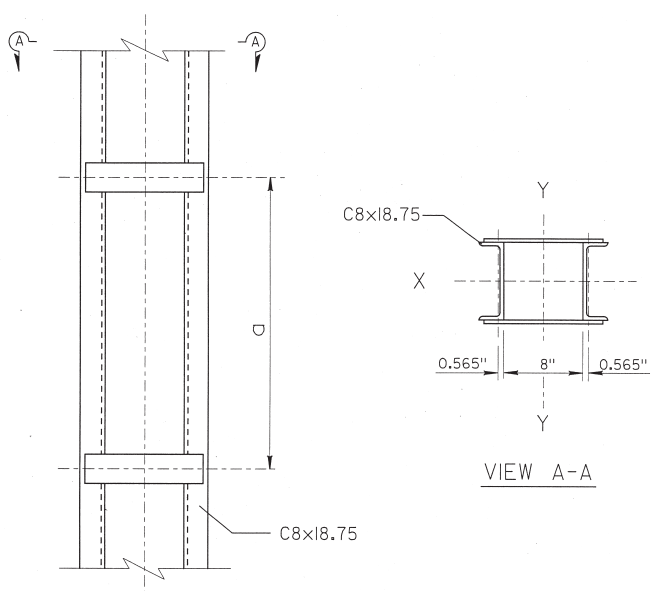

Fig. 9. Details of design example.

$$F_{cr} = \left(0.658^{\lambda_c^2}\right)F_y = 34.95 \text{ ksi} \qquad \text{(AISC E2-2)}$$

$$\phi_c P_n = 0.85A_g F_{cr} = 0.85(11.02)(34.95)$$
$$= 327 \text{ kips} > P_u = 320 \text{ kips} \qquad \textbf{OK}$$

Select batten plate spacing a = 42 in. The slenderness ratio of the flange component between connections (l/r_y = 42/0.6 = 70) does not exceed the slenderness ratio of the built-up member as a whole. Thus, the AISC LRFD requirement is satisfied.

4. Proposed Procedure—check axial load-carrying capacity and determine lacing spacing a

a) Try batten plate spacing a = 42 in. and assume δ_o/a = 1/1000

$P_G/P_L = 1.0$

$$a = \frac{h}{2r_f} = \frac{9}{2(0.6)} = 7.5$$

The β value is equal to 1.12 for KL/r = 70 (see Figure 8b). Therefore,

$$\phi_c P_n = 0.85A_g F_{cr} = 0.85(11.02)(31.91)$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} = \frac{1.12 \times 70}{\pi}\sqrt{\frac{50}{29{,}000}} = 1.036 \qquad \text{(AISC E2-4)}$$

$$F_{cr} = \left(0.658^{\lambda_c^2}\right)F_y = 31.91 \text{ ksi} \qquad \text{(AISC E2-2)}$$

$$= 299 \text{ kips} < P_u = 320 \text{ kips} \qquad \textbf{NG}$$

b) Try batten plate spacing a = 30 in. and assume δ_o/a = 1/1000

$P_G/P_L = 0.51$

$$a = \frac{h}{2r_f} = \frac{9}{2(0.6)} = 7.5$$

The β value is almost equal to 1.0 for KL/r = 70 (see Figure 8b). The effect of compound buckling, therefore, can be safely ignored.

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} = \frac{70}{\pi}\sqrt{\frac{50}{29{,}000}} = 0.925 \qquad \text{(AISC E2-4)}$$

$$F_{cr} = \left(0.658^{\lambda_c^2}\right)F_y = 34.95 \text{ ksi} \qquad \text{(AISC E2-2)}$$

$$\phi_c P_n = 0.85A_g F_{cr} = 0.85(11.02)(34.95)$$
$$= 327 \text{ kips} > P_u = 320 \text{ kips} \qquad \textbf{OK}$$

CONCLUSIONS

Two types of built-up members are commonly used for steel construction. Laced or battened members with widely spaced flange components fall in the first type, and closely spaced steel shapes interconnected at intervals by welds or connectors form the second type. The compressive strength of both types of members is affected by the shearing effect. For the first type, the shearing effect results from the deformation of flanges and laces, while for the second type the effect is caused by the shearing of intermediate connectors. The LRFD Specification (AISC, 1999) considers the shearing effect of the second type, but not the first type.

The compressive strength of built-up members may also be affected by the "compound" buckling due to the interaction between the global buckling mode of the member and the localized flange buckling mode between lacing points or intermediate connectors. In this paper, a factor β was developed to consider the effect of compound buckling. Numerical values of β, that are a function of the global slenderness ratio, local slenderness ratio of flange components, out-of-straightness ratio, and separation factor, were presented in charts. For the second type of built-up members, it was shown that the LRFD approach of limiting the slenderness ratio of flange components to three-quarters of the global slenderness ratio effectively mitigates the effect of compound buckling. For the first type of built-up members, however, no similar limit is specified in the LRFD Specification (AISC 1999); thus, "naive optimization" may result. Therefore, it is recommended that the AISC LRFD Specification statement of:

> Lacing, including flat bars, angles, channels, or other shapes employed as lacing, shall be so spaced that l/r of the flange included between their connections shall not exceed the governing slenderness ratio for the member as a whole.

be revised as:

> Lacing, including flat bars, angles, channels, or other shapes employed as lacing, or batten plates shall be so spaced that l/r of the flange included between their connections shall not exceed three-fourths times the governing slenderness ratio for the laced member as a whole.

Charts that are developed for the β value can be used in evaluating existing structures.

NOTATIONS

The following symbols are used in this paper:

a = length of each laced panel (see Figure 3)
A_f = cross section area of individual flange component
EA_f^* = effective axial stiffness

h = distance between centroids of individual components perpendicular to the member axis of buckling
I_f = moment of inertia of individual flange component relative to its centroidal axis parallel to member axis of buckling
K = Effective length factor of a built-up compression member as a whole unit
L = laterally unsupported length of a built-up member in buckling plane
P_e = elastic buckling load
P_G = elastic global buckling load of built-up member
P_L = elastic buckling load taken about the axis parallel to member axis of buckling of two flange components between laced panel
S_v = shear stiffness of the member (= shear force required to produce a unit shear deformation)
r = radius of gyration of built-up section about axis of buckling acting as a whole unit
r_f = radius of gyration of individual flange component relative to its centroidal axis parallel to member axis of buckling = $\sqrt{I_f / A_f}$
w_1 = deformed shape of local flange component buckling
w_o = initial imperfection deflection shape
α = separation factor = $h/2r_f$
α_v = shearing factor for built-up members
δ_o = out-of-straightness (see Figure 3)
δ_1 = maximum deformation of local flange component
β = buckling mode interaction factor

REFERENCES

AISC (2001), *Load and Resistance Factor Design Manual of Steel Construction*, 3rd ed., American Institute of Steel Construction, Chicago, IL.

AISC (1999), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, December 27, 1999, American Institute of Steel Construction, Chicago, IL.

AISC (1993), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, December 1, 1993, American Institute of Steel Construction, Chicago, IL.

AISC (1986), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, September 1, 1986, American Institute of Steel Construction, Chicago, IL.

Aslani, F. and Goel, S. C. (1991), "An Analytical Criteria for Buckling Strength of Built-up Compression Members," *Engineering Journal*, AISC, Vol. 28, No. 4, pp. 159-168.

Aslani, F. and Goel, S. C. (1992), "Analytical Criteria for Stitch Strength of Built-up Compression Members," *Engineering Journal*, AISC, Vol. 29, No. 3, pp. 102-110.

Astaneh-Asl, A., Goel, S. C., and Hanson, R. D. (1985), "Cyclic Out-of-Plane Buckling for Double-Angle Bracing," *Journal of Structural Engineering*, ASCE, Vol. 111, No. 5, pp. 1135-1153.

Bazant, Z. P. and Cedolin, L. (1991), *Stability of Structures*, Oxford University Press, New York, NY.

Beedle, L. S. (1991), *Stability of Metal Structures: A World View*, SSRC, Lehigh University, Bethlehem, PA.

Bjorhovde, R. (1972), "Deterministic and Probabilistic Approaches to the Strength of Steel Columns," *Ph.D. Dissertation*, Lehigh University, Bethlehem, PA.

Bleich, F. (1952), *Buckling Strength of Metal Structures*, McGraw-Hill Book Company, New York, NY.

Galambos, T. V. (1998), *Guide to Stability Design Criteria for Metal Structures*, 5th ed., Structural Stability Research Council, John Wiley & Sons, New York, NY.

Koiter, W. T. and Kuilken, D. D. C. (1971), "The Interaction between Local Buckling and Overall Buckling on the Behavior of Built-Up Columns," Report No. 447, Laboratory of Engineering Mechanics, Delft.

Temple, M. C. and Elmahdy, G. (1996), "Slenderness Ratio of Main Members between Interconnectors of Built-up Compression Members," *Canadian Journal of Civil Engineering*, Vol. 23, pp. 1295-1302.

Temple, M. C. and Elmahdy, G. (1995), "Local Effective Length Factor in Equivalent Slenderness Ration", *Canadian Journal of Civil Engineering*, Vol. 22, pp. 1164-1170.

Temple, M. C. and Elmahdy, G. (1993), "Buckling of Built-up Compression Members in the Plane of the Connectors", *Canadian Journal of Civil Engineering*, Vol. 20, pp. 895-909.

Temple, M. C. and Elmahdy, G. (1992), "Equivalent Slenderness Ratio for Built-up Members", *Canadian Journal of Civil* Engineering, Vol. 20, pp. 708-711.

Thompson, J. M. T. and Hunt, G. W. (1973), *A General Theory of Elastic Stability*, John Wiley & Sons, Chichester-New York, NY.

Thompson, J. M. T. and Supple, W. J. (1973), "Erosion of Optimum Designs by Compound Branching Phenomena," *J. Mech. Phys. Solids*, Vol. 21, 1973.

Timoshenko, S. P. and Gere, J. M. (1961), *Theory of Elastic Stability*, 2nd Ed., McGraw-Hill Book Company, New York, NY.

Zahn, C. J. and Haaijer, G. (1988), "Effect of Connector Spacing and Flexural-Torsional Buckling on Double-Angle Compressive Strength," *Engineering Journal*, AISC, Vol. 25, No. 3, pp. 109-118.

Zandonini, R. (1985), "Stability of Compact Built-up Struts: Experimental Investigation and Numerical Simulation," Construczione Metalliche, No. 4, Milan, Italy, pp. 202-224.