Design of Mill Building Columns Using Notional Loads

JON A. SCHMIDT

INTRODUCTION

Structural engineers have long recognized that mill building columns constitute a unique design situation. These columns typically support both the roof of the building at the top and one or more crane rails at an intermediate elevation. They accomplish this by taking the form of one of four common configurations (Fisher, 1993): a uniform column with a cantilevered bracket, a stepped column, separate laced columns, or separate battened columns (Figure 1). The last three options involve the use of a stronger and stiffer column section below the crane rail elevation by providing a larger section or a combination of two sections tied together for composite action. The presence of applied crane loads and, often, a transition of section properties at a point within the in-plane unbraced length of the overall column presents special challenges to the designer attempting to assess the member's strength and stability.

Traditionally, determination of the load capacity of a steel column requires the calculation of an effective length factor, K. This value, when multiplied by the column's actual unbraced length, estimates the length of an equivalent pin-ended column with the same buckling load as the actual column. To be reasonably accurate, the effective length factor must account for various influences on the column's behavior, including initial material and geometrical imperfections, inelasticity, end restraint characteristics, and both horizontal and vertical interaction among the columns in the frame being analyzed. Over the years, engineers have developed several special procedures for establishing effective length factors for mill building columns that reflect their unique loading and geometry.

An alternative to the effective length approach is emerging in the United States following its successful application in other countries. The notional load method involves the application to a steel frame of lateral loads that are proportional to the corresponding gravity loads at the same elevations. The ratio of the lateral loads to the gravity loads is primarily a function of material properties and the number of columns in the frame, calibrated to account properly for initial out-of-plumbness and inelasticity. Subsequent second-order elastic analysis of the structure incorporating these loads provides for connection behavior and frame interaction, including additional column moments due to the notional loads. Consequently, the designer can check each column using an axial capacity corresponding to an effective length factor of 1.0.

The purpose of this paper is to propose the application of the notional load method to the design of mill building columns and provide examples demonstrating its use.

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BACKGROUND

Accurate design of columns requires accurate approximation of their actual behavior. Material imperfections (especially residual stresses due to hot-rolling) and geometrical imperfections (especially initial out-of-straightness and initial out-of-plumbness) are inevitable consequences of the process by which the steel shapes used for columns are produced and put in place (Clarke and Bridge, 1992). Researchers developed the strength curve for the design of columns in the AISC *Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 1999) using pin-ended sections with common residual stress patterns and a typical average initial out-of-straightness of about L/1,500 at mid-height (Galambos, 1998). The implied initial out-of-straightness and out-of-plumbness for a column design are therefore dependent upon its effective length factor (Schmidt, 1999). Consequently, these values

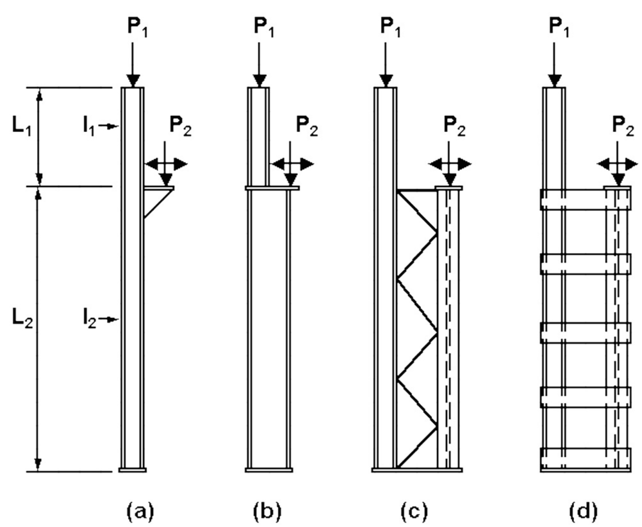

Fig. 1. Typical mill building column types: (a) uniform bracketed, (b) stepped, (c) laced, (d) battened (Fisher, 1993).

are not directly related to the actual geometrical imperfections, which are usually governed by AISC fabrication and erection tolerances of $L/1{,}000$ and $L/500$, respectively (AISC, 2000). Even so, the intuitive nature of the effective length concept and its successful application over the course of many years testify to its adequacy for most ordinary situations encountered in structural design, especially when appropriate refinements are utilized.

Huang (1968) first outlined a procedure for determining the effective length of a stepped column for use in obtaining the axial capacities of its upper and lower segments. He recognized the dependence of this calculation on the ratios of the two segments' lengths, moments of inertia, and applied axial loads, and provided graphs for the effective length factor based on these relationships and the assumption of a fixed base and pinned top. Anderson and Woodward (1972) extended the approach to include other end conditions, including pinned-pinned, fixed-free, fixed-slider (translation permitted but rotation restrained), and fixed-fixed, but did not provide design aids beyond flow charts for their computer programs. Agrawal and Stafiej (1980) corrected the fixed-fixed case, added pinned-fixed and pinned-slider end conditions, and provided comprehensive tables of effective length factors for all seven combinations. All of these researchers had to solve complex transcendental equations in order to obtain their results. The Association of Iron and Steel Engineers (AISE) Technical Report No. 13, *Guide for the Design and Construction of Mill Buildings* (AISE, 1997), recommends a similar column design philosophy and provides corresponding effective length factor tables.

Obviously, a key consideration when using any of these approaches is the assumed location of and connection type at the "top" of the column. Most mill building columns are connected to roof trusses at two points, either at a knee brace and at the bottom chord level, or at the top and bottom chord levels, depending on the truss geometry. Anderson and Woodward (1972) proposed using the following as the effective "top": (1) for a pinned top, midway between the two truss connection points; and (2) for a slider top, the bottom connection point. On the other hand, Bendapudi (1994) suggested that when no knee brace is present, the bottom chord level should always be treated as the column top, since this is the elevation at which horizontal bracing is typically provided. He recommended treating the column top connection as a pin for crane loading, since in this case the bracing will distribute the lateral force to several frames, but as a slider for wind loading, since under this condition all of the frames will sway together. Bendapudi also provided other helpful tips, including procedures for designing lacing systems and fixed bases.

With the increased availability of computers to perform structural analysis, it is no longer absolutely necessary to approximate the restraint provided to mill building columns by roof members or trusses. Instead, designers can now model explicitly an entire plane frame, or even a complete three-dimensional structure. Fraser (1989) developed an alternative method for the design of pin-based uniform columns with brackets in unbraced frames that involves obtaining an effective length factor from the well-known alignment chart (AISC, 1999) and then modifying it to account for the discontinuity of the applied load. He also demonstrated the inaccuracy of simplifying a mill building frame by treating it as though the roof member is located at the crane rail elevation. Fraser (1990) later proposed a direct solution procedure for pin-based stepped columns in unbraced frames and provided several graphs to facilitate its implementation. Lui and Sun (1995) applied an approach developed earlier by Lui (1992) to determine effective length factors for columns with any combination of end conditions. This procedure is intended to approximate the behavior of a frame as it buckles by utilizing its lateral stiffness and distribution of moments when it is subjected to first-order elastic analysis under lateral loads that are an arbitrary but consistent fraction of the gravity loads applied to the frame at the corresponding elevations.

NOTIONAL LOADS

Similarly, the notional load method involves the application of lateral loads to the frame being analyzed. However, in this case, their magnitudes are far from arbitrary. In fact, notional loads actually provide engineers with a means of incorporating realistic column imperfections into their designs without having to model them explicitly. The LRFD (AISC, 1999) column curve already accounts for residual stresses and initial out-of-straightness when the actual unbraced length of the column is used in place of an effective length. The application of appropriately proportioned lateral loads to a gravity-loaded frame will produce forces and moments in the members consistent with those that would result from an initial lateral displacement of one end relative to the other (out-of-plumbness). While the magnitude of such a load should intuitively be directly related to the magnitude of the erection tolerance (Figure 2), this is only true if the subsequent structural analysis will include the effects of inelastic behavior (Kim and Chen, 1996). Since most design offices currently use second-order elastic analysis, it is necessary to adjust the ratio of the notional loads to the gravity loads so as to calibrate the results of such an analysis with those obtained by a more rigorous method.

A recent ASCE report (ASCE, 1997) documents a thorough effort to determine the appropriate notional loads $N_u = \xi P_u$ for use with the LRFD column curves and interaction equations (AISC, 1999). This process incorporated a commonly assumed residual stress distribution

(Galambos and Ketter, 1959), an initial out-of-straightness of L/1,000, and an initial out-of-plumbness of L/500. Subsequent correlation of second-order elastic analyses using notional loads with plastic zone analyses that modeled these imperfections explicitly led to the development of the following simple expression for the notional load parameter (Schmidt, 1999 and ASCE, 1997):

$$\xi = \sqrt{\frac{F_y}{165E}\left(1+\frac{2}{c}\right)}, \qquad c \geq 2 \tag{1}$$

where at a given level of the structure—crane rail or roof for mill buildings—ξ is the ratio of the notional lateral load to the applied gravity load; F_y and E are the yield stress and elastic modulus, respectively, of the material used for the columns; and c is the number of columns in the plane frame that corresponds to the direction of the notional load. Using common values of $F_y = 50$ ksi, $E = 29{,}000$ ksi, and $c = 2$, the notional load parameter $\xi = 0.0046$, which is more than double the value attributable to out-of-plumbness alone (1/500, or 0.002).

Mill building columns tend to have relatively high in-plane effective length factors, resulting in high effective slenderness ratios. The ASCE report (ASCE, 1997) proposes an additional adjustment factor for the notional load parameter to account for this situation, which can be expressed as follows:

$$k_\lambda = \frac{2}{\pi c_r}\sum_r \frac{L}{r}\sqrt{F_y \sum \frac{3I}{L^2}\frac{\Delta_f}{\sum\left(M_{fb}+M_{ft}\right)}} \tag{2}$$

where c_r is the number of restrained columns in a plane frame story and the L/r summation includes only these columns. A first-order elastic analysis of the frame subjected only to the unmodified notional loads provides Δ_f, which is the inter-story drift, and M_{fb} and M_{ft}, which are the moments at the bottom and top of each column, respectively. These moments are considered additive when the column is bent in reverse curvature and subtractive when the column is bent in single curvature. Note that for mill building columns, the designer should treat the crane girder and roof elevations as separate "stories" and calculate different values of k_λ and, hence, the modified notional load $N_u = k_\lambda \xi P_u$ for each. When the frame consists of two restrained columns of identical length and cross-section, Equation (2) can be simplified to

$$k_\lambda = 1.56\sqrt{\frac{P_y \Delta_f}{\sum\left(M_{fb}+M_{ft}\right)}} \tag{3}$$

An obvious disadvantage of this modification, even in its simplified form, is that it requires the designer to know the column and girder or truss sizes. Therefore, it is best used as a refinement to the design after initial proportioning of members using notional loads based on Equation (1) alone. In some cases, the effective slenderness of the columns will be small enough to cause k_λ to be less than one. While it is acceptable to reduce the notional loads accordingly, it is not necessary since the basic notional loads are conservative in this situation.

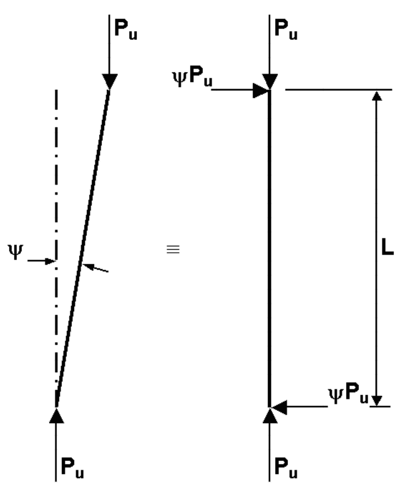

Fig. 2. Equivalent approaches for modeling out-of-plumbness imperfection (ASCE, 1997).

DESIGN CHECKS

For accurate design, the second-order elastic analysis of the structure must incorporate the notional loads into all load combinations, including separate gravity load cases with the notional loads applied in each principal lateral direction unless the geometry and loading are completely symmetrical. For two-dimensional analysis of cases with crane, wind, seismic, or other actual lateral loads, the designer needs only to apply notional loads in the same direction as the real loads. While lateral crane loads will usually be shared with adjacent frames through plan bracing, it is reasonable and conservative to apply the notional loads to each frame at full magnitude. When an engineer employs three-dimensional frame analysis, the ASCE report (ASCE, 1997) recommends that "notional loads be applied simultaneously (at full magnitude) in both principal orthogonal directions of the frame," meaning concurrent with and in each direction perpendicular to any lateral loads except in cases of complete symmetry. The designer can then check each member in the frame for the forces and moments obtained from the two- or three-dimensional second-order elastic analysis using the LRFD (AISC, 1999) beam-column interaction equations, expressed as follows:

$$\frac{P_u}{\phi_c P_{n(L)}} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \le 1$$

$$\frac{P_u}{\phi_c P_{n(L)}} \ge \frac{2}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \tag{4}$$

and

$$\frac{P_u}{2\phi_c P_{n(L)}} + \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \le 1$$

$$\frac{P_u}{\phi_c P_{n(L)}} < \frac{2}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \tag{5}$$

where $P_{n(L)}$ is the axial capacity of the member based on its actual length ($K = 1.0$), and M_{ux} and M_{uy} are the strong- and weak-axis bending moments in the member, respectively, with the effects of the notional loads included. Note that the criterion for whether Equation (4) or Equation (5) applies is different from that given for LRFD Equations H1-1a and H1-1b. The reason for this is illustrated in Figure 3.

Suppose that a column is proportioned to correspond to point A using the effective length approach. Since $P_u/\phi_c P_n$ is greater than 0.2, LRFD Equation H1-1a governs, and the result is equal to the ratio of the segment lengths OA/OA′. The notional load method increases the value of the moment term to offset the decrease in the axial force term

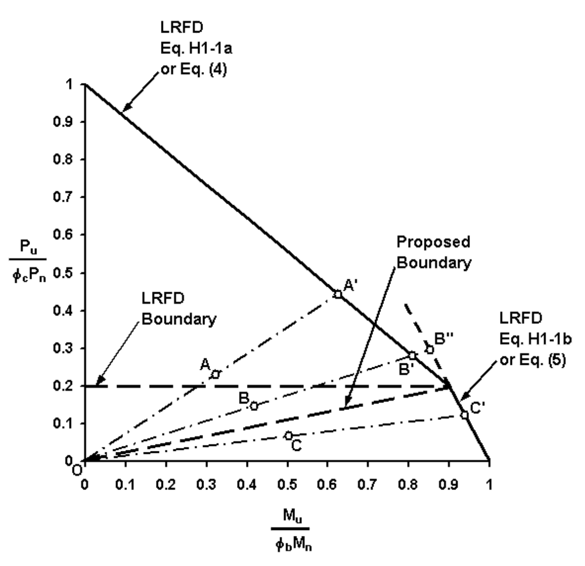

Fig. 3. Beam-column interaction equations (AISC, 1999).

caused by the use of the column's actual unbraced length, rather than its effective length, to calculate axial capacity. This is reflected by a shift on Figure 3 to point B. Since $P_u/\phi_c P_n$ is now less than 0.2, LRFD Equation H1-1b governs, and the result is equal to the ratio OB/OB″. While this correctly shows that the column is still adequate, it does not accurately reflect the reserve strength of the member, resulting in an interaction ratio that is lower than that obtained using the effective length approach. The correct ratio comes from Equation (4) and is equal to OB/OB′, which will be comparable to OA/OA′. Similar results occur for a column that corresponds to point B using effective length and shifts to point C using notional loads if the check is based on LRFD Equation H1-1a for the first case (OB/OB′) and Equation (5) for the second case (OC/OC′).

Some advantages of the notional load method for mill building column design when compared with the effective length approach are readily apparent. It is not necessary to make an assumption as to where the effective "top" of the column is located and whether this point represents a pinned, slider, or fixed connection; the designer simply models the actual frame geometry, which rarely fits precisely into one of these idealized categories. Unless the adjustment factor k_λ from Equation (2) or (3) is significantly greater than one, the notional load parameter is independent of the geometry and section properties of the members in the frame model, simplifying the iterative design process. In addition, the notional load method eliminates the need to incorporate inelastic stiffness reductions into the effective length calculations to improve their accuracy. The latter approach would usually require a separate series of iterative analysis runs for each individual load case to account for the variation in the magnitude of the axial force in each column (ASCE, 1997).

It is important to recognize that the notional load method is calibrated to an ultimate strength (LRFD) design philosophy and is not appropriate for use in Allowable Stress Design (ASD). Also, there is some question as to whether the design of beams, connections, and foundations, as well as deflection evaluation, should include the additional forces created by the notional loads. The imperfections that notional loads are intended to account for really do exist, but are significant primarily at or near failure of the frame. Therefore, it seems logical to include notional load effects in factored load design of beams and connections, but not in foundation design or deflection checks based on service loads. Further research on this issue is needed, not just for mill buildings, but also for any kind of steel frame structure.

SUMMARY

The design of mill building columns using notional loads to account for stability effects is carried out as follows:

1. Determine the applied vertical loads P_u on each column at the crane rail and roof elevations for the various gravity load cases.
2. Calculate the notional load parameter ξ using Equation (1).
3. Apply the lateral notional loads $N_u = \xi P_u$ to the columns at the crane rail and roof elevations, and combine them with the applied gravity and lateral loads as required to account for asymmetrical geometry and/or loading.
4. Perform a two- or three-dimensional second-order elastic analysis of the frame under these load combinations, with the notional loads included.
5. Proportion members based on the forces and moments obtained from this analysis, using Equations (4) and (5).
6. Check the modification factor k_λ from Equation (2) or (3) for the "stories" above and below the crane rail elevation. If $k_\lambda < 1$ or $k_\lambda \approx 1$ for both "stories", then the design can be considered complete. If $k_\lambda > 1$ for a "story", increase the notional loads at that "story" to $N_u = k_\lambda \xi P_u$, analyze the frame again, and revise member sizes as required.

DESIGN EXAMPLES

All of the design examples discussed below incorporate the following assumptions:

1. The steel used for the members has a yield strength of 50 ksi and an elastic modulus of 29,000 ksi. Therefore, with two columns in each frame, Equation (1) gives $\xi = 0.0046$.
2. All loads given are already factored for LRFD design.
3. Accounting for some distribution to adjacent frames, the total lateral crane load carried by the frame being designed is 5 percent of the total vertical crane load and is equally distributed to the two columns. The notional loads are applied to the frame in addition to and in the same direction as this load.
4. Effective length factors are referenced to the overall in-plane unbraced length of each column (base to roof girder or truss), rather than the lengths of the individual segments.
5. Out-of-plane bracing for the column as a whole is present at a maximum of 5 feet on center to ensure that in-plane behavior governs axial capacity, so that comparisons of interaction ratios are meaningful. In actual mill buildings, this will not usually be the case, so that the use of the notional load method will often cause the axial capacity to be governed by out-of-plane behavior. This introduces a number of considerations that merit further study, most notably whether notional loads in one direction should be included in the evaluation of capacity in the orthogonal direction.
6. The compression flange of each column is braced at the base, crane girder, and roof girder or truss levels, and $C_b = 1.0$.
7. "Corrected" effective length factors account for inelastic stiffness reduction where such behavior is indicated by iterative calculations (ASCE, 1997), in which τ is determined from LRFD Equations E2-2, E2-3, and E2-4 (AISC, 1999). In addition, the "corrected" values for the upper column segments are based on treating them as a second story in accordance with Lui's method (Lui, 1992), instead of adhering to the common assumption that their critical loads are proportional to those of the corresponding lower column segments.
8. "Refined" notional loads incorporate k_λ values obtained from Equation (3).
9. The "difference" tabulated for each column reflects the accuracy of the interaction ratio obtained using the "refined" notional loads relative to the ratio obtained using the "corrected" effective length factor.

Examples 1, 2, and 3

Figures 4 and 5 depict pinned-based rigid frames utilized by both Fraser (1989 and 1990) and Lui and Sun (1995) to demonstrate their proposed methods of calculating effective length factors for uniform bracketed and stepped mill building columns, respectively. The various eccentricities shown are arbitrary, since none of these references dealt with actually checking the adequacy of the columns under the applied loads. For the uniform bracketed columns in Figure 4, the basic notional loads are 0.72 kips at the left crane girder, 0.21 kips at the right crane girder, and 0.24 kips at each column at the roof level. For the stepped columns in Figure 5, the corresponding basic notional loads are 1.38 kips, 0.64 kips, and 0.24 kips.

Tables 1 and 2 compare the interaction ratios obtained for each segment of each column using the tables provided by Agrawal and Stefiej (1980) for the pinned-slider case, Fraser's methods (Fraser, 1989 and 1990), Lui and Sun's method (Lui and Sun, 1995), and the notional load method. The "corrected" effective length factors for the frame in Figure 4 (Table 1) indicate a stiffness reduction factor for the left lower column only of 0.86, while inelasticity had no effect on the effective length factors for the frame in Figure 5 (Table 2). The "refined" notional loads result from k_λ values for the crane girder and roof levels of 1.39 and 1.66 for the frame in Figure 4 (Table 1), and 2.17 and 2.69 for the frame in Figure 5 (Table 2).

The results for both frames demonstrate the inaccuracy of using effective length factors that do not account for frame action and the conservatism of using elastic effective length factors instead of inelastic values. Table 1 shows the general agreement of interaction ratios obtained using the

Table 1. Interaction Ratios for Frame in Figure 4

Column	Left Lower		Left Upper		Right Lower		Right Upper	
Method	*K*	Ratio	*K*	Ratio	*K*	Ratio	*K*	Ratio
Agrawal & Stafiej (1980)	2.06	0.830	4.15	0.395	2.01	0.763	2.99	0.294
Fraser (1989)	2.13	0.838	4.25	0.403	3.12	0.848	4.25	0.415
Lui & Sun (1995)	2.14	0.839	4.27	0.404	3.13	0.850	4.27	0.417
Lui & Sun (1995), Corrected	2.03	0.827	2.91	0.315	3.22	0.863	2.91	0.288
Basic Notional Load	1.00	0.835	1.00	0.303	1.00	0.800	1.00	0.252
Refined Notional Load	1.00	0.837	1.00	0.313	1.00	0.826	1.00	0.253
% Difference[a]		1.2%		-0.6%		-4.3%		-12%

[a]Refined Notional Load vs. Lui & Sun (1995), Corrected

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Lui & Sun (1995)	2.14	0.839	4.27	0.404	3.13	0.850	4.27	0.417
Lui & Sun (1995), Corrected	2.03	0.827	2.91	0.315	3.22	0.863	2.91	0.288
Basic Notional Load	1.00	0.835	1.00	0.303	1.00	0.800	1.00	0.252
Refined Notional Load	1.00	0.837	1.00	0.313	1.00	0.826	1.00	0.253
% Difference[a]		1.2%		-0.6%		-4.3%		-12%

[a]Refined Notional Load vs. Lui & Sun (1995), Corrected

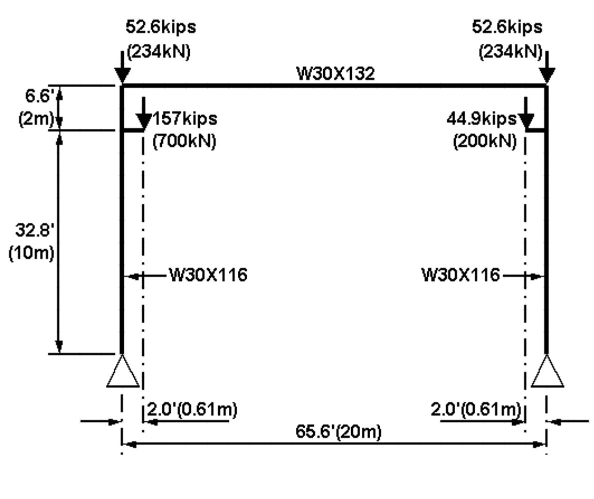

Fig. 4. Example 1: Mill building frame with uniform bracketed columns and roof girder (Fraser, 1989; Lui and Sun, 1995).

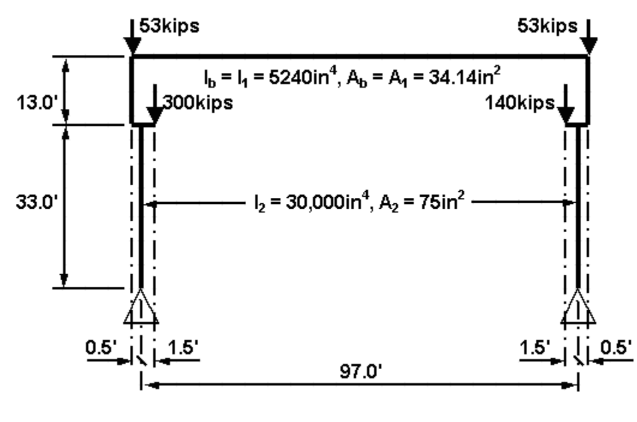

Fig. 5. Examples 2 and 3: Mill building frame with stepped columns and roof girder (Fraser, 1990; Lui and Sun, 1995).

Table 3. Load Cases for Frame in Figure 5 with Fixed-Based Columns				
Load Case	Crane Loads (kips)		Roof Loads (kips)	
	Left Column	Right Column	Left Column	Right Column
I	440	0	53	53
II	330	110	53	53
III	220	220	53	53

notional load method with those provided by the inelastic effective length approach for the frame in Figure 4. This is true whether or not the notional loads are refined using k_λ from Equation (3). However, Table 2 shows significant differences between the interaction ratios from the two procedures, especially for the lower, larger column segments, even when the k_λ values are applied to the notional loads. This is likely due to the dependence of the frame on its lighter upper column segments and roof girder for lateral stiffness, since the crane girder elevation is not actually a "story" with a rigidly connected member between the columns. It may also be a reflection of the fact that the deformation pattern of the pinned-base columns does not closely match that assumed in the calibration of the notional load parameter (ASCE, 1997). Further research on the applicability of the notional load method to pinned-based stepped mill building columns is needed.

In practice, most mill building columns are fixed at the base in accordance with the recommendation of AISE Technical Report No. 13 (AISE, 1997). Lui and Sun (1995) extended their method of determining effective length factors to a frame identical to that in Figure 5, except with fixed column bases and varying crane loads (Table 3). Table 4 provides the interaction ratios for the three different crane load cases using the methods of Agrawal and Stefiej (1980) for the fixed-slider case, Lui and Sun (1995), and notional loads. Table 5 provides the stiffness reduction factors calculated iteratively for the various column segments and used to determine the "corrected" effective length factors. Once again, there is good agreement between the notional load method and the effective length approach. It should be noted that for this particular example, the ratios are relatively insensitive to effective length; hence, uncorrected values of both the effective length factors and the notional load parameters would probably be sufficiently accurate for design use. In fact, for all three cases, Equation (3) provided k_λ values less than one, indicating that the basic notional loads are actually somewhat conservative.

Example 4

Figure 6 depicts a fixed-based rigid frame with stepped columns and a roof truss, rather than a roof girder, providing an opportunity for a comprehensive examination of the various approaches outlined above for design of the columns. The results are provided in Table 8. Agrawal and Stefiej (1980) require calculation of the ratios I_1/I_2 = 999/17,300 = 0.058 ≈ 0.1, L_2/L_T = 50/60 = 0.833, $(P_2/P_T)_L$ = 480/570 = 0.842 ≈ 0.8, and $(P_2/P_T)_R$ = 120/210 = 0.571 ≈ 0.6. Their tables for the fixed-slider case require interpolation between L_2/L_T values of 0.7 and 0.9 to obtain the K values shown. Lui and Sun (1995) require first-order analysis of the frame under lateral loads proportional to the gravity loads. For this example, the basic notional loads will be used for this purpose and are equal to 2.21 kips at the left crane girder, 0.55 kips at the right crane girder, and 0.41 kips at the top of each column. The various parameters required to determine effective length factors using this approach are shown in Tables 6 and 7 for the elastic and inelastic cases, respectively. Note that the designer must go through several iterations of the necessary calculations, repeatedly revising the analysis to reflect new stiffness reduction factors (τ), before the values converge to those shown in Table 7. Since the k_λ values for the two "stories" of this frame are 1.02 and 1.00, there is minimal change in the interaction ratios when using the refined notional load method.

The results again show good agreement between the notional load method and the inelastic effective length approach. In fact, notional loads produce rather conserva-

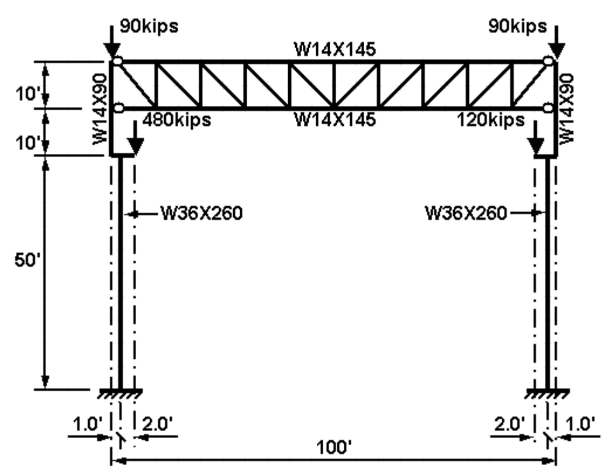

Fig. 6. Example 4: Mill building frame with stepped columns and roof truss.

L C	Column Method	Left Lower K	Left Lower Ratio	Left Upper K	Left Upper Ratio	Right Lower K	Right Lower Ratio	Right Upper K	Right Upper Ratio
I	Agrawal & Stafiej (1980)	1.23	0.310	1.23	0.187	1.38	0.204	0.62	0.112
	Lui & Sun (1995)	1.03	0.305	1.34	0.189	3.16	0.209	1.34	0.119
	Lui & Sun (1995) (Corrected)	0.72	0.314	1.38	0.195	4.28	0.221	1.38	0.126
	Notional Load	1.00	0.317	1.00	0.191	1.00	0.213	1.00	0.118
	Notional Load (Refined)	1.00	0.312	1.00	0.188	1.00	0.209	1.00	0.116
	% Difference[a]		-0.6%		-3.6%		-5.4%		-7.9%
II	Agrawal & Stafiej (1980)	1.23	0.236	1.23	0.165	1.25	0.194	1.07	0.120
	Lui & Sun (1995)	1.17	0.235	1.34	0.167	1.80	0.199	1.34	0.124
	Lui & Sun (1995) (Corrected)	0.98	0.232	1.58	0.172	2.08	0.203	1.58	0.129
	Notional Load	1.00	0.244	1.00	0.166	1.00	0.201	1.00	0.118
	Not. Load (Refined)	1.00	0.239	1.00	0.164	1.00	0.198	1.00	0.118
	% Difference[a]		3.0%		-4.7%		-2.5%		-8.5%
III	Agrawal & Stafiej (1980)	1.24	0.196	1.24	0.144	1.24	0.196	1.24	0.144
	Lui & Sun (1995)	1.38	0.198	1.33	0.145	1.38	0.198	1.33	0.145
	Lui & Sun (1995) (Corrected)	1.36	0.197	1.66	0.152	1.36	0.197	1.66	0.152
	Notional Load	1.00	0.203	1.00	0.142	1.00	0.203	1.00	0.142
	Not. Load (Ref.)	1.00	0.199	1.00	0.141	1.00	0.199	1.00	0.141
	% Difference[a]		1.0%		-7.2%		1.0%		-7.2%

Table 4. Interaction Ratios for Frame in Figure 5 with Fixed-Based Columns

[a]Notional Load (Refined) vs. Lui & Sun (1995) (Corrected)

tive interaction ratios for the upper column segments of this frame, since their effective lengths are less than the actual unbraced lengths. Even so, unless wind or seismic load cases control the design, it would appear that the columns could be reduced in size as a design refinement. Determining inelastic effective length factors for the new configuration would require revising all of the tedious calculations outlined above. However, the basic notional loads remain unchanged, allowing the designer to check the new member sizes much more quickly.

It is of interest to investigate the impact of removing assumption (5) above in favor of a more realistic out-of-plane bracing arrangement—at the roof, truss bottom chord, and crane rail elevations, and midway between the base and the crane rail elevation (25-ft unbraced length). For this situation, out-of-plane behavior governs the axial strength of the left lower column segment for either the notional load method or the effective length approach. Consequently, the interaction ratio obtained for this segment with the in-plane notional loads included in the analysis (0.754) is somewhat conservative (by 6.5 percent) when compared with that obtained using the effective length approach (0.708). In-plane behavior still governs the axial strength of the right lower column segment using the inelastic effective length factor of 2.36, but out-of-plane behavior governs when $K = 1$ for the notional load method. The interaction ratios turn out to be very close (0.770 and 0.778, respectively, a 1.0 percent difference). The results for the two upper segments are unchanged since in-plane behavior still governs their design using either procedure.

CONCLUSION

The notional load method provides engineers with a practical new procedure for the structural design of steel columns in frames, particularly under unique design conditions such as those that exist for mill building columns. Notional loads account for initial imperfections more directly than effective lengths. Unlike the effective length factor in most methods, the notional load parameter is not necessarily dependent on the relative stiffnesses of frame members and therefore need not usually be recalculated every time section properties are changed. Notional loads also accurately incorporate inelastic effects without the need for iteration to determine stiffness reductions. Areas that require further study include whether notional loads should be incorporated into beam, connection, foundation, and/or deflection checks; the applicability of in-plane notional loads on columns controlled by out-of-plane stability considerations; and the use of notional loads to design pinned-based stepped mill building columns. Even so, future editions of AISE Technical Report No. 13 (AISE, 1997) should include the notional load method as an alternative for mill building column design.

Table 5. Stiffness Reduction Factors for Frame in Figure 5 with Fixed-Based Columns				
Load Case	**Left Lower**	**Left Upper**	**Right Lower**	**Right Upper**
I	0.27	0.70	1.00	0.70
II	0.36	0.78	0.69	0.78
III	0.48	0.82	0.48	0.82

Table 6. Lui & Sun (1995) (Elastic) Parameters for Frame in Figure 6				
Segment	**Lower**		**Upper**	
Parameter	**Left Column**	**Right Column**	**Left Column**	**Right Column**
M_s (kip-ft)	9.44	0.94	7.29	1.22
M_L (kip-ft)	89.0	79.7	9.71	11.8
Curvature	Reverse	Reverse	Single	Single
η (kips/in.)	8.26	7.10	29.6	42.7
Δ_i (in)	0.239		0.048	
$\sum P/L$ (kips/in.)	1.30		1.50	
$\sum H$ (kips)	3.58		0.82	

Table 7. Lui & Sun (1995) (Inelastic) Parameters for Frame in Figure 6				
Segment	**Lower**		**Upper**	
Parameter	**Left Column**	**Right Column**	**Left Column**	**Right Column**
τ	0.67	1.00	0.93	0.93
M_s (kip-ft)	11.8	0.14	7.06	0.43
M_L (kip-ft)	73.8	93.3	12.1	13.6
Curvature	Reverse	Reverse	Single	Single
η (kips/in.)	6.04	6.98	25.5	44.7
Δ_i (in.)	0.284		0.056	
$\sum P/L$ (kips/in.)	1.30		1.50	
$\sum H$ (kips)	3.58		0.82	

Table 8. Interaction Ratios for Frame in Figure 6								
Column	**Left Lower**		**Left Upper**		**Right Lower**		**Right Upper**	
Method	***K***	**Ratio**	***K***	**Ratio**	***K***	**Ratio**	***K***	**Ratio**
Agrawal & Stafiej (1980)	1.39	0.672	0.98	0.565	1.41	0.724	0.71	0.380
Lui & Sun (1995) (Elastic)	1.32	0.664	0.75	0.497	2.17	0.751	0.75	0.388
Lui & Sun (1995) (Inelastic)	1.18	0.651	0.78	0.504	2.36	0.770	0.78	0.395
Basic Notional Load	1.00	0.683	1.00	0.588	1.00	0.764	1.00	0.489
Refined Notional Load	1.00	0.684	1.00	0.589	1.00	0.765	1.00	0.490
% Difference[a]		5.1%		17%		-0.6%		24%
[a]Refined Notional Load vs. Lui & Sun (1995) (Inelastic)								

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