Cantilever Beam Framing Systems

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INTRODUCTION

Cantilever beam framing is a multiple-span system of framing which is more efficient than simple beam framing, but simpler to design and erect than continuous beam systems. It involves cantilevering the beam from one span past a support a short distance into the next span, where it supports the end of the next beam using a simple shear connection. This is shown in Figure 1 for two-span and three-span configurations. The configurations are identified as SC (suspended span/cantilever), CSC (cantilever/suspended span/cantilever), and SCS (suspended span/cantilever/suspended span).

This type of system is advantageous because the positive moment of the cantilevered beam is reduced by the negative moment induced over the support, and the moment in the suspended beam is reduced by the shorter effective span. Because the beams usually run over the tops of columns, this framing scheme is best suited to roof and mezzanine support. It is most used in warehouse and industrial roof framing. Cantilever systems are also used occasionally in steel bridge construction. Cantilever framing is usually executed in steel or light timber, due to the need for a simple and economical "hinge" connection. Typical hinge connections (shear splices) are shown in Figure 2. The detail utilizing a pair of plates is the simplest, cheapest, and most used. The detail comprising angles bolted back to back has the advantage of having no loose field pieces to get lost. The choice of detail is usually left to the fabricator. It is important to note that the detail shown for light timber framing should not be replicated in steel, especially for wide-flange beams.

ADVANTAGES

The primary advantages of cantilever framing are economy, stiffness, and ease of erection. Compared to simple beam framing, the use of cantilever framing may result in fifteen to thirty percent savings in the roof beams, on the order of \$0.15 to \$0.30 per square foot of roof. On large roofs, this can amount to considerable savings. Larger bays and longer spans tend to increase the advantage. Cantilever systems also tend to be slightly more economical than continuous beam systems in lengths long enough to require separate field pieces (the actual lengths of beam shipped to the site) with splices. Continuous beams usually require moment splices. These are more expensive than simple shear splices, and, more importantly, require all field pieces to be the same nominal depth. Cantilever systems can and often do vary the depth of different field pieces, reducing the total steel weight.

The second advantage of cantilever framing is stiffness. A typical cantilever system designed on the basis of flexural strength typically has about half the deflection of an equivalent simple span system, and is almost as stiff as a continuous beam system. This stiffness is important for roof ponding considerations. In many cases where a simple-span system is governed by deflection and ponding criteria, a cantilever-beam system is stiff enough to be governed by flexural strength, thus requiring less steel.

The third major advantage of cantilever systems is ease of fabrication and erection. The connections and erection are no more difficult than simple framing, and avoid the complexity and high cost of full moment splices needed for continuous beam construction, especially when these systems are designed using plastic analysis. Compared to continuous framing, a cantilever beam is far easier and more forgiving to erect. Because the system is statically determinate, small errors in the length of support columns are easily accommodated. If continuous beams are erected on supports whose elevations are imperfect by typical construction tolerances, the beam has to be forced into position by the erector, which may be difficult and dangerous.

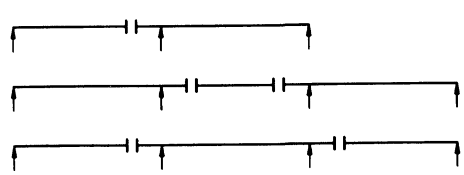

Fig. 1. Configurations.

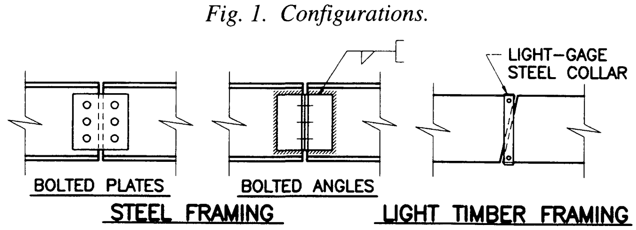

Fig. 2. Shear Splices.

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(Many engineers consider this same support problem to favor a statically determinate system. They feel that stresses "locked in" to the continuous beam, either by erection on out-of-position supports or by unequal support settlement under load, reduce its strength. While this may be true in timber construction, the ductility of steel construction renders this "advantage" moot in most cases.)

DISADVANTAGES

Many engineers favor simple span framing in spite of its added weight and cost because it is quick and simple to design, and because existing roof framing is readily analyzed for changed loads, e.g., the addition of rooftop mechanical units, material handling systems, or catwalks, to name a few examples. This does not just apply to retrofitting of old buildings. In the usual design office setting, structural design occurs concurrently with mechanical system design. Practically speaking, it is very unusual to have the luxury of a completely defined set of mechanical loads when designing the structural framing. Thus, the easily corrected simple framing has a very real advantage for the design engineer in the usual situation of "design, draw, get actual loads, redesign." This advantage of simple framing is compounded in the design of foundations, because the analysis of reactions is more involved with cantilever systems than with simple framing.

Another practical consideration affected by the decision to use cantilever beam roof framing schemes is drainage. Again, the roof drainage layout is usually designed concurrently with the structure. In structures with the larger bays which favor cantilever construction, the usual means of providing roof slope is to slope the roof structure itself rather than through the use of tapered insulation, which is typical for smaller drainage areas in cold parts of the country. Because the piping into which roof drains feed is almost invariably located at columns, the low point of the roof is best located at a column. This is easily accomplished with simple span framing, but can be difficult with some cantilever layouts, requiring additional plumbing or the shifting of high and low points. Extra construction cost can be avoided through early coordination with the architect and mechanical designer, but they are often hard to pin down on such details early enough in the process to benefit the structural engineer.

A third disadvantage of cantilever beams, one shared by continuous framing, is the relative difficulty of analyzing and designing such framing compared to simple-span beams. The analysis of cantilever beams is not handled well by all computer software packages (due to the hinges), and a rigorous hand analysis of such a system can be daunting. In addition, a proper analysis of cantilever or continuous beams requires careful attention to live load patterning, because the maximum effect in one span is typically seen when balancing loads in adjacent spans are minimized. (This subject is covered in more depth subsequently.) More designers than would care to admit it are put off by this complexity, both for the increased work and the increased risk of error. Rooftop mechanical units and other non-uniform loads, especially if applied to suspended spans, add to the complexity. To a pragmatic design group leader attempting to get reliable work out of young engineers in a reasonable time, simple framing has an obvious appeal. With a little experience, most engineers are capable of executing a cantilever beam design properly, but are not willing to spend the time. Detailed design of cantilever beams will never be as fast as picking simple span beams out of a table.

A fourth issue with cantilever and continuous framing is that the designer has to pay more attention to bracing than with simple span beams. A simple span beam with clip angle end connections and joists welded to its top flange requires only a check of allowable unbraced length against joist spacing. Beams with negative moments require more careful attention to bracing; preferably both flanges will be braced at least at the columns.

However, the biggest disadvantage of cantilever beam systems, again shared by continuous framing, is the difficulty found in modifying such systems. Frequently, as industrial processes change in response to market or regulatory forces, modifications may become necessary in the building structure that houses these processes. Often, this involves adding large air-handling or pollution-control equipment on the roof, or cutting away beams and columns to allow for ductwork or new equipment. Because each span of a cantilever (or continuous) beam depends on the adjacent spans for balancing negative moments, changes in the loading or strength of a span affect not only that span but also adjacent ones. Thus, both the analysis and retrofit of such systems are considerably more complicated and less certain than those for simple span framing. An owner of a five-year-old building may be less impressed with saving $0.30 per square foot five years ago when he or she is faced with the prospect of a building which cannot easily be modified or strengthened for five times that amount now. Many engineers consider it a good investment to spend part of the savings realized by cantilever systems; building in a judicious amount of overcapacity.

Another issue sometimes cited as an argument against both cantilever and simple framing is the redundancy and reserve strength inherent in continuous framing. While this is true, the deflections needed to mobilize this capacity usually would render the structure unusable under normal loading conditions and are usually considered appropriate only in the event of extreme accidental overload. The design methodology that tries to exploit this property, plastic design of continuous beams, needs careful checking for deflection and permanent distortion under service loads. It also carries a cost penalty in additional detailing (stiffeners, bracing, and full moment splices) required.

A final disadvantage that can occasionally occur is the risk of support uplift. In situations with a high ratio of temporary load to permanent load, and especially during construction, it is possible for a support at the opposite end of a span from the cantilever to experience uplift. Uneven spans will exacerbate this situation. While this may not seem like a large risk in most steel construction, with anchor bolts extending into concrete footings, some unexpected problems can occur. Occasionally, builders will misplace or forget the cast-in-place anchor bolts, and then quietly substitute smaller, shorter drilled-in expansion bolts with questionable capacity.

SUMMARY OF TYPICAL USES

For all the above reasons, and others less significant, cantilever framing tends to find most use in two areas. The most common use is light industrial, warehouse, school, and office construction as primary steel roof framing, carrying bar joists and metal deck. These types of buildings share the attributes of relatively flat roofs, evenly distributed loads, simple roof drainage patterns, large regular bay and framing layouts, low probability of change in design loading, and a strong desire for initial economy at the risk of higher cost to retrofit later.

The other primary area where cantilever framing is found is in light timber floor and roof joists, especially in light commercial, office, and school construction. These applications benefit from reduced deflection (always a concern with wood framing), reduced strength requirements, and the ability to use shorter pieces of wood, when compared with simple framing. The main concerns with cantilever construction in timber framing are uplift and negative moment strength of pieces graded for bending in the positive moment direction only. Timber framing is not considered further herein.

Cantilever systems are also used occasionally in highway bridge construction, both for economy and for the very real benefit of moving the deck expansion joint away from the supports. Because a frequent cause of bridge substructure deterioration is from chloride-laden water leaking through bridge joints, this is a significant consideration. Highway bridges typically have a much higher ratio of maximum to permanent load than buildings, and railroad bridges are higher still. For this reason the cantilevers on highway bridges are usually quite short, and uplift and fatigue must be checked carefully. They are seldom if ever used on railroad bridges due to these concerns. It is worth noting that the hinge details usually used on cantilever highway bridges are made fairly complex by the need to provide a movable joint; they are thus expensive. The rules and recommendations given here are intended for building structures; bridge structures are not considered further.

DESIGN CONSIDERATIONS

Once a workable bay size and roof drainage pattern have been established, and any required rooftop mechanical units located and defined (or guessed at) with reasonable accuracy, the most important design considerations are bracing, live load patterning, and cantilever configuration. These are examined in detail below. Almost all of the following is predicated on the assumption of uniform load on uniform spans. In typical steel framing, the load actually is brought to bear on a beam by joists which have discrete reaction locations. The assumption of uniform load is very good for seven or more joist spaces per span, and is usually acceptable for fewer spaces. The following comparison is for a three span cantilever-suspended span-cantilever (C/S/C) configuration (from trial roof designs evaluated for this paper) carrying uniform load and various joist spacings per span. The trial designs are for an interior beam line of a roof with 40-foot wide bays. The column spacing is 35 feet in the direction of the beams.

Loading Pattern	Suspended Span Moment	%	Cantilever Moment	%	Backspan Moment	%
Uniform	164	100	422.8	100	446	100
7 Spaces	162.8	99	411.9	97	457.2	102.5
6 Spaces	179.8	110	407.0	96	451.2	101
5 Spaces	154.2	94	409.1	97	454.3	102
4 Spaces	171.8	105	414.9	98	448.6	100.6
3 Spaces	152.8	93	368.8	87	439.7	99

Several things can be noted in this example. The maximum overload is 10 percent above that calculated assuming uniform load, occurring in the suspended span. An even number of joist spaces results in a joist at the center of the suspended span, increasing the moment above that for a uniform load. Thus, it might be prudent as a general rule to overdesign the suspended span. (Often, due to the flange width required to carry joist seats from two bays, this beam ends up with a reasonable amount of overcapacity in any case.) The cantilever moment is always less with actual joist loads than with distributed loads, because the discrete joists gather load and deposit it harmlessly on top of the column. The backspan moment is barely affected.

LIVE LOAD PATTERNING

The issue of live load patterning requires a good deal of thought and judgment on the part of the engineer, a fact that is reinforced by the difficulty of building any consensus on the subject. Patterning of floor load is relatively straightforward and agreed upon; most engineers accept the possibility of one span being occupied (and thus, if not thought about too hard, being subject to the full Code design live load) while the span next to it is unoccupied. Thus, to many designers, the patterning of 100 percent of floor live load (plus any removable dead load, such as partition load) is quite reasonable. (It is also spelled out quite clearly in the UBC (ICBO, 1997).) Thus, for the relatively rare design of a cantilever beam *floor* system (in steel construction, almost always occurring in this country as a partial-bay mezzanine, due to the large expense of running a beam through a column rather than over it), most designers accept the need to pattern their floor loads.

For roof design, agreement is less universal. On a flat roof with no significant topographical features (such as rooftop mechanical units), many engineers consider the possibility of an uneven snow load of any significance to be negligible. This view is supported (intentionally or otherwise) by design handbooks such as the AISC *Manual of Steel Construction* (AISC, 1989; AISC, 1994) which contain tables of moment and reaction constants for cantilever beam systems under uniform load. These tables greatly simplify the manual calculation of cantilever beams, and many engineers reject out of hand any thought of designing such beams by any more rigorous (and more difficult) method. Unfortunately, it is difficult to compile such tables for anything other than uniform (non-patterned) load and equal spans. Many otherwise intelligent engineers consider the very existence of such tables to constitute an implicit acceptance of non-patterned live loads.

Nonetheless, the reasonable patterning of live loads is essential to a safe and consistent design philosophy for cantilever or continuous framing systems (cantilever more so than continuous). This is borne out by the requirements of many Building Codes, from the rather vague advice of the UBC (ICBO, 1997) (section 1603.4) that "...the loading conditions which would cause maximum shear and bending moments along the member shall be investigated," and (section 1605.4) "Snow loads full or unbalanced shall be considered... where such loading will result in larger members or connections;" to the far more demanding requirement of the British Standard BS 8110 (section 3.2.1.2.2) which not only mandates patterned loading of live load, but also requires full factored loads, balanced by only *unfactored* permanent loads in alternate spans.

On typical roof designs with snow loading, it seems prudent to reduce the snow considered on balancing spans to, at most, half the full design value. This is a requirement, for example, of the applicable codes of Canada (NRCC, 1990), Chicago (City of Chicago, 1990), and the State of Wisconsin (Wisconsin Department of Industry, Labor, and Human Relations, 1994).

Although live loads should be patterned for both continuous and cantilever designs, it is more important for cantilever systems, because the balancing negative moments which are at the heart of cantilever system design are derived entirely from vertical load, whereas a continuous beam system has some negative moment capacity regardless of vertical load in adjacent spans.

The following table compares the effect of load patterning on three-span systems of continuous and cantilever (C/S/C) beams from the roof designs evaluated in the trial designs prepared for this paper. Note that load patterning affects only the backspan (positive) moments of cantilever beams, not the cantilever (negative) moments or suspended span moments. It affects all moments of continuous beams, but to a lesser degree. Note also that cantilever beam load patterning very rarely requires "checkerboard" loading (only for checking moment reversal in a beam with a cantilever at each end) because the beams are statically determinate. The continuous beam positive moments tabulated below are due to checkerboard loading, but are little changed by using only single span load patterning (and don't govern in any case).

The parenthetical numbers are the ratio of the moment in question to the "0 percent patterning" value (i.e., full uniform load everywhere), and show the sensitivity of the beam to load patterning.

Load Patterning	Contin. Beam Pos. Moment	Contin. Beam Neg. Moment	C/S/C Pos. Backspan Moment
None (0%)	375.5 ft-k (1.00)	469.4 ft-k (1.00)	375.4 ft-k (1.00)
50%	408.9 ft-k (1.09)	495.6 ft-k (1.06)	446.0 ft-k (1.19)
100%	442.3 ft-k (1.18)	521.8 ft-k (1.11)	516.6 ft-k (1.38)

Thus, in this example, a continuous beam in the governing negative moment region has a moment range of 11 percent depending on the degree of live load patterning used, while the cantilever beam backspan moment varies 38 percent.

CANTILEVER CONFIGURATION

There are several general rules of thumb to keep in mind when laying out a cantilever framing scheme. They are given in no particular order.

1. The designer should try to pick a cantilever length that will optimize one or more of the design parameters, the most obvious being reactions, moment, and deflection. Typically, moment is chosen, because cantilever framing is usually stiff enough without being optimized for deflection, and because the cost of the supports taken all together will not be affected much by minor differences in configuration. Optimization for moment is covered in some detail below.
2. Once a cantilever length has been selected, it is often found that a joist happens to fall right on the splice. Typically the splice is then moved, to provide about 6 inches (center to center) to the joist. If K-series joists are being used, the widest seat made is about 5 inches, so the splice can be located closer to the joist if needed. If the splice needs to be moved, some designers feel it is better to shorten rather than lengthen the cantilever, both for deflection and uplift, although this may result in slightly heavier beams. However, if the cantilever is made slightly longer, then a bottom-chord extension can be welded to the joist to brace the cantilever tip. This is desirable. This issue is discussed further under Rule Eight.

3. For uneven spans, the splice should be located in the shorter span. Primarily this is done out of fear of uplift at the support opposite the cantilever. Another reason to do this is that if the splice is located in the longer span, the backspan beam typically ends up being smaller than the suspended beam. To many engineers, this "just doesn't look right," stress checks or not. It also can generate a lot of concerned telephone calls from clients.
4. This rule, a rather specific one, is that for three-span systems it is more economical, as well as better looking, to have one suspended span in the middle (C/S/C) than one on each end (S/C/S). If the AISC (AISC, 1989; AISC, 1994) tabulated parameters or those at the end of this paper are used to set the cantilever length and design the beams, the C/S/C system will be about 15 percent lighter than the S/C/S system. These tabulated values attempt to balance cantilever and backspan moments, thus optimizing the center span beam of a S/C/S system. If instead the entire beam line is optimized by balancing the cantilever and suspended span moments, the S/C/S system will be virtually the same weight and cost as the C/S/C system, but with increased concern for moment reversal and ponding. Thus, for three-span systems the C/S/C configuration is preferred. This rule is violated only for very uneven span lengths (shorter end spans) or where concentrated loads require a large beam in the center span.
5. If a beam has a cantilever on one end, the other end must land on a support rather than being suspended from another cantilever. Such a system (one column under each field piece) is tempting in long (multi-bay) layouts because all the beams except the first and the last are the same length and experience the same forces under uniform load. This provides economy and simplicity in fabrication and erection. However, a failure in any member or splice (or a problem during erection) can initiate a catastrophic failure of the entire beam line. While some engineers consider this possibility negligible, such a system is subject to progressive failure in the event of an accident or unforeseen load at nearly any location in its length. This rule probably should have been the first one, because of the not-particularly-obvious danger such a system holds. (This rule does not preclude a beam with a cantilever on each end, but rather a series of field pieces each of which has only one end on a support.)
6. It is usually best to configure the beams such that concentrated or heavier loads (rooftop mechanical units being the usual example) fall on a backspan rather than on a suspended span. First of all, just one beam needs be reinforced or made heavier (the positive moment zone of the backspan) rather than two (the suspended span and the cantilever, which thus usually involves making the backspan beam heavier anyway). Secondly, no extra load falls on the splice between the beams. Also, the reactions are more easily figured, and there is no risk of increased uplift. This is an important consideration when retrofitting rooftop units onto an existing roof.
7. If a column connection needs to be designed (i.e., if the end of a beam doesn't just land on a column cap plate), a reaction should be shown on the plans. If this occurs, it typically happens at the end of a beam, and would thus be safe if a fabricator treated it as a simple span beam per usual practice. However, it is much less trouble to show the reaction on the plans than it is to revisit the design when the fabricator calls.
8. The stability of the beam on top of the column must be assured. The preferred method of doing this, if a joist falls on top of the column, is to weld a small angle from the first bottom panel point of the joist on one side of the beam (see the next paragraph) to the bottom flange of the beam. If this is not feasible, bearing stiffeners should be added to the beam (regardless of whether the web strength equations require it). For economy in fabrication, these may be stopped one inch or so short of the top flange. The connection of beam to column then must be sufficiently strong and stiff to prevent the beam from tipping over. This method is both more expensive and less certain than bracing to the bottom chord of a joist.

   Similarly, the lateral-torsional stability of the beam in the negative moment region must be assured. This issue is discussed in an article by Essa and Kennedy (Essa and Kennedy, 1995). Typically, stability is achieved by welding the same small bracing angle to the bottom flange of the beam at the joists on each side of the column. This must be carefully called out on the plans. Note that at interior beam lines (those with joists coming from both sides), the angle braces should all go to joists in one bay, not both bays or alternating bays. If braces are brought to joists on both sides of a beam, an accidental continuity is created, with resultant unintended compression in the joist bottom chord.
9. It is often preferable to configure the beams to produce slightly smaller negative than positive moments. In steel construction, with the usual case of steel joists bearing on the cantilever beam system, it is desirable and typical to place a joist at each column for stability of the framework during erection. The bottom flange of the beam is typically bolted to a column cap plate, and OSHA requires that the joists at columns be bolted to the top flange of the beam. The bolt holes in the beam flanges often result in less moment capacity at the support than at midspan, where the joists are typically welded to the beam.

CANTILEVER LENGTH OPTIMIZATION

The ratio of lengths of the cantilever to the main span, or backspan as it is called, should be selected to optimize the size of beam required for any ratio of temporary (maximum) to permanent load, and for any desired ratio of negative (support) moment to positive (backspan) moment. Reasonably simple equations can be derived which will give the optimum cantilever lengths for a given beam layout. A simplifying assumption is made that the maximum moment in a backspan is at midspan, and is calculated as $wL^2/8 - M_c/2$, where M_c is the cantilever moment. The error due to this assumption is small; in the C/S/C example in the trial designs it amounted to only 1.9 percent.

In the following, these terms are used:

α = ratio of cantilever length to backspan length
μ = ratio of desired negative moment to positive moment
λ = ratio of permanent load to maximum load
L = span length (all spans assumed the same)

For example, for a roof framing system with Dead Load = 25 psf and Snow Load = 40 psf, with the previously recommended value of 50 percent of the live load considered for balancing moments:

$$\lambda = \frac{(25 + 0.50 \times 40)}{25 + 40} = 0.69 \text{ for Working Stress Design}$$

$$\lambda = \frac{(25 \times 1.2 + 0.50 \times 40 \times 1.6)}{(25 \times 1.2 + 40 \times 1.6)} = 0.66 \text{ for LRFD}$$

Further notation is used to describe the layout of the spans and hinges. See Figure 1 for further clarification.

- SC — A two-span system with a suspended beam and a cantilever-and-backspan.
- CSC — A three-span system with a cantilever-and-backspan beam at each end, and a suspended beam in the center span.
- SCS — A three-span system with a suspended beam in each end span and a cantilever on each end of the center span.
- SCSC — A four-span system with a suspended beam in the first and third spans.

A beam line with more than four spans can easily be designed as overlapping sets of these layouts, and would be quite difficult to optimize rigorously (in fact, the four span SCSC system, as will be seen, required some simplification); thus, configurations of more than four spans will not be considered further. This should not be considered as a recommendation against such systems, which in fact work quite well.

TWO-SPAN SC SYSTEMS

The suspended span reaction $p = \dfrac{(1 - \alpha)L\lambda w}{2}$

The center reaction = $(1 + \alpha)wL$

The backspan reaction $= \dfrac{(1 - \lambda\alpha)wL}{2}$ (max)

$= \dfrac{(\lambda - \alpha)(wL)}{2}$ (min)

The cantilever moment $M_c = p\alpha L + \dfrac{(\alpha L)^2 \lambda w}{2} = \dfrac{\alpha L^2 \lambda w}{2}$

The maximum cantilever moment $M_{c-max} = \dfrac{\alpha L^2 w}{2}$

(M_c with $\lambda = 1$)

The maximum backspan moment $M_b = \dfrac{wL^2}{8} - \dfrac{\lambda M_{c-max}}{2}$

(with $\lambda < 1$)

Equating, $M_b = \mu M_{c-\max}$; $\dfrac{wL^2}{8} - \dfrac{\lambda\alpha L^2 w}{4} = \dfrac{\mu\alpha L^2 w}{2}$

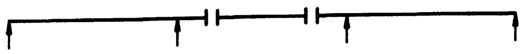
A.l.a. (after a little algebra),

$$\alpha = \frac{\mu}{(4 + 2\lambda\mu)}$$

Thus, for μ (= M_{c-max}/M_b) in the usual range of 0.9 to 1.0, and λ between 0.6 and 1.0, the cantilever length with respect to the span varies from about 0.15 to 0.19. See below for a tabulation of this equation for a range of λ and μ.

It is important to note that this equation optimizes only the cantilevering beam, balancing the cantilever moment against the backspan moment. The suspended beam's moment never enters into the algebra. As it happens, for a two span system with $\lambda = \mu = 1.0$ (the values implicit, for example, in the AISC cantilever tables (AISC, 1989; AISC, 1994)), the above equations lead to an optimum cantilever length of $0.167L$, which gives a cantilever and backspan moment of 0.0833 wL^2, and a suspended span moment of 0.0868 wL^2 (4 percent greater). The AISC value of $0.172L$ more nearly balances all three moments (although not quite), with a tabulated moment value of 0.086 wL^2. Thus, it safely allows the use of the same section for both spans. The AISC tables only attempt this further (one moment value) simplification for the two-span system.

THREE-SPAN CSC SYSTEMS



The same definitions apply for CSC systems as for the previously discussed SC system. Note that the suspended span length is $(1 - 2\alpha)L$. It can be seen that:

The suspended span reaction $p = \dfrac{(1 - 2\alpha)\lambda wL}{2}$

The center column reaction $= \dfrac{(1 + \alpha)(2 - \alpha)wL}{2}$

The end column reaction $= \dfrac{(1 - \lambda(\alpha - \alpha^2))wL}{2}$ (max)

$= \dfrac{(\lambda - (\alpha - \alpha^2))wL}{2}$ (min)

Thus, if $(\alpha - \alpha^2) > \lambda$, uplift results. A value of λ this low would be extremely rare in normal practice.

The suspended span moment $M_s = \dfrac{wL^2(1-2\alpha)^2}{8}$

The cantilever moment $M_c = p\alpha L + \dfrac{(\alpha L)^2\lambda w}{2}$

$= \dfrac{(\alpha - \alpha^2)L^2\lambda w}{2}$

The maximum cantilever moment

$M_{c-\max} = \dfrac{(\alpha - \alpha^2)L^2 w}{2}$ (M_c with $\lambda = 1$)

The maximum backspan moment $M_b = \dfrac{wL^2}{8} - \dfrac{\lambda M_{c-\max}}{2}$
(with $\lambda < 1$)

Following a similar derivation, it is then found that

$$(\alpha - \alpha^2) = \frac{\mu}{[4 + 2\lambda\mu]} = K \quad \text{(defining } K\text{)}$$

and thus

$$\alpha = \frac{(1 - \sqrt{(1-4K)})}{2}$$

See below for a tabulation of this equation for a range of λ and μ.

THREE-SPAN SCS SYSTEMS

This system has a suspended beam of length $(1 - \alpha)L$ in each end span.

The suspended span reaction $p = \dfrac{(1 - \alpha)\lambda wL}{2}$

The center column reaction $= \left(1 + \alpha - \dfrac{\lambda\alpha}{2}\right)wL$ (max)

The cantilever moment $M_c = p\alpha L + \dfrac{(\alpha L)^2\lambda w}{2} = \dfrac{\alpha\lambda wL^2}{2}$

The maximum cantilever moment $M_{c-\max} = \dfrac{\alpha wL^2}{2}$
(M_c with $\lambda = 1$)

The maximum center span moment $M_b = \dfrac{wL^2}{8} - \dfrac{\lambda M_{c-\max}}{2}$
(with $\lambda < 1$)

$= wL^2\left(\tfrac{1}{8} - \dfrac{\lambda\alpha}{2}\right)$

The suspended span moment $M_s = \dfrac{wL^2(1 - \alpha)^2}{8}$

A derivation similar to the preceding gives

$$\alpha = \frac{\mu}{[4+4\lambda\mu]}$$

to balance the cantilever moment vs. center span moment. This is based on balancing the cantilever and backspan moment.

However, the entire system is better optimized if the cantilever moment is balanced with the suspended span moment, which results in a cantilever length (αL) of 0.1716 × Span. This value is not dependent on live load patterning. It results in cantilever lengths greater than the above analysis would indicate, and forces greater negative than positive moments in the center span. In fact, for λ (the ratio of permanent to maximum load) less than 0.69, the center span will suffer moment reversal at midspan, with negative moment over the entire span. Thus, ponding in the end spans is a distinct concern (because a level center span will dump water to the end spans). For end spans longer than the center span, there is a possibility of uplift at the *interior* supports for low ratios of permanent to maximum load. Because the optimized SCS system configured in this way is typically the same weight as an equivalent CSC system (which has fewer concerns), the CSC system is usually a better choice for designs with relatively uniform loads (values of λ close to 1).

Where large rooftop mechanical units or other concentrated loads can be placed on the center span with a reasonable assurance of permanence, however, an SCS system may be well suited. Such a situation allows the large reserve of unused positive moment strength to be utilized at no cost, and with reduced risk of ponding in the end spans.

The above equation for α is tabulated below for a range of λ and μ. As noted above, an SCS system designed to this parameter will seldom be as economical as a CSC system.

Table 1. Ratio Of Cantilever Length To Backspan Length, α									
	SC			CSC			SCS		
λ	$\mu = 0.9$	$\mu = 0.95$	$\mu = 1.0$	$\mu = 0.9$	$\mu = 0.95$	$\mu = 1.0$	$\mu = 0.9$	$\mu = 0.95$	$\mu = 1.0$
1.0	0.1552	0.1610	0.1667	0.1921	0.2017	0.2113	0.1184	0.1218	0.1250
0.9	0.1601	0.1664	0.1724	0.2002	0.2108	0.2215	0.1243	0.1280	0.1316
0.8	0.1654	0.1721	0.1786	0.2092	0.2209	0.2327	0.1308	0.1349	0.1389
0.7	0.1711	0.1782	0.1852	0.2191	0.2321	0.2454	0.1380	0.1426	0.1471
0.6	0.1772	0.1848	0.1923	0.2301	0.2447	0.2598	0.1461	0.1513	0.1563
0.5	0.1837	0.1919	0.2000	0.2425	0.2590	0.2764	0.1552	0.1610	0.1667

FOUR-SPAN SCSC SYSTEMS

The problem in optimizing a four-span system is in deciding which part to try to optimize, and then wading through the algebra to arrive at a solution. An optimal solution for an SCSC system (where the first and third spans contain suspended beams) involves three different cantilever lengths, rather than just one as in the past three examples. The algebra can become rather daunting. A solution which is as good practically, and easier to tabulate or program, is to consider such a system as an overlap of an SCS system and a CSC system. The cantilever carrying the first span suspended beam can be sized using the SCS equation. The cantilever coming from the fourth span carrying the third span suspended beam can be sized using the CSC equation. The cantilever coming from the second span carrying the third span suspended beam is then sized using the average value from these two equations. This results in a very well balanced system with little underutilized capacity.

NUMERICAL SUMMARY

To simplify future reference, these definitions are repeated:

α = ratio of cantilever length to backspan length
μ = ratio of desired negative moment to positive moment
λ = ratio of permanent load to maximum load
L = span length (all spans assumed the same)

SC A two-span system with a suspended beam and a cantilever-and-backspan.
CSC A three-span system with a cantilever-and-back-span beam at each end, and a suspended beam in the center span.
SCS A three-span system with a suspended beam in each end span and a cantilever on each end of the center span.

RATIO OF CANTILEVER LENGTH TO BACKSPAN LENGTH, α

See Table 1.

DESIGN EXAMPLES AND COMPARISONS

For purposes of comparison, an interior beam line for a three bay building is designed with the following configurations:

Simple span
Cantilever (CSC)
Cantilever (SCS)
Continuous beam
Continuous beam (plastic design)

The building is to have a bar joist roof with 40-foot joist span, and 35-foot bays in the direction of the beam span. Included in the comparison are footing and column costs. The columns are 20-feet tall, designed in square tube shapes. ASTM A572 Grade 50 steel is used for the beams, with ASTM A500 Grade B steel (F_y = 46 ksi) in the columns. LRFD design methodology is used for all elements except joists and footings, with footings sized for a service bearing pressure of 2,500 psf. Roof DL is 25 psf plus beams, and Roof LL (Snow) is 40 psf. Maximum desired deflection is L/240 (1.75 inches) for total load, although some of the designs exceed this. Strength and deflection are calculated using half the snow load in balancing spans. Costs are for one beam line of three bays for construction in the Midwest in 1998. The designs are summarized in Table 2.

From the trial designs, several points can be noted:

The cantilever designs are the most economical, with detailing costs penalizing the continuous designs in spite of their relatively low steel weights.

Of the two cantilever designs, the CSC configuration offers better deflection performance at a slightly lower cost. In addition, its shallow suspended span beam may offer advantages for mechanical runs.

The SCS and Plastic Design alternates suffer from large deflections and will probably require redesign if ponding is judged an issue. In particular, the SCS alternate is problematic because of the potential for uplift (moment rever-

Table 2.					
	Simple	CSC	SCS	Elastic Contin.	Plastic Contin.
Outer span beams	W24×68	W24×55	W21×50	W24×55	W21×50
Center span beam	W24×68	W16×26	W21×50	W24×55	W21×50
Total beam weight (lbs.)	7,140	5,239	5,250	5,775	5,250
Max. deflection (in.)	1.69	1.60	1.86	1.36	1.86
Total beam cost (incl. splice plates)	$4,430 ($0.62/lb)	$3,780 ($0.72/lb)	$3,860 ($0.74/lb)	$4,230 ($0.73/lb)	$4,080 ($0.78)
Outer columns	TS6×6×3⁄16	TS6×6×3⁄16	TS6×6×3⁄16	TS6×6×3⁄16	TS6×6×3⁄16
Inner columns	TS8×8×1⁄4	TS8×8×1⁄4	TS8×8×1⁄4	TS8×8×1⁄4	TS8×8×1⁄4
Total column weight (lbs.)	1,614	1,614	1,614	1,614	1,614
Total column cost (incl. base plates)	$1,800 ($1.00/lb)	$1,800 ($1.00/lb)	$1,800 ($1.00/lb)	$1,800 ($1.00/lb)	$1,800 ($1.00/lb)
Outer footings	4'-6"×1'-3"	4'-6"×1'-3"	4'-6"×1'-3"	4'-0"×1'-0"	4'-0"×1'-0"
Inner footings	6'-6"×1'-9"	6'-6"×1'-9"	6'-6"×1'-9"	7'-0"×2'-0"	7"-0"×2'-0"
Total footing vol. (c.y.)	7.35	7.35	7.35	8.44	8.44
Total footing cost	$1,470	$1,470	$1,470	$1,690	$1,690
Joist and Deck cost	$10,500	$10,500	$10,500	$10,500	$10,500
Total Roof Struct.	$18,200	$17,550	$17,630	$18,220	$18,070

sal) in the center span. This increases the contributary area to the end spans for ponding by 50 percent and renders the usual simplified ponding analysis incorrect.

CONCLUSIONS

Cantilever framing systems are a reasonable and economical alternative to simple span and continuous beam framing systems. While more demanding analytically than simple span framing, in buildings with large bays they will typically be the most economical system available. In large buildings with regular repetitive layouts, this additional design effort is offset by the construction savings realized. Using the simplified methods and equations described in this paper, these systems are viable for many routine designs.

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