Ponding Calculations in LRFD and ASD

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INTRODUCTION

The AISC *LRFD Specification for Structural Steel Buildings* (AISC, 1993) contains two alternative solutions for ponding calculations:

1. a conservative approximation in Chapter K2; and
2. a more exact alternative in Appendix K2.

Similar provisions exist in the ASD Specification. The alternative method for design against ponding instability of roofs in both the AISC LRFD Specification Appendix K2 and ASD Commentary Section K2 is based upon Marino (1966). Recently, two equations were suggested as a replacement for ASD Commentary Section K2 and Figures C-K2.1 & C-K2.2 (Milbradt, 1995). It is expected that such a calculation-based solution would be desirable as an alternative to the current graphical solution.

In this paper, the equations suggested by Milbradt are evaluated. In addition, an anomaly in the formulation of LRFD ponding calculations is identified. Accordingly, a modification is proposed for the definitions of the stress indices U_p and U_s in LRFD Specification Appendix K2.

EVALUATION OF THE EQUATIONS SUGGESTED BY MILBRADT

The simplified equations suggested by Milbradt are as follows. To check the primary member, given f_0 and C_s:

$$C_p \leq 1.04 - 0.97 C_s - 1.27 \frac{f_0}{F_y} \tag{1}$$

To check the secondary member, given f_0 and C_p:

$$C_s \leq 1 - 1.07 C_p - 1.25 \frac{f_0}{F_y} \tag{2}$$

where f_0 is as defined in Table 1 and the flexibility constants C_p and C_s are defined as follows:

$$C_p = \frac{32 L_s L_p^4}{10^7 I_p}$$

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$$C_s = \frac{32 S L_s^4}{10^7 I_s}$$

In the above equations:

L_p = column spacing in direction of girder (length of primary members), ft

L_s = column spacing perpendicular to direction of girders (length of secondary members), ft

S = spacing of secondary members, ft

I_p = moment of inertia of primary members, in^4

I_s = moment of inertia of secondary members, in^4

F_y = yield stress, ksi

The graphical solution in LRFD Specification Figures A-K2.1 and A-K2.2, which are identical to ASD Specification Figures C-K2.1 and C-K2.2, gives the relationship between critical values for the flexibility constant C and the stress index U, which represents the available elastic bending strength to resist ponding; the subscripts p and s are used with both C and U to indicate primary and secondary framing, respectively. These curves are plotted from Equations 7 and 8 in Marino (1966), where, for the primary member:

$$U_p \geq \frac{\alpha_p \left[1 + \frac{\pi}{4} \alpha_s + \frac{\pi}{4} \rho (1 + \alpha_s) \right]}{1 - \frac{\pi}{4} \alpha_p \alpha_s}$$

For the secondary member:

$$U_s \geq \frac{\alpha_s \left[1 + \frac{\pi^3}{32} \alpha_p + \frac{\pi^2}{8\rho} (1 + \alpha_p) + 0.185 \alpha_s \alpha_p \right]}{1 - \frac{\pi}{4} \alpha_p \alpha_s}$$

In the above equations:

$$\alpha_p = \frac{C_p}{1 - C_p}$$

$$\alpha_s = \frac{C_s}{1 - C_s}$$

$$\rho = \frac{C_s}{C_p}$$

The stress index U is defined in Table 1. As noted in Table 1, Marino recommended the use of F.S. = 1.25. Thus, although the form is different, the 1989 ASD Specification definition

Table 1.
Comparison of the Definitions of f_0 and U_p, U_s

	Marion (1966)	1989 ASD	1993 LRFD
Definition of f_0	f_0 is extreme fiber flexural stress in a member at onset of ponding	f_0 is the computed bending stress in the member due to the supported loads	f_0 is the stress due to $1.2D + 1.2R$
U_p, U_s	$\left[\dfrac{1}{F.S.}\dfrac{F_y}{f_0} - 1\right]^a$	$\left[\dfrac{0.8F_y - f_0}{f_0}\right]$	$\left[\dfrac{F_y - f_0}{f_0}\right]$

[a] F.S. = 1.25 recommended for design office use

of U is identical to that proposed by Marino. However, it is apparent that the definition of U used in the 1993 LRFD Specification is not equivalent to that in Marino's paper. This will be discussed later.

The Marino curves are compared with those plotted from Milbradt's simplified equations in Figures 1 and 2. As shown in Figure 1 for primary members, when C_s is equal to 0.2, Milbradt's solution is nearly identical to Marino's. A slight variation is noted as C_s increases above or decreases below 0.2. As shown in Figure 2 for secondary members, Milbradt's solution is nearly identical to Marino's when C_p is equal to zero. A slight variation in curvature is noted as C_p increases.

Equation 1 provides a reasonable check of primary members for ponding with results that are generally within ten percent of the Marino solution. Equation 2 exhibits higher variation at the extremes as illustrated in Figure 2. The foregoing variations are well within the accuracy of the methods involved however final designs still must satisfy Marino's solution, which is the basis of the AISC Specification.

PONDING CALCULATIONS IN LRFD

The ponding provisions in both LRFD and ASD are generally consistent in format and intent, with minor deviations in the definition of variables. As such, they should yield similar results. However, a simple comparison of equivalent designs illustrates that the 1993 LRFD ponding provisions do not yield the same results as the ASD provisions from which they were derived.

For example, using $F_y = 50$ ksi, a comparison of values of U is given in Table 2, where f_D and f_R are the stresses due to dead load D and rain load R, respectively. These comparative U values were determined as:

$$U = \frac{0.8F_y - (f_D + f_R)}{(f_D + f_R)} \text{ for ASD} \tag{5}$$

and

$$U = \frac{PF_y - 1.2(f_D + f_R)}{1.2(f_D + f_R)} \text{ for LRFD} \tag{6}$$

where the implied value of P in Equation 6 is 1.0. Because the stress index is unitless, U_{LRFD} should equal U_{ASD}. Thus,

$$\frac{PF_y - 1.2(f_D + f_R)}{1.2(f_D + f_R)} = \frac{0.8F_y - (f_D + f_R)}{(f_D + f_R)} \tag{7}$$

Equation 7 can be reduced to

$$P = 1.2 \ (0.8) = 0.96$$

and it can be seen that the theoretically comparable value of P is 0.96. However, to maintain the traditional accuracy implied in the AISC LRFD Specification, $P = 0.95$ is recommended[1]. Accordingly, it is proposed that the stress index be calculated in LRFD as:

$$U = \frac{PF_y - f_0}{f_0}$$

where $P = 0.95$. This proposed revision provides excellent agreement between LRFD and ASD ponding calculations.

SUMMARY

The definitions of stress indexes U_p and U_s in the 1993 LRFD Specification and the 1989 ASD Specification do not result in similar designs. A modification is suggested for the definitions of the stress indexes U_p and U_s in AISC LRFD Appendix K2 to eliminate this anomaly. Simplified equations for the flexibility constants C_p and C_s suggested by Milbradt are also validated for preliminary design.

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[1] 0.96 and 0.95 have the same numerical accuracy, but LRFD resistance factors have traditionally been rounded to the nearest increment of 0.05

Table 2.
Comparison of Stress Indices in LRFD and ASD

$f_D + f_R$ (ksi)	U	
	1989 ASD	1993 LRFD
10	3.00	3.17
20	1.00	1.08
30	0.333	0.389
40	0	0.042

REFERENCES

American Institute of Steel Construction (1993), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, Chicago, IL.

American Institute of Steel Construction (1989), *Specification for Structural Steel Buildings-Allowable Stress Design and Plastic Design*, Chicago, IL.

Marino, Frank J. (1966), "Ponding of Two-Way Roof Systems," *Engineering Journal*, AISC, Vol. 3, No. 3, Chicago, IL, July.

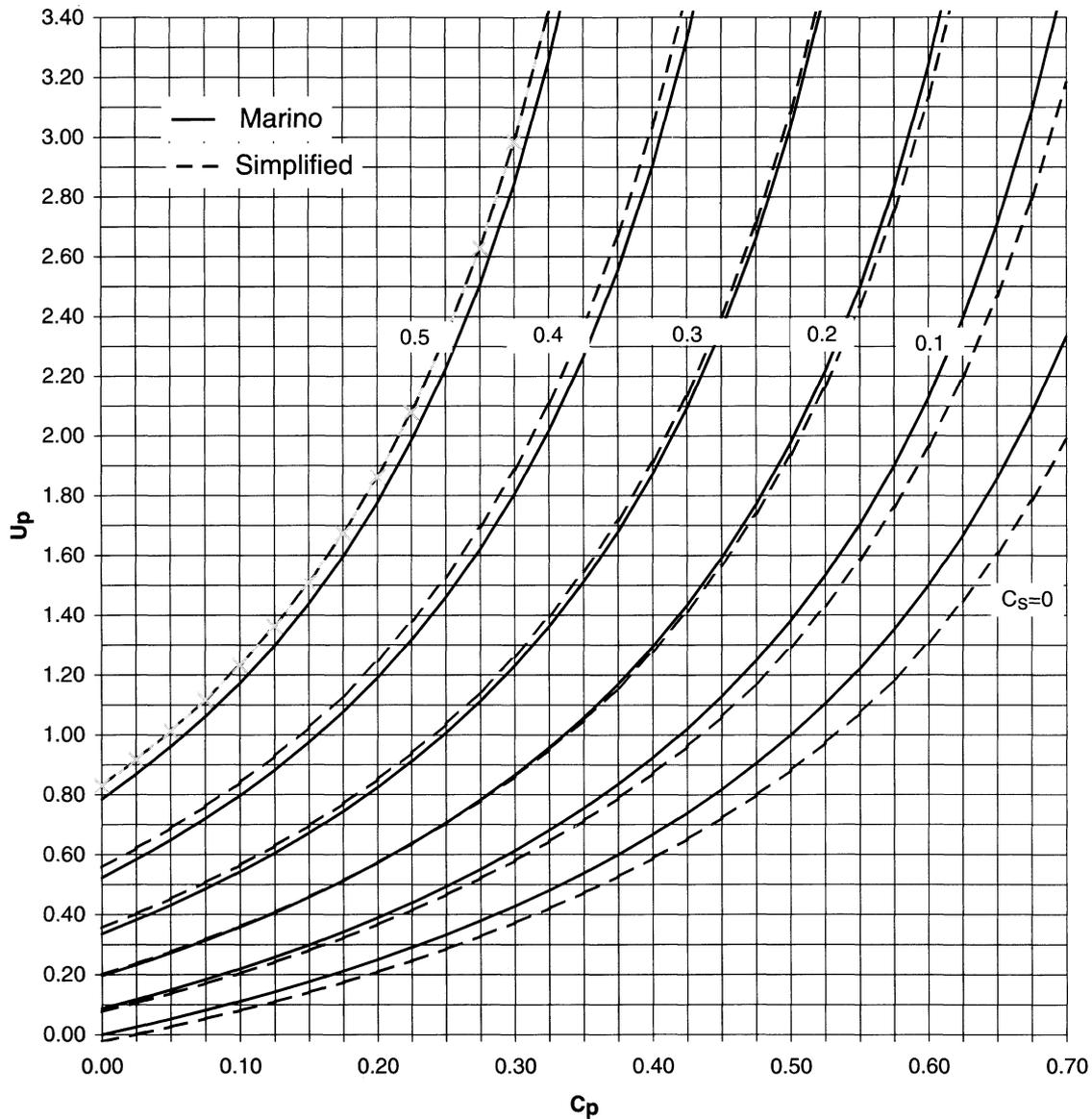Milbradt, K. P. (1995), Letter to Mr. Nestor Iwankiw, AISC, February.

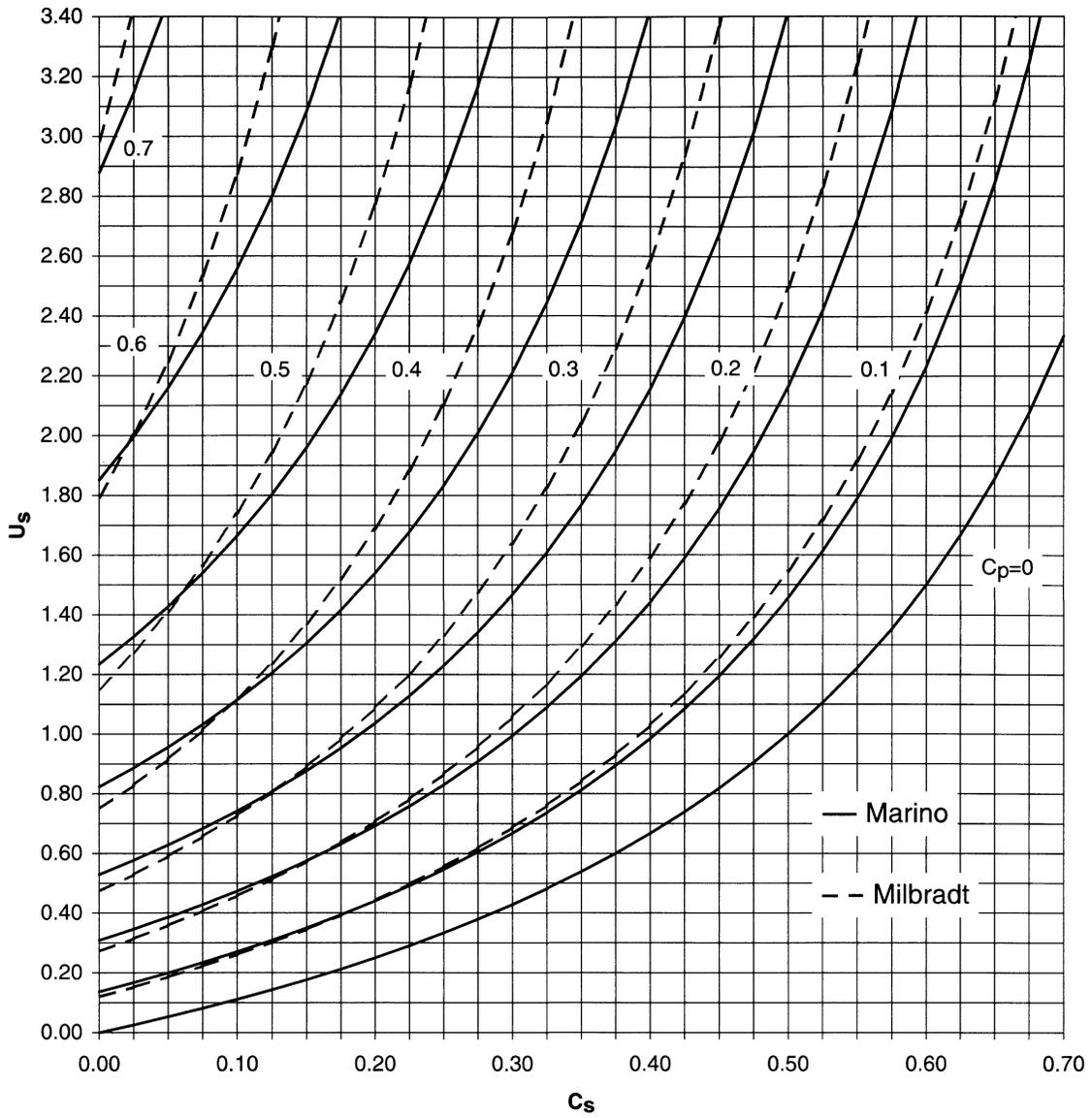*Fig. 1. Comparison of Original and Simplified Formulas ($C_p - U_p$).*

Fig. 2. Comparison of Marino and Milbradt Formulas ($C_s - U_s$).