Semi-Rigid Frame Design Methods for Practicing Engineers

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ABSTRACT

Design of semi-rigid (PR) frames focuses on behavior characteristics of non-linear connections, including their substantially different loading and unloading characteristics. Moment-rotation connection representations, such as the three-parameter power model, facilitate the calculation of stiffness data required for frame analysis. In this paper, the connection characteristics are described in terms of linearized connection stiffnesses that are calculated on the basis of expected connection loads. This allows for the use of first-order analysis to determine structural stability, serviceability and member load effects. The design method detailed in this paper includes the concurrent selection of connections and member sizes. The LRFD approach of AISC is utilized, including the provisions that rely on amplification factors to account for second-order effects. Member section checks are made with unbraced length K-factors determined from the alignment charts, using modified relative distribution factors to account for connection flexibility. New connection limit states are quantified by the useful range of expected connection deformation.

1.0 INTRODUCTION

The AISC Specification[1] identifies two construction types: FR and PR, which refer to the degree of restraint that the connections offer in structural steel frames, namely fully restrained and partially restrained, respectively. While FR construction represents building frames with "rigid" connections, PR construction represents frames with simple "pinned" connections as well as semi-rigid connections. The distinction between these two types of PR construction (simple and semi-rigid) is that the connections in a semi-rigid frame are able to develop moments and rotational deformations that significantly affect the member forces and displacements, while the moment response of connections in simple frames is negligible. Thus, while all semi-rigid frames are considered as PR construction, not all PR construction exhibit semi-rigid behavior. This distinction becomes evident from the study of the connection moment-rotation relationships.

Significant benefits can be gained by incorporating connection behavior into the design of partially restrained (PR) building frames when the connections have semi-rigid properties. These benefits offer the potential for reducing costs. The economy of semi-rigid frames is the result of reduced beam moments, reduced beam deflections, and increased restraint at column ends. The effect is most pronounced in braced (sway-prevented) frames, where requirements for beam and column sections are reduced as a result of the more realistic connection behavior. The case for unbraced (sway-permitted) frames is not as clear, because the increased connection flexibility leads to increased drift and reduced column restraint. This can result in higher material costs, but there is still a potential for improved economy. This is particularly due to the fact that semi-rigid connections utilize simpler details which are more economical to fabricate and erect.

The difficulty in designing semi-rigid frames lies primarily in the non-linear connection behavior, where the stiffness depends on the connection deformation. Significant progress has been made in solving this problem with the introduction of advanced methods of frame analysis.[7,13] However, these methods require the use of computer programs that are not yet suitable for office use, nor are they readily available. Also, at this time Chen's methods do not include the unloading behavior of the connections, which is radically different from their loading behavior.

A method based on using expected connection loads to estimate equivalent linearized connection stiffnesses is presented here to provide design and analysis techniques for semi-rigid frames. Using the estimated linearized connection stiffness values allows the use of simple, first-order analysis techniques, which are familiar to practicing engineers. The approach also allows the use of computer software that is based on linear-elastic elements, and currently in common use and familiar to most design offices.

A second major problem associated with designing semi-rigid frames is how to quantify the strength limit

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state, or connection moment capacity. Tests[18,24] have demonstrated that the ultimate connection moment is often well beyond the useful range of expected connection deformation, far exceeding serviceability limits. A new definition of connection limit states that is related to serviceability requirements is presented in this paper as a rational approach to this issue. The limit states are based on a practical expected limit to connection deformation.

With these problems addressed and satisfied, it is possible to use the AISC LRFD format, following the approximate method that relies on amplification factors to account for second-order effects. This method is particularly suited to the need to account for the abrupt change in connection stiffness that is associated with load reversal.

In part, since North American design codes address the use of semi-rigid connections in only a limited way, design engineers have been reluctant to use them. However, with continued research, code criteria for semi-rigid frames are likely to become more detailed. Although more research is required to explore fully the behavior of semi-rigid frames, enough information is available now to implement the technology in normal design practice.

2.0 CHARACTERISTICS OF SEMI-RIGID CONNECTIONS

The databases of beam-to-column connection tests[18,24] indicate a generalized, non-linear behavior as illustrated in Figure 1, where point a represents an expected connection moment, M_e, and corresponding deformation, ϕ_e. This particular M-ϕ curve has been constructed from test data for a flush end-plate beam-to-column connection.[24] The illustration indicates the key connection stiffness parameters:

- Initial stiffness, R_{ki}—the initial slope of the M-ϕ curve, which is nearly constant for the first few data points. However, some connections exhibit M-ϕ relationships that make a unique definition of R_{ki} very difficult.
- Secant stiffness, R_{ks}—the effective stiffness of the connection based on an expected moment. $R_{ks} = M_e/\phi_e$, where M_e and ϕ_e are illustrated in Figure 1.
- Tangent stiffness, R_{kt}—the instantaneous stiffness, which usually decreases as the moment increases. Such behavior does not apply for the full range of M-ϕ response when "work hardening" affects the connections.[6]
- Unloading stiffness (reversing loads), R_{ku}—the unloading rate that is approximately linear until reaching zero moment. It is the result of subsequent, superimposed loads that produce rotations opposite in direction to the initial response. This phenomenon has been observed by Popov and Pinkney[28] and others; the unloading stiffness is commonly assumed to equal the initial stiffness, R_{ki}.

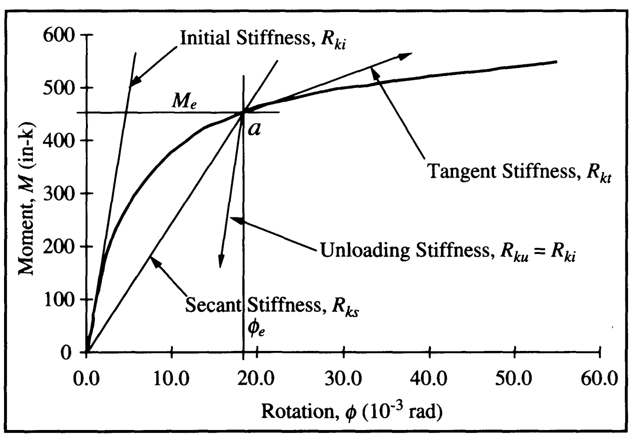


Fig. 1. Moment vs. Rotation for a Flush End-Plate Connection[24]

Figure 1 also illustrates the dependence of the secant stiffness, R_{ks}, and the tangent stiffness, R_{kt}, on the expected connection moment and deformation, as represented by point a.

2.1 Connection Models

Since it is not practical to depend on the availability of test data for a myriad of connection details, analytical models are a convenient substitute. In an early attempt to resolve this problem, Kennedy[22] derived a polynomial expression for the flexible end-plate connection. Later, Frye and Morris[16] presented a polynomial model to represent the loading behavior of a variety of connection types. Although suitable for many connections, the problem with the Frye and Morris model is that it gives negative stiffness values for certain connection types and ranges of M-ϕ. Since this is physically unacceptable, Jones et al.[20] achieved an accurate correction, substituting a B-spline curve to fit experimental M-ϕ data.

Kishi et al.[25,26] presented the three-parameter power model to represent the M-ϕ behavior of connections utilizing bolted angle elements. The M-ϕ curve is defined by Equation (1).

$$M = \frac{R_{ki}\phi}{\left[1 + \left(\frac{\phi}{\phi_0}\right)^n\right]^{1/n}} \tag{1}$$

where

R_{ki} = initial stiffness of the connection
n = shape factor
ϕ_0 = reference plastic rotation, calculated as $\phi_0 = M_u/R_{ki}$
M_u = ultimate moment capacity of the connection

The tangent stiffness is the first derivative of Equation (1):

$$R_{kt} = \frac{R_{ki}}{\left[1 + (\phi/\phi_0)^n\right]^{\frac{n+1}{n}}} \tag{2}$$

Thus, the loading portion of the M-ϕ curve for these types of connections may be mathematically expressed in terms of three simple parameters, which may be calculated from the material properties and detail dimensions by using methods such as those presented by Chen et al.[7] These parameters are the initial stiffness, the ultimate moment capacity, and the shape factor for the M-ϕ curve.

A key problem associated with the three-parameter power model is that for many connection details, the ultimate moment capacity, M_u, is often not practical to define as a strength limit state. This is because these connections do not exhibit a clearly defined plateau in the M-ϕ curve.

2.2 Connection Classification

Beam-to-column connections are produced in many types and sizes with a wide range of responses. This offers opportunities for the designer to satisfy a variety of structural needs and conditions. It also presents a problem in determining when a particular connection should be treated as a simple pin, because of its high flexibility; as a rigid connection, because of its high stiffness and strength; or as a semi-rigid connection, because of a combination of stiffness and strength that differs significantly from "rigid" or "pinned." With the advent of the semi-rigid category in design practice, there is a need for simple criteria to determine how stiff is "rigid," and how flexible is "pinned." Bjorhovde et al.[6] offered a simple test, illustrated in Figure 2(a), to classify the properties of a connection as "rigid," "semi-rigid," or "flexible." This is a non-dimensional system that uses the strength and stiffness of the connection in comparison to the beam properties. The illustration of Figure 2(a) indicates a connection in the semi-rigid category, where the curve lies in the area between the "rigid" and "flexible" boundaries. The implementation of this system uses terms that are defined as follows:

1. The non-dimensional characteristic length factor for the initial connection stiffness is given as

$$\alpha_i = \frac{EI}{R_{ki}d} \tag{3}$$

where

E = modulus of elasticity
I = moment of inertia of the beam
R_{ki} = initial connection stiffness
d = beam depth

2. The connection moment is given as a non-dimensional ratio, compared to the fully plastic moment of the beam:

$$\overline{M} = M/M_p \tag{4}$$

where

M = connection moment
M_p = plastic moment of the beam

3. The non-dimensionalized connection deformation is given by Equation (5)

$$\overline{\phi} = \phi/\phi_p \tag{5}$$

where ϕ = connection deformation, and ϕ_P is defined by

$$\phi_p = \frac{5dM_p}{EI} \tag{6}$$

It is the plastic bending rotation of the beam, illustrated in Figure 2(b).

The product ($\alpha_i\, d$) is termed the characteristic length, or the length of the beam that will have the same stiffness as the initial connection stiffness. The value of five in Equation (6) was chosen because it represented the middle of the semi-rigid range for reference length, as determined from a review of the Kishi and Chen[24] database.[6]

Eurocode 3[12] includes a connection classification system that requires evaluations to determine what frame behavior to consider in the analysis: simple, rigid or semi-rigid. Identifying these frame behavior characteristics are similar to the objectives of the nondimensional system presented in Figure 2(a), but the EC3 system can result in a different conclusion.

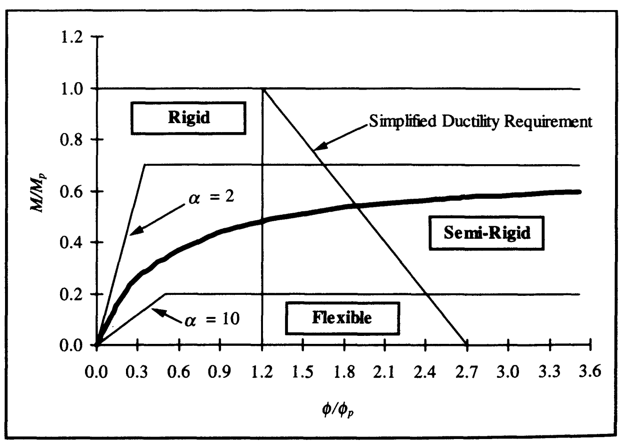


Fig. 2(a). Non-Dimensional Classification of Connections[6]

Using either the Bjorhovde system or the EC3 system offers the following benefits:

- A means to determine if semi-rigid design and analysis techniques are appropriate and necessary.
- The ability to assess the connection behavior in relation to member behavior.
- The opportunity to select semi-rigid connections concurrently with member sizes during the design process.

The term α, the general term for the effective characteristic length factor, is also useful in calculating member forces and column stability, since it provides a way to quantify the effective connection flexibility at various degrees of connection loading. For example, the characteristic length factor associated with the connection secant stiffness is given as

$$\alpha_s = \frac{EI}{R_{ks}d} \tag{7}$$

2.3 Connection Limit States

Examination of many M-ϕ test curves does not indicate a clear point for defining the ultimate connection strength. The M-ϕ curves for many of the connections are still rising at the end of the test. For this reason Christopher[9] identified a practical connection deformation, and then defined the corresponding connection moment as the strength limit state. Considering the connection deformation for a simple beam with a span-to-depth ratio of 30 or less as an extreme case, the upper bound of the expected beam end rotation, θ_R, including beam flexibility and inter-story drift, is given as

$$\theta_R = 0.0008F_y + \delta/h \tag{8}$$

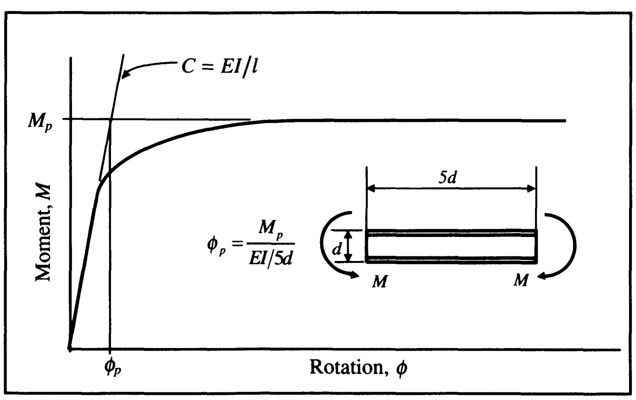


Fig. 2(b). Beam Moment-Rotation Curve

where

F_y = the yield strength of the beam steel (ksi)
δ/h = the specified inter-story drift ratio

In Equation (8) the value of $0.0008F_y$ reflects the simple beam end rotation resulting from a uniformly distributed load that causes a plastic moment to develop at mid-span.

If θ_R is the practical limit of beam end rotation, then a semi-rigid connection should be expected to have the ability to deform safely throughout the range of θ_R while resisting moment and shear. Thus, the connection deformation requirement, or in effect the ductility requirement, ϕ_{uj}, should equal θ_R. Using Equation (8), connections supporting steel beams with a yield stress of $F_y = 36$ ksi should expect to see a limit of about 0.03 radians, and with $F_y = 50$ ksi, 0.040 radians of rotational deformation. As a comparison to these numbers, Thornton[31,32] assumed an end rotation as 0.03 radians for a maximum expected value for most cases, and considered 0.07 radians as very large. Also, Deierlein[13] noted that the moment at $\theta = 0.02$ radians corresponds fairly well to the nominal connection strength. As a comparison to the Bjorhovde classification system, the results of Equation (8) indicate a connection deformation about 30% less than the simplified ductility requirement illustrated in Figure 2(a).

Substituting $\phi_{uj} = \theta_R$, from Equation (8) into the three-parameter power model equation gives the strength limit state as

$$M_{uj} = \frac{R_{ki}\phi_{uj}}{\left[1 + \left(\frac{\phi_{uj}}{\phi_0}\right)^n\right]^{1/n}} \tag{9}$$

This new identification of the strength limit state removes a major obstacle to implementing semi-rigid frame design.

2.4 Some Reliability Issues

Variation in the values of the initial connection stiffness, the connection moment capacity and the shape factor used in the three-parameter model affect the reliability of expected connection strength and behavior. Thus, the use of the three-parameter power model itself, or any mathematical substitution for actual connection data, affects reliability. Sellier et al.[30] present approximate coefficients of variation for each of these parameters, as given in Table 1. These data are approximate because certain assumptions were made regarding connection property statistics. Very limited statistical test data are available; the study by Rauscher and Gerstle[29] is one of very few.

These sources of variation are of concern for two reasons:

1. The concern for the individual connection performance and capacity.

2. The effect of these variations on member forces as they are calculated from analyses that rely on the interaction of connection stiffness and member stiffness.

Further investigation is therefore required to verify these data, and to determine if there is a need to modify the LRFD resistance factors when designing a semi-rigid frame. Until this work is complete, it is assumed that a resistance factor of 0.9 is appropriate for the connections in a semi-rigid frame design, and the practical limit state for semi-rigid connection design strength, M'_n, is

$$M'_n = 0.9M_{uj} \tag{10}$$

2.5 Frame Sensitivity to History and Direction of Connection Loads

If expected values of connection moments can be accurately estimated, then linearized connection stiffness values can be calculated and used in a frame analysis to determine member forces, displacements and column stability. However, observation of the typical M-ϕ curve represented by Figure 1 suggests that the sequence and direction of the connection loads are needed in addition to the estimate of expected connection moments. For example, consider an unbraced (sway-permitted) semi-rigid frame that is first subjected to gravity loads, and subsequently to lateral loads due to wind or seismic action. In this case the connections at the beam ends rotate as they are first loaded by the gravity loads. With the application of lateral load, the leeward beam ends continue to rotate in the same direction, while the windward beam ends begin to unload. During the subsequent loading stage of the frame, the connection behavior is significantly affected, in the sense that the loading rate, R_{kt}, at the leeward beam end is much less than the unloading rate, R_{ki}, at the windward beam end.

Disque[14] recognized that, in general, the loading characteristics of semi-rigid connections are much different from their unloading characteristics, and he proposed the method of "directional moment connections" for design of unbraced frames. This method considers that moment connection properties depend on the loading direction and history. The moment connections in Disque's model develop a plastic hinge when they are loaded in one direction, and respond elastically when unloaded in the opposite direction.

Clearly, connection behavior is affected by the history and direction of sequentially applied loads. Therefore, it is necessary to use the representative connection stiffnesses for each loading stage for calculating member forces in the analysis of semi-rigid frames.

A similar case can be made when considering semi-rigid frame stability. Column behavior in semi-rigid frames is affected by the degree of end-restraint offered by the connection resistance to column rotation. For example, consider a braced (sway-prevented) frame, with beam loads gradually applied until column buckling occurs. As the beams are loaded, their end rotations develop connection moments. When column buckling develops from additional load, column rotation occurs at the beams. Examination of these beam-to-column connections indicates that at the initiation of buckling, those on one side of the columns continue to load, while those on the opposite side will unload.[5] Thus, evaluating frame stability also requires consideration of connection loading history and direction, i.e. the initial loading from beam end rotation, and the subsequent loading caused by column buckling. Chen and Lui[8] confirmed this observation by the analysis of a semi-rigid braced "tee" sub-assemblage subjected to sequential loading.

Since braced frame buckling is characterized by single curvature of the columns, there is a regular, alternating pattern of connection loading-unloading behavior as illustrated by Figure 3. The following observations apply to a semi-rigid braced frame with repeating patterns of

Table 1 Coefficients of Variation for Connection Parameters[30]	
Connection Parameter	**COV**
Ultimate Moment Capacity, M_u	18%
Initial Stiffness, R_{ki}	25%
Shape Factor, n	15%

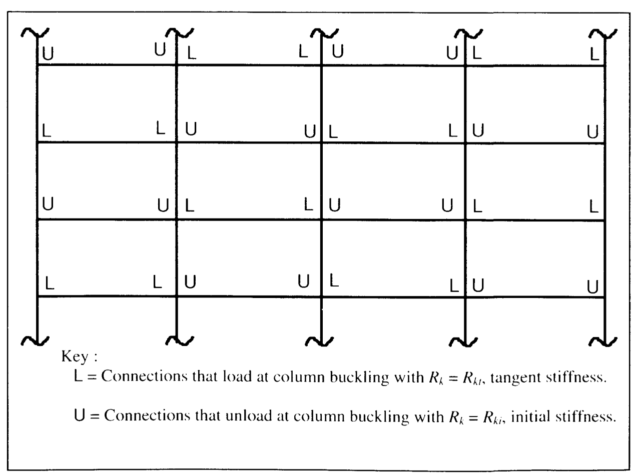


Fig. 3. Connection Behavior at Column Buckling in a Braced Frame

member sizes:[9]

1. All columns are offered restraint from symmetrically supported beams.
2. Beams offer restraint based on the combined stiffness of the beam and the connection in alternating positions. This stiffness is either the initial or the tangent stiffness for the connection.
3. Exterior columns are restrained unequally at the top and bottom, i.e. at one end the restraint is based on the tangent stiffness, and at the other end the initial stiffness applies.
4. Interior columns are restrained equally at the top and bottom, i.e. the restraint is the sum of that based on tangent stiffness on one side of the column, and initial stiffness on the other side.

The results of a similar study of an unbraced semi-rigid frame also indicate a pattern to connection behavior that is sensitive to the history and direction of connection loads.[9] This study utilized the analysis of an unbraced "tee" subassemblage as illustrated in Figure 4. The initial connection load due to gravity loads on the beams is represented by Figure 4(a), and the subsequent vertical and lateral loads are shown in Figure 4(b). After the application of the initial, first-stage gravity load, w, a second-stage lateral load, H, and proportional axial load, P, were applied gradually until failure occurred. The results indicated that the connection on the windward beam end unloaded at the higher initial stiffness rate, while the connection on the leeward beam end continued to load at the lower reduced rate. Since unbraced frame buckling is characterized by double curvature of the columns, this behavior results in a regular, repeating pattern of connection loading-unloading behavior as illustrated in Figure 5.

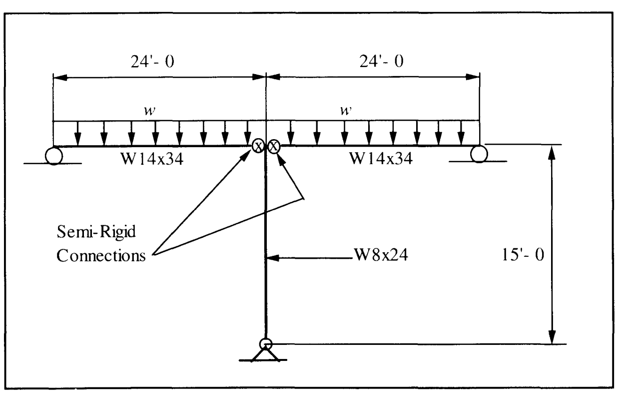


Fig. 4(a). Unbraced "Tee"—Load Sequence 1

3.0 EXPECTED VALUES OF CONNECTION MOMENTS

Expected values of connection moments can be determined from beam-line diagrams such as the one shown in Figure 6. The basic assumption of this illustration is that the beam end rotation, θ, is equal to the connection deformation, ϕ. Theoretically, this assumption is violated by rotations of the supporting columns, but in most cases the column rotation is small, and the assumption will result in good initial estimates of the connection deformation. These estimates provide ways to calculate beam end moments and connection stiffness values.[17] The beam-line diagram is therefore a useful tool for selecting preliminary beam sections and beam-to-column connections concurrently.

A frame analysis, using linearized connection stiffness values determined from the expected connection moments, provides a means to refine the preliminary design.

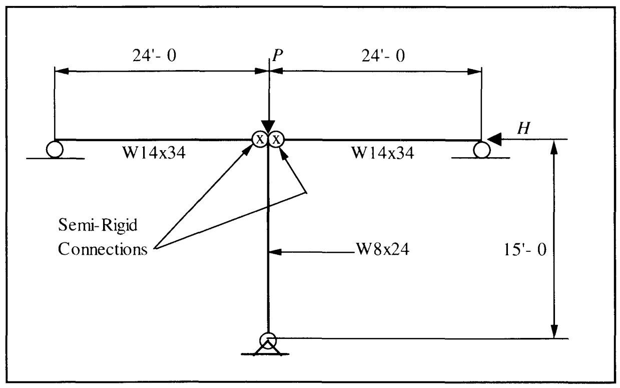
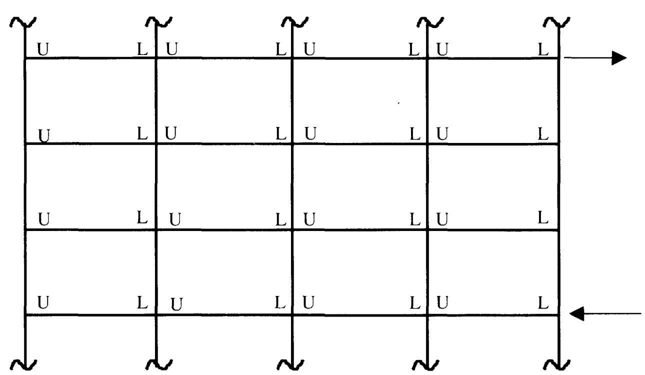


Fig. 4(b). Unbraced "Tee"—Load Sequence 2



Key :
L = Connections that load at column buckling with $R_k = R_{kL}$, reduced stiffness.

U = Connections that unload at column buckling with $R_k = R_{ki}$, initial stiffness.

Fig. 5. Connection Behavior at Column Buckling in an Unbraced Frame

This analysis accounts for the column rotations in calculating the connection deformations. An additional subsequent frame analysis serves to verify the expected connection moment by demonstrating that the connection deformations are consistent with the connection moments determined from the M-ϕ curves. This process of concurrent member and connection design serves to shorten the iterative design and analysis process by providing an accurate initial estimate of connection moments.

3.1 Using the Beam-Line Diagram to Estimate Connection Moments

The connection flexibility ratio, u, which links connection behavior and beam stiffness, was recognized by Geschwindner[17] to simplify the expression for the connection moment in term of the beam stiffness. It is a convenient tool for estimating beam end moments, and it is defined by

$$u = \frac{EI}{R_k l} = \frac{\alpha d}{l} \tag{11}$$

where

- α = EI/dR_k = the characteristic length factor for connection flexibility
- E = modulus of elasticity
- d = beam depth
- l = span length
- I = moment of inertia of the beam
- R_k = stiffness of the connection

Utilizing the intersection of the beam-line and the M-ϕ curve gives

$$\phi = \frac{M_s}{R_k} = \frac{M_s \alpha d}{EI} \tag{12}$$

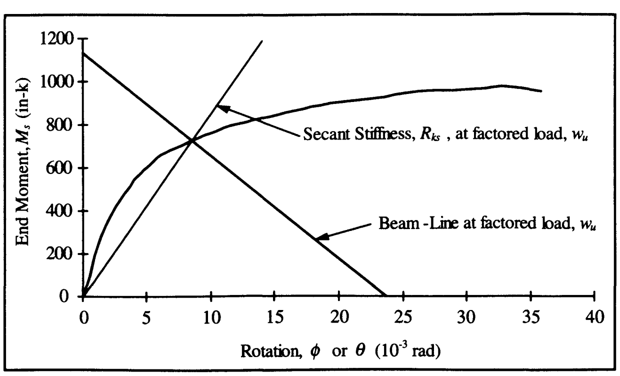


Fig. 6. Beam-Line Diagram with M-ϕ Curve[10]

where M_s = the reactive moment at the beam end that is equal to the connection moment.

The equation of the beam-line for a symmetrically supported beam with a uniformly distributed load is given as

$$\theta = \frac{wl^3}{24EI} - \frac{M_s l}{2EI} \tag{13}$$

Substituting $\phi = \theta$ from Equation (12) into Equation (13) results in expressions for the beam moments in terms of the applied load and the connection stiffness:

$$M_s = m_s w l^2 \tag{14}$$

$$M_f = m_f w l^2 \tag{15}$$

where M_f = the moment at the beam mid-span and where

$$m_s = \frac{1}{12(1 + 2u)} \tag{16}$$

$$m_f = 0.125 - m_s \tag{17}$$

Equations (16) and (17) are plotted in Figure 7 to illustrate the usefulness of the connection flexibility ratio in the design process, where the connection can be selected to minimize the beam moments. Using the fixed-end moment as a reference point, Geschwindner presented a diagram that is similar to Figure 7. Kotlyar[27] gives equations in terms of the connection flexibility ratio for beams with other loading conditions and semi-rigid connection support configurations in a useful and convenient table.

3.2 Concurrent Selection of Connections and Beam Sizes

If reasonable assumptions can be made for the connection stiffness values, in terms of the connection flexibility ratio, u, then the beam moments may be calculated, and

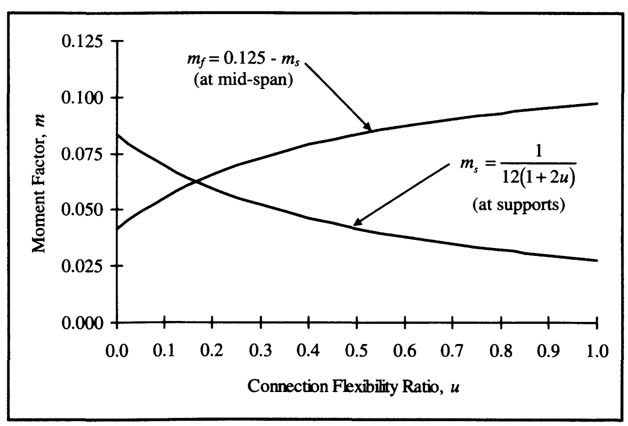


Fig. 7. Optimum Connection Flexibility (Symmetrically Supported Beam)

preliminary beam sections may be selected. If gravity loads are assumed to govern the beam sizes, then u_s, the connection flexibility ratio corresponding to the connection secant stiffness, R_{ks}, should be assumed. The problem, however, is knowing what values of u_s are reasonable assumptions for this purpose. As in the design of any statically indeterminate structure, the selection of one component affects the behavior and resulting load effects of the adjacent components. This phenomenon requires a trial and adjustment process. Semi-rigid braced frames are also statically indeterminate, and the selection of the connection property, u_s, in the design process also requires trial and adjustment.

To help in the process of identifying reasonable assumptions for u_s, an example is provided in Figure 8, which shows a family of M-ϕ curves for the same type of semi-rigid connection supporting a beam with a uniformly distributed load represented by the beam-line. For this example, the beam is a 25 ft long W21$\times$44 carrying a total (factored) distributed load of 4.25 k/ft. Each M-ϕ curve, generated by the three-parameter power model, represents a top and seat angle connection with double web clip angles. The web clip angles are L4 $\times$ $3\frac{1}{2}$ $\times$ $\frac{5}{16}$ $\times$ 0'-5$\frac{1}{2}$", and the top and seat angles are L4 $\times$ 4 of variable thickness and 0'-6$\frac{1}{2}$" long. Thus, the intersections of the M-ϕ curves and the beam-line represent the expected connection moments and deformations, depending on the top and seat angle thickness, and assuming no column rotation. Table 2 presents the expected beam moments, M_f at the span center, and M_s at the supports, and end rotations, ϕ_s, as a response to the variation in top and seat angle thickness, t. Table 3 provides the connection stiffness values, R_k, and the corresponding values of the connection flexibility ratios, u, which are calculated from the expected connection moments given in Table 2.

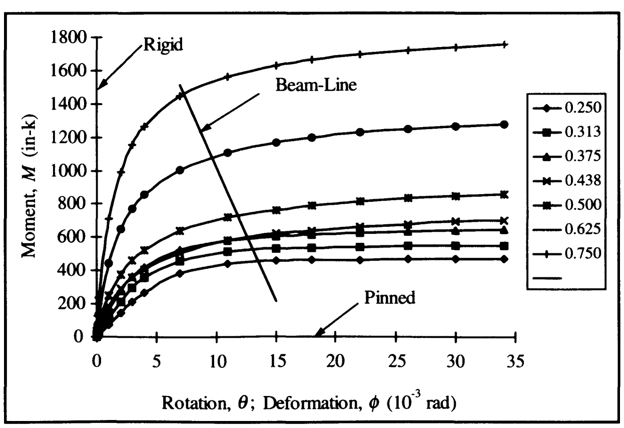


Fig. 8. Influence of Connection Behavior on Beam Response[11]

This example serves as a guide to make reasonable assumptions for values of u_s to use for concurrently selecting preliminary beam and connection sizes.

3.3 Structural Analysis Using Expected Connection Loads

First-order structural analysis of semi-rigid frames depends on using connection stiffness values which are consistent with the M-ϕ behavior. Generally, the semi-rigid connection behavior affects braced and unbraced frames in significantly different ways.

3.3.1 Braced Frames

The secant stiffness of the connections provides the means to distribute the applied loads to the members. Calculations for member forces do not need to consider loading history, because the connections are always loaded in the same direction until column buckling occurs. A simple first-order analysis which includes the approximate connection stiffness therefore will provide reasonably accurate values for the member forces.

This approach works because there is very little column rotation at the interior beam-to-column connections. However, in semi-rigid braced frames, significant column rotation occurs at the connections to the exterior columns. This can generally be ignored in a preliminary analysis, and in a later analysis the connection stiffness can be adjusted upward to account for the column rotation and the associated decrease in connection moment.

Christopher[9] demonstrated that preliminary selection of beams framed to exterior columns should be made by assuming pinned connections at the exterior ends, while the interior ends are supported with semi-rigid connections. This agrees with the column stability analysis developed by Bjorhovde[5]. The interior beams should be selected by assuming symmetrical supports (equal connection stiffness at each end). The connection design should take place concurrently with the beam selection.

This approach is possible by assuming a value for u_s, which corresponds to the connection secant stiffness. As noted from the above example, the range of $0.4 < u_s < 1.0$ for a top and seat angle with double clip angles represents a relatively stiff connection. However, this depends on the beam section and span length. Higher values of u_s correspond to less connection stiffness. Generally, u_s will affect the resulting beam end moments, which may then be calculated to permit the selection of preliminary beam sizes. Connection element details can be selected that result in a value of u_s that is close to the assumed value.

Selection of preliminary column sections may be made by simply assuming unbraced length K-factors of 1.0 for exterior columns and 0.85 for interior columns. A more accurate frame stability evaluation to determine unbraced length K-factors is appropriate after connection stiffnesses are adjusted for the selected connection details to account for column rotations in a frame analysis.

With the preliminary selection of beams, connections, and columns, this analysis can be performed by making a linear-elastic computer model of the frame. The connection behavior is best represented using zero length elements, but where the software does not include such elements in its library, adjunct (dummy) members can represent the connection stiffness in a variety of possible arrangements. An example is shown in Figure 9. The results of this preliminary analysis provide the means to evaluate the connections' performance, for ductility demand, for comparison of expected connection moments to those calculated in the analysis, for member section checks, and finally for stability of the frame. A final verification analysis is required to account for refining adjustments in the connection stiffness values and changes in the member sizes.

3.3.2 Unbraced Frames

The sensitivity of semi-rigid unbraced frames to directional changes of the connection moments in the loading process requires a different approach to the analysis. If a loading sequence is assumed to start with gravity loads and end with lateral wind or seismic loads, then there is a change in connection stiffness with each loading sequence, as some connections continue to load while others unload. This behavior is illustrated by Figures 10(a), (b) and (c), where the initial connection loading is represented by point "a" resulting in M_i and ϕ_i on the M-ϕ diagram. Point "b" in Figure 10(a) illustrates the response following a subsequent increased loading. The *effective connection stiffness*

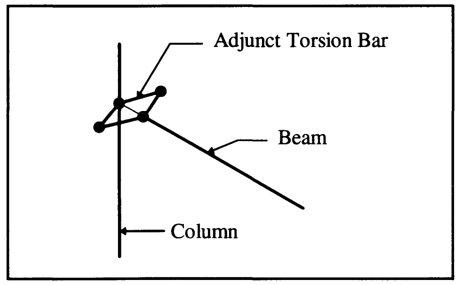


Fig. 9. Connection Model Using Torsion Elements

Table 3 Variation of Connection Behavior with Top and Seat Angle Thickness,[11] t						
Angle Thickness t (in)	Stiffness, R_k (in-k/rad)			Flexibility Ratio, u		
	4.25 k/ft			4.25 k/ft		
	R_{ks}	R_{kL}	R_{ki}	u_s	u_L	u_i
0.7500	198461	11777	1523175	0.411	6.919	0.054
0.6250	111697	8606	836525	0.730	9.469	0.097
0.5000	61445	6188	414105	1.326	13.169	0.197
0.4375	47137	5171	278375	1.729	15.758	0.293
0.3750	46617	2658	181492	1.748	30.656	0.449
0.3125	39781	1410	115693	2.048	57.812	0.704
0.2500	33264	1005	73949	2.450	81.064	1.102

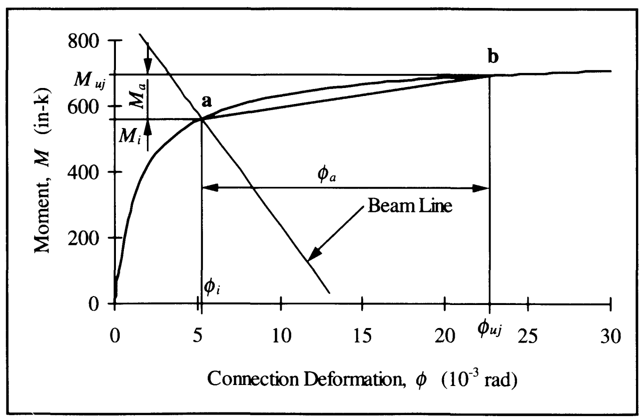


Fig. 10(a). Loading After Initial Deformation

Table 2 Expected Floor Beam Response To Connection Behavior[11]									
Top & Seat Angle Thickness, t (in)	Rigid	0.750	0.625	0.500	0.438	0.375	0.313	0.250	Pinned
M_s (in-k)	2656	1459	1080	727	596	591	521	450	0
M_f (in-k)	1328	2526	2904	3257	3388	3394	3463	3534	3984
ϕ_s (×1000 Rad)	0.00	7.35	9.67	11.84	12.64	12.67	13.10	13.54	16.30

associated with the increase of load from "a" to "b" is determined as

$$R_{kL} = \frac{M_a}{\phi_a} = \frac{M_{uj} - M_i}{\phi_{uj} - \phi_i} \tag{18}$$

where M_a and ϕ_a are defined in the figure, and M_{uj} (Equation 9) is defined as the practical limit state. Equation (18) also implies that the connection limit state is reached at the end of the second sequence, when the additional moment reaches M_a. Thus, $M_i + M_a = M_{uj}$.

Point "c" in Figure 10(b) indicates a subsequent decreased loading. The effective connection stiffness associated with a loading reversal from "a" to "c" is the same as the initial connection stiffness, R_{ki}. In some cases it may be possible for the unloading connections (at windward beam ends) to unload completely and then continue to load in the opposite direction, as illustrated in Figure 10(c). Because of this possibility, a simple test is

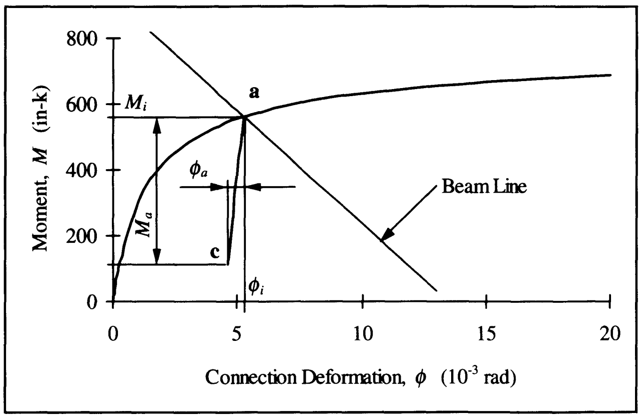


Fig. 10(b). Unloading After Initial Deformation ($|M_a| < |M_i|$)

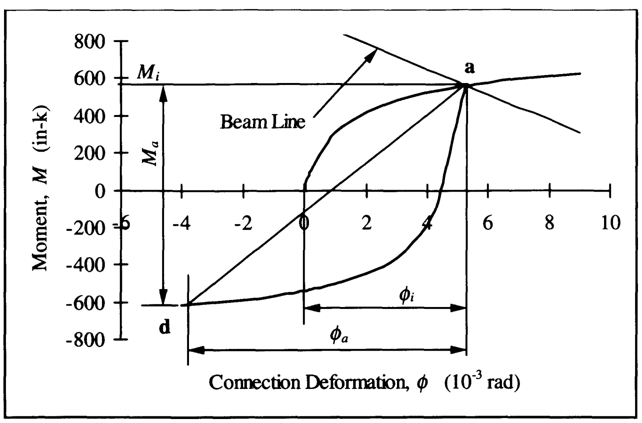


Fig. 10(c). Unloading After Initial Deformation ($|M_a| > |M_i|$)

necessary to confirm the correct use of R_{ki} as the operative connection stiffness for connections at the windward beam ends. This test is to determine which is greater in magnitude—the initial connection moment resulting from gravity loads, M_i, or the subsequent change in connection moment, M_a. Thus, the unloading stiffness at windward beam ends is equal to R_{ki}, if and only if, $|M_a| < |M_i|$.

Although possible, from the experience of the authors, it is unlikely that $|M_a| > |M_i|$, even for small values of initial gravity load. However, for such cases the conservative approach is simply to ignore the (diminished) benefit of considering R_{ki} in these connections.

Implied in this approach is the need to superimpose the analysis for gravity load with the analysis for lateral load. Superposition of load effects on non-linear structures is not legitimate because compatibility of force and deformation is violated. However, if the load-deformation relationship is modified for each loading event, superposition of non-linear structures can be achieved, or approximated, using linearized stiffness values based on the expected loads. Barakat and Chen[3,4] recognized this concept when they presented a first-order elastic analysis method consistent with the AISC LRFD approach, although their procedure neglects the effect of load reversal.

3.4 Evaluating Semi-Rigid Frame Stability Using Expected Connection Loads

The solutions provided by Julian and Lawrence[21] to determine column unbraced length factors are the basis for the well-known alignment charts for braced and unbraced frames.[2] The use of these solutions in building column design is well established common practice. However, structural configurations often conflict with the assumptions used in the Julian and Lawrence solutions. This is particularly true for semi-rigid frames when the beams are not rigidly connected to the columns. The relative stiffness distribution factor, given as

$$G = \frac{\sum(EI_c/l_c)}{\sum(EI_g/l_g)} \tag{19}$$

can be modified to accommodate these violations of the assumptions required for the original solution. Such modifications have been presented by Yura[33] and Johnston[19] for certain structural configurations, by considering the effective stiffness of the beams framing into the column ends. If the effective stiffness of a beam and its semi-rigid connections can be determined from the expected connection loads, this same approach to modify G can be utilized for semi-rigid frames. This approach was developed by Bjorhovde[5] for semi-rigid frames and presented by Christopher,[9] using Driscoll's[15] solution for the effective stiffness of a beam with semi-rigid connections at each end.

A list of effective stiffness values are presented in Table 4,[9] based on the equation for the modified relative stiffness distribution factor, given as

$$G = \frac{\sum(EI_c/l_c)(\mathrm{SRF})\mu_c}{\sum C^*} \tag{20}$$

where

SRF = the Stiffness Reduction Factor[2] to account for inelastic column behavior
μ_c = column stiffness modification factor in Table 4, to account for pinned or fixed column ends
C^* = modified beam stiffness in Table 4, representing the effective stiffness of an assembly of a beam and its connections

The values in the table are expressed in terms of the connection flexibility ratio, u.

4.0 LIMITATIONS AND RESEARCH NEEDS

4.1 Connection Types

The research to develop the use of the three-parameter power model to represent connection behavior is presently limited to connections that have bolted connection elements which are attached to the column flange. Similar work needs to be undertaken to include other connection arrangements. In particular, the database includes few connection arrangements which connect to the column web.

4.2 Reliability

Reliability studies are required to investigate and calculate a statistically based resistance factor. The suggestion by the authors of using $\phi = 0.9$ is based on current practice for connection design, but it does not necessarily account for the statistical variation of connection properties, model accuracy, and the resulting associated variation in member load effects. Such statistical data are very sparse.

The semi-rigid connection properties are sensitive to the connection detail dimensions. This fact is evident from the procedure presented by Chen et al.[7] to calculate M_u, R_{ki}, and n, the parameters that define connection behavior for the three-parameter power model. Sellier et al.[30] presented the results of studies that show the sensitivity of connection behavior from variation of connection features. This sensitivity indicates the need for the design engineer to maintain control of the connection details during the preparation of shop fabrication drawings. The usual shop drawing approval review is an appropriate opportunity to compare the connection dimensions with those used

Table 4
Modifications to *G*, The Relative Stiffness Distribution Factor

	Frame Configuration	Braced Frame	Unbraced Frame
a)	Semi-rigid connections of equal stiffness at each end.	$C^* = \frac{EI_g/l_g}{1+2u}$	$C^* = \frac{EI_g/l_g}{1+6u}$
b)	Semi-rigid connection at near end, pin at far end.	$C^* = \left(\frac{3}{2}\right)\left(\frac{EI_g/l_g}{1+3u}\right)$	$C^* = \left(\frac{1}{2}\right)\left(\frac{EI_g/l_g}{1+3u}\right)$
c)	Semi-rigid connection at near end, fixed at far end.	$C^* = \left(\frac{1}{2}\right)\left(\frac{EI_g/l_g}{1+4u}\right)$	$C^* = \left(\frac{2}{3}\right)\left(\frac{EI_g/l_g}{1+4u}\right)$
d)	Semi-rigid connections of unequal stiffness at each end.	N/A	See Note 1.
e)	Column pinned at far end.	$\mu_c = 1.5$	$\mu_c = 0.5$
f)	Column fixed at far end.	$\mu_c = 2.0$	$\mu_c = 0.667$

Notes:

1. $C^* = \frac{(2u_B+1)EI_g/l_g}{12u_Au_B+4u_A+4u_B+1}$
where "*A*" refers to the near end of the beam, and "*B*" refers to the far end of the beam.
2. $u = \frac{EI_g/l_g}{R_k}$ where R_k = Effective connection stiffness.
3. $u = \infty$ for a pinned connection.
4. $u = 0$ for a fixed (rigid) connection.

in the connection property calculations, and then make corrections if necessary.

4.3 Standardized Connections

The use of standard connection design can be a significant asset in the implementation of semi-rigid frame design. Standard connection dimensions and shear capacities are already included in the AISC Manual.[2] In an attempt to fill the need for semi-rigid connection information, Kim and Chen[23] have provided a table of properties for top and seat-angle connections with double web clip angles using the terms of the three-parameter power model, M_u, R_{ki}, and n. Additional, similar work is necessary for other connection arrangements. To better utilize the design procedure outlined in Appendix A, additional information is needed to present a list of characteristic length factor values for a_i, for the initial connection stiffness, and that for the average, expected or relative values of α_s, for the secant connection stiffness. The availability of this information will reduce the trial and adjustment process associated with selecting a connection detail. This information will also enable selection of connections that will have the engineer's anticipated effect on member forces and frame stability.

5.0 SUMMARY AND CONCLUSIONS

1. Connection behavior is estimated using mathematical models which are calculated from the material properties and detail dimensions. The three-parameter power model is appropriate and practical for this purpose.
2. New connection limit states are defined by the practical, maximum expected connection deformation, as given by Equation (8). The required connection ductility should be assured by tests of similar connection types as recorded in the databases.
3. Initial, preliminary member section sizes are selected using the expected values of connection moments determined by the beam-line diagrams superimposed on the connection M-ϕ curves.
4. Connections are concurrently selected with the member section sizes to reduce the iterative verification process for the connection deformations. This verification process ensures that the deformations determined in the frame analysis are consistent with those determined from the M-ϕ curves.
5. Identifying expected values of connection moments serves to estimate effective linearized connection stiffness values. These can be used in the structural analysis to determine member forces.
6. Column stability is evaluated using the effective end restraint at the beam-to-column connections. The combined stiffness of the beam and semi-rigid connection assembly, based on the expected connection moment, is used to calculate the effective column restraint.
7. The loading sequence which results in connection load reversal is considered in the analysis to determine member forces and frame stability.
8. The LRFD approach of AISC, which relies on amplification factors to account for second-order effects, is appropriate for design and analysis of semi-rigid frames because of the following considerations:

   - The design and analysis technique of separately evaluating M_{nt} and M_{lt} load effects allows superposition of sequentially applied loads when the semi-rigid frame behavior is non-linear.
   - The criteria for the LRFD Specification are satisfied.

These criteria make it possible for practicing engineers to implement the use of semi-rigid frame technology into design practice using tools and procedures already in place. As an aid to the proper use of these design techniques, recommendations for connection stiffness values for the design of braced and unbraced semi-rigid frames are listed in Table 5. The design procedure for semi-rigid unbraced frames is provided in Appendix A.

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Table 5
Linearized Stiffness Values

Calculation Objective	Braced Frame	Unbraced Frame: Non-Sway M_{nt}	Unbraced Frame: Sway M_{lt}
P_u and M_u (Note 1) Member Forces	R_{ks}	R_{ks}	R_{ki} and R_{kL} (Note 2)
P_{e1} Euler Buckling Load for B_1	R_{ki} and R_{kt} (Note 3)	R_{ki} and R_{kt} (Note 3)	
P_{e2} (Note 4) Euler Buckling Load for B_2			R_{ki} and R_{kL} (Note 2)
P_n Nominal Compressive Strength	R_{ki} and R_{kt} (Note 3)		R_{ki} and R_{kL} (Note 2)

Notes:
1. R_{ks} is calculated from the beam-line diagram or expected moment value.
2. Refer to the repeating pattern of connection stiffness resulting from double column curvature. (Fig. 5)
3. Refer to the alternating pattern of connection stiffness resulting from single column curvature. (Fig. 3)
4. Actual connection loading stiffness is R_{kt} at Euler buckling, but R_{kL} is more convenient and conservative.

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APPENDIX A
DESIGN PROCEDURE

The following procedure, which incorporates the criteria outlined in this paper, gives the necessary design steps for an *unbraced* (*sway permitted*) semi-rigid frame such as that shown in Figure 11.

1. Select span-to-depth ratios for the beams and columns to accommodate architectural requirements.
2. Assume a value for u_s, the connection flexibility ratio based on the connection secant stiffness R_{ks}, for each semi-rigid connection. Decreasing u_s corresponds to increasing the connection stiffness. For guidance in assuming u_s, refer to the example in Section 3.2. For example, assuming $u_s = 0.25$ and 0.70 for the roof and floor beam connections, respectively, for the frame illustrated in Figure 11 will provide relatively stiff connections to limit drift.
3. Calculate beam moments (both interior and exterior spans) using Figure 7 and Equations (14) to (17) for the assumed values of u_s, and select preliminary beam sizes.
4. Design connections to accommodate beam shear reactions, and to behave according to the assumed values of u_s.
   - Use Table 9-2 in Vol. II of the AISC Manual[2] to select web clip angles.
   - By trial and adjustment, select top and seat angle sizes and dimensions that will result in secant stiffness values that are consistent with the assumed values of u_s for the loading combination that produces the maximum gravity load for the beams. The assumptions for u_s in the example of step 2 above correspond to the following connection details:
   - Connection for W14 roof beams—($1.2D + 0.5L + 1.6L_r$)

     Top and seat angles—L6 × 6 × ½ × 0 ft.-6 in. with a gage distance of 2 in.

     Web clip angles—2-L4 × 3½ × ¼ × 0 ft.-5½ in. with a gage distance of 2 in.

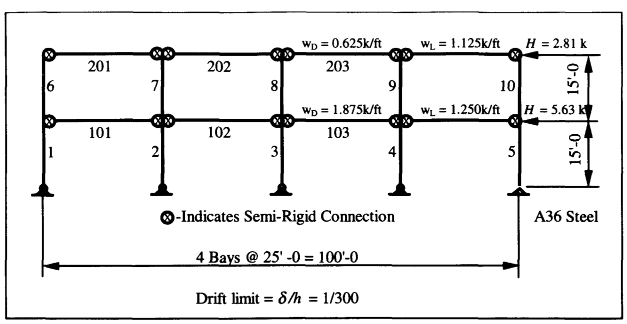

Fig. 11. Semi-Rigid Unbraced Frame Example[13]

- Connection for W21 floor beams—$(1.2D + 1.6L + 0.5L_r)$
  Top and seat angles—L6 × 6 × ½ × 0 ft.-6 in. with a gage distance of 2 in.
  Web clip angles—2-L4 × 3½ × ¼ × 0 ft.-11½ in. with a gage distance of 2 in.
- Determine the M-ϕ characteristics of the connections by using the procedures outlined by Chen et al.[7] for the three-parameter power model.
- Check connection behavior for its classification as rigid, semi-rigid, or flexible, according to the Bjorhovde et al.[6] classification method represented by Figure 2(a).
- Calculate the connection limit states using Equations 8, 9 and 10.
- Calculate the linearized stiffness values, R_{ks}, R_{kt} and R_{kL}, for the connections. Note that there is one set of values for each loading combination to be considered in the analysis.

5. Select preliminary column sizes using estimated axial loads and moments resulting from beam reactions as follows:

   - Assume the beams are rigidly connected to the columns at their "windward" ends, and pinned at their "leeward" ends, as illustrated in Figure 12.
   - Assume leeward exterior columns are governed by gravity load combinations.
   - Assume windward exterior columns and interior columns are governed by wind and gravity load combinations, and consider drift limits as well as strength requirements.

6. Create connection models using adjunct (dummy) members with properties calculated according to connection stiffness. Using the adjunct torsion bar system illustrated by Figure 9, the torsional stiffness of the adjunct members are calculated as

$$I_X = \frac{R_k L}{G} \tag{A1}$$

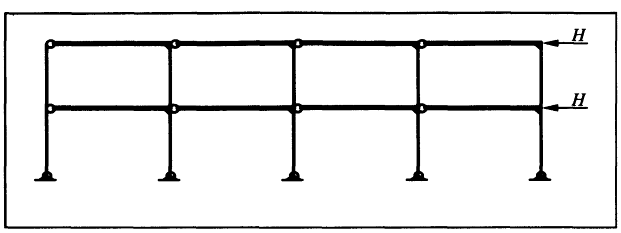

Fig. 12. Unbraced Frame with Wind Load

where

I_X = adjunct member torsion constant
L = assumed adjunct member length
G = shear modulus

Identify adjunct member property, I_X, for each of the connection stiffness values representing the following:

- Initial stiffness
- Secant stiffness, for each gravity load combination such as

$$(1.2D + 1.6L + 0.5L_r)$$
$$(1.2D + 0.5L + 1.6L_r)$$

- For reduced stiffness using:

$$R_{kL} = \frac{M_{uj} - M_i}{\phi_{uj} - \phi_i} \tag{18}$$

For example, the initial stiffness of the floor beam connection, R_{ki}, noted above in step number 4 is 1,551,000 in-k/rad for a W21 beam. This stiffness was calculated from Chen et al.[7] The corresponding stiffness of the adjunct members in Figure 9 is 1391 in.^{4} when their length is assumed to be 10 in.

Other examples of calculated dummy member properties representing these connection stiffness values is given in Table A1 for L = 10 in. and G = 11150 ksi. Flexural moment of inertia values I_Y and I_Z are assumed as 1000 in^4 to minimize errors caused by displacements of the beam ends due to flexure of the dummy members.

7. Assemble a first-order linear-elastic computer model of the frame, incorporating the connection models with adjunct members as determined in Step 6. The use of any commercially available 3-D analysis program is suitable. Prepare the model for two loading conditions, utilizing the approximate method outlined in the AISC LRFD Specification to account for second-order effects:

   - Case I, the *non-sway frame* condition—subjected to gravity loads to determine M_{nt} loads using the connection secant stiffness, R_{ks}, for all semi-rigid connections in the frame.
   - Case II, the *sway frame* condition—subjected to lateral loads to determine M_{lt} loads using the initial connection stiffness, R_{ki}, for all "windward" beam end connections, and R_{kL} for all "leeward" beam end connections.

For example, primary load cases and subsequent factored load combinations that are appropriate for the design problem illustrated by Figure 11 are listed as follows:

- Gravity load 1—Applied to the non-sway frame ($1.2D + 1.6L + 0.5L_r$)
- Gravity load 2—Applied to the non-sway frame ($1.2D + 0.5L + 1.6L_r$)
- Gravity load 3—Applied to the non-sway frame ($1.2D + 0.5L + 0.5L_r$)
- Wind load—Applied to the sway frame (W)

Case I—Gravity load (1 or 2)
Case II—Combined wind and gravity load ($1.2D + 0.5L + 0.5L_r + 1.3W$)
$(1.2D + 0.5L + 0.5L_r)B_1 + (1.3W)B_2$

8. Evaluate connection response using computer model results as follows:
   - Determine if connections are modeled correctly by checking connection stiffness as R_{ki}, R_{ks} or $R_{kL} = (M_{beam})/(\theta_{Beam} - \theta_{Column})$.
   - Show that connections do not exceed the strength limit state, M'_n.
   - Check connection deformation with the ductility limit, ϕ_{uj}. If the factored connection load effect, $M_u < M_{uj}$, then there is also adequate ductility, and no need to check the connection deformation against a connection ductility limit. Review the database for tests of similar connection types to assure that ϕ_{uj} can be reached.
   - Verify that the connection unloading stiffness is equal to the initial connection stiffness, R_{ki}, by showing that the initial connection moments, M_i, due to gravity loads are greater than subsequent wind moments, M_a, at each of the "windward" beam ends. If $|M_i| < |M_a|$, unloading connection stiffness must be adjusted to insure compatibility between connection moment and connection deformation, considering the connection loading history.
9. Evaluate story drift using the amplification factor,

$$B_2 = \frac{1}{1 - \frac{\sum P}{\sum P_{e2}}}$$

where $\sum P = \sum(D + L)$ at service loads.

Thus, the amplified drift, $\delta = B_2\delta'_x$, where δ'_x = story drift from a first-order elastic analysis. Note that connection properties used to calculate δ'_x and P_{e2} should be calculated from beam-line diagrams using service level loads.

10. Review beam response using the computer model results to evaluate strength and serviceability limit states.
11. Evaluate column sizes and frame stability using computer model results. Determine unbraced length factors from the alignment charts shown on page 6-186 of the AISC Manual.[2] Modify the relative stiffness distribution factor, G, to account for column end restraint offered by the beams using Table 4 and

$$G = \frac{\sum(EI_c/l_c)(\text{SRF})\mu_c}{\sum C^*} \tag{20}$$

- Calculate P_{e1} for the *non-sway frame:*
  – At interior columns (see Figure 3):

$$\sum C^* = \frac{EI_g/l_g}{1 + 2u_t} + \frac{EI_g/l_g}{1 + 2u_i} \qquad \text{(Table 4)}$$

Table A1
Summary of Adjunct Member Properties

Adjunct Members	R_k in-k/rad	A in²	I_x in⁴	I_y in⁴	I_z in⁴	Model (Type)
Roof Beams						
($1.2D + 1.6L + 0.5L_r$)						
R_{ks}	164600	100	148	1000	1000	Non-sway
R_{kL} (leeward ends)	9548	100	8.56	1000	1000	Sway
R_{ki} (windward ends)	712300	100	639	1000	1000	Sway
($1.2D + 0.5L + 1.6L_r$)						
R_{ks}	58260	100	52.3	1000	1000	Non-sway
Roof Beams						
($1.2D + 1.6L + 0.5L_r$)						
R_{ks}	129800	100	116	1000	1000	Non-sway
R_{kL} (leeward ends)	9949	100	8.92	1000	1000	Sway
R_{ki} (windward ends)	1551000	100	1391	1000	1000	Sway
($1.2D + 0.5L + 1.6L_r$)						
R_{ks}	227300	100	204	1000	1000	Non-sway

– At exterior columns (see Figure 3), noting the pattern of connection stiffness at alternating levels:

$$\sum C^* = \frac{EI_g/l_g}{1 + 2u_i} \qquad \text{(Table 4)}$$

or

$$\sum C^* = \frac{EI_g/l_g}{1 + 2u_t} \qquad \text{(Table 4)}$$

where

$u_t = \dfrac{EI_g}{R_{kt} l_g}$, corresponding to tangent stiffness, and

$u_i = \dfrac{EI_g}{R_{ki} l_g}$, corresponding to unloading stiffness.

– Determine unbraced length factors.
– Calculate amplification factor, B_1 and column strength, P_n, for each of the columns using the unbraced length factors determined from above.

• Calculate P_{e2} and P_n for the *sway frame*. Consider the unequal connection stiffness at each end of the restraining beams. Use column end restraint as:

$$\sum C^* = \sum \left[\frac{(2u_B + 1)EI_g/l_g}{12u_A u_B + 4u_A + 4u_B + 1} \right] \qquad \text{(Table 4)}$$

where A refers to the near end, and B refers to the far end of the beam for either initial stiffness, R_{ki} or reduced connection stiffness, R_{kL} as indicated by Figure 5.

• Determine combined column moments from Case I and Case II analysis, by

$$M_u = B_1 M_{nt} + B_2 M_{lt} \qquad \text{AISC Eq. (C1-1)}$$

• Apply the AISC LRFD interaction Equations (H1-1a) and (H1-1b) to evaluate column strength and stability.

12. Modify preliminary member sizes and connection details of the frame to meet limit states requirements and/or to improve economy. This involves repeating steps required to calculate the modified connection properties for the computer model.
13. Review the modified computer model results to verify the frame design using steps 8 through 12 to evaluate connection response, story drift, beam response, column strength and stability. This step is also necessary to verify the design by showing that the calculated connection moments are close to the expected moments used in the beam-line diagrams.