Simplified Inelastic Design of Steel Girder Bridges

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ABSTRACT

Past and present building and bridge specifications for inelastic design are discussed. The present inelastic bridge specifications apply only to compact girders and specify the mechanism method for satisfying the strength limit state. The theoretical inelastic behavior of noncompact girders subjected to moving loads is described. If these loads exceed the shakedown load, which is generally below the ultimate load predicted by the mechanism method, permanent deflections progressively increase without limit. New inelastic procedures are proposed for checking the strength and permanent-deflection limit states used in the LRFD bridge specifications. The strength check is based on the shakedown load and the permanent-deflection check is made by limiting positive-bending stresses after yielding has occurred at interior supports and caused inelastic redistribution of moments. The proposed procedures apply to both compact and noncompact continuous-span girders and are much simpler than present procedures.

INTRODUCTION

Building Specifications

For many years, research on the inelastic behavior of buildings has been conducted in the United States and abroad (ASCE, 1971). This research showed that the full strength of structures, especially statically indeterminate structures, can only be determined by considering inelastic behavior. Suitable procedures for calculating this strength for buildings were developed and included in the AISC specifications (AISC, 1993). As a result, plastic-design procedures for buildings are now well established.

These procedures apply specifically to compact members that can sustain sufficient plastic rotations to form mechanisms, which limit the ultimate strength of the structure. Generally the members have a constant cross section for their full length or have only one or two changes in cross section within that length. Usually buildings need not be designed for moving loads, although theoretical procedures (ASCE, 1971) are available to calculate the shakedown load at which permanent deflections stabilize under a set of moving loads. This shakedown load is usually somewhat lower than the ultimate load for the structure and, therefore, is the appropriate limit defining strength under moving loads.

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Bridge Specifications

Although the basic principles on which the building specifications (AISC, 1993) are based apply to bridges as well as buildings, two significant differences must be considered when applying the methods to bridges. First, buildings generally can be designed for static loads, but bridges must be designed for moving loads. Second, buildings generally can utilize compact members while bridges often utilize noncompact girders with slender webs. Also, longer span bridge girders are likely to have many more changes in cross section than typical building members. Because of these differences, the plastic-design procedures for buildings are not sufficient for bridges.

The increase in strength provided by inelastic behavior was first incorporated into the AASHTO specifications for highway bridges in a limited empirical way. Specifically, two simple provisions were incorporated into the Load Factor Design (LFD) specifications (AASHTO, 1973). First, the elastic moment caused at a compact section by the design loads was permitted to equal the plastic-moment capacity of the section rather than being limited to the yield-moment capacity. Second, 10 percent of the peak negative elastic moments in continuous-span girders were permitted to be shifted to positive-bending regions before the bending strengths at these locations were checked. This second provision was intended to account in an approximate way for the redistribution of moments that actually occurs due to inelastic action.

Comprehensive inelastic design procedures were first permitted for highway bridges with the adoption of guide specifications for Alternate Load Factor Design (ALFD) in 1986 (AASHTO, 1986). These design procedures, which were originally called autostress design procedures, are applicable only to compact sections. They specify a strength check by the plastic-design mechanism method at Maximum Load and a permanent-deflection check by inelastic procedures at Overload. In this latter check, yielding is allowed to occur at peak negative-moment locations (piers) and the resulting redistribution moments (automoments) are calculated. The stresses in positive-bending regions due to the combined applied and redistribution moments are limited to 95 percent and 80 percent of the yield stress in composite and noncomposite girders, respectively. These same stress limits are imposed at both positive- and negative-bending locations in the LFD specifications (AASHTO, 1992) and are assumed to prevent objectionable permanent deflections. In 1994, the inelastic design procedures from the guide specifications (AASHTO, 1986) were incorporated into the new AASHTO Load and Resistance Factor Design (LRFD) bridge specifications (AASHTO, 1994) with minor modifications and additions. These inelastic design procedures are alternative provisions that apply only to continuous-span bridges with compact sections at piers.

In 1993, inelastic rating procedures were proposed for highway bridges (Galambos et al., 1993). These procedures utilize the same rating vehicles and load and resistance factors as the alternative load factor rating procedures adopted in 1989 guide specifications (AASHTO, 1989), but define the strength limit state as either shakedown (deflection stability) or a specified maximum permanent deflection.

Present Study

The present paper first describes the inelastic behavior of highway bridges under moving loads. Specifically, the following aspects of inelastic behavior are discussed: (a) plastic rotations due to yielding at critical locations, (b) redistribution moments and permanent deflections resulting from these plastic rotations, (c) shakedown due to sequential loadings, and (d) methods of calculating inelastic behavior of compact and noncompact members.

The paper then proposes simplified inelastic methods of checking the strength and permanent-deflection limit states used in the LRFD bridge specifications (AASHTO, 1994). The simplified strength check is based on the shakedown load and the simplified permanent-deflection check is made by limiting positive-bending stresses after yielding has occurred at piers. The paper also explains how a rigorous inelastic analysis can be used directly in checking the strength and permanent-deflection limit states. The proposed procedures apply to both compact and noncompact sections and are much simpler than present procedures. Corresponding proposed specification provisions and commentary suitable for inclusion in the LRFD bridge specifications are presented elsewhere (Schilling et al., 1996).

Throughout this paper, compact and noncompact mean compact and noncompact according to the LRFD bridge specifications (AASHTO, 1994).

INELASTIC BEHAVIOR

Plastic Rotation

Typical rotation curves for compact and noncompact sections are shown in Figure 1. Such curves are obtained by applying a concentrated load at midspan of a simple beam. The total rotation at any load is the sum of the slopes at the ends of the beam and is composed of elastic and plastic components as illustrated. Elastic rotation occurs along the entire length of the member and can be accurately calculated by beam theory. The plastic rotation, in contrast, is concentrated in a short region at midspan and is usually determined by subtracting the calculated elastic rotation from the measured total rotation in a test.

If the loading is completely removed, the plastic rotation remains as illustrated by the unloading curve in the figure. If load is then reapplied, the rotation increases along the previous unloading curve and no additional plastic rotation results unless the original load is exceeded. Although plastic rotation actually occurs over a finite yield length, it is usually assumed to occur at a single cross section (infinitesimal length) to simplify calculation procedures for plastic design (ASCE, 1971). Thus, the member is assumed to be elastic over its entire length and to have all of the plastic rotation

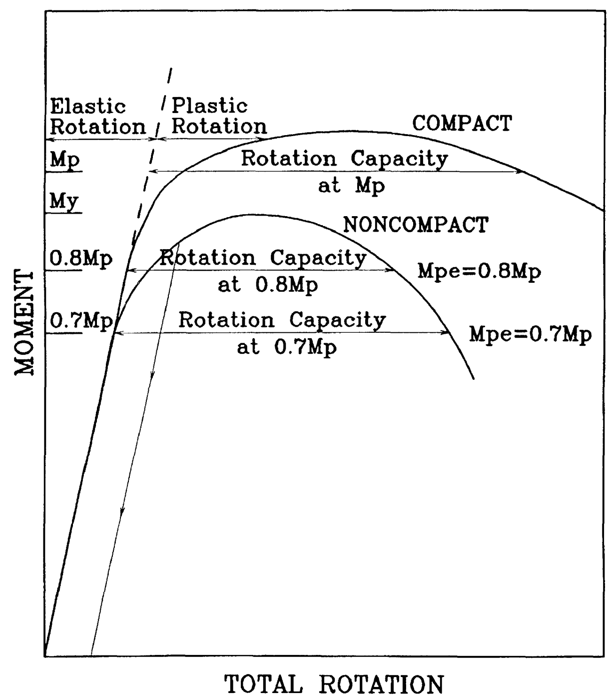

Fig 1. Typical rotation curves for compact and noncompact sections.

concentrated in a single angular discontinuity. This discontinuity is equivalent to the angular discontinuity created by cutting the ends of two beams slightly off square and then welding them together end to end. Thus, plastic rotations caused by yielding have the same effect on subsequent structural behavior as angular discontinuities (kinks) that could be built into a member by slight angular mismatches at splices.

For compact sections, the rotation curve rises above the plastic moment, remains above that moment until a large amount of rotation has been imposed, and then decreases below the plastic moment if additional rotation is imposed. The plastic rotation over which the curve remains above M_p is the rotation capacity of the section.

For noncompact sections, the rotation curve does not reach the plastic moment and starts decreasing at lower rotations than the curve for compact sections. As illustrated in the figure, however, combinations of effective plastic moments, and corresponding rotation capacities, can be defined in a manner similar to that of the full plastic moment and corresponding rotation capacity. Specifically, the rotation capacity for a given effective plastic moment, M_{pe}, is the plastic rotation over which the moment exceeds M_{pe}. The effective plastic moment can be used in place of the full plastic moment in plastic-design procedures, such as the mechanism method, provided that the required rotation capacity for a section does not exceed the actual rotation capacity corresponding to that effective plastic moment.

Redistribution Moments and Permanent Deflections

When plastic rotations occur in continuous spans, redistribution moments and permanent deflections develop as illustrated in Figure 2 for a two-span girder that has yielded at the pier (Schilling, 1993). This yielding causes a permanent angular discontinuity that can be simulated by cutting the ends of two beams slightly off square and then welding them together end to end as illustrated in an exaggerated way in the figure. When the spliced beam is placed on the abutments and held down against the pier (either by a downward reaction at the pier or by deadweight), redistribution moments occur along the beam as illustrated. These redistribution moments remain after all loading has been removed from the structure and cause permanent deflections along the girder.

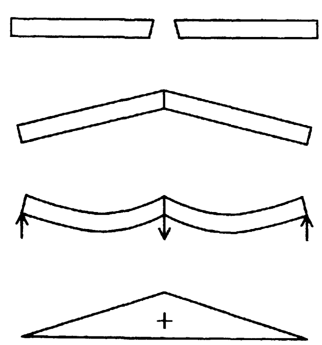

Fig. 2. Plastic-rotation analogy.

If the amount of plastic rotation at a pier is known, the magnitude of the resulting redistribution moments and permanent deflections can be calculated by classical methods of indeterminate analysis. In these methods, the continuous span is treated as a series of simple spans and the end moments necessary to restore continuity with the known angular discontinuity in place are determined. The end moment required to cause a given end rotation depends on the stiffness of the adjacent span. Thus, the magnitude of the redistribution moments depends on the magnitude of the plastic rotation and on the stiffness properties of the girder.

Redistribution moments are held in equilibrium by the reactions at piers and abutments. Therefore, the redistribution moments must peak at pier locations and vary linearly between reactions. If yielding and plastic rotations occur at more than one pier, the redistribution moments caused by the plastic rotation at each pier can be calculated separately and summed to get the total redistribution moments. Subsequent loading may cause additional yielding and thereby modify the redistribution moments.

Yielding at peak-moment and flange-transition locations within a span also causes plastic rotations, redistribution moments, and permanent deflections (Schilling, 1993). The redistribution moments, which can be calculated in a manner similar to that for yielding at piers, again are held in equilibrium by reactions at piers and abutments and must vary linearly between such supports.

Shakedown

If moving loads are applied to a continuous-span girder bridge, the redistribution moments and permanent deflections caused by yielding at one location when the loads are in a particular position may be changed by additional yielding at that location when the loads are moved to a new position. For example, consider a symmetrical three-span girder loaded by a set of two concentrated loads representing a truck as explained in detail elsewhere (Schilling, 1996).

First, the loads are positioned to straddle Pier 1 and cause yielding and plastic rotation at that pier. The resulting redistribution moments have a positive peak at Pier 1 and a lower negative peak at Pier 2. Next, the loads are positioned to straddle Pier 2 and cause plastic rotation at that pier. The resulting redistribution moments have a positive peak at Pier 2 and a lower negative peak at Pier 1. Next, the loads are returned to straddle Pier 1 and cause additional plastic rotation because of the negative redistribution moment remaining at that pier from the previous load position.

Under continued repositioning of the loads, the plastic rotations, redistribution moments, and permanent deflections will eventually stabilize if these loads do not exceed the shakedown loads. Otherwise, the plastic rotations and permanent deflections will continue to increase without limit and incremental collapse will occur. If a girder shakes down to an equilibrium condition it will behave elastically under all subsequent loadings that do not exceed the original loading.

According to plastic-design theory (ASCE, 1971), a structure will shakedown to an equilibrium condition if a valid pattern of redistribution moments can be found such that the algebraic sum of these redistribution moments and the elastic moments for any loading position does not exceed the plastic-moment capacity at any cross section. This theorem was developed for compact members, but can be applied to noncompact members if a reduced effective-plastic-moment capacity is used instead of the full plastic-moment capacity. This effective-plastic-moment capacity must be based on a rotation capacity sufficient to achieve shakedown. Thus, the required rotation capacity at each pier is the plastic rotation at that pier at shakedown.

Inelastic Analysis of Noncompact Members

Classical plastic-design theory was developed specifically for compact members but can be applied to noncompact members if an effective plastic moment based on an appropriate rotation capacity is used in place of the full plastic moment. An inelastic analysis of either type of member is performed by assuming that all yielding is concentrated at plastic hinges of zero length and that the moment at each plastic hinge remains constant at the full or effective plastic moment as loading is applied to the structure (ASCE, 1971).

Actually the moments at yield locations vary as loading is applied; therefore, the classical method of analysis is approximate. The variation is much larger for noncompact members than for compact members because of the shapes of the respective rotation curves. Consequently, inelastic behavior, especially of noncompact members, can be defined more precisely by two similar new methods that account for the variation of moment at yield locations: the unified autostress method (Schilling, 1989; Schilling, 1993) and the residual deformation method (Dishongh, 1990; Dishongh and Galambos, 1992).

Like the classical method, both new methods assume that all yielding is concentrated at plastic hinges of zero length, which can be represented as angular discontinuities. The moment at each yield location is uniquely defined by simultaneously satisfying two relationships: a continuity relationship and a rotation relationship.

The continuity relationship interrelates the total moments at all pier locations and the plastic rotations at all yield locations; it depends on the stiffness properties of the girder. The total moment at each pier is equal to the algebraic sum of the elastic moment and the redistribution moments due to plastic rotations at all yield locations. The rotation relationship interrelates the plastic rotation and total moment at each yield location; thus, it is the rotation curve for the section. Typical rotation curves for various types of sections are available from several sources for use in these two methods (Schilling et al., 1996).

In both methods, a solution is obtained by conceptually cutting the continuous span into simple spans and then calculating the end moments necessary to restore continuity with all plastic rotations (angular discontinuities) in place. The unknown inelastic quantities are the redistribution moments at all piers and the plastic rotations at all yield locations. A unique solution is achieved by satisfying the continuity relationship at each pier and the rotation relationship at each yield location. The number of these relationships matches the number of unknowns. Once the inelastic unknowns have been determined, the redistribution moments and permanent deflections throughout the girder can be determined by well-known elastic analytical methods.

Both analytical methods were developed specifically to calculate the redistribution moments and permanent deflections in a continuous-span girder due to a given loading. By substituting an elastic moment envelope for the elastic moment diagram for the given loading, however, the methods can also be used to calculate the redistribution moments and permanent deflections due to the combination of stationary and moving loads that define the elastic moment envelope. In other words, the methods predict the final redistribution moments and permanent deflections that remain after live loads are successively placed at all possible positions to simulate passage of one or more trucks across the bridge.

Simultaneous equations or iterative procedures are usually required for a solution. If the simultaneous equations do not provide a valid solution or the iterative procedure does not converge, this means that the given loading exceeds the ultimate strength of the girder or that the combination of static and moving loads defining the elastic moment envelope exceeds the shakedown loading.

STRENGTH LIMIT STATE

General

Proposed Limit

Shakedown is the appropriate strength limit for continuous-span bridge girders subjected to moving loads (Galambos et al., 1993; Schilling et al., 1996) since incremental collapse can theoretically occur at loadings up to 15 percent below the ultimate strength of the girder as defined by the mechanism method (ASCE, 1971). Therefore, two alternative shakedown checks are proposed to satisfy the strength limit state in the LRFD bridge specifications: a simplified check or a rigorous check. These alternative checks are described later. Recent tests have confirmed shakedown behavior for bridge girders subjected to moving loads (Barker et al., 1996).

The simplified shakedown check has the major advantage that it is much easier to determine than ultimate strength calculated by the mechanism method. In checking shakedown by the simplified method, the entire girder is checked in one simple operation; no simultaneous equations or iterative procedures are required. In the mechanism method, in contrast, all possible mechanisms must be individually identified and checked. This can be tricky and involves considerable work if there are many flange transitions and/or unsymmetric spans (Schilling et al., 1996).

Resistance Factor

Several circumstances tend to reduce the risk of incremental collapse in actual bridges. Tests have shown that shakedown almost always occurs at higher sequential loadings than predicted by theory because of strain hardening (ASCE, 1971). These results are for static tests in which loads and deflections were allowed to stabilize after each loading in the sequence. Because of dynamic yielding effects discussed elsewhere (Schilling et al., 1996), only a small amount of the yielding theoretically predicted for a given truck will occur during a single passage of that truck across the bridge. This is true even if the truck moves at a relatively slow speed since several minutes are required to fully stabilize loads and deflections after a load application in the inelastic range. Thus, higher loadings than theoretically predicted, and many repetitions of these loadings, are required to produce incremental collapse in bridges.

Furthermore, inelastic lateral redistribution of moments among adjacent girders provides an additional reserve strength not accounted for in a single-girder check (Frangopol and Nakib, 1991; Moses et al., 1993). The magnitude of this additional reserve strength depends on the strength and stiffness of the deck and any transverse members present. Because of these mitigating factors, and because ample visual warning of incremental collapse is provided by the progressively increasing permanent deflections, it is proposed that a resistance factor, ϕ_{sd}, of 1.1 be used for this limit state.

Compression-Flange Bracing

The bracing equation specified for inelastic design in the LRFD bridge specifications (Equation 6.10.11.1.1c-1) can be used with the proposed new procedures to prevent lateral buckling of the compression flange in negative-bending regions. The total moments, elastic plus redistribution, should be used in this equation. A check of compression-flange bracing is not required in positive-bending regions because it is assumed that the deck provides adequate support to the compression flange in that region.

Limits of Applicability

In line with present inelastic procedures (AASHTO, 1991; AASHTO, 1994), it is proposed to limit the new inelastic design procedures to constant-depth I girders of steels with yield stresses not exceeding 50 ksi. Although inelastic behavior of other types of girders, including variable-depth girders, box girders, and girders of stronger steels, is expected to be similar, sufficient tests have not yet been performed to validate the proposed new procedures for such girders. Test and analytical studies are in progress at the University of Nebraska and Georgia Tech University to determine the rotation behavior of high performance steels with yield stresses of 70 and 100 ksi. This is the key information required to validate the simplified inelastic procedures for such steels.

Similarly, it is proposed to limit the new procedures to girders with web slenderness ratios not exceeding the maximum permitted in Article 6.10.5.3.2b of the LRFD bridge specifications (AASHTO, 1994) for webs without a longitudinal stiffener. Sufficient studies have not been performed to develop inelastic design procedures for girders with longitudinal stiffeners.

Simplified Check

Bending Resistance

According to the shakedown theorem discussed previously (ASCE, 1971), shakedown can be checked for a given sequential loading by simply assuming valid redistribution moments and checking that the algebraic sum of these redistribution moments and the elastic moment envelope does not exceed the effective-plastic-moment capacity at any location. This approach can fit conveniently into the format of the LRFD bridge specifications (AASHTO, 1994) by defining the bending resistance, M_r, as

$$M_r = \phi_{sd} M_{pe} - M_{rd} \geq 0 \tag{1}$$

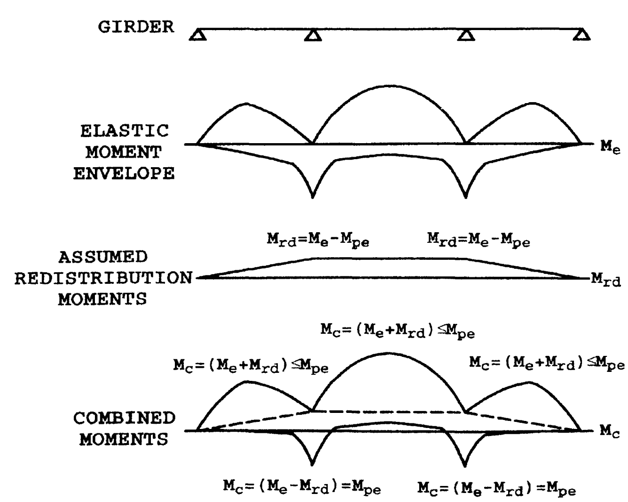

Fig. 3. Shakedown check.

where M_{pe} is the effective plastic moment of the section, M_{rd} is the redistribution moment at the section, and ϕ_{sd} is the resistance factor for shakedown to be taken as 1.1. The elastic moments at all sections due to the factored loadings must not exceed this bending resistance as illustrated in Figure 3. For composite girders, the elastic moments are the combined moments for loads applied before and after the slab has hardened.

If Equation 1 is not satisfied at a particular positive-bending location, the girder cross section can be changed either at that location or at adjacent pier locations to satisfy the deficiency. Similarly, if the check shows that particular positive-bending regions are over-designed, changes can be made either at those locations or at adjacent pier locations to improve the economy of the girder. If the equation is not satisfied at flange-transition locations in negative-bending regions, these locations must be moved.

Redistribution Moments

It is proposed that the redistribution moments at piers be taken as

$$M_{rd} = \phi_{sd} M_{pe} - M_e \geq 0 \tag{2}$$

where M_e is the total elastic moment at the pier due to loads applied before and after the slab has hardened. This is the smallest value of M_{rd} that satisfies the shakedown theorem at the pier; a larger M_{rd} would satisfy this theorem at the pier but would make it harder to satisfy the theorem in positive-bending regions. In the equation, the correct sign must be used for M_e,and M_{pe} is for bending in the same direction as M_e; normally, M_e and M_{pe} are negative and M_{rd} is positive. If M_{pe} is numerically larger than M_e, no redistribution of moment occurs and M_{rd} is zero; this means that the pier section is over-designed for this limit state.

Since the redistribution moments must vary linearly between reactions as discussed earlier, the full redistribution moment diagram can be obtained by connecting the pier moments by straight lines and extending these lines from the first and last piers to the zero moments at adjacent abutments as illustrated in Figure 3. These redistribution moments are the final redistribution moments caused by all yielding at positive- and negative-bending locations due to the loadings defined by the elastic moment envelope.

Effective Plastic Moment for Negative-Bending Sections

Equations defining the effective plastic moment, M_{pe}, for negative-bending sections are developed in Appendix I and given below for (a) ultracompact-flange sections and (b) all other sections. These equations are intended to assure that the sections provide a rotation capacity of at least 30 mrad, which is a conservative estimate of the required capacity for negative-bending sections in a shakedown check as discussed in the appendix.

For ultracompact-flange sections, the web may be noncompact but the compression flange must satisfy:

$$\frac{b_c}{2t_c} \leq 0.291\sqrt{\frac{E}{F_{yc}}} \tag{3}$$

where b_c is the compression-flange width, t_c is the compression-flange thickness, E is the modulus of elasticity, and F_{yc} is the compression-flange yield stress. For composite or noncomposite sections satisfying Equation 3:

If

$$\frac{2D_{cp}}{t_w} \leq 3.76\sqrt{\frac{E}{F_{yc}}}, \text{ then:}$$

$$M_{pe} = M_p \tag{4}$$

If

$$3.76\sqrt{\frac{E}{F_{yc}}} < \frac{2D_{cp}}{t_w} \leq 5.05\sqrt{\frac{E}{F_{yc}}} \text{ then:}$$

$$M_{pe} = M_y \tag{5}$$

If

$$\frac{2D_{cp}}{t_w} > 5.05\sqrt{\frac{E}{F_{yc}}}, \text{ then:}$$

$$M_{pe} = \left[1.56 - 0.111\left(\frac{2D_{cp}}{t_w}\right)\sqrt{\frac{F_{yc}}{E}}\right]M_y \tag{6}$$

where D_{cp} is the depth of web in compression at the plastic moment, t_w is the web thickness, M_p is the plastic moment, M_{pe} is the effective plastic moment, and M_y is the yield moment. For composite sections (steel plus rebars), M_y is the total moment due to loads applied before and after the slab has hardened that causes a stress in one or both flanges equal to the yield stress.

Ultracompact-flange sections at piers must be provided with a transverse stiffener placed a distance of one-half the web depth on each side of the pier (Schilling and Morcos, 1988; Schilling, 1993). If the stiffeners are placed on only one side of the web, they must be welded to the compression flange. These stiffeners are needed at piers to improve the rotation capacity, but are not required at negative-bending flange-transition locations because smaller rotation capacities are required at such locations.

For negative-bending sections that do not qualify as ultracompact-flange sections, the effective plastic moment should be calculated by substituting the following effective yield stresses for the actual yield stresses in the usual plastic-moment calculations:

$$F_{yec} = 0.0845E\left(\frac{2t_c}{b_c}\right)^2 \leq F_{yc} \tag{7}$$

$$F_{yet} = F_{yec} \leq F_{yt} \tag{8}$$

$$F_{yew} = 1.32E\left(\frac{t_w}{D_{cp}}\right)^2 \leq F_{yw} \tag{9}$$

where F_{yec}, F_{yet}, and F_{yew} are the effective yield stresses for the compression flange, tension flange, and web, respectively, and F_{yc}, F_{yt} and F_{yw} are the corresponding actual yield stresses. If rebars are included in the section, the full yield stress should be used for these rebars. Additional transverse stiffeners placed on both sides of piers are not required for such negative-bending sections.

Equations 7, 8, and 9 are essentially the same as equations included in the inelastic provisions of the LRFD bridge specifications (AASHTO, 1994). If both the following equation and Equation 3 are satisfied, the section is fully effective and M_{pe} equals M_p:

$$\frac{2D_{cp}}{t_w} \leq 2.30\sqrt{\frac{E}{F_{yw}}} \tag{10}$$

Effective Plastic Moment for Positive-Bending Sections

For composite sections in positive bending, M_{pe} equals M_p if the web is compact since the slab resists local buckling of the compression flange. The web is usually compact because only a small portion of it is in compression. Such compact sections provide an adequate rotation capacity for positive-bending locations (Schilling et al., 1996). For noncomposite sections in positive bending, the effective plastic moment can be conservatively calculated by the procedures proposed for negative-bending sections.

Rigorous Check

As an alternative to the simplified check, shakedown can be directly checked by a rigorous inelastic analysis by the unified autostress method or the residual deformation method. The elastic moment envelope for the specified loadings should be used in this analysis to determine the final permanent deflections. If a valid solution can be obtained, the girder will shakedown to an equilibrium condition under repeated applications of the specified loadings. To be consistent with the simplified check, the rotation curves used in the analysis must be modified by the resistance factor, ϕ_{sd}. Specifically, the modified curve should be obtained by multiplying the moment corresponding to a given rotation by ϕ_{sd}.

PERMANENT-DEFLECTION LIMIT STATE

General

Proposed Limits

This serviceability limit state is intended to avoid objectionable permanent deflections that could adversely affect riding quality. Two alternative checks are proposed to accomplish this: (a) a simplified check in which yielding is permitted at piers but the positive-bending stresses that occur after redistribution of moments are limited to specified percentages of the yield stress and (b) a rigorous check in which the actual permanent deflections calculated by inelastic methods are limited to a specified value. The first approach is similar to that used in present inelastic specifications (AASHTO, 1991; AASHTO, 1994), but the proposed calculation procedures are much simpler.

Resistance Factor

A resistance factor of 1.0 is generally used with serviceability limit states (AASHTO, 1994). Therefore, it is conservatively proposed to use a resistance factor of 1.0 even though the many mitigating circumstances discussed in subsequent paragraphs suggest that a higher resistance factor, or a lower loading than is presently specified (AASHTO, 1994), could safely be used for this limit state. In the future, it might even be possible to show by a reliability analysis that the risk of objectionable permanent deflections is acceptably low for girders that satisfy the proposed strength limit state check. In that case, the permanent-deflection limit state check could be eliminated.

The moments caused by the factored live loading specified for this limit state (AASHTO, 1994) approximate the maximum moments expected during the life of the bridge (Nowak, 1995; Schilling et al., 1996). Because of dynamic yielding effects, many loading cycles (truck passages) would be required to develop the full theoretical permanent deflections; this effect was illustrated in the AASHO road tests (TRB, 1962). The moments caused by these additional load cycles must be lower than the maximum since bridges are subjected to a continuous spectrum of moments of varying magnitudes and the specified loadings correspond to the maximum of these.

Numerous field measurements have shown that the actual stresses in bridges under traffic loading are almost always well below those calculated by normal design procedures (Moses et al., 1987). Thus, the actual amount of yielding and the permanent deflections are expected to be less than calculated. Many circumstances contribute to the difference between actual and calculated stresses including: (a) unintended composite action, (b) contributions to strength from nonstructural elements, such as parapets, (c) unintended partial end fixity at abutments, (d) catenary tension forces due to "frozen" joints or rigid end supports, (e) longitudinal distribution of moment, and (f) direct transfer of load through the slab to the supports.

The specified limiting positive-bending stresses (95 percent and 80 percent of yield stress) are considered to be conservative, especially for noncomposite sections, since numerous static beam tests have shown that permanent deflections caused by yielding below the yield moment are small enough to be neglected (ASCE, 1971). This suggests that the yield stress, rather than 80 percent to 95 percent of the yield stress, would be an appropriate limit for the permanent-deflection check.

The consequences of violating the permanent-deflection limit state are much smaller than the consequences of violating the strength limit state. Therefore, a considerably lower reliability factor is justified for the permanent-deflection check.

Little or no evidence of objectionable permanent deflections in steel bridges subjected to normal traffic loading for many years has been reported. This includes many bridges designed for lower loadings than are now allowed. Over 70,000 bridges are now older than 50 years and about 1200 bridges older than 100 years.

Simplified Check

Positive-Bending Stresses

In the proposed simplified check, yielding is permitted at pier sections and stresses need not be checked within a length extending to the nearest flange transition or point of contraflexure, whichever is closer, on each side of the pier. At all other locations, the flange stresses after redistribution of moments must satisfy:

$$|f_{ef} + f_{rd}| \leq |\alpha F_{yf}| \tag{11}$$

where f_{ef} is the elastic stress in the flange due to the specified factored loading, f_{rd} is the flange stress due to the redistribution moment, F_{yf} is the flange yield stress, and α is 0.95 for composite sections and 0.80 for noncomposite sections. For composite sections, f_{ef} is the sum of the flange stresses caused by moments applied before and after the slab has hardened and f_{rd} is the flange stress caused by the redistribution moment applied to the composite section. Normally, f_{ef} and f_{rd} have the same sign in positive-bending regions.

The stress limits are the same as the positive-bending stress limits applied in the original inelastic bridge design specifications (AASHTO, 1986); they indirectly limit the permanent deflections that can occur as a result of yielding at pier sections. As mentioned earlier, these stress limits are quite conservative compared with numerous tests made over the years and probably could be changed to 100 percent of the yield stress for both composite and noncomposite sections. To be conservative and to limit the length over which negative-bending yielding occurs, these positive-bending stress limits are also applied at negative-bending flange transitions. A mrad is one thousandth of a radian.

Redistribution Moment

It is proposed that the redistribution moments be determined in the same way as in the strength check. Specifically, the redistribution moments at pier locations can be determined from

$$M_{rd} = M_{pe} - M_e \geq 0 \tag{12}$$

and the rest of the redistribution-moment diagram can be obtained by connecting these moments by straight lines. M_{pe} is the effective plastic moment defined under the next heading and M_e is the elastic moment for the factored loading specified for this limit state.

The proposed procedure is based on the assumption that any yielding that occurs below the effective plastic moment is small enough to be neglected (Schilling et al., 1996). This assumption is consistent with proposed inelastic rating procedures (Galambos et al., 1993) and with plastic-design procedures for buildings (AISC, 1993; ASCE, 1971). It is further justified by the conservative aspects of the permanent-deflection check discussed earlier.

Since it utilizes elastic moment envelopes and effective plastic moments, the proposed procedure gives the redistribution moments that will remain after the specified loadings have been repeatedly applied at different locations as explained in more detail elsewhere (Schilling et al., 1996). Thus, the present AASHTO requirement (AASHTO, 1994) that "the two spans adjacent to each interior support shall be successively loaded until the resulting redistribution moments converge within acceptable limits" is not needed with the proposed procedure.

Effective Plastic Moment at Pier Sections

The effective plastic moments are needed at pier sections to permit calculation of the redistribution stresses, f_{rd}, in Equation 10, but are not needed for other sections. It is proposed that the effective plastic moments for pier sections be based on a required rotation capacity of 9 mrad (Schilling et al., 1996), which is conservative for the permanent-deflection limit state as discussed in Appendix I.

For compact sections,

$$M_{pe} = M_p \tag{13}$$

where M_p is the full plastic moment. The rotation capacity of such sections is well above 9 mrad.

For sections with ultracompact compression flanges and noncompact webs,

$$M_{pe} = M_y \tag{14}$$

since tests (Schilling and Morcos, 1988; Schilling, 1990) showed that such sections can sustain M_y through a rotation exceeding 9 mrad as indicated in Appendix I.

For noncompact sections

$$M_{pe} = 0.8M_y \tag{15}$$

since tests (Schilling, 1985; Schilling, 1988) showed that such sections can sustain a moment equal to $0.8M_y$ through a rotation of 9 mrad as indicated in Appendix I.

Rigorous Check

As an alternative to the simplified check, the permanent deflection can be calculated by a rigorous inelastic analysis and limited to a specified value. The elastic moment envelope for the specified factored loading must be used in the analysis to get the final permanent deflection as described earlier. Typical rotation curves for various types of sections are available elsewhere (Schilling et al., 1996) and should be used without modification since the resistance factor for this limit state is 1.0.

It is usually appropriate to define deflection limits as a fraction of the span length (AASHTO, 1992; AASHTO, 1994). For the permanent-deflection limit state, it is proposed that the maximum permanent deflection calculated within a span of length, L, be limited to $L/300$. This is the limit above which deflections become visually noticeable (Galambos and Ellingwood, 1986) and was suggested to be the highest limit suitable for inelastic rating (Galambos et al., 1993). $L/600$ was suggested as an alternative, more conservative, limit for inelastic rating. It corresponds to the maximum permanent deflections observed in the AASHO road tests (TRB, 1962) for beams subjected to stresses not exceeding the 95 percent and 80 percent stress limits discussed earlier. Since the choice of the specified limit has a major influence on the permanent-deflection check, and hence on the economy of the design, a limit based specifically on riding quality should be developed in the future.

CONCLUSIONS

The following conclusions were drawn from the present study.

Present plastic-design procedures for buildings are not sufficient for bridges, which are subjected to moving loads and often utilize noncompact members with many changes in cross section. The inelastic behavior of such members under stationary or moving loads, however, can be satisfactorily analyzed by methods such as the unified autostress method or the residual deformation method. In these methods, all yielding is assumed to be concentrated in plastic hinges of zero length and a unique solution is obtained by simultaneously satisfying rotation relationships at all yield locations and continuity relationships at all pier locations.

Present inelastic design specifications for bridges apply only to compact girders. They specify the mechanism method, or the unified autostress method, for checking the strength limit state and the beam-line method, or the unified autostress method, for checking the permanent-deflection limit state. The mechanism method, which gives the ultimate load for the girder, is straightforward for simple cases but can be tricky for unsymmetrical spans with many flange transitions. Similarly, the beam-line and unified autostress methods are straightforward for simple cases but usually involve an iterative procedure and other complications for more complex cases.

The simplified procedure proposed herein for checking the strength limit state applies to both compact and noncompact girders, and assures that permanent deflections caused by repeated applications of the specified loading will eventually shakedown to an equilibrium condition. Thus, the proposed procedure is based on the shakedown load, which is somewhat smaller than the ultimate load and is the appropriate strength limit for girders subjected to moving loads. The proposed procedure utilizes elastic moment envelopes and does not require any simultaneous equations or iterative procedures even for the most complex cases. Therefore, it is generally much simpler to apply than the present procedures.

The simplified procedure proposed herein for checking the permanent-deflection limit state applies to both compact and noncompact girders and assures that objectionable permanent deflections that could adversely affect riding quality will not occur under the specified loading. Specifically, yielding is permitted at piers, but the resulting permanent deflections are indirectly limited by restricting the positive-bending stresses after redistribution of moments due to pier yielding. The same stress restriction is imposed in present procedures, but a simplified approach, rather than the beam-line or unified autostress method, is used in the proposed procedure to calculate these positive-bending stresses. The simplified approach is similar to that proposed for the strength check; it utilizes elastic moment envelopes and does not require any simultaneous equations or iterative procedures. Therefore, it is generally much simpler to apply than the present procedure.

As an alternative to the proposed simplified procedures, a rigorous inelastic analysis can be used to directly check the strength and permanent-deflection limit states for both compact and noncompact girders. The elastic moment envelopes due to the specified loading are used in the check of either limit state. The girder satisfies the strength limit state if a valid solution can be found for this elastic moment envelope since a valid solution is not possible if the shakedown load is exceeded. In that case, the girder will fail by incremental collapse. To satisfy the permanent-deflection limit state, the calculated permanent deflection must not exceed a specified value, perhaps $L/300$.

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APPENDIX I. DERIVATIONS OF NEGATIVE-BENDING EFFECTIVE PLASTIC MOMENT RELATIONSHIPS

Strength Limit State

Required Rotation Capacity

For negative-bending sections in the strength check, it is proposed that the effective plastic moment be based on a required rotation capacity of 30 mrad (1 mrad = 0.001 radian). Available data suggest that this value is conservative for a mechanism check, and is even more conservative for a shakedown check because the plastic rotations occurring at shakedown are less than those required to form a mechanism (Schilling et al., 1996).

Specifically, in 50 preliminary autostress designs made for a wide range of design parameters, the plastic rotations at pier sections in a mechanism check at Maximum Load ranged up to 29 mrad, but were usually much less (Schilling, 1986). Also, the required rotation capacity for the pier section of a two-span continuous girder analyzed by the mechanism method in a previous autostress study was only 11 mrad (Carskaddan, 1976). These data suggest that 30 mrad is a suitable conservative value for the required rotation capacity for negative-bending sections in the strength check. Further trial designs, however, would be desirable to determine whether a lower required rotation capacity would be sufficient.

Ultracompact-Flange Sections

For sections with an ultracompact compression flange, tests showed that the moment, M_{pe}, that can be sustained through a specified plastic rotation, R, is defined by

$$M_{pe} / M_{max} = 1.0 - 0.0092(R - RL) \tag{16}$$

where M_{max} is the maximum moment capacity of the section and RL is the limiting plastic rotation at which M_{pe} equals M_{max} and is a function of the web slenderness, D/t_w (Schilling, 1993; Schilling and Morcos, 1988).

The test results defined (Schilling, 1993) the following values of RL for specific web slenderness ratios and these values were used in Equation 16 to calculate corresponding values of M_{pe} / M_{max} for a plastic rotation of 30 mrad:

D/t_w	RL	M_{pe} / M_{max}
160	10.7	0.822
140	20.2	0.910
120	30.8	1.007

A straight line was fit to these results to get the following equation defining M_{pe} / M_{max} as a function of D/t_w:

$$M_{pe} / M_{max} = 1.562 - 0.004625 D/t_w \tag{17}$$

This equation is specifically for symmetrical sections of 50 ksi steel. It was generalized to apply to unsymmetrical sections by substituting D_{cp} for D and to apply to other steels by multiplying by:

$$\sqrt{\left(\frac{50}{F_{yc}}\right)\left(\frac{E}{29{,}000}\right)}$$

The following equation resulted:

$$\frac{M_{pe}}{M_{max}} = 1.56 - 0.111\left[\frac{2D_{cp}}{t_w}\sqrt{\frac{F_{yc}}{E}}\right] \tag{18}$$

where D_{cp} is the depth of web in compression at M_p, F_{yc} is the yield stress of the compression flange, and E is the modulus of elasticity. In this equation, M_{pe} equals M_{max} when

$$\frac{2D_{cp}}{t_w} \leq 5.05\sqrt{\frac{E}{F_{yc}}} \tag{19}$$

Furthermore, M_{max} equals M_p when the web is compact and M_y when it is not. Thus, Equations 4, 5, and 6 apply.

All Other Sections

Equations 7, 8, and 9, which are proposed for calculating the effective plastic moment, M_{pe}, of sections that do not qualify as ultracompact-flange sections, are empirical equations that were originally developed (Haaijer et al., 1987) for use with compact sections only. Experimental data showed that the rotation capacity corresponding to this empirical M_{pe} was at least 60 mrad for compact sections. In later tests of noncompact sections (Schilling, 1988; Schilling, 1990), the rotation capacity corresponding to the M_{pe} from the empirical equations ranged from 34 to 70 mrad (Schilling et al., 1996). Since the required rotation capacity is 30 mrad, the empirical equations can be used for all sections that do not qualify as ultracompact flange sections.

Permanent-Deflection Limit State

Required Rotation Capacity

A required rotation capacity of 9 mrad is conservative for the permanent-deflection limit state (Schilling et al, 1996) because the loading specified for this limit state almost always causes plastic rotations that fall on the ascending portion of the presently used rotation curve (AASHTO, 1991; AASHTO, 1994), which reaches M_p at about 6 mrad. This suggests that the required rotation capacity is usually below 6 mrad. Furthermore, the plastic rotation at the pier at overload is only 4.1 mrad in the design example in the ALFD guide specifications (AASHTO, 1991). Thus, it may be possible to show through trial designs that a required rotation capacity less than 9 mrad would be adequate for the permanent-deflection check and could be used in the future.

Compact Sections

For compact sections, M_{pe} can be taken as M_p since the rotation capacity at M_p greatly exceeds 9 mrad.

Ultracompact-Flange Sections

Tests (Schilling, 1990; Schilling, 1993) showed that for ultracompact-flange sections with web slenderness ratios not exceeding the maximum permitted for inelastic design, M_{pe} equals M_{max} if the required rotation capacity corresponding to M_{pe} does not exceed 9 mrad (Schilling et al., 1996). If the web is noncompact, M_{max} equals the yield moment, M_y. Therefore, it is proposed that M_{pe} be taken as M_y for ultracompact-flange sections. Sections with an ultracompact compression flange and compact web, of course, qualify as compact sections for which M_{pe} equals M_p.

Noncompact Sections

The M / M_{max} corresponding to 9 mrad on the lower-bound rotation curve for noncompact sections (Schilling, 1985; Schilling, 1988) is about 0.8. Since M_{max} equals M_y for such sections, it is proposed that M_{pe} be taken as $0.8M_y$ for noncompact sections.

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APPPENDIX III. NOTATION

The following symbols are used in this paper:

b_c = Compression-flange width
D = Web depth
D_{cp} = Depth of web in compression at the plastic moment
E = Modulus of elasticity
f_{ef} = Elastic flange stress due to the specified loading; for composite sections, it is the sum of the stresses caused by loads applied before and after the slab has hardened
f_{rd} = Flange stress due to the redistribution moment; for composite sections, it is calculated by applying the entire redistribution moment to the composite section
F_{yc} = Compression-flange yield stress
F_{yf} = Flange yield stress
F_{yt} = Tension-flange yield stress
F_{yw} = Web yield stress
F_{yec} = Compression-flange effective yield stress; used to calculate M_{pe}
F_{yet} = Tension-flange effective yield stress; used to calculate M_{pe}
F_{yew} = Web effective yield stress; used to calculate M_{pe}
M = Moment
M_e = Elastic moment due to specified loading; for composite sections, it is the total elastic moment due to loads applied before and after the slab has hardened
M_r = Bending resistance
M_p = Plastic moment
M_y = Yield moment
M_{max} = Maximum moment; normally it is equal to M_p for compact sections and M_y for noncompact sections
M_{pe} = Effective plastic moment
M_{rd} = Redistribution moment
R = Plastic rotation
RL = Limiting plastic rotation at which $M_{pe} = M_{max}$
t_c = Compression-flange thickness
t_w = Tension-flange thickness
α = Factor defining the permissible positive-bending stress for the permanent-deflection limit state; it is 0.95 for composite sections and 0.80 for noncomposite sections
ϕ = Resistance factor for shakedown; proposed to be 1.1