K Factors for Unbraced Frames: Alignment Chart Accuracy for Practical Frame Variations

W. M. KIM RODDIS, HAZLAN ABDUL HAMID, and CHELSEA Q. GUO

ABSTRACT

For design of steel columns in unbraced frames, the current AISC specification commentaries from both LRFD and ASD contain an alignment chart to determine the K factor for a particular column. The K factor is based on the effective length concept where K factors are used to equate the strength of a compression member of length L to an equivalent pin-ended member of length KL subjected to axial load only. The unbraced frame alignment chart is a graphical representation in nomograph form of a transcendental equation of a buckling solution of a subassemblage. This solution involves several assumptions not necessarily satisfied in a particular practical situation.

The goals of this paper are to point out the practical limitations of the unbraced frame alignment chart effective length approach and to encourage the use of story-based effective length factors. This is done in the following manner. A parametric study shows the limits of accuracy and applicability of the unbraced frame alignment chart results from a linear buckling analysis. The benefits of the AISC Commentary equation C-C2–5 story-based effective length factor are shown through comparison to the alignment chart and linear buckling analysis results. The parametric study shows sufficient examples to encourage practitioners to use story-based techniques for stability assessment.

The parametric study investigates variations in bay width, column moment of inertia, loading, and column height. All nomograph results reported include correction of girder factors to account for unequal girder end moments with non-centerspan inflection point.

The alignment chart performance is relatively insensitive to bay width variation. Variations in column moment of inertia and column loading lead to large inaccuracies in the alignment chart K factor values but C-C2-5 better handles these cases. The alignment chart performance is most sensitive to column height variation. Configurations with large variation in column height require frame stability analysis to obtain accurate K factors.

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INTRODUCTION

Problem Statement

In the design of steel columns in unbraced frames, the current American Institute of Steel Construction (AISC) commentaries for both the Load and Resistance Design Factor (LRFD) Specification [AISC 1993] and the Allowable Stress Design (ASD) Specification [AISC 1989] contain an alignment chart to determine the K factor of a particular column. The K factor is based on the effective length concept where K factors are used to equate the strength of a compression member of length L to an equivalent pin-ended member of length KL subjected to axial load only. The alignment chart is deemed to satisfy the analysis requirement of LRFD-C2.2 to get adequate K factor values of columns to determine their effective lengths. The LRFD and ASD commentaries recommend its use instead of frame buckling analysis to compute K factors.

The alignment chart is widely used because of its straight forward method of obtaining the effective length of a column [Shanmugam and Chen 1995]. The unbraced frame alignment chart is a graphical representation of a transcendental equation of a buckling solution of a subassemblage. This solution involves several assumptions limiting the use of the alignment chart to idealized cases not necessarily satisfying a particular practical situation. This study uses a parametric approach to observe the accuracy of the alignment chart K values as well as story-based K values. Results are presented graphically to assist practicing engineers in developing a feel for the effect of a parametric change on the stability behavior of the unbraced frame.

The Effective Length Method

The effective length approach is an approximate method used for column stability evaluation. It examines an individual member instead of the framed structure as a whole. This is necessary in order to simplify the analysis to a level that is practical for use in routine design.

A compression member in a frame interacts not only with other members which are adjacent to it (horizontally) but it also interacts with members in other stories (vertically). In order to adequately represent these interactions the analytical process can become very complex and in the general case requires a full frame stability analysis.

Estimation of the interaction effects of the total frame on an individual compression member is the essence of the effective length concept. The K factor is used to equate the strength of an individual framed compression member of length L to an equivalent pin-ended compression member of length KL subjected to axial load only. Although it is well known that the effective length approach introduces inaccuracies into the process, the simplicity of examining an individual member is likely to make the approach an important part of American framed column design in the foreseeable future [Hellesland and Bjorhovde 1996].

This paper is restricted to the effective length K factor approach to column design. This approach is discussed in both LRFD and ASD Specifications of AISC. Other approaches to address stability, such as the equivalent imperfection approach used by most other countries, are of course valid for column design, but are excluded from the scope of this study.

The Alignment Chart

The alignment chart is based on the buckling of the subassemblage consisting of the column under investigation and its immediately adjacent members (column above, column below, girders framing to top of column, and girders framing to bottom of column). The resulting transcendental equation of the buckling solution for the unbraced subassemblage is represented graphically in the sidesway permitted alignment chart [AISC 1989, AISC 1993]. The alignment chart is a nomograph, that is to say, a set of scales for the variables in a problem which are so distorted and so placed that a straight line connecting the known values on some scales will provide the unknown values at its intersections with other scales [Webster 1970].

The alignment chart is a widely accepted method of obtaining the K factor to be used in the design process. However it should be realized that in obtaining the alignment chart various simplifications and assumptions were used. Violation of these simplifications and assumptions can lead to inaccurate K factors.

The alignment chart as developed by O. J. Julian and L. S. Lawrence is presented in detail by Kavanaugh [1962]. They prepared the currently used chart with the following assumptions [AISC 1993]:

1. Behavior is purely elastic.
2. All members have constant cross section.
3. All joints are rigid.
4. For braced frames, rotations at opposite ends of restraining beams are equal in magnitude and opposite in sign, producing single curvature bending.
5. For unbraced frames, rotations at the far ends of the restraining members are equal in magnitude and opposite in sign, producing reverse-curvature bending.
6. The column stiffness parameter $\phi = L\sqrt{P/EI}$ must be identical for all columns.
7. Joint restraint is distributed to the column above and below the joint in proportion to I/L of the two columns.
8. All columns buckle simultaneously.
9. No significant axial compression force exists in the girders.

The LRFD Commentary [AISC 1993] section C-C2 on frame stability notes that these assumptions and simplifications are based on idealized conditions, rarely existing in practice. It goes on to state that when the assumptions are violated, unrealistic design may result. The *Guide to Structural Stability Design Criteria for Metal Structures* [Galambos 1988] suggests that the chart is applicable to symmetrical frames, symmetrically loaded, and gives reliable results for frames where the stiffness is approximately proportional to the loading.

Various researchers have dealt with the problem of the performance of the alignment chart when the assumptions on which it is based are violated. Most studies presented situations in which the alignment chart gives unrealistic K values and presents solutions to them. Some [Duan and Chen 1989, Bridge and Fraser 1987, Yura 1971] modify the stiffness ratio, i.e. the G factor, some [LeMessurier 1977, Chu and Chow 1969] introduce corrections to values from the alignment chart, some [Aristizabal-Ochoa 1994, Lui 1992] give new equations to obtain the K value, and some [Cheong-Siat-Moy 1986] propose the elimination of K factor use in the design process.

PARAMETRIC STUDY

Comparing K Factors from Alignment Chart and Frame Stability Analysis

To observe the performance of the alignment chart when assumptions of regularity of frame structure and loading are violated, a parametric study is carried out to determine the difference between K factor values obtained from the alignment chart and a total frame stability analysis. Frames considered are unbraced (sidesway uninhibited).

This study only covers buckling in the elastic range. When the alignment chart is used to evaluate K factors it is implicitly assumed that elastic buckling controls. The problem of inelasticity is well covered by Yura [1971] and Salmon and Johnson

[1996]. This study does not investigate simply connected leaning columns since they are well covered elsewhere (AISC 1993, Hajjar and White 1994). The research reported in this paper mostly revolves around the assumption that the column stiffness parameter, $\phi = \sqrt{P/EI}$, is identical for all the columns in a particular story. In practical situations, column dimensions and loadings may vary thus varying the column stiffness parameters. For the frames investigated in this parametric study, the girder inflection point is not at the midspan of the girder. Therefore, the G factors are corrected to account for the differing girder end moments as discussed in LRFD Commentary Section C2 [AISC 1993]. All nomograph results reported in this paper include this G factor correction. Nomograph results without this correction appear in an earlier research report (Hamid & Roddis 1997). G factors for fixed bases are taken as 1.

In order to compare the difference of the K factor values between a total frame buckling analysis and the alignment chart, a structural analysis software package ROBOT V6 [Metrosoft 1996] is used. This study uses the linear buckling analysis capabilities of the package. The buckling analysis produces results of critical buckling loads and effective lengths. The effective lengths from the analysis are divided by the actual column length and used to obtain the K factor values from the frame buckling analysis.

K values from the alignment chart are obtained by solving the transcendental equation that the alignment chart is based upon instead of a visual inspection of the chart itself. The comparison of the alignment chart and full frame buckling analysis is done in the form of graphs of K values against the parameter studied.

The parameters studied are:

- bay width
- column moment of inertia
- loading
- column height

The column moment of inertia and the column height affect the stiffness of the framed columns and thus the buckling strength of the columns. The bay width sets the girder length affecting the stiffness of the girder with respect to the column and thus the column buckling strength. The loading affects column buckling strength by the leaning effects from other columns in the story. Symmetric and unsymmetric loading are considered. As mentioned earlier, the loading effect of material nonlinearity introduced by inelasticity due to the applied load is not considered in this study.

This study uses as a baseline structure a three story two bay unbraced frame, fixed at the column footings (see Figure 1). The structure is taken from an article by Shanmugam and Chen [1995]. From the reference structure the parameters to be studied are varied. The increment and ranges of the parameters used in this study are given in Table 1. The variations are run through Robot V6 and their buckling lengths and K factor values are obtained. The K factor values from the alignment chart are then obtained by solving the transcendental equation.

Story-Based Effective Length Factors

In addition to investigating the performance of the alignment chart through this parametric study, practitioners need to know what other methods are available when the alignment chart performs badly.

As already stated, the alignment chart is based on the buckling of the subassemblage consisting of the column under investigation and its immediately adjacent members. Story-based effective length factor approaches attempt to cover more general behavior than the locally restricted alignment chart approach without requiring the complexity of a full frame stability analysis. The LRFD Commentary [AISC 1993] presents two methods of determining story-based effective length factors. Of these two methods, the formulation given in equation C-C2-5 is indicated as being simple to use although at some reduction in design values when leaning effects are minimal.

The story-based method given in LRFD C-C2-5 is quite similar to the methods given in [LeMessurier 1977] and [Lui 1992]. All three are based on the use of a first-order sidesway deflection to approximate the buckled shape of the story. This is used to obtain an expression for a story-based effective length factor. These methods only require a first-order frame analysis to determine the horizontal deflection at every story level. A straightforward application of the respective formula then yields the K factor.

In [LeMessurier 1977] and [Lui 1992], an empirical term is used to account for the P-δ effect. LRFD C-C2-5 takes a simpler approach by approximating the P-δ effect with the term $(0.85 + 0.15RL)$ rather than calculating it explicitly per

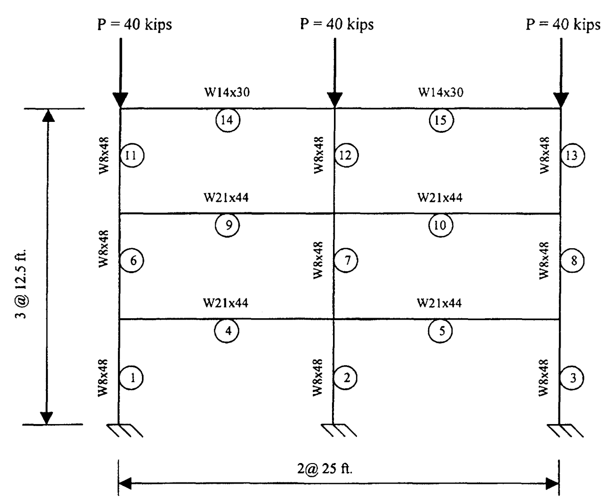

Fig. 1. Baseline unbraced frame for parametric study.

column. For the context of this particular parametric study, both [Lui 1992] and LRFD C-C2-5 were investigated. Parametric study results for Lui's equation appear in an earlier research report [Hamid & Roddis 1997]. C-C2-5 was found to be simpler to apply at the cost of a modest conserveratism when compared to Lui's equation. Only results for LRFD C-C2-5 are reported in this paper.

All story-based effective length procedures must be capped to insure that undue unconservative error is not introduced by a failure mode not detected by story-based buckling procedures. For example, a weak column buckling in a braced mode will not be identified by a story-based procedure. For the approach used in this paper, equation C-C2-5a contains the expression based on a story-based buckling procedure from which effective lengths may be computed. Equation C-C2-5b contains the capping provision from which a minimum permissible value for each effective length factor may be calculated. The values reported in this study were the minimum *K* factor computed jointly from equations C-C2-5a and C-C2-5b. This is indicated by using the designation C-C2-5 to refer to the combined use of C-C2-5a and C-C2-5b. In almost all cases, C-C2-5a is the governing equation. C-C2-5b does govern for a few extreme cases, for example for very large differences in moment of inertia of adjacent columns or for very large variations in column loads.

Organization of Parametric Study Presentation

The following sections examine the *K* factor behavior of the subject frame as the parameters of bay width, column moment of inertia, loading, and column height are varied in turn. Each section presents the parametric range, gives *K* factors determined from the alignment chart followed by *K* factors determined from frame buckling analysis, compares the difference in *K* factors obtained by these two methods, applies C-C2-5 to a selected column, and infers limitations on alignment chart use.

BAY WIDTH

Bay Width Variation

The bay width is varied in increments of 5 feet from the baseline width of 25 feet until the width is doubled to 50 ft (see Table 1 and Figure 2). Only the right bay of the structure is varied, with the left bay held constant at 25 ft. This increase in right bay width violates the alignment chart assumption that the structure is symmetric.

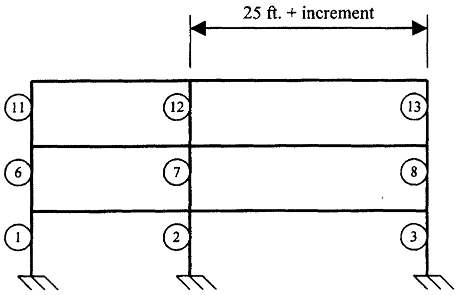

Fig. 2. Variation in bay width.

Table 1. Increments and Ranges of Parameters

Parameter	Baseline	Increment	Range
Bay width	25 ft.	5 ft.	25 ~ 50 ft.
Col. Mom. Inertia	184 in.^{4}	184 in.^{4}	184 ~ 1900 in.^{4}
Loading	40 kips	40 kips	40 ~ 200 kips
Col. Height	12.5 ft.	2.5 ft.	12.5 ~ 25 ft.

Alignment Chart *K* Factors

K factors for the left hand column tier (columns 1, 6, and 11) from the alignment chart vary only slightly with changes in right bay width since the chart uses a local approach considering only those members directly joined to the column. The slight variation is due to the *G* factor correction for the non-centerspan girder inflection point location.

K factors for the columns in the middle tier (columns 2, 7, and 12) increase slightly in value as the bay width increases. This is expected since widening the bay reduces the stiffness of the girder and thus increases the *G* value of the two ends of the column which increases the *K* factor value of the connected column.

K factors for the right column tier (columns 3, 8, and 13) increase (10 percent column 3, 20 percent column 8, 21 percent column 13) as the bay width increases. The increase in *K* values for the right column tier is more pronounced than for the middle column tier since only one girder frames into the ends of the columns and thus the *G* factor for the ends of the columns are more sensitive to the changes in girder stiffness.

Frame Buckling *K* Factors

K factors for all the columns increase less than 7 percent as the right bay width doubles. The widening of the bay reduces the overall structure stiffness and thus increases the *K* factor value for the whole structure. The *K* factor for all the columns are the same throughout the bay width variation.

Difference in *K* Factors

Comparison of the alignment chart *K* factors to the frame buckling *K* factors shows that the alignment chart *K* factors for bay width variation are quite accurate. The most unconservative *K* factor (13 percent) occurs in the middle column tier middle story, column 7. The most conservative *K* factor occurs in the right column tier upper story, column 13.

Table 2. *K* Factors for Variation in Bay Width										
Span (ft)	RV6	column 1 Nomo	column 2 Nomo	column 3 Nomo	column 6 Nomo	column 7 Nomo	column 8 Nomo	column 11 Nomo	column 12 Nomo	column 13 Nomo
25	1.256	1.340	1.200	1.340	1.350	1.090	1.350	1.400	1.130	1.400
30	1.276	1.330	1.210	1.350	1.330	1.110	1.470	1.380	1.160	1.480
35	1.294	1.330	1.220	1.400	1.330	1.130	1.490	1.380	1.180	1.520
40	1.310	1.320	1.230	1.440	1.320	1.140	1.530	1.370	1.190	1.600
45	1.325	1.320	1.240	1.460	1.320	1.160	1.570	1.370	1.210	1.650
50	1.337	1.320	1.250	1.480	1.320	1.180	1.620	1.370	1.230	1.700

Application of C-C2-5

Since column 7 gave the most unconservative alignment chart *K* factor values when compared to frame buckling values, C-C2-5 was also used to find its *K* factor values and the results are compared in Table 3 and in Figure 3.

Limitation of Alignment Chart Use

From the results obtained by comparing the *K* factor values from the alignment chart and frame buckling analysis, the most unconservative value obtained is 13 percent when the span is doubled. The alignment chart in the case of reasonable span variation gives sufficiently accurate values for practical purposes.

COLUMN MOMENT OF INERTIA

Column Moment of Inertia Variation

The moment of inertia of the right hand column tier (columns 3, 8, and 13) of the structure is increased to about 10 times the ratio of the baseline moment of inertia (see Table 4 and Figure 4). The increments are done by picking an existing rolled section that has a moment of inertia value close to the desired value.

The variation in moment of inertia of right column tier (columns 3, 8, and 13) violates the assumption that the column stiffness parameter $\phi = \sqrt{P/EI}$ for all the columns within the story is the same. This is also contrary to the guideline that the alignment chart be used when the structure is symmetric.

Results from both the alignment chart and frame buckling analysis are shown in Table 5.

Alignment Chart *K* Factor Values

K factors from the alignment chart remain constant for both the left column tier (column 1, 6, and 11) and middle column tier (column 2, 7, and 12) due to the local approach of the alignment chart. The *G* factor correction for girder end moments causes the middle column tier (column 2, 7, and 12) *K* factors to vary. *K* factors for the columns in the right hand tier (column 3, 8, and 13) increase significantly (46 percent column 3, 141 percent column 8, 143 percent column 13) in value as the column moment of inertia increases through the variation. This is expected since the increase in the column moment of inertia reduces the *G* of the respective joints and thus reduces the *K* factor of the connected columns.

Frame Buckling *K* Factors

As the moment of inertia of the right column tier increases,

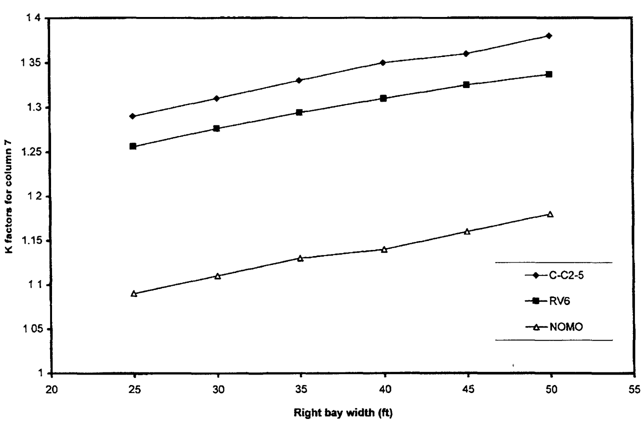

Fig. 3. K factors from C-C2-5 compared to RV6 and alignment chart for column 7—bay width variation.

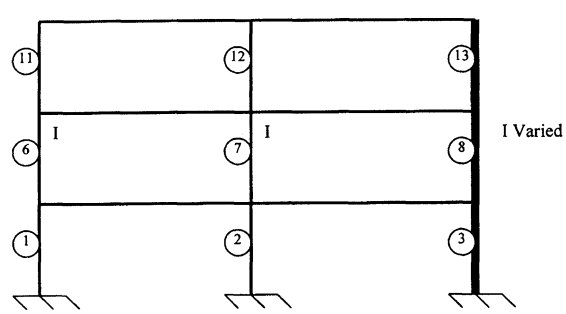

Fig. 4. Variation in moment of inertia.

Table 3. *K* Factors for C-C2-5 Compared to RV6 and Alignment Chart for Column 7—Right Bay Width Variation			
Span (ft)	RV6	Nomo	C-C2-5
25	1.256	1.090	1.290
30	1.276	1.110	1.310
35	1.294	1.130	1.330
40	1.310	1.140	1.350
45	1.325	1.160	1.360
50	1.337	1.180	1.380

Table 4. Column Sections Used in Moment of Inertia Variation		
Ratio	Section Used	I_x value (in.^{4})
I	W8×48	184
2*I*	W10×60	341
3*I*	W10×88	534
4*I*	W12×87	740
5*I*	W12×106	933
6*I*	W14×99	1110
7*I*	W14×109	1240
8*I*	W14×132	1530
9*I*	W14×145	1710
10*I*	W14×159	1900

the *K* factors of the columns in the left column tier and middle column tier decrease 13 percent, while those values of the columns in the right column tier increase 178 percent. This is due to the interaction effect between the various members of the unbraced frame. The columns in the right column tier become stronger as their moment of inertia is increased. This enables them to brace the weaker columns (left and middle tier), increasing the *K* factor of the stronger column and decreasing the *K* factor of the weaker columns. This phenomena is also observed by Hajjar and White [1994] and Lui [1992]. *K* factors of less then 1.0 would result if the moment of inertia value were further increased. *K* factors of less than 1.0 can now be used to design columns according to the LRFD Specification [AISC 1993].

Differences in *K* Factor Values

Due to the failure of the alignment chart to capture the full effect of the interaction between components of an unbraced frame, differences between *K* factors from the alignment chart and frame buckling ranges from being overly conservative to being overly unconservative. As shown in Figure 5, for columns in the left column tier, values from the alignment chart are conservative, with column 11 having the highest value of 28 percent conservative. As shown in Figure 6, columns in the right column tier give mostly unconservative values of up to 44 percent for column 3.

Application of C-C2-5

Since column 3 gave the most unconservative alignment chart *K* factor values when compared to frame buckling values, C-C2-5 was also used to find its *K* factor values and the results are compared in Table 6 and Figure 7. At the end of the variation values from C-C2-5 are unconservative by 30 percent as compared to 44 percent for values from the alignment chart.

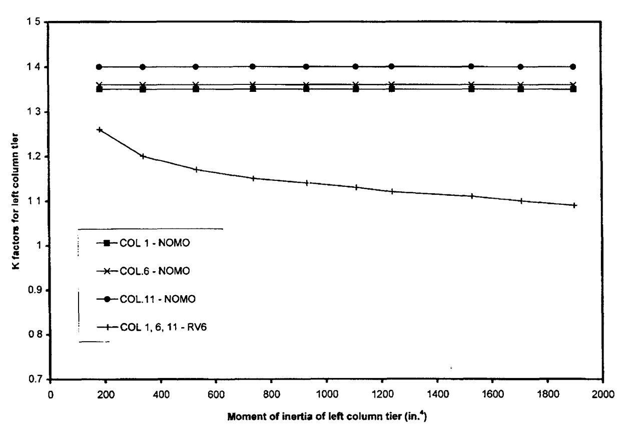

Fig. 5. K factors for left column tier moment of inertia variation.

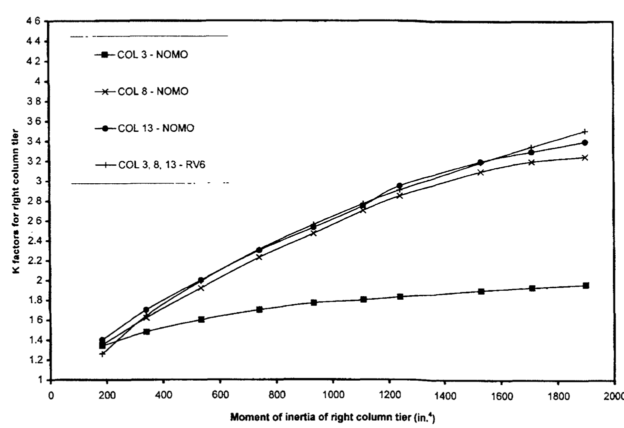

Fig. 6. K factors for right column tier moment of inertia variation.

I_x of 3, 8, 13 (in.^{4})	column 1		column 2		column 3		column 6		column 7		column 8		column 11		column 12		column 13	
	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
184	1.26	1.35	1.26	1.20	1.26	1.34	1.26	1.36	1.26	1.09	1.26	1.35	1.26	1.40	1.26	1.13	1.26	1.40
341	1.20	1.35	1.21	1.20	1.64	1.48	1.20	1.36	1.20	1.09	1.64	1.62	1.20	1.40	1.20	1.12	1.64	1.70
534	1.17	1.35	1.18	1.18	1.99	1.60	1.17	1.36	1.18	1.07	2.00	1.92	1.17	1.40	1.17	1.11	2.00	2.00
740	1.15	1.35	1.16	1.17	2.31	1.70	1.15	1.36	1.16	1.06	2.31	2.23	1.15	1.40	1.15	1.11	2.31	2.30
933	1.14	1.35	1.14	1.17	2.56	1.77	1.14	1.36	1.14	1.05	2.56	2.47	1.14	1.40	1.14	1.09	2.56	2.53
1110	1.13	1.35	1.13	1.16	2.77	1.80	1.13	1.36	1.13	1.04	2.77	2.70	1.13	1.40	1.13	1.09	2.77	2.75
1240	1.12	1.35	1.13	1.16	2.91	1.83	1.12	1.36	1.12	1.04	2.91	2.85	1.12	1.40	1.12	1.08	2.91	2.95
1530	1.11	1.35	1.11	1.16	3.19	1.90	1.11	1.36	1.11	1.04	3.19	3.10	1.11	1.40	1.11	1.07	3.19	3.20
1710	1.10	1.35	1.10	1.16	3.35	1.93	1.10	1.36	1.10	1.03	3.35	3.20	1.10	1.40	1.10	1.06	3.35	3.30
1900	1.09	1.35	1.10	1.16	3.50	1.96	1.09	1.36	1.10	1.03	3.51	3.25	1.09	1.40	1.09	1.06	3.51	3.40

Table 5.
K Factors for Moment of Inertia Variation

Limitation of Alignment Chart Use

In the case of moment of inertia variation, in order to keep values from being unconservative by more than 10 percent, the moment of inertia of frame column members should not vary by more than 2. Using variation of up to 51 could lead to unconservative errors of up to 30 percent and variation of 101 could lead to unconservative errors of up to 44 percent.

LOADING

Loading Variation

The baseline column load of 40 kip is increased to 200 kip in 40 kip increments. First, the right hand column tier is incrementally loaded while the left and middle column tier loads are held constant at 40 kip. Loading the right columns gives both asymmetry of loading and violation of the uniform column stiffness parameter. Next, the middle column tier is incrementally loaded while the outside column loads are held constant at 40 kip. Varying the load on the middle column tier maintains symmetry of loading but violates the uniform story column stiffness parameter assumption (see Table 1 and Figure 8).

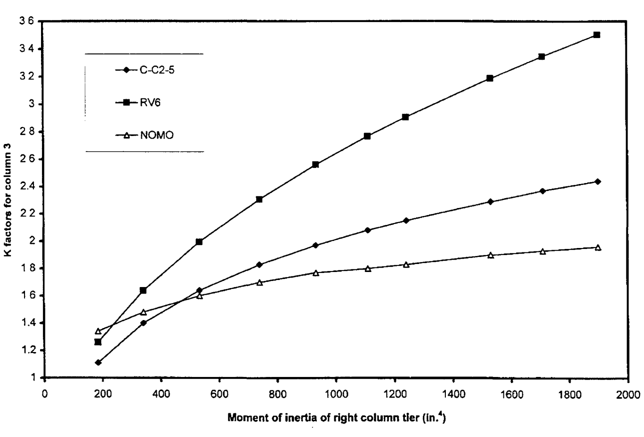

Fig. 7. K factors from C-C2-5 compared to RV6 and alignment chart for column 3—right column tier moment of inertia variation.

Alignment Chart *K* Factors

K factors from the alignment chart remain constant throughout both loading variations for all the column tiers since load is not a factor in the alignment chart equation. The effect of symmetry also does not affect the alignment chart *K* factor values as both symmetric and asymmetric configurations of loading give the same values.

Frame Buckling *K* Factors

When the right column tier was loaded, *K* factors for the

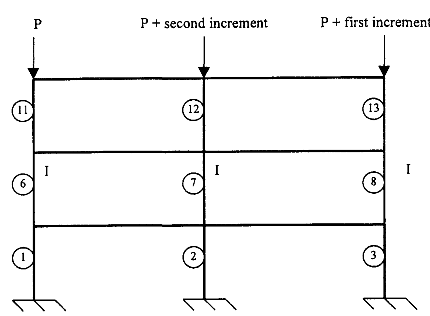

Fig. 8. Variation in loading.

Table 6.
***K* Factors from C-C2-5 Compared to RV6 and Alignment Chart for Column 3—Right Column Tier Moment of Inertia Variation**

I_x of 3, 8, 13 (in.^{4})	RV6	Nomo	C-C2-5
184	1.26	1.34	1.11
341	1.64	1.48	1.40
534	2.00	1.60	1.64
740	2.31	1.70	1.83
933	2.56	1.77	1.97
1110	2.77	1.80	2.08
1240	2.91	1.83	2.15
1530	3.19	1.90	2.29
1710	3.35	1.93	2.37
1900	3.50	1.96	2.44

columns in the right column tier decrease 32 percent as the loading is increased (Figure 9). The *K* factors for the middle (Figure 10) and left column tiers behave similarly, increasing 50 percent.

When the middle column tier was loaded, *K* factor values for the columns in the middle column tier decrease 31 percent (Figure 11). The values for the right and left (Figure 12) column tier are the same due to symmetry and increase 50 percent.

Decrease of *K* factors of the columns under increased load can be attributed to the fact that columns where loadings are held constant become relatively stronger as compared to the column under increased load. As a result the weaker columns reach their buckling load earlier and thus "lean" on the stronger columns. This results in an increase in *K* factor value of the columns whose loadings were held constant. Columns that were loaded drop their *K* factor value to less than 1.0 which is now allowed to be used in the design of unbraced frames [AISC 1993]. These facts were also observed by Lui [1992] and Hajjar and White [1994].

Differences in *K* Factor Values

For the column tiers where the loading was increased, the *K* factors from the alignment chart are conservative as compared to values from frame buckling. For the columns where the loading was held constant, values are unconservative. When the right column tier was loaded, column 13 gives a conservative value of 63 percent with other columns in the tier being similar. Among the columns whose loadings were held constant, column 7 gives the most unconservative value of 42 percent. When the middle column tier was loaded, column 2 gives a conservative value of 36 percent. Among the columns held constant, column 8 is unconservative by 29 percent.

Application of C-C2-5

Since column 7 with the load applied to the right column tier gave the most unconservative alignment chart *K* factors, C-C2-5 was also used to find its *K* factors. The results (Table 8 and Figure 13) show C-C2-5 captures the effect of load variation very well.

Limitation of Alignment Chart Use

In order to limit the unconservative error due to load variation to 25 percent, load on columns should not vary by more than 2*P*. Variation of up to 5*P* would lead to unconservative errors of up to 42 percent.

COLUMN HEIGHT

Column Height Variation One

Column height of the bottom and top stories are varied

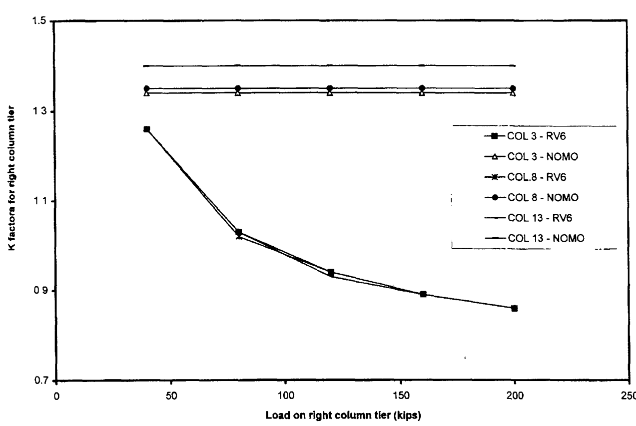

Fig. 9. K factors for right column tier for load variation—right column tier loaded.

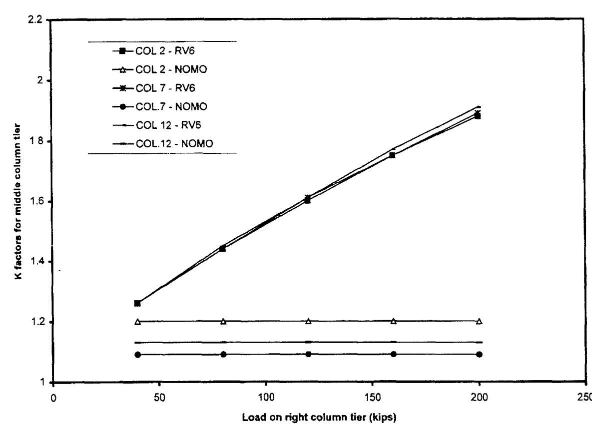

Fig. 10. K factors for middle column tier for load variation—right column tier loaded.

Table 7a.
***K* Factors for Load Variation—Right Column Tier Loaded**

Load on 3, 8, 13 (kips)	column 1		column 2		column 3		column 6		column 7		column 8		column 11		column 12		column 13	
	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
40	1.26	1.34	1.26	1.20	1.26	1.34	1.26	1.35	1.26	1.09	1.26	1.35	1.26	1.40	1.26	1.13	1.26	1.40
80	1.45	1.34	1.44	1.20	1.03	1.34	1.45	1.35	1.44	1.09	1.02	1.35	1.45	1.40	1.45	1.13	1.02	1.40
120	1.63	1.34	1.60	1.20	0.94	1.34	1.62	1.35	1.61	1.09	0.94	1.35	1.62	1.40	1.61	1.13	0.93	1.40
160	1.79	1.34	1.75	1.20	0.89	1.34	1.78	1.35	1.75	1.09	0.89	1.35	1.78	1.40	1.77	1.13	0.89	1.40
200	1.94	1.34	1.88	1.20	0.86	1.34	1.93	1.35	1.89	1.09	0.86	1.35	1.92	1.40	1.91	1.13	0.86	1.40

Table 7b.
***K* Factors for Load Variation—Middle Column Tier Loaded**

Load on 2, 7, 12 (kips)	column 1		column 2		column 3		column 6		column 7		column 8		column 11		column 12		column 13	
	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
40	1.26	1.34	1.26	1.20	1.26	1.34	1.26	1.35	1.26	1.09	1.26	1.35	1.26	1.40	1.26	1.13	1.26	1.40
80	1.45	1.34	1.04	1.20	1.45	1.34	1.45	1.35	1.03	1.09	1.45	1.35	1.46	1.40	1.03	1.13	1.46	1.40
120	1.62	1.34	0.95	1.20	1.62	1.34	1.62	1.35	0.95	1.09	1.62	1.35	1.63	1.40	0.95	1.13	1.63	1.40
160	1.77	1.34	0.90	1.20	1.77	1.34	1.77	1.35	0.90	1.09	1.77	1.35	1.79	1.40	0.90	1.13	1.79	1.40
200	1.90	1.34	0.88	1.20	1.90	1.34	1.91	1.35	0.87	1.09	1.91	1.35	1.93	1.40	0.87	1.13	1.93	1.40

separately. Column height is varied from 12.5 ft. to 25 ft. in 2.5 ft. increments (see Table 1 and Figure 14).

Alignment Chart *K* Factors

Based on the alignment chart, the change in column height decreases the *G* value of one end of the column and thus decreases the value of the *K* factor value of the column considered. The *K* factor for the top story is not affected by the column height changes to the columns in the bottom floor, and the *K* factor for the bottom story is not affected by the column height changes to the columns in the top floor since the chart uses a local approach considering only those members directly joined to the column.

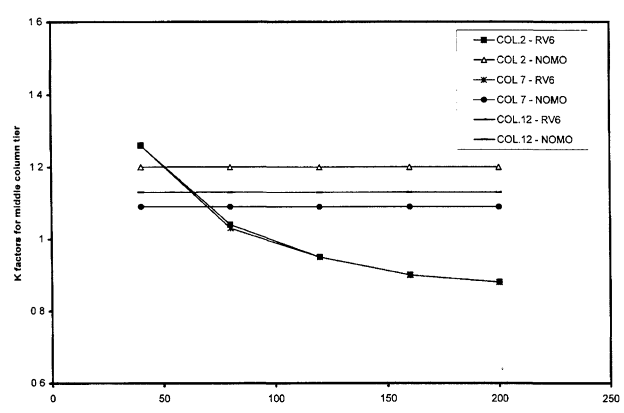

Fig. 11. K factors for middle column tier for load variation—middle column tier loaded.

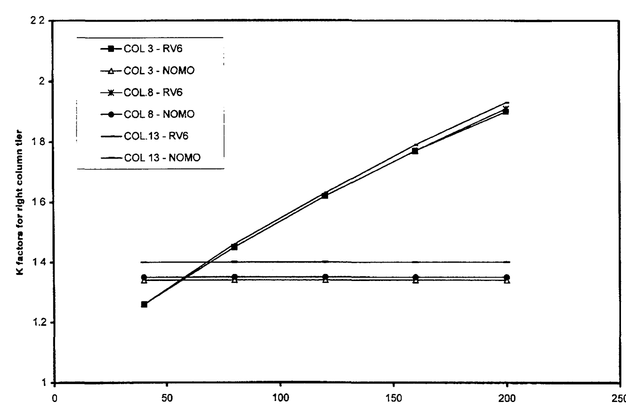

Fig. 12. K factors for right column tier for load variation—middle column tier loaded.

Table 8. K Factors from C-C2-5 Compared to RV6 and Alignment Chart for Column 7—Right Column Tier Load Variation			
Loads on 3, 8, 13 (kips)	RV6	Nomo	C-C2-5
40	1.256	1.090	1.290
80	1.442	1.090	1.480
120	1.606	1.090	1.650
160	1.753	1.090	1.800
200	1.886	1.090	1.960

Frame Buckling *K* Factors

When the bottom story height increases, the *K* factor for the columns in the bottom floor (Figure 15) drops 83 percent. The *K* factor for the middle and top stories behave much the same so only the top story graph is shown (Figure 16). The *K* factor for the upper stories (Figure 16) increases 64 percent. The *K* factor values for the bottom story decrease to a story height of about 15 ft. remain flat to a story height of about 15 ft. and then flatten out, while the *K* factor values for the upper stories remain flat to a story height of about 15 ft. and then increases in value.

When the top story height was increased, the *K* factors for the columns in the lower stories increase 95 percent. The *K* factors for the bottom and second stories behave much the same, so only the bottom story graph is shown (Figure 17). For the columns in the top story (Figure 18), the *K* factor values decrease 81 percent.

The variation in column height exhibits the "leaning column" effect as an inter-story phenomena. The lengthened columns are increasing in slenderness and thus getting weaker as compared to columns that are held constant. The weaker columns will then "lean" onto the columns that are stronger, thus increasing the *K* factor value of the stronger columns, i.e., columns in the stories that are held constant.

Application of C-C2-5

K factor values for column 7 with the bottom columns varied were obtained using C-C2-5. Alignment chart values were 48 percent unconservative when compared to frame buckling values. Results are shown in Table 10 and Figure 19.

From the results obtained, C-C2-5 gives better agreement with buckling analysis results, but it still fails to capture the effect of story height variation.

Limitation of Alignment Chart Use

In order to limit unconservative error to about 10 percent, height variation should not be more than 15 ft. or 1.2*H*. Height variation of up to 2*H* could lead to unconservative errors of up to 50 percent.

CONCLUSIONS

This study was carried out with the aim of observing the performance of the alignment chart when the assumptions

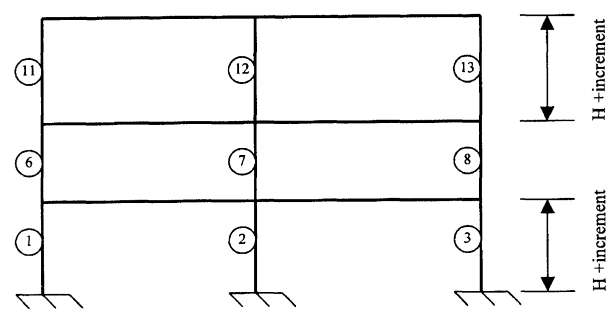

Fig. 14. Variation in height.

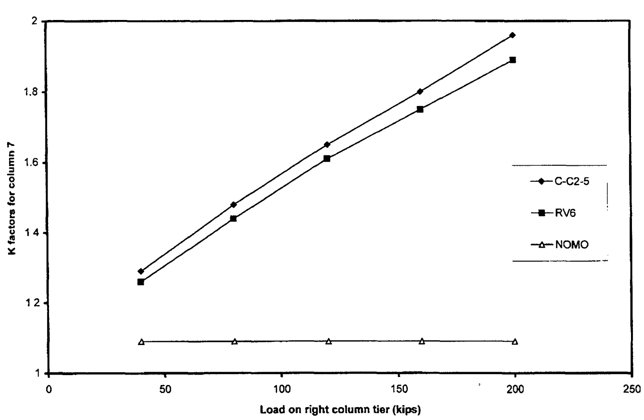

Fig. 13. K factors from C-C2-5 compared to RV6 and alignment chart for column 7—right column tier load variation.

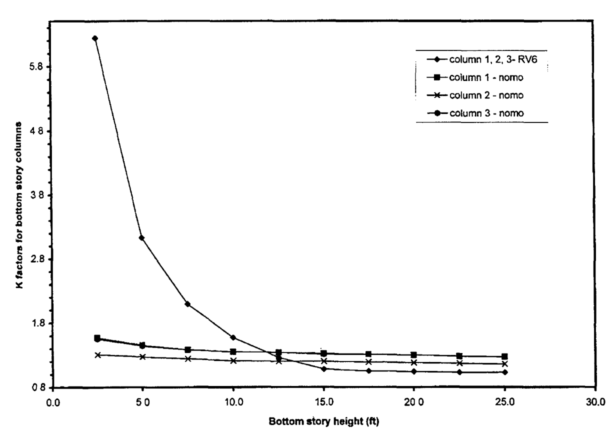

Fig. 15. K factors for bottom story columns—bottom story height variation.

Table 9a. *K* Factors for Height Variation—Bottom Story Varied																		
Height of 1st floor (ft)	**column 1**		**column 2**		**column 3**		**column 6**		**column 7**		**column 8**		**column 11**		**column 12**		**column 13**	
	RV6	**Nomo**	**RV6**	**Nomo**	**RV6**	**Nomo**	**RV6**	**Nomo**	**RV6**	**Nomo**	**RV6**	**Nomo**	**RV6**	**Nomo**	**RV6**	**Nomo**	**RV6**	**Nomo**
2.5	6.24	1.57	6.24	1.31	6.24	1.55	1.25	1.57	1.25	1.20	1.25	1.57	1.25	1.40	1.25	1.12	1.25	1.38
5.0	3.13	1.45	3.13	1.27	3.13	1.44	1.25	1.44	1.25	1.15	1.25	1.43	1.25	1.40	1.25	1.12	1.25	1.38
7.5	2.09	1.38	2.09	1.24	2.09	1.38	1.25	1.38	1.25	1.13	1.25	1.39	1.25	1.40	1.25	1.12	1.25	1.38
10.0	1.57	1.35	1.57	1.21	1.57	1.35	1.25	1.37	1.25	1.10	1.25	1.37	1.25	1.39	1.25	1.13	1.25	1.38
12.5	1.26	1.34	1.26	1.20	1.26	1.34	1.26	1.35	1.26	1.09	1.26	1.35	1.26	1.39	1.26	1.13	1.26	1.38
15.0	1.08	1.32	1.08	1.20	1.08	1.31	1.29	1.33	1.29	1.09	1.29	1.33	1.29	1.38	1.29	1.14	1.29	1.37
17.5	1.05	1.31	1.05	1.19	1.05	1.31	1.47	1.31	1.47	1.08	1.47	1.31	1.47	1.37	1.47	1.15	1.47	1.37
20.0	1.04	1.30	1.04	1.18	1.04	1.30	1.66	1.30	1.66	1.07	1.66	1.30	1.66	1.37	1.66	1.15	1.66	1.37
22.5	1.03	1.28	1.03	1.17	1.03	1.28	1.86	1.28	1.86	1.06	1.86	1.28	1.86	1.36	1.86	1.15	1.86	1.36
25.0	1.03	1.27	1.03	1.16	1.03	1.27	2.05	1.27	2.05	1.06	2.05	1.27	2.05	1.36	2.05	1.16	2.05	1.36

underlying its development are violated. From the results of the parametric study on the unbraced frame structure used in this study, the following observances and recommendations can be made.

Bay Width Variation

From the comparison made between the *K* factors obtained by using the alignment chart and frame buckling analysis the most unconservative error of the alignment chart value is 13 percent even though the bay width was doubled. Therefore, the alignment chart in the case of practical bay width variation should be able to give adequately accurate values. Use of C-C2-5 in this case gives more accurate *K* factor values than the alignment chart.

Moment of Inertia Variation

The most unconservative error of the alignment chart values is 44 percent over a 10 time increase in column moment of inertia. In order to keep values from being unconservative by more than 10 percent, the moment of inertia of the framed column members should not vary by more than a factor of 2. The use of C-C2-5 in this case gives more accurate *K* factor values than the alignment chart, however, system buckling analysis should be used for extreme variations of moments of inertia.

Loading Variation

The unconservative error of the alignment chart value is 42 percent when column loads varied by a factor of 5. Even when

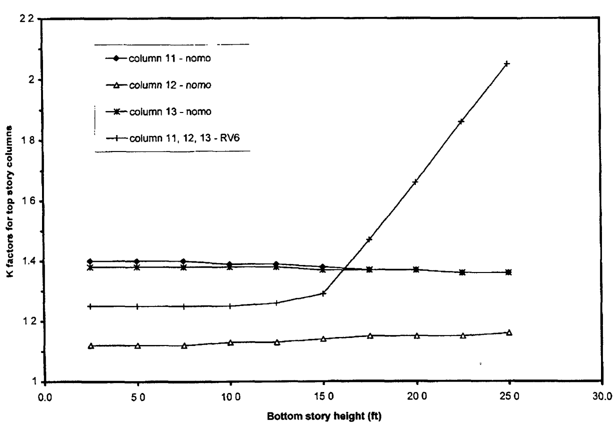

Fig. 16. K factors for top story columns—bottom story height variation.

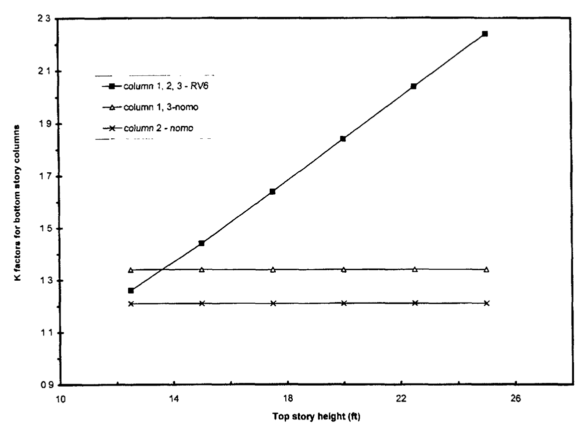

Fig. 17. K factors for bottom story columns—top story height variation.

Table 9b.
***K* Factors for Height Variation—Top Story Varied**

Height of 3rd floor (ft)	column 1		column 2		column 3		column 6		column 7		column 8		column 11		column 12		column 13	
	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo	RV6	Nomo
2.5	1.15	1.34	1.15	1.20	1.15	1.34	1.15	1.57	1.15	1.22	1.15	1.57	5.76	2.20	5.76	1.57	5.76	2.15
5.0	1.16	1.34	1.16	1.20	1.16	1.34	1.16	1.44	1.16	1.15	1.16	1.44	2.89	1.73	2.89	1.33	2.89	1.70
7.5	1.16	1.34	1.16	1.20	1.16	1.34	1.16	1.39	1.16	1.13	1.16	1.39	1.93	1.54	1.93	1.25	1.93	1.54
10.0	1.17	1.34	1.17	1.20	1.17	1.34	1.17	1.36	1.17	1.11	1.17	1.36	1.47	1.47	1.47	1.18	1.47	1.47
12.5	1.26	1.34	1.26	1.20	1.26	1.34	1.26	1.35	1.26	1.09	1.26	1.35	1.26	1.39	1.26	1.13	1.26	1.39
15.0	1.44	1.34	1.44	1.21	1.44	1.34	1.44	1.34	1.44	1.10	1.44	1.34	1.20	1.37	1.20	1.11	1.20	1.37
17.5	1.64	1.34	1.64	1.21	1.64	1.34	1.64	1.32	1.64	1.00	1.64	1.32	1.17	1.33	1.17	1.10	1.17	1.33
20.0	1.84	1.34	1.84	1.21	1.84	1.34	1.84	1.32	1.84	0.99	1.84	1.32	1.15	1.32	1.15	1.09	1.15	1.32
22.5	2.04	1.34	2.04	1.21	2.04	1.34	2.04	1.31	2.04	0.98	2.04	1.31	1.13	1.30	1.13	1.08	1.13	1.30
25.0	2.24	1.34	2.24	1.21	2.24	1.34	2.24	1.30	2.24	0.97	2.24	1.30	1.12	1.27	1.12	1.06	1.12	1.27

the load variation is limited to $2P$, the unconservative error was 25 percent. The alignment chart does not reflect differences of symmetry and anti-symmetry of loading, nor does it capture the "leaning column" phenomena. C-C2-5 is a very good alternative in obtaining the K factor. For practical applications, this example of loading variation shows the usefulness of C-C2-5 for cases of unequal column loads such as a perimeter column. Note that this example *only* varied loading, while actual perimeter columns would see a decrease in I with a decrease in P so that the variation in column stiffness parameter from interior to exterior columns would not be as marked as shown for this parametric case.

Variation in Column Height

The most unconservative error of the alignment chart values is 48 percent when column height was doubled. Limiting the variation to about $1.2H$ should give unconservative values of less than 10 percent. The alignment chart fails to capture the interaction between the components of the unbraced frame member. The results from frame buckling analysis show that there is also a "leaning column" effect in varying the column heights. But this effect is more of an inter-story effect. Columns which are shorter become the stronger columns and the longer columns which are weaker "lean" on to the shorter columns causing the shorter columns to increase their K factor values. Note that this parametric example *only* varied height, while actual practical occurrences of column height increase, such as office building bottom stories, also normally have an

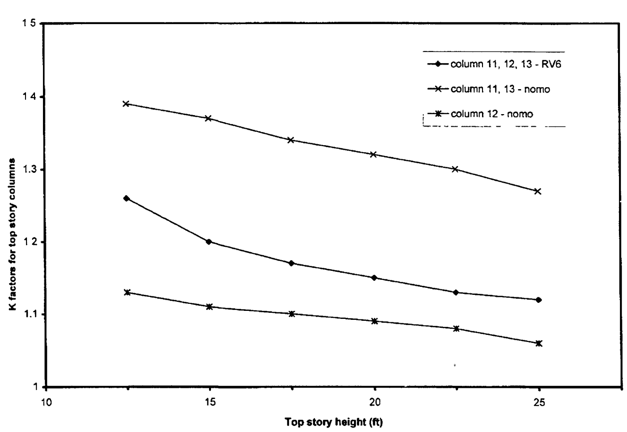

Fig. 18. K factors for top story columns— top story height variation.

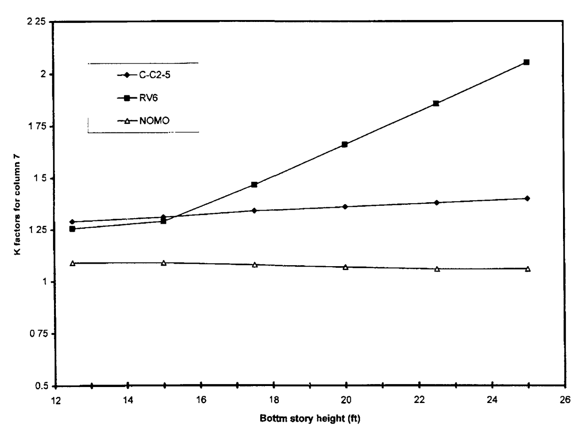

Fig. 19. K factors from C-C2-5 compared to RV6 and alignment chart for column 7— bottom story height variation.

Table 10.
***K* Factors from C-C2-5 Compared to RV6 and Alignment Chart for Column 7—Bottom Story Height Variation**

Height of 1st floor (ft)	RV6	Nomo	C-C2-5
12.5	1.256	1.090	1.290
15.0	1.290	1.090	1.310
17.5	1.466	1.080	1.340
20.0	1.660	1.070	1.360
22.5	1.857	1.060	1.380
25.0	2.054	1.060	1.400

accompanying increase in *I* which mitigates the effect of the increase in *h*.

C-C2-5 does to some extent give better *K* factor values for variation in column height. But it also fails to capture the full effect of column height variation. Variation in column height was the parameter most degrading to the accuracy of the alignment chart. Configurations with large variation in column height require frame stability analysis to obtain accurate *K* factors.

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