# Steel Cable Creates Novel Structural Space Systems

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STRUCTURAL ENGINEERING is inherently a creative profession. Nevertheless, as often practiced, structural engineers are involved mainly in the design of a few well known structural systems, while research and development are usually directed towards greater economy of material and labor in those systems.

However, creativeness in structural engineering should be measured by the evolving of structural systems which offer more flexibility for architectural planning and which could be constructed more economically than structural systems of yesterday. Flexibility could be measured by greater span lengths and aesthetics of the structural systems. Such structural systems could be left exposed and thus save in exterior finishes.

The theoretical principles which would govern such structural systems have to be based on the following considerations:

1. Avoid flexural members as far as practicable.
2. Attempt to utilize the material either in direct tension or direct compression.
3. Utilize geometry of the structure in resisting the loads.
4. Design the entire structure as a whole to resist the load; i.e., the characteristics of the entire structure as a whole entity should be considered in the structural analysis, rather than that of individual members (leaving to chance the interaction between the members).

Structural systems that could be evolved on the basis of these principles are bound to result in significant savings of construction cost. They would offer tremendous flexibility in architectural layouts and would contribute to the aesthetics of a structure. These principles would offer an infinite number of shapes, new forms and would open new horizons in the construction planning. Such an approach to engineering would indeed contribute to the progress of construction.

Application of the above mentioned principles would generally result in space structural systems; namely, systems which carry the superimposed load in three dimensions, as opposed to plane structural members which carry the load in two dimensions only.

One of the most beneficial methods of achieving exciting structural systems, particularly in the United States with its advanced construction techniques and high ratio of labor to material costs, is the high strength cable, particularly the bridge strand.

One of the high dividends of using strands is that one can create a structure where the roof, rather than being supported by the structure below, actually holds the entire structure together, resulting in significant economy in the superstructure, the supporting elements and the foundations. It will be noted that space structures with strands would be subjected essentially to tensile stresses. The big difference between a conventional space structure, and a "tension" space structure, is that the stresses in the component members of a "tension" space structure could be predetermined by the designer. A simplified summary is that a tension space structure is equivalent to a structure prestressed in its entirety.

This article deals with the theoretical aspects of suspension cables and elimination of their elastic instability. Some of the applications of dampened suspension systems are also discussed.

## STRUCTURAL BEHAVIOR OF A SUSPENDED CABLE; DYNAMIC INSTABILITY

Modern high strength cables and strands offer a material which is four to six times stronger than, and at a cost that is only twice as much as, ordinary structural steel. Furthermore since a cable is subjected to tension only, the possibility of buckling is precluded and, hence, its entire cross sectional area may be utilized at the maximum permissible stress. The resulting structure is not only light but also effects further economy in the foundations.

Because of these advantages, the use of cables as the principal structural members for suspension roofs for intermediate and large spans stands to reason. Indeed, many attempts have been made by the engineers of many countries in this direction. One obvious structural solu-

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tion was to translate the principles of suspension bridges to buildings.

However, suspension roofs in buildings present engineers with new, previously unexperienced phenomena: aerodynamic instability, or flutter. Thus, a suspension structure should not only be designed to resist static loads, but should also be investigated for the effect of dynamic loads (wind, vibrations transmitted through the ground, sonic waves, etc.).

This paper presents means and design methods to achieve structural systems for suspension roofs which do not exhibit the phenomenon of aerodynamic instability. It does not deal with, nor attempt to determine, the behavior of a suspension roof during flutter. For practical purposes, if factors contributing to self-exciting vibrations are eliminated from a structural system to begin with, investigation of, and design for, flutter becomes irrelevant.

The structural system presented in this paper consists of cables interconnected in such a manner that there is always flow of energy from a cable which tends to flutter to other cables which are at a lower energy level. In other words, each cable is sufficiently dampened during erection of the roof so as not to develop dynamic instability.

For easier understanding of this design method, the following simple case of static loads on a single suspended cable is presented as a transition to the main topic of the paper.

**Design for Static Loads**—Statically, one layer of suspension cables is adequate to support any commonly prescribed dead and live loads. Indeed, many designs of suspension roofs were and are of this type.

The following example illustrates a one-layer cable suspension roof and has been extremely simplified. Though such simplification might not apply in a practical design, it does help illustrate the basic principles. From these basic principles, the gap to practical design is very narrow. The reader is urged to note particularly the vibration characteristics of this roof.

*Data:* (See (Fig. 1)

| | |
|---|---|
| Rectangular building: | 700 x 300 ft |
| Span: | 300 ft |
| Dead load: | 20 lbs per sq ft of horizontal projection (including weight of cable) |
| Live load: | 30 lbs per sq ft of horizontal projection |
| Spacing of cables: | 4 ft-0 in. o.c. |

*Solution:*

The first step is to determine the sag of cables, as erected, i.e., before the application of dead and live loads. (For this step, the cable may be assumed to be weightless.)

The magnitude of the sag has both architectural and structural implications. In suspension bridges, the sag-to-span ratio is about 1:8. Architecturally, a smaller sag is always desirable. As will be discussed later in this presentation, since elimination of flutter requires cable tensions in excess of those induced by superimposed dead and live loads, it is possible within the economical range of construction to meet architectural requirements and to provide relatively small sag-to-span ratios.

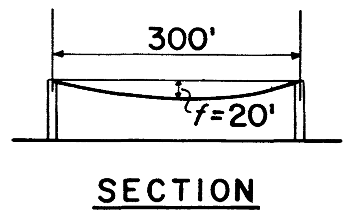


*Figure 1*

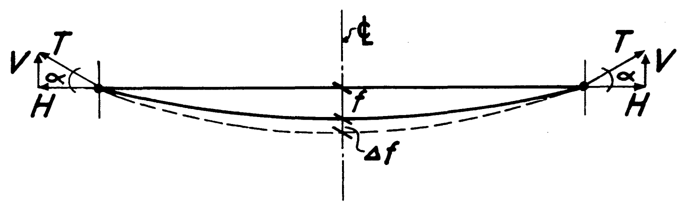


*Figure 2*

In this example, the sag (before application of load) will be assumed to be $\frac{1}{15}$ of span or, for the 300 ft span, a sag of 20 ft.

The applied dead and live load per linear foot of horizontal projection is

$$q = (20 + 30) \times 4 = 200 \text{ lbs}$$

Load $q$ will subject the cable to tension. The maximum value of this tension $T$ is given by

$$T = \frac{ql^2}{8f} \sqrt{1 + 16(f/l)^2} \qquad (1)$$

The sag of the cable after the application of $q$ will be larger than the initial $f$. However, the difference is very small for practical purposes, as will be illustrated later, and hence in equation (1) above, one may use the initial value of sag $f$. Thus

$$T = \frac{200(300)^2}{8 \times 20} \sqrt{1 + 16(\tfrac{1}{15})^2} = 116 \text{ kips} \qquad (2)$$

Use a 2-in. diameter galvanized bridge strand with cross sectional area of 2.43 sq in., weighing 8.48 lbs per linear foot. Factors of safety and allowable design stresses in strands are beyond the scope of this presentation.

Angle $\alpha$ (see Fig. 2) of the cable at the anchorage point is given by

$$\tan \alpha = 4f/l = 4/15 = 0.267 \quad (3)$$

$$\alpha = 0.2618 \text{ radians } (15^\circ) \quad (4)$$

Vertical reaction $V$ is

$$V = T \sin \alpha = 116 \times 0.2588 = 30 \text{ kips} \quad (5)$$

Horizontal reaction $H$ is

$$H = T \cos \alpha = 116 \times 0.9659 = 112 \text{ kips} \quad (6)$$

It will be noted that the horizontal component of $H$ at the anchorage point also represents the tension at midspan of the cable.

The actual tension along the cable varies from maximum value $T_i = 116$ kips at the anchorage point to the minimum value $H = 112$ kips at center of span. For small ratios of sag-to-span, it may be assumed that the cable is subjected to a uniform tension $T_i = 116$ kips.

For small sag-to-span ratio, the approximate initial length of the cable (before the application of $q$) is given by

$$L = 1\ [1 + (8/3)\ (f/l)^2] = 303.5 \text{ ft} \quad (7)$$

Elastic elongation of the cable is

$$\Delta L = T_i L/EA \quad (7a)$$

where $E$ = modulus of elasticity (approximately 24,000 ksi) and $A$ = cross sectional area of cable (2.43 sq in. for 2 in. diameter strand).

Therefore, from (2) and (7)

$$\Delta L = 0.605 \text{ ft} = 7.25 \text{ in.} \quad (8)$$

Increase in sag, $\Delta f$, due to the cable elongation of $\Delta L$ is

$$\Delta f = \frac{\Delta L}{(16/15)\ (f/l)[5 - 24\ (f/l)^2]} = 1.75 \text{ ft} \quad (9)$$

*Discussion of results:*

Total deflection due to combined dead and live loads is not of particular interest. Of interest is the live load deflection. Since in static design the behavior of cables is directly proportional to the superimposed load, live load deflection for our problem is $1.75 \times (30/50) = 1.05$ ft. This gives a ratio of deflection to span of $1.05/300 = 1/286$.

The tension in the cable depends solely on the magnitude of $q$ and the geometry of the cable, i.e., its shape (in our case parabola), span and sag. In accordance with (8), the total sag of the cable under $q$ is $20 + 1.75 = 21.75$ ft.

In accordance with (1), as the sag increases, the tension decreases; for a sag of 21.75 ft the tension drops approximately from 116 kips to $116(20/21.75) = 107$ kips. Actually, under a tension of 107, the sag will be slightly less than 21.75 ft; at equilibrium, the tension in the cable will be somewhere between 116 and 107 kips (closer to 107 kips) and the total sag will be between 20 ft and 21.75 ft (closer to 21.75 ft).

This example of the decrease of the tension in the cable illustrates an important characteristic of suspension roofs; destructive internal forces and reactions on abutments are reduced when deflections are increased. This characteristic should be utilized in investigating the actual factor of safety of suspension roofs. In general, at incipient collapse, the forces which tend to destroy a suspension roof are being gradually reduced, and this stabilizes the structure.

**Dynamic Behavior**—A description of the phenomenon of flutter and of self-exciting vibrations in a suspension roof would entail a lengthy mathematical treatise. However, elimination of flutter could be explained through consideration of natural frequencies of the individual cables. For this reason, natural frequencies of the cable in the above example will be computed, and a few observations made.

Natural frequency of a suspended cable depends on the load attached to it, and the tension in the cable. The natural frequencies of a tight cable are given by

$$W_n = n(\pi/l)\sqrt{T/(q/g)} \quad (10)$$

where $g$ = acceleration due to gravity; $n$ = any integer; other terms as defined previously.

The difference between $l$ and $L$, as shown before, is small; hence, equation (10) could be used.

Since $T$ is proportional to the applied load $q$, the natural frequencies do not depend on the magnitude of load applied to the cable. It should be noted, however, that this independence of natural frequencies from the load hold only if: (a) tension $T$ is computed on the basis of the initial sag $f$ (in our example, $f = 20$ ft) and (b) there is no other tension in the cable except that due to the *uniformly distributed* load only. Condition (a) is of minor significance and is accurate enough for practical purposes. Condition (b), on the other hand, is of major significance. If the tension $T$ in the cable is due to a combination of uniformly distributed loads and a series of concentrated loads, the natural frequencies of the cable would depend on the locations and relative magnitudes of the concentrated loads. This latter fact should be borne in mind later in this presentation when dampening is discussed.

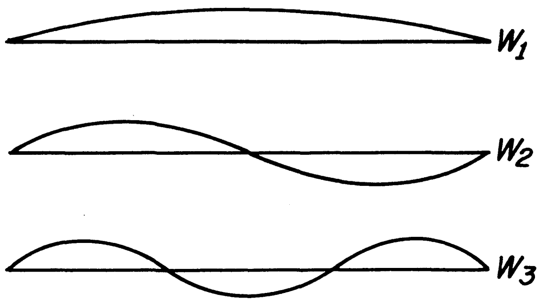


*Figure 3*

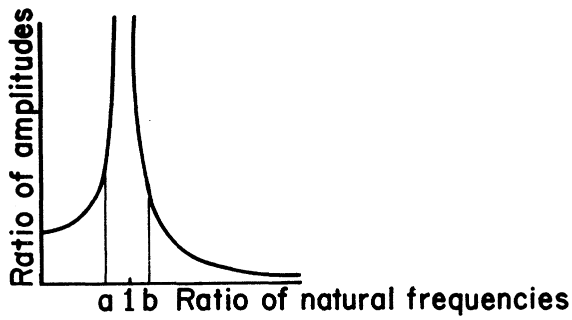


*Figure 4*

The first three natural frequencies of the cable under consideration are

$$\begin{aligned} W_1 &= 1.430 \ \text{rad/sec} \\ W_2 &= 2.860 \ \text{rad/sec} \\ W_3 &= 4.290 \ \text{rad/sec} \end{aligned} \tag{10a}$$

Modes of vibration (the fundamental modes) corresponding to each $W$ are shown in Fig. 3.

Obviously, there are an infinite number of natural frequencies of this cable, and an infinite number of modes of vibration corresponding to each integer value of $n$ in equation (10). Furthermore there are natural frequencies of the roof as a whole which, as was stated previously, are difficult to determine.

Resonance in the roof shown in Fig. 1 would occur when the externally applied dynamic load has a frequency $W_e$ equal to $W_1$, or to any other higher natural frequencies, $W_2$, $W_3$, etc., of the roof. In other words, resonance and subsequent destructive effects on the roof are to be expected when the ratio of $W_e/W$ falls within the approximate range a–b in Fig. 4.

It is reasonable to conclude that in order to eliminate the destructive effects of resonance, the designer should create a structural suspension system with such a property that the ratio $W_e/W$ will be outside of the a–b range in Fig. 4. One further look, however, would reveal the futility of an attempt to design a suspension roof with the clear assurance that the $W_e/W$ ratio would fall outside the a–b range. The reasons are obvious: on the one hand, the accurate value of range of values could not be ascertained; on the other hand, the value of $W_e$ is indeterminate. One could possibly guess, or limit statistically, the range of values of $W$ and $W_e$. But even the most elaborate mathematical and statistical work would not preclude occurrence of $W_e$ values outside the design range, since $W_e$ is beyond anybody's control.

In a number of suspension roofs built throughout the world, attempts have been made to combat flutter by addition of a continuous mass—such as cast-in-place concrete or precast concrete plank over the cables—or provision of anchor guy cables tying the suspension roof to the ground.

Although, undoubtedly, these methods helped to stabilize suspension roofs, in the opinion of the writer such a solution is not a proper one nor does it solve the problem of flutter designwise, since criteria used on one suspension roof could not be applied to another suspension roof with different dimensions and characteristics.

Among the greatest disadvantages, however, of stabilizing suspension roofs by the addition of a continuous mass or anchor guy-cables, are:

a. Additional significant cost. The additional cost involves not only additional material, but also additional weight on the building and foundations. In some cases, the added cast-in-place concrete on the suspension roof could have spanned the constructed roof as a thin shell, without any need for the cables.
b. This method still does not assure that some kind of unexpected externally applied dynamic forces would not cause the suspension roof to flutter.

### ELIMINATION OF FLUTTER— DOUBLE LAYERS OF CABLE

Figure 5 is the same as Fig. 4, except that it also includes the effect of dampening shown by the dotted line.

It is evident that a dampened suspension roof will not vibrate no matter what its natural frequencies and no matter what the frequencies of the externally applied loads.

Fortunately, it is relatively easy and economical to achieve full dampening of a suspension roof by constructing the suspension roof with interconnected cables in such a manner that the entire assembly would comprise an internally self-dampening system.

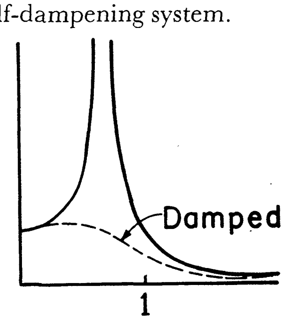


*Figure 5*

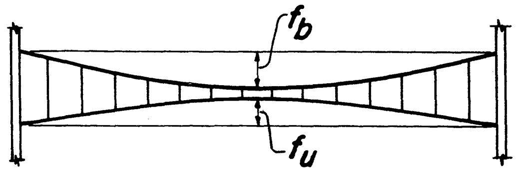


*Figure 6*

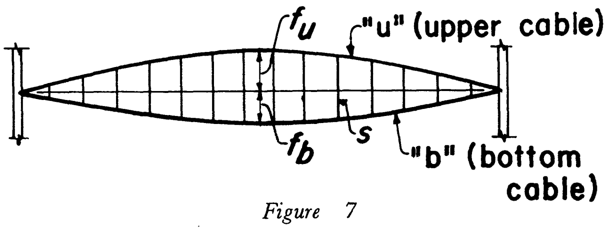


*Figure 7*

Examples are shown in Figs. 6 and 7, but the following discussion will be limited to Fig. 7. The suspension roof is composed of two layers of cables. Cables **b** in the lower layer are similar to the primary cables in the previously solved example; the upper layer of cables **u** contains one cable corresponding to each cable **b** in the lower layer. Each cable **b** in the lower layer is connected to the corresponding cable **u** in the upper layer by struts **s** in Fig 7.

Cables **b** and **u** are erected with initial tension (prestress) $T_b$ and $T_u$, respectively. The magnitude of these tensions depends on the spread $(f_u + f_b)$ (Fig. 7), number and location of struts and the size and weight of cables **b** and **u**. When thus erected, the only vertical load that the cables carry are their own weight and the weight of the struts.

As dead load (e.g., roof deck) and live load are applied, the assembly of the two cables (with the struts) acts essentially as a beam with a span $l$. Consequently, the tension on the top cable **u** is decreased by amount $\Delta T_u$, while the tension in the bottom cable **b** is increased by amount $\Delta T_b$. Magnitudes of $\Delta T_u$ and $\Delta T_b$ depend on the magnitude and distribution of the applied dead and live loads and sizes of cables. If, under the most critical combination of dead and live loads, the value of $\Delta T_u$ is less than $T_u$, while $T_b + \Delta T_b$ is less than the design capacity of the lower cable **b**, both cables **b** and **u** will remain under tension without overstress. Also, the following should be noted:

a. Values of $\Delta T_u$ and $\Delta T_b$ could vary within a wide range during the service life of the roof.
b. The value of $T_u - \Delta T_u$, i.e., the residual tension in the upper cable at any time should not be too small to cause undesirable sag of the upper cable between struts (the upper cables may be supporting the roof deck).
c. Residual tension $T_u - \Delta T_u$ and $T_b + \Delta T_b$ in the upper and lower cables respectively should be such that the deflection of the assembly of Fig. 7 is not excessive.

Let us now consider the dynamic properties of the two individual cables **b** and **u** in Fig. 7, under any superimposed load which causes decrease $\Delta T_u$ in tension of the upper cable and increase $\Delta T_b$ in tension of the lower cable.

Referring to equation (10): Natural frequencies of the bottom cable are

$$W_b = n\frac{\pi}{l}\sqrt{\frac{T_b + \Delta T_b}{q_b/g}} \quad (11)$$

Natural frequencies of the upper cable are

$$W_u = n\frac{\pi}{l}\sqrt{\frac{T_u - \Delta T_u}{q_u/g}} \quad (12)$$

where in (11) and (12), $q_b$ and $q_u$ are the weights per linear foot of the bottom and upper cables, respectively.

It will be noted that $T_b + \Delta T_b$ increases with the load, while $T_u - \Delta T_u$ decreases with the load.

When the complete dead load has been applied, natural frequencies are:

for the lower cable:

$$W_b = n\frac{\pi}{l}\sqrt{\frac{T_b + \Delta T_{bd}}{q_b/g}} \quad (13)$$

for the upper cable:

$$W_u = n\frac{\pi}{l}\sqrt{\frac{T_u - \Delta T_{ud}}{q_u/g}} \quad (14)$$

Therefore, if under dead load, $T_b$ is always made to be greater than $T_u$, it is seen from (13) and (14) that the natural frequencies of the lower cable, $W_b$, and the upper cable, $W_u$, corresponding to a particular value on the integer $n$ will always have different values at any magnitude of the live load.

When a cable vibrates, its actual geometry is a superposition of several of its fundamental modes, as well as of the modes due to forced vibration (i.e., due to the frequency of the externally applied dynamic load). From the preceding discussion, therefore, it follows that under a given externally applied dynamic force, the geometry of vibration of the lower cable will always tend to be different from that of the upper cable, as shown in Fig. 8, which shows the two cables in an imaginary situation without the struts.

In the suspension roof shown in Fig. 7, the lower cable is connected to the upper cable by struts **s**. If one of the cables, say the lower one, tends to be excited by an externally applied dynamic load and, therefore. tends to assume a certain geometry at a particular instant, the upper cable, due to its different characteristic, would tend to assume a different geometric configuration.

Thus, there will be a flow of energy from one cable to the other cable, and one cable will dampen the vibrations of the other cable in the same assembly. As in a classic shock absorber where energy from a vibrating system is transmitted into the dash-pot, the entire suspension roof consisting of a series of pairs of cables, as in Fig. 7, constitutes a giant internal shock absorber.

**Design Example**—A suspension roof consisting of the double layer of cables discussed in this section will be illustrated for the roof in Fig. 1.

In order not to complicate this presentation mathematically, it will be assumed that both the lower and the upper cables have parabolic shapes, i.e., that all applied loads are uniformly distributed on horizontal projection along both the lower and the upper cables. This also means that all the struts together are equivalent to a continuous diaphragm having the necessary properties to satisfy the assumption. This will not be so in reality, since most of the load will be distributed through the struts at concentrated points; therefore, shapes of the cables will not be parabolic and actual tensions will be different than in parabolic cables subjected to the same total load. This approximation, however, would not detract from the illustration or from the principle of the design approach.

All data for Fig. 1 will hold for this illustrative example. It will be assumed that the combined weight of the two layers of cables and the struts is $q_w = 25$ lbs per linear foot of horizontal projection. Hence, the superimposed loads (per linear foot of horizontal projection) are:

$$\begin{aligned} &\text{Dead load, } q_d = 20 \times 4 - 25 = 55 \text{ lbs per lin ft} \\ &\text{Live load, } q_l = 30 \times 4 = 120 \text{ lbs per lin ft} \\ &\text{Total load, } q = 55 + 120 = 175 \text{ lbs per lin ft} \end{aligned} \qquad (15)$$

Cross section of the roof, as erected, is shown in Fig. 9. The rise of the upper cable is $f_u = 20$ ft, or the same as the sag $f_b$ of the lower cable. In the assembly as erected with the geometry shown in Fig. 9, let the tensions in the upper and the lower cables be $T_{iu}$ and $T_{ib}$, respectively. Correspondingly, the "equivalent diaphragm" (i.e., the struts) exerts a uniformly distributed load $q_i$ on the upper and lower cables. (Note $q_i$ is force "exerted" by the diaphragm, not "transmitted" by it. In Fig. 9, the force transmitted to the lower cable is $q_w$, which is in addition to $q_i$).

The problem is to determine the size of cables, and the initial tensions as erected, so as to carry safely the superimposed loads $q_d$ and $q_l$ and so that the natural frequencies of the upper and lower cables are different at all times.

In practice, the designer should also choose proper values for $f_u$ and $f_b$ which do not necessarily have to be equal. As a matter of fact, there are many advantages,

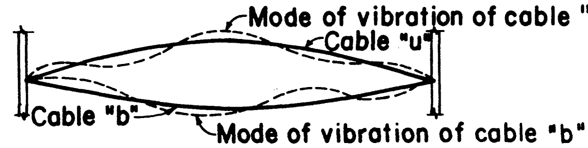

*Figure 8*

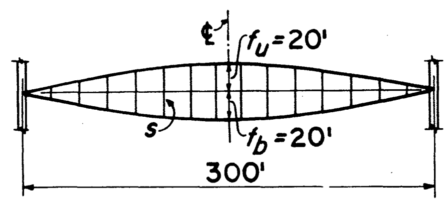

*Figure 9*

both structurally and architecturally, to having the sag unequal to the rise. In this example, the two are assumed equal for simplicity.

Before proceeding with the calculations, it is worthwhile to observe the structural characteristics of the assembly in Fig. 9:

a. If under some load the bottom cable deflects $\Delta f$, the upper cable would deflect the same amount. (This holds true even if $f_b$ is unequal to $f_u$.)
b. When the assembly deflects, the gain in tension of the bottom cable $\Delta T_{ib}$ is not generally equal to the loss in tension $\Delta T_{iu}$ of the upper cable.
c. In general, the assembly in Fig. 9 could not be considered as a simply-supported beam in which the bending moment $M_p$ due to some superimposed load $p$ (applied either to the top or the bottom cable) would be resisted by a couple consisting of changes in cable tensions. In other words, generally the relationship

$$\Delta T_{ib}(f_b + f_u) = \Delta T_{iu}(f_b + f_u) = M_p$$

does not hold.
d. Generally, when the superimposed load $p$ is applied to either the upper or the lower cable, the tension in the lower cable would be less than the combined tension due to $q_i$ and $p$ (namely, less than if $q_i$ and $p$ were assumed to be applied to the lower cable suspended by itself).
e. Under a superimposed uniformly distributed load $p$, the force $q_i$ exerted by the "equivalent diaphragm" will be reduced by $\Delta q_i$. When $f_b = f_u$, the value of $\Delta q_i$ is given by the following equation:

$$\Delta q_i = p \frac{A_u}{A_u + A_b} \qquad (16)$$

where $A_u$ and $A_b$ are cross sectional areas of the upper and lower cables, respectively. Thus, the lower cable has to be designed for a tension caused by a uniformly distributed load

$$q_w + q_i + (p - \Delta q_i) \quad (17)$$

while the upper cable has to be designed for a tension caused by a uniformly distributed load

$$q_i - \Delta q_i \quad (18)$$

f. In practice, there must be enough residual tension left in the upper cable under the most critical superimposed load, to keep its sag between consecutive struts to permissible maximum, depending on the type of roof deck used.

Equation (16) together with (17) and (18) are very important to the designer in the initial stages of choosing size of cables and the amount of the initial tension $T_{iu}$. (In practice, a designer should use an equation similar to (16), except that $q_i$ is given in terms of $A_u$ and $A_b$, as well as $f_u$ and $f_b$; this will give him insight into the choice of desirable values for $f_u$ and $f_b$.)

It will be noted from (16) that if $A_u = A_b$, the lower cable will carry a tension due to $q_i$ plus $\frac{1}{2}p$; if $A_u$ is considerably greater than $A_b$, the additional tension in the lower cable will be that due to but a fraction of $p$; if $A_b$ is considerably greater than $A_u$, the additional tension in the lower cable will be due to almost complete intensity of $p$.

It is hoped that this discussion will give the designer enough background and general understanding of the load resistance characteristics of a suspended roof consisting of two layers of prestressed cables to enable him to choose sizes of, and initial tensions in, the cables. This step will, therefore, be eliminated from this presentation and we shall proceed with the calculations of tensions and dynamic characteristics of our example, assuming that the assembly of Fig. 9 consists of the following:

Lower cable: $2^1/_4$ dia. strand, $A_b = 3.11$ sq in., weighing 10.83 lbs per lin ft

Top cable: $1^7/_8$ dia. strand, $A_u = 2.14$ sq in., weighing 7.42 lbs per lin ft

Initial tension in the upper cable, $T_{iu} = 90$ kips. Therefore:

$$q_i = 200(90/116) = 155 \text{ lbs per lin ft}$$
$$q_i + q_w = 155 + 25 = 180 \text{ lbs per lin ft}$$

Initial tension in bottom cable:

$$T_{ib} = (180/155) \times 90 = 104.2 \text{ kips}$$

Natural frequencies:

$$W_u = 6.52,\ 13.04,\ 19.56$$
$$W_b = 5.83,\ 11.66,\ 17.49$$

Also:

$$\Delta q_i = p\frac{2.14}{2.14 + 3.11} = p \times 0.408$$

In connection with the natural frequencies, the following reasoning was pursued:

a. In a single suspended cable, as in Fig. 1, the tension is always proportional to the load. Hence, in accordance with equation (10), the natural frequency of a suspended cable under its own weight or in conjunction with a superimposed load stays the same.

b. Natural frequency of either the bottom of the upper cable in Fig. 9, where they are separated by struts and where most of the load is exerted and transmitted through the struts, could be calculated only after a lengthy mathematical procedure.

c. In the opinion of the writer, however, the actual natural frequency of each of the cables is not of practical importance. As long as the natural frequencies of each of the cables are computed using the same common condition, and as long as the frequencies thus computed are different from each other, there is sufficient assurance that the two cables would tend to vibrate in different modes (under one specific externally applied dynamic force), and thus flutter would be eliminated.

d. To satisfy requirement (c) above, it is adequate to compute the natural frequencies by equation (10), using the total span of the cable, the actual existing tension, and only the weight of the cable itself. (Note the condition of the bottom cable between two consecutive struts, when the roof deck is applied to the upper cable.)

e. Several rude test models with double layer of cables were subjected to various pulsating loads but, as expected, did not exhibit any flutter tendencies. A double layer suspension roof designed by the writer per above-mentioned principles was completed in 1959 in the northern part of the United States. During the years since its completion this roof has exhibited no tendency to flutter even though, in addition to heavy winds, mechanical and air conditioning equipment, billboards and other items were placed between the two layers of cables or hung from them. To the best knowledge of the writer, all other single layer suspension roofs without rigid membrane or guy-anchors, did exhibit flutter tendencies within such a time span.

f. In choosing the initial tensions and geometrical configuration in Fig. 9, the designer should ascertain that at the application of the entire dead load the natural frequencies of the upper and lower cables are different and that as the load increases, the natural frequencies diverge further. This will assure that at no time and under no live load would the natural frequencies of the two cables coincide.

Proceeding with the numerical example:

Condition under dead load, $p = q_d$:

$$\Delta q_i = 55 \times 0.408 = 22.4 \text{ lbs per lin ft}$$
$$p - \Delta q_i = 55 - 22.4 = 32.6 \text{ lbs per lin ft}$$
$$q_i - \Delta q_i = 155 - 22.4 = 132.6 \text{ lbs per lin ft}$$

Tension in upper cable:

$$T_{ud} = 90(132.6/155) = 77.0 \text{ kips}$$
$$(q_w + q_i) + (p - \Delta q_i) = 180 + 32.6 = 212.6 \text{ lbs per lin ft}$$

Tension in bottom cable:

$$T_{bd} = 104.2(212.6/180) = 122.0 \text{ kips}$$
$$W_u = 6.02, 12.06, 18.09$$
$$W_b = 6.30, 12.60, 18.90$$

Deflection $\Delta f$ (from (9)) $= 1.75\ (32.6/200)(2.43/3.11)$ $= 0.223$ ft or 1/1350 of the span

Condition under dead and live loads, $p = q$:

$$\Delta q_i = 175 \times 0.408 = 71.5 \text{ lbs per lin ft}$$
$$p - \Delta q_i = 175 - 71.5 = 103.5 \text{ lbs per lin ft}$$
$$q_i - \Delta q_i = 155.0 - 71.5 = 83.5 \text{ lbs per lin ft}$$

Tension in the upper cable:

$$T_u = 77.0\ (83.5/132.6) = 48.7 \text{ kips}$$
$$(q_w + q_i) + (p - \Delta q_i) = 180 + 103.5 = 283.5 \text{ lbs per lin ft}$$

Tension in bottom cable:

$$T_b = 122(283.5/212.6) = 162.0 \text{ kips}$$
$$W_u = 4.80, 9.60, 14.40$$
$$W_b = 7.30, 14.60, 21.90$$

$\Delta f$ (dead and live load) $= 0.223\ (103.5/32.6) = 0.707$ ft

$\Delta f$ (live load only) $= 0.223\ [(103.5 - 32.6)/32.6] = 0.484$ ft or 1/620 of the span

**Conclusions**—The design method presented in this section dampens vibrations — and hence prevents aerodynamic instability — of the individual cables. The entire roof as such, however, will have its own fundamental frequencies and, if subjected to pulsating load, will behave as any other structure constructed of conventional materials.

The numerical example illustrated in this presentation contained many arithmetical simplifications. In practice the design procedure will be somewhat longer. Particularly:

a. Loads are transmitted onto cables at concentrated points; therefore, the curve of the cable is not a parabola. One has to determine the shape accurately. The designer should include in his computations the effect of unsymmetrical loading and upward suction of wind. Unsymmetrical loads might have significant effects on deflection. In this respect, the fact that cables are loaded through the struts is an advantage. By properly proportioning lengths of the struts and, hence, the angle $\alpha$ of the cable at the anchorage point, the designer may reduce deflections—which are not as easily controllable in freely suspended cables, as in Fig. 1.

b. Final choice of cable sizes should be such as to provide maximum economy.
A suspended roof with two layers of cables requires more cables and anchorages than a roof with one layer of cables. However, in the United States, the additional cost of cables and anchorages—particularly in a large span roof—is much less than the cost of a rigid membrane, such as concrete, placed over the cables. Fittings rather than cables are usually the costly item. By using continuous cables around the abutments, many fittings could be eliminated thus reducing the cost of the suspended roof.

c. The cost of abutments in any suspension roof is a significant item; sometimes the abutments are more expensive than the suspended roof itself. There are, however, several ways to reduce the cost of the abutment.
In a circular building, an exterior ring could be provided into which all cables are anchored. Such a ring is self-contained structurally. Because of high initial tensions in the cables, bending moments in the ring are relatively small. A ring under compression from the cables has no tendency to buckle; hence, the material in the ring could be fully utilized. A radial pattern of cables in a circular suspension roof is also helpful in reducing deflections at the center of the roof since the superimposed load is smaller in the middle portion of the cable than towards the ends.

Another way to considerably reduce the cost of a suspension roof is to utilize the tension of the cables to support large portions of the building.

## APPLICATIONS

After completion of erection, the system of the double layer pretensioned cables described in this presentation behaves and actually constitutes a rigid structural system, comparable to, and in many cases stiffer than, conventional steel trusses designed for the same superimposed loads.

In examples A and B which follow, the dampened pretensioned cable system was used only for the large span roofs of these projects. In each case, the tension of the cables was transmitted to an inert ring resting on top of columns. As was mentioned previously, however, the tension in the cables could be used advantageously in the structure below the roof, effecting large economies in the construction of the entire structure. Succeeding examples C and D show this application.

Structures shown in this presentation are round or partly round. The principles described, however, could be used on rectangular or on any other shape buildings.

**A. Utica Municipal Auditorium—Utica, N. Y.**—An example of a suspended roof with a double layer of cables is the roof for the Municipal Auditorium in Utica, New York (Figs. 10 and 11).

This 250-ft span building was completed in 1959. It proved that a two-layer prestressed cable suspension system described previously eliminates flutter entirely. Throughout its service the roof structure behaved as a typical rigid steel construction, in spite of severe dynamic and vibratory forces to which it has been subjected.

Besides the inherent economy due to the lightness of the roof and, therefore, the need for a minimum number of columns to support such a roof, further economies have been achieved in maintenance and space: the curvature of the roof is much shallower than that of a dome, hence there is less space to heat and air condition; the space between the two layers of cables is occupied by mechanical and air conditioning equipment, which would otherwise have required other space in the building.

The entire roof was prefabricated and erected in three weeks, using only one tower as scaffolding. Prestressing of the cables was achieved by jacking apart the two central steel rings. Optimum curvature of the cables required to eliminate flutter and to minimize deflections due to eccentric loads was obtained by the vertical struts between the two layers of cables.

**B. New York State Pavilion—1964–1965 New York World's Fair**—The 350-ft span of the roof of the main pavilion consists of a pretensioned cable system similar to that discussed previously. In this case, however, the suspension system is shallow at the center and deep at the ends. The principle is shown in Figs. 12 and 13. The roof has an oval shape in plan with a major axis of 350 ft and a minor axis of 260 ft. The cables are anchored into a steel ring. The entire roof "floats" on lubrite plates supported on steel brackets which protrude from 16 exterior towers.

The ring and the cable suspended system were all erected on the ground. The entire suspension system of cables was assembled and prestressed within a few days. After completion of prestressing, the entire roof was lifted into its final position.

**C. Travelers Insurance Company Pavilion—1964-1965 New York World's Fair**— How engineering can contribute to flexibility and economy of a structure is exemplified in the Travelers Insurance Company pavilion.

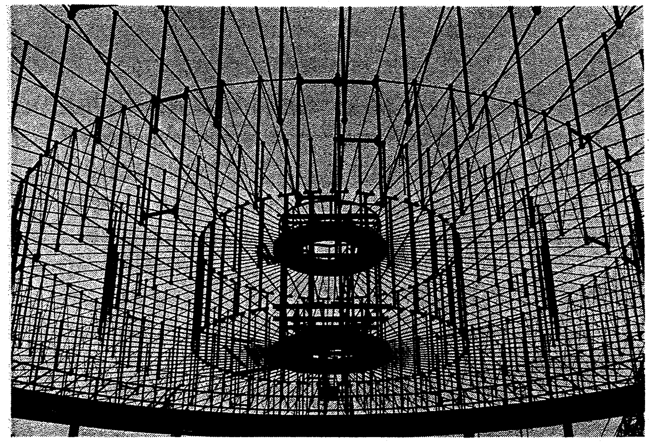

*Fig. 10. View of cable suspended roof, Utica Municipal Auditorium*

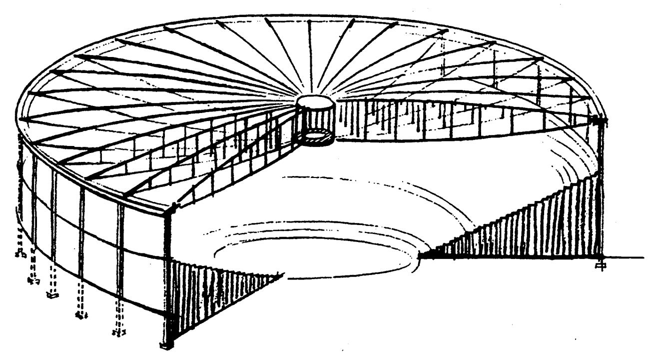

*Fig. 11. Framing schematic, Utica Municipal Auditorium*

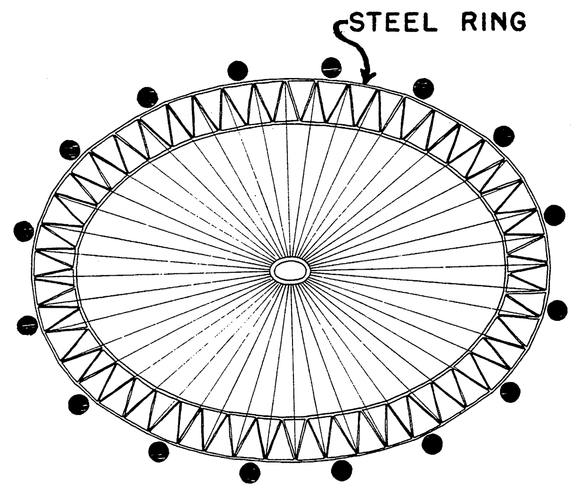


*Fig. 12. Schematic roof plan, N. Y. State Pavilion*

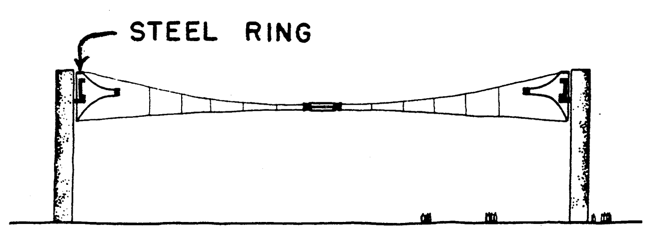


*Fig. 13. Schematic section, N. Y. State Pavilion*

The symbol of the Travelers Insurance Company being an umbrella, the architectural concept for the building was to simulate a 160-ft diameter umbrella. The interior of the building had to be a closed, lightless space unobstructed by interior columns, in the shape of a compressed doughnut.

One reasonable structural approach to framing this building would be as shown in Fig. 14. Such conventional steel framing would consist of cantilevered trusses topped by a dome. The cantilevers would be 6 ft deep at the base, while the dome would be approximately 5 ft deep at the apex. Such construction would have required scaffolding for erection and would have entailed relatively expensive labor expenditure per pound of fabricated steel. In this scheme, the dome would be essentially in compression, accompanied by bending, while bending moment would predominate in the cantilevers.

Instead, the structural system of the Travelers Insurance Company pavilion, as finally designed and built, consisted of a steel tension space structure as shown in Figs. 15 and 16.

This system resulted in much thinner members, great economies of material and fast fabrication and erection without any need for scaffolding.

The structural system consisted of twenty-four "boomerang-shaped" prefabricated steel ribs. These ribs were assembled in place, with a single temporary support at the outer edge of the boomerang. The tops of the boomerangs were connected by a double layer of cables, described in this paper.

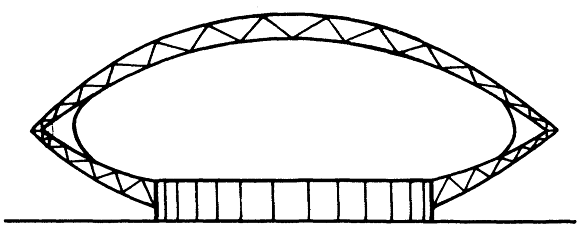

*Figure 14*

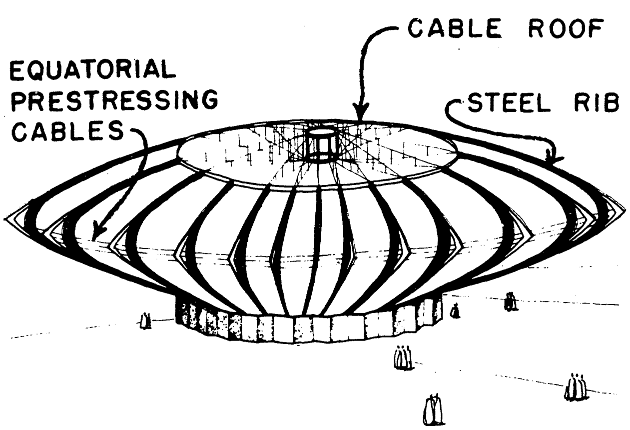


*Fig. 15. Schematic view, Travelers Insurance Pavilion*

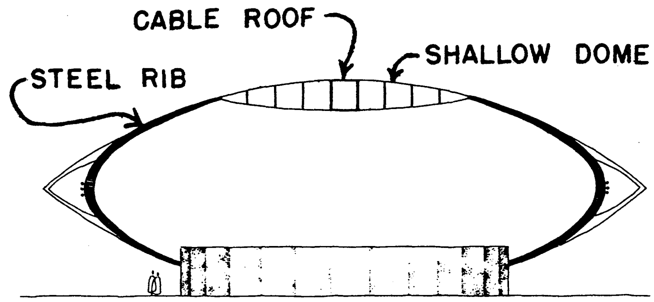


*Fig. 16. Schematic section, Travelers Insurance Pavilion*

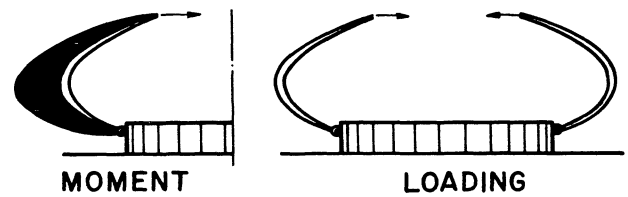


*Fig. 17. Before application of equatorial cables*

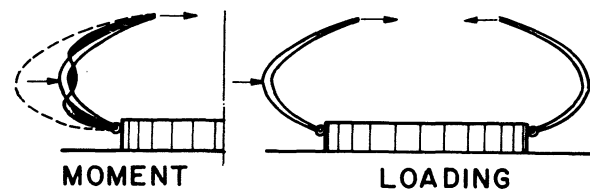


*Fig. 18. After application of equatorial cables*

The tension required to keep the cables dynamically stable was adequate to lift the ribs off their exterior supports. This loading condition and the bending moments due to the cable tension plus the applied load are shown in Fig. 17. To effect further economies and stiffness, equatorial cables were wrapped around the ribs. Spreading the cables away from the ribs at the equator induced a horizontal force in the ribs. The resulting loading condition and bending moments in the ribs are shown in Fig. 18.

A view of the upper portion of the as-built structure is shown in Fig. 19. Each rib ties into the compression ring. Each cable is socketed to the top of one rib, runs through a saddle connected to a tension plate, and returns to an adjoining rib. Purlins on the cable material are only for the support of roofing material.

In the adopted as-built structural system, the distribution of the material is different from a conventional framing, adding more useable headroom inside the main structure as well as larger clearances on the outside between the ground and the projecting structure. Further, it contributes to the architectural flexibility and aesthetics of the pavilion.

*Fig. 19. View of cable roof, Travelers Insurance Pavilion*

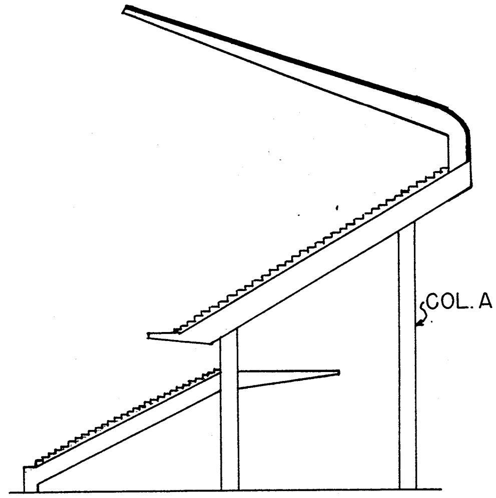


*Figure 20*

The resulting structure is a space tension structure. Its stress distribution pattern is entirely different from that shown in Fig. 14. This stress pattern and the method of construction reduced the weight of steel to a fraction of the framing shown in Fig. 14, and speeded up erection since no scaffolding was required in its construction.

**D. Open Baseball Stadium**—In conventional open-type stadiums with a roof over the seats, a column such as column A in Fig. 20 is always found to be required since, without that column, cantilevering of the seats becomes an economically insuperable problem. Furthermore, attempts to bring the edge of the roof to the front seats, which are the best seats in a stadium, are not successful because of economic and headroom problems.

Another approach to stadium design is indicated in Fig. 21. The structural considerations in this scheme are as follows: Imagine that beam B, which holds the upper tier of seats, is hinged at the column support. Under superimposed load this beam, unless restrained, would rotate as shown by the dotted line. After such rotation, the ends **a** of all these beams would lie on the dotted line shown in Fig. 22. (Note that for the curved line **a–a** to flatten out as shown, columns should be placed farther from the edge at the end tiers.)

To eliminate such rotation, either the connection between the beam and the column should be made strong, which is expensive, or the ends **a** of all beams may be held together to prevent displacements towards the dotted line. The latter was done in this stadium.

All beams are connected by a shelf (see Fig. 21). As shown in Fig. 22, this shelf looks like a C-clamp held against spreading by cables, and this in turn prevents beams B from rotating. The action of the C-clamp is equivalent to a horizontal force *P* shown in Figure 21.

Although the building looks like a cantilevered structure from the outside, it is actually a space structure with a large cantilever effect, achieved at a lower cost than that of a pure cantilever.

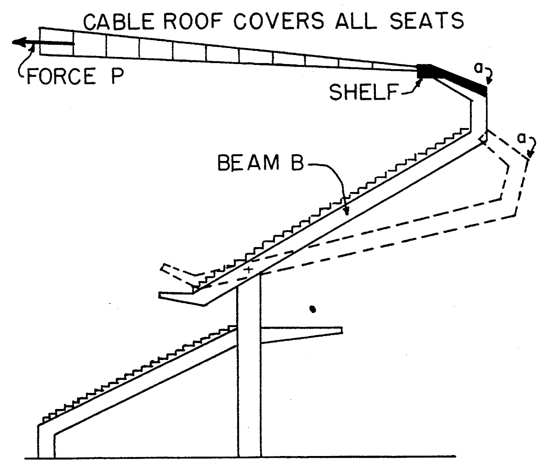


*Figure 21*

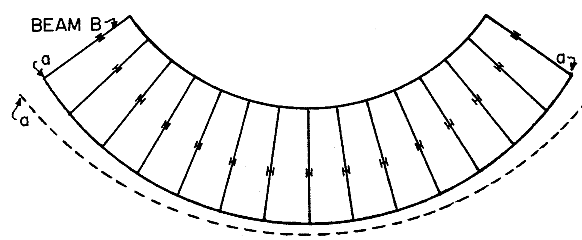


*Figure 22*

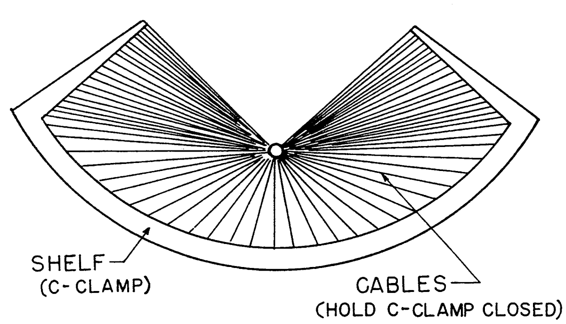


*Figure 23*