Web Shear Strength of Flexural Members

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INTRODUCTION

The shear strength of webs of flexural members is given in the AISC LRFD Specification (AISC 1993) in three separate places: Chapter F, Beams and Other Flexural Members; Appendix F; and Appendix G, Plate Girders. Chapter F covers only unstiffened webs with no tension-field action, Appendix F covers both stiffened and unstiffened webs with no tension-field action, and Appendix G covers stiffened and unstiffened webs, with or without tension-field action. A total of nine equations are given for V_n the nominal shear strength. It will be shown here that with a slight modification of the variable C_v (the ratio of critical web shear buckling stress to shear yield stress), these nine equations can be reduced to two equations that cover all conditions—stiffened or unstiffened webs, tension field or no tension field.

Although the grouping of the shear strength equations in the AISC Specification is efficient for design purposes, it can obscure the equivalence of the equations. It is hoped that the condensation presented here will promote a clearer understanding of the Specification provisions.

SPECIFICATION APPENDIX G

Appendix G, Plate Girders, is the most general provision with regard to shear strength. Web shear strength is a function of two variables: h/t_w, the web width-thickness ratio, and a/h, the aspect ratio of web panels formed by the placement of vertical web stiffeners, where

h = web depth, measured from inside of flange to inside of flange in a built-up welded member or from toe of fillet to toe of fillet in a hot-rolled member
t_w = web thickness
a = clear spacing of the vertical web stiffeners

In the following equations for the nominal shear strength, the variable k_v is used, where

$$k_v = 5 + \frac{5}{(a/h)^2}$$

$$= 5 \text{ for } a/h > 3 \qquad \text{(AISC Eq. A-G3-4)}$$

$$= 5 \text{ for } a/h > \left(\frac{260}{h/t_w}\right)^2$$

The limit states for web shear strength are web shear yielding and web shear buckling:

$$\text{If } \frac{h}{t_w} \leq 187\sqrt{\frac{k_v}{F_{yw}}},$$

the strength is based on shear yielding, and

$$V_n = 0.6A_wF_{yw} \qquad \text{(AISC Eq. A-G3-1)}$$

where

A_w= cross-sectional area of the web
F_{yw}= tensile yield stress of the web

$$\text{If } \frac{h}{t_w} > 187\sqrt{\frac{k_v}{F_{yw}}},$$

the shear strength is based on buckling, and

$$V_n = 0.6A_wF_{yw}\left[C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}}\right] \text{(AISC Eq. A-G3-2)}$$

AISC equation A-G3-2 can also be written as follows:

$$V_n = 0.6A_wF_{yw}C_v + 0.6A_wF_{yw}\left[\frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}}\right] \qquad (1)$$

The first term in Equation 1 is the shear buckling strength; the second term is the post-buckling strength deriving from tension-field action.

The variable C_v is the ratio of the critical buckling strength to the yield strength, and it is defined as follows:

$$\text{For } 187\sqrt{\frac{k_v}{F_{yw}}} < \frac{h}{t_w} \leq 234\sqrt{\frac{k_v}{F_{yw}}},$$

$$C_v = \frac{187\sqrt{k_v/F_{yw}}}{(h/t_w)} \qquad \text{(AISC Eq. A-G3-5)}$$

$$\text{For } \frac{h}{t_w} > 234\sqrt{\frac{k_v}{F_{yw}}},$$

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$$C_v = \frac{44{,}000k_v}{(h/t_w)^2 F_{yw}} \qquad \text{(AISC Eq. A-G3-6)}$$

Tension-field action cannot be used in certain situations. In the cases that follow, the second term in Equation 1 must be dropped, and $V_n = 0.6A_w F_{yw} C_v$:

- In end panels
- In hybrid girders
- When $a/h > 3$
- When $a/h > \left(\frac{260}{h/t_w}\right)^2$

SPECIFICATION APPENDIX F2

Appendix F2 gives the shear strength of stiffened or unstiffened webs with no tension-field action. These provisions are identical to those of Appendix G with the tension field term of AISC Equation A-G3-2 omitted. To demonstrate this, each provision of Appendix F2 will be given, followed by the corresponding provision of Appendix G.

- For $\frac{h}{t_w} \le 187\sqrt{\frac{k_v}{F_{yw}}}$, $V_n = 0.6F_{yw}A_w$ (AISC Eq. A-F2-1)

where k_v is defined as in Appendix G. This provision is identical to the corresponding one in Appendix G.

- For $187\sqrt{\frac{k_v}{F_{yw}}} < \frac{h}{t_w} \le 234\sqrt{\frac{k_v}{F_{yw}}}$,

$$V_n = 0.6F_{yw}A_w \frac{187\sqrt{k_v/F_{yw}}}{(h/t_w)} \qquad \text{(AISC Eq. A-F2-2)}$$

From Appendix G, when

$$\frac{h}{t_w} > 187\sqrt{\frac{k_v}{F_{yw}}},$$

$$V_n = 0.6A_w F_{yw} C_v$$

(without tension-field action) and for

$$187\sqrt{\frac{k_v}{F_{yw}}} < \frac{h}{t_w} \le 234\sqrt{\frac{k_v}{F_{yw}}},$$

$$C_v = \frac{187\sqrt{k_v/F_{yw}}}{(h/t_w)}$$

so the Appendix G provision for this range of h/t_w is

$$V_n = 0.6A_w F_{yw} C_v = 0.6A_w F_{yw} \frac{187\sqrt{k_v/F_{yw}}}{(h/t_w)},$$

which is the same as AISC Equation A-F2-2.

- For $\frac{h}{t_w} > 234\sqrt{\frac{k_v}{F_{yw}}}$,

$$V_n = \frac{A_w(26{,}000k_v)}{(h/t_w)^2} \qquad \text{(AISC Eq. A-F2-3)}$$

From Appendix G, when

$$\frac{h}{t_w} > 187\sqrt{\frac{k_v}{F_{yw}}},$$

$$V_n = 0.6A_w F_{yw} C_v$$

(without tension-field action) and for

$$\frac{h}{t_w} > 234\sqrt{\frac{k_v}{F_{yw}}},$$

$$C_v = \frac{44{,}000k_v}{(h/t_w)^2 F_{yw}}$$

Therefore,

$$V_n = 0.6A_w F_{yw} C_v = 0.6A_w F_{yw} \frac{44{,}000k_v}{(h/t_w)^2 F_{yw}} = \frac{A_w(26{,}000k_v)}{(h/t_w)^2},$$

which is the same as AISC Equation A-F2-3.

SPECIFICATION CHAPTER F

The shear strength provisions of Chapter F of the AISC Specification are a further specialization of Appendices G and F. Chapter F gives the shear strength of beams with no tension-field action and unstiffened webs only. With no stiffeners,

$$\frac{a}{h} > 3 \text{ and } k_v = 5$$

This means that the limiting values of h/t_w from the appendixes take the form

$$187\sqrt{\frac{k_v}{F_{yw}}} = 187\sqrt{\frac{5}{F_{yw}}} = \frac{418}{\sqrt{F_{yw}}}$$

and

$$234\sqrt{\frac{k_v}{F_{yw}}} = 234\sqrt{\frac{5}{F_{yw}}} = \frac{523}{\sqrt{F_{yw}}}$$

The provisions of Chapter F are related to Appendix F as follows:

- For $\frac{h}{t_w} \le \frac{418}{\sqrt{F_{yw}}}$, $V_n = 0.6F_{yw}A_w$ (AISC Eq. F2-1)

This is the same as the provision from Appendix F, with

$$187\sqrt{\frac{k_v}{F_{yw}}} = \frac{418}{\sqrt{F_{yw}}}$$

- For $\frac{418}{\sqrt{F_{yw}}} < \frac{h}{t_w} \le \frac{523}{\sqrt{F_{yw}}}$,

$$V_n = 0.6F_{yw}A_w\frac{187\sqrt{k_v/F_{yw}}}{(h/t_w)} \qquad \text{(AISC Eq. F2-2)}$$

This is the same as the provision from Appendix F, with

$$187\sqrt{\frac{k_v}{F_{yw}}} = \frac{418}{\sqrt{F_{yw}}} \text{ and } 234\sqrt{\frac{k_v}{F_{yw}}} = \frac{523}{\sqrt{F_{yw}}}$$

- For $\frac{523}{\sqrt{F_{yw}}} < \frac{h}{t_w} \le 260,\ V_n = \frac{132{,}000A_w}{(h/t_w)^2}$ (AISC Eq. F2-3)

From Appendix F,

$$\text{For } \frac{h}{t_w} > 234\sqrt{\frac{k_v}{F_{yw}}},\quad V_n\frac{A_w(26{,}000k_v)}{(h/t_w)^2}$$

For $k_v = 5$, this becomes

$$\text{For } \frac{h}{t_w} > \frac{523}{\sqrt{F_{yw}}},$$

$$V_n = \frac{A_w(26{,}000k_v)}{(h/t_w)^2} = \frac{A_w(26{,}000)(5)}{(h/t_w)^2} = \frac{132{,}000A_w}{(h/t_w)^2}$$

which is the same as the provision from Chapter F, except that an upper limit of $h/t_w = 260$ is imposed for AISC Equation F2-3. This is a long-standing limit on the web width-thickness ratio for *unstiffened* webs.

CONSOLIDATION OF PROVISIONS

If the definition of C_v is changed from "the critical *web buckling stress* divided by the yield stress" to "the critical *web stress* divided by the yield stress," then the provisions of the Specification can be reduced to the following two equations: *without* tension-field action,

$$V_n = 0.6A_wF_{yw}C_v \qquad (2)$$

With tension-field action,

$$V_n = 0.6A_wF_{yw}C_v + 0.6A_wF_{yw}\left[\frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}}\right] \qquad (3)$$

where C_v is defined as follows:

$$\text{For } \frac{h}{t_w} \le 187\sqrt{\frac{k_v}{F_{yw}}},\quad C_v = 1$$

$$\text{For } 187\sqrt{\frac{k_v}{F_{yw}}} < \frac{h}{t_w} \le 234\sqrt{\frac{k_v}{F_{yw}}},\quad C_v = \frac{187\sqrt{k_v/F_{yw}}}{h/t_w}$$

$$\text{For } \frac{h}{t_w} > 234\sqrt{\frac{k_v}{F_{yw}}},\quad C_v\frac{44{,}000k_v}{(h/t_w)^2F_{yw}}$$

If stiffeners are not used, use Equation 2 and $k_v = 5$. For all other cases,

$$k_v = 5 + \frac{5}{(a/h)^2}$$

RECOMMENDATIONS

The consolidation of web shear strength provisions can be incorporated into the Specification in such a way as to maintain a consistency among Chapter F, Appendix F, and Appendix G. In the following, recommended modifications of the relevant portions are shown (without equation numbers).

CHAPTER F:

F2. Design for Shear

2. Design Shear Strength

The design shear strength of unstiffened webs with $h/t_w \le 260$ is ϕ_vV_n, where

$$\phi_v = 0.90$$

$$V_n = 0.6A_wF_{yw}C_v$$

where C_v is the critical web shear stress divided by the shear yield stress and is defined as follows for unstiffened webs:

$$\text{For } \frac{h}{t_w} \le \frac{418}{\sqrt{F_{yw}}},\quad C_v = 1$$

$$\text{For } \frac{418}{\sqrt{F_{yw}}} < \frac{h}{t_w} \le \frac{523}{\sqrt{F_{yw}}},\quad C_v = \frac{418\sqrt{F_{yw}}}{h/t_w}$$

$$\text{For } \frac{523}{\sqrt{F_{yw}}} < \frac{h}{t_w} \le 260,\quad C_v = \frac{220{,}000}{(h/t_w)^2F_{yw}}$$

APPENDIX F:

F2. Design for Shear

2. Design Shear Strength

The design shear strength of stiffened or unstiffened webs is ϕ_vV_n, where

$$\phi_v = 0.90$$

$$V_n = 0.6A_wF_{yw}C_v$$

where C_v is the critical web shear stress divided by the shear yield stress and is defined as follows for stiffened or unstiffened webs:

$$\text{For } \frac{h}{t_w} \le 187\sqrt{\frac{k_v}{F_{yw}}}, \quad C_v = 1$$

$$\text{For } 187\sqrt{\frac{k_v}{F_{yw}}} < \frac{h}{t_w} \le 234\sqrt{\frac{k_v}{F_{yw}}}, \quad C_v = \frac{187\sqrt{k_v/F_{yw}}}{h/t_w}$$

$$\text{For } \frac{h}{t_w} > 234\sqrt{\frac{k_v}{F_{yw}}}, \quad C_v = \frac{44{,}000k_v}{(h/t_w)^2F_{yw}}$$

where

$$k_v = 5 + \frac{5}{(a/h)^2}$$

$$= 5 \text{ for } a/h > 3$$

$$= 5 \text{ for } a/h > \left(\frac{260}{h/t_w}\right)^2$$

APPENDIX G:

G3. Design Shear Strength

The design shear strength shall be $\phi_v V_n$, where $\phi_v = 0.90$ and V_n is determined as follows:

Without tension-field action,

$$V_n = 0.6A_wF_{yw}C_v$$

With tension-field action,

$$V_n = 0.6A_wF_{yw}C_v + 0.6A_wF_{yw}\left[\frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}}\right]$$

where C_v is the critical web shear stress divided by the shear yield stress and is defined as follows for stiffened or unstiffened webs:

$$\text{For } \frac{h}{t_w} \le 187\sqrt{\frac{k_v}{F_{yw}}}, \quad C_v = 1$$

$$\text{For } 187\sqrt{\frac{k_v}{F_{yw}}} < \frac{h}{t_w} \le 234\sqrt{\frac{k_v}{F_{yw}}}, \quad C_v = \frac{187\sqrt{k_v/F_{yw}}}{h/t_w}$$

$$\text{For } \frac{h}{t_w} > 234\sqrt{\frac{k_v}{F_{yw}}}, \quad C_v = \frac{44{,}000k_v}{(h/t_w)^2F_{yw}}$$

If stiffeners are not used, use Equation A-G3-1 and $k_v = 5$. For all other cases,

$$k_v = 5 + \frac{5}{(a/h)^2}$$

$$= 5 \text{ for } a/h > 3$$

$$= 5 \text{ for } a/h > \left(\frac{260}{h/t_w}\right)^2$$

SUMMARY

It has been demonstrated here that the AISC Specification provisions for web shear strength can be reduced to two equations that cover all situations with regard to stiffeners and tension-field behavior. This consolidation can be incorporated into the Specification while still maintaining the categories defined by Chapter F, Appendix F, and Appendix G.

REFERENCE

American Institute of Steel Construction, *Load and Resistance Factor Design Specification for Structural Steel Buildings*, Chicago, IL, 1993.