# The Behavior and Load-Carrying Capacity of Unstiffened Seated-Beam Connections

W. H. YANG, W. F. CHEN and M. D. BOWMAN

## ABSTRACT

Unstiffened seated-beam connections are often used to connect a beam to a column. For many years this connection was designed by using tabular methods in the AISC Design Manuals, including the most recent LRFD Manual. The manual tables are generated based on the *required* bearing length method developed in 1940s. This paper investigates the validity of this method by examining the formulations of the model and analyzing the connection behavior. The interactions between connection components are discussed and more rational and accurate models are developed for connections consisting of a flexible angle and a stiff beam. Through comparison of various results, the current LRFD procedures are assessed and practical implications of this research are summarized.

## INTRODUCTION

An often used simple connection between a beam and column web is the unstiffened seated-beam connection, where the end reaction is supported by an unstiffened angle bolted or welded to the column as shown in Figure 1. The angle connecting the upper portion of the beam to the column, optionally located either on the top flange or the upper part of the web of the beam, is only for stability considerations. The entire load is assumed to be transmitted through the bottom or "seat" angle.

In both the 4th and 9th edition of the AISC Manual,[1-2] the procedure for the design of a seated connection is the so-called *required* bearing length method developed in 1940s. In this procedure, the location of the reaction is assumed to be at the center of the effective bearing length $N$ required for beam web local yielding (Figure 2). The bending moment on the critical section of the angle, assumed at the base of the fillet on the outstanding leg (Figure 2), is determined by taking the beam reaction times the distance to the critical section. The thickness of the seat angle is then determined by limiting the flexural stress on this critical section. This procedure is also incorporated in the latest AISC LRFD Manual.[3]

Seated connection tests conducted 50 years ago[4-6] were probably the experimental foundation for the AISC Manual design procedures. Most of the experiments on seated connections reported after 1980s[7-11] studied the moment-rotation characteristics of the seated connections as semi-rigid connections other than their load-carrying capacity as simple connections. Recently reported experiments by Ellifritt et al.[42-44] addressed stiffened seated connections only, where the strength of column web or the welding between seat and column often control the behavior of connections.

Although extensive theoretical analysis on the behavior of seated-beam connection can be found in the literature, almost all of them focused on the influence of connection flexibility on the performance of columns or frames as a whole,[12-23] or on modeling connection moment-rotation relations,[24-27] or both topics.[28-34] Prior analytical research efforts on the load-carrying capacity of seated connections were very limited,[35-37] probably because this type of connections has had a very good performance record for many years. While the current AISC/LRFD procedures, translated from the previous ASD procedures,[38] have the merit of simple formulations and appear to be reasonably conservative, experimental research by Roeder and Dailey[39] raised a number of questions regarding the design procedure because it was found that the safety of seated connections was not provided by factors suggested in the design calculation.

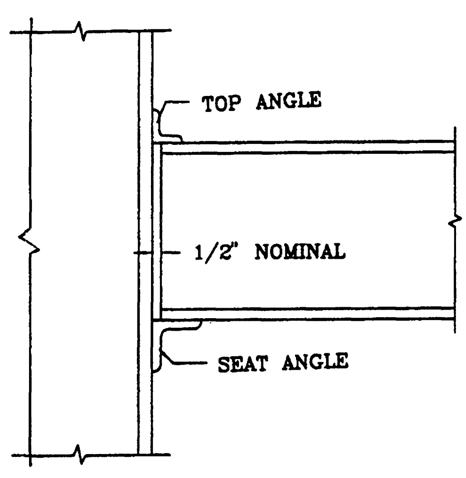



*Fig. 1. Unstiffened seated-beam connection.*

W. H. Yang, W. F. Chen, and M. D. Bowman are with the School of Civil Engineering, Purdue University, West Lafayette, IN.

This paper studies the behavior and load-carrying capacity of unstiffened seated beam connections consisting of a flexible angle and a strong beam, where the strength of the seat angle itself will control the structural behavior of the connection (the bolts' related strength are assumed to be adequate). Connections involving a thick angle and a weak beam will not be the main topic of the present study, because the limit states of beam web crippling or local yielding would be more critical in that case.

The first part of this paper briefly reviews the AISC LRFD procedures. Focusing on the effects of beam-bottom-flange to seat-angle attaching bolts, the second part studies the behavior of the connections by analyzing the interactions between the seat-angle and the supported-beam, from which simplified models for the calculation of the load-carrying capacity of the seated connections will be justified. Then, the load-carrying capacity of the seated connections, for both no-bolt and bolted cases, are determined from a failure mechanism analysis that uses a plastic hinge method.[40] Finally, through the comparison of various methods, the validity of the LRFD procedures are evaluated and assessed and the practical implications of this research emphasized.

## REVIEW OF THE AISC/LRFD PROCEDURES

The design loads for the AISC/LRFD Manual[3] tables are based on two possible limit states: (1) excessive bending stress on the seat angle, and (2) local web yielding of the supported beam. When computing the load-carrying capacity of the connection, both limit states must be considered and the governing condition is the one which provides the most conservative results.

The basic assumptions used in the procedures are that: (1) the critical section is located at the toe of the fillet of the outstanding leg; (2) the reaction occurs at the center of the effective bearing length $N$ required for the local web yielding limit state; (3) the angles fillet radius is ⅜-in. and the beam setback is ¾-in., which is ¼-in. more than the nominal setback to allow for possible mill underrun in beam length.

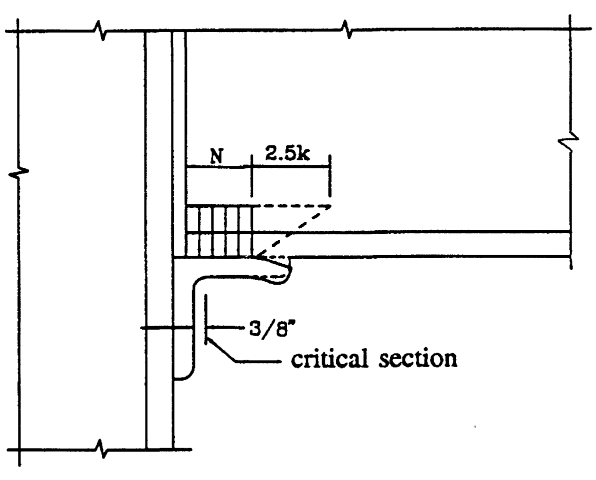



*Fig. 2. AISC assumption for bearing stress.*

The plastic moment of the angles outstanding leg is:

$$M_p = \frac{F_{y\text{-}angle} L t_a^2}{4} \tag{1}$$

The flexural design strength, based on a nominal resistance factor of 0.9, is:

$$\phi_b M_n = 0.9 M_p = 0.225 F_{y\text{-}angle} L t_a^2 \tag{2}$$

The design strength can be expressed in terms of the reaction:

$$\phi_b M_n = \phi_b (eR) \tag{3}$$

Thus, the equation for the limit state of flexure is:

$$\phi_b R = \frac{0.225 F_{y\text{-}angle} L t_a^2}{e} \tag{4}$$

The equation for the limit state of local web yielding is:

$$F_{y\text{-}beam} \geq \frac{\phi_b R}{t_w (N + 2.5k)} \tag{5}$$

where

- $\phi_b R$ = magnitude of the reaction corresponding to the design flexural strength of the angle
- $e$ = distance from the angle fillet to the location of reaction
- $t_a$ = angle leg thickness
- $L$ = angle length
- $t_w$ = beam web thickness
- $F_{y\text{-}angle}$ = yield stress of angle
- $F_{y\text{-}beam}$ = yield stress of beam
- $k$ = distance from outer face of flange to the web toe of fillet
- $N$ = beam bearing length

Three cases are considered, depending on the relative values of $N$, $k$ and the angle leg dimension.

### Case I—Basic Case ($2.5k \leq N \leq 3.25$ inches)

The following equation relating the eccentricity of $R$ and the bearing length is obtained by examining the geometry in Figure 3(a):

$$\frac{N}{2} + \frac{3}{4} = t_a + \frac{3}{8} + e \tag{6}$$

There are three equations (4, 5 and 6) and three unknowns ($\phi_b R$, $N$ and $e$), which can be solved to provide the following expression for $\phi_b R$ and $N$ by assuming the flexural strength and web yielding limit states are achieved simultaneously:

$$\frac{(\phi_b R)^2}{2t_w F_{y\text{-}beam}} + \phi_b R\left(\frac{3}{8} - t_a - \frac{5}{4}k\right) - 0.225F_{y\text{-}angle}Lt_a^2 = 0 \quad (7)$$

$$N = \frac{\phi_b R}{t_w F_{y\text{-}beam}} - 2.5k \quad (8)$$

**Case II—$N$ is less than 2.5$k$**

If the value of $N$ resulting from the solution of Equation 8 is less than 2.5$k$, the reaction is assumed to be located at one-fourth of the distance ($N$ + 2.5$k$) as shown in Figure 3(b). The geometrical equation is rewritten as:

$$\frac{N + 2.5k}{4} + \frac{3}{4} = t_a + \frac{3}{8} + e \quad (9)$$

Resolving Equations 4, 5 and 9 gives the following quadratic in $\phi_b R$:

$$\frac{(\phi_b R)^2}{4t_w F_{y\text{-}beam}} + \phi_b R\left(\frac{3}{8} - t_a\right) - 0.225F_{y\text{-}angle}Lt_a^2 = 0 \quad (10)$$

**Case III—$N$ is greater than 3.25 inches**

If the value of $N$ resulting from the solution of Equation 8 is greater than 3.25 inches, which is the maximum possible bearing length for a 4-in. angle, $N$ is assumed to be 3.25 inches, as shown in Figure 3(c). In this case, the governing limit state is local web yielding and the resulting maximum load capacity is:

$$\phi_b R = (3.25 + 2.5k)t_w F_{y\text{-}beam} \quad (11)$$

In summary, the LRFD procedures for computing the maximum load-carrying capacity of an unstiffened seat angle are as follows:

1. Assume Case I applies, and use Equation 7 and Equation 8 to compute, respectively, the values of $\phi_b R$ and $N$;
2. If $N$ is greater than 2.5$k$ and less than 3.25 inches, the value for $\phi_b R$ is valid and the solution process stops;
3. If $N$ is less than 2.5$k$, Case II applies and Equation 10 should be used to calculate a new value for $\phi_b R$; and
4. If $N$ from Step (1) is greater than 3.25 inches, Case III applies and Equation 11 should be used to calculate a new value for $\phi_b R$.

This algorithm was used to generate Table 9-6 found on page 9-136 of the AISC LRFD Manual of Steel Construction.[3] In generating the tables, the following approximations for $k$ are used for simplicity:

$$k = 2.5t_w \text{ for } t_w < 5/16 \quad (12a)$$

$$k = 2.75t_w \text{ for } t_w \geq 5/16 \quad (12b)$$

**Example 1**

Using the LRFD procedures to compute the design strength of an unstiffened seated connection.

*Given*

$t_w$ = 9/16-in., $t_a$ = ½-in., $L$ = 8-in.

*Assume*

Beam set back = ¾-in., angle outstanding leg length = 4 in.

Use ASTM A36 steel for both the seat angle and the supported beam.

*Solution*

From Equation 12b:

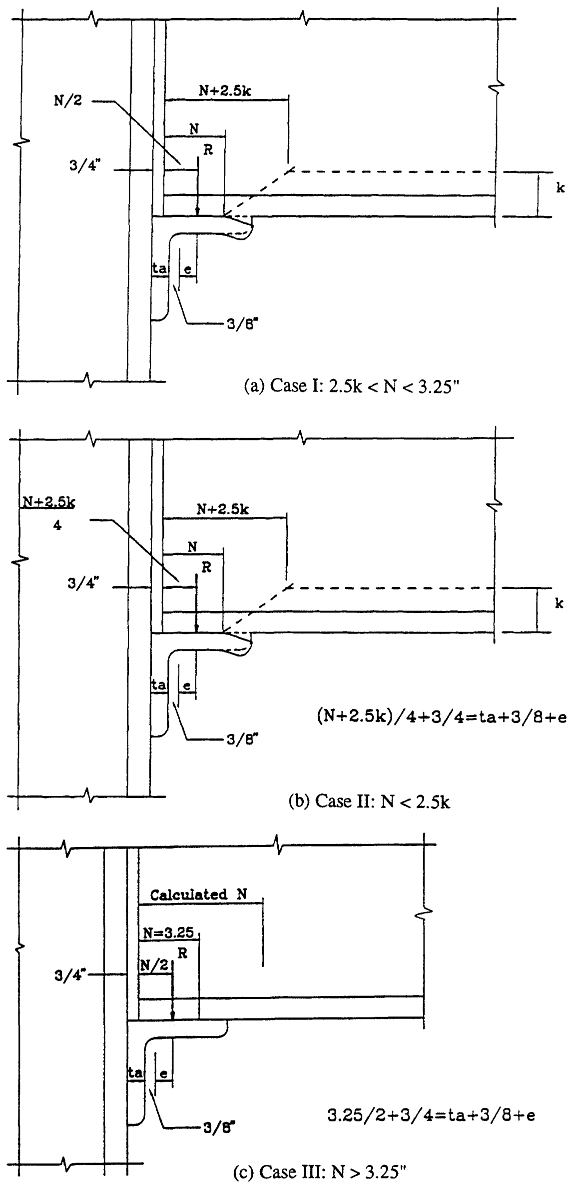



*Fig. 3. LRFD procedures.*

$$k = 2.75t_w = (2.75)\left(\frac{9}{16}\right) = 1.547 \text{ in.}$$

Substitute $t_w = 9/16$-in., $t_a = 1/2$-in., $L = 8$ in., $k = 1.547$ in. and $F_{y\text{-}beam} = F_{y\text{-}angle}$ into Equation 7:

$$\frac{(\phi_b R)^2}{40.5} - 2.06\phi_b R - 16.2 = 0$$

Solving the quadratic equation gives:

$$\phi_b R = 90.6 \text{ kips}$$

Using Equation 8:

$$N = \frac{\phi_b R}{t_w F_{y\text{-}beam}} - 2.5k = \frac{90.6}{(9/16)(36)} - (2.5)(1.547)$$

$$N = 0.608 \text{ in.} < 2.5k = (2.5)(1.547) = 3.867 \text{ in.}$$

Since $N$ is less than $2.5k$, then Case II applies.

Substitute $t_w = 9/16$-in., $t_a = 1/2$-in., $L = 8$ in., and $F_{y\text{-}beam} = F_{y\text{-}angle}$ into Equation 10:

$$\frac{(\phi_b R)^2}{81} - 0.125\phi_b R - 16.2 = 0$$

Solving the quadratic equation yields the following:

$$\phi_b R = 41.6 \text{ kips}$$

## INCONSISTENCIES IN THE LRFD PROCEDURES

The above LRFD procedures are essentially derived from prior ASD procedures[1,2] developed as early as the 1940s. The assumed distribution of forces within the connection appear to be highly idealized and somewhat unrealistic. Some inconsistencies in the formulations of the procedures are described in the forthcoming sections.

### Unrealistic Values of Effective Bearing Length *N*

For a particular seat angle size, the load-carrying capacity of the connection should not exceed some limit value regardless of the supported beam size. However, the resultant design strength from the LRFD procedures could increase without bound, for a seat angle of a given thickness, should the beam web thickness keep increasing. At this point, the LRFD procedures are inconsistent when applied to connections consisting of a flexible-angle and thick-web-beam.

If the beam web were made thicker, the effective bearing length $N$ required based on the local web yielding limit state would decrease. If the beam-web thickness $t_w$ is large enough, the required bearing length $N$ may become zero, or even a negative value. In these cases, the limit state of excessive bending stress on the seat angle should be the sole control condition, and the equation for the limit state of local web yield,

$$F_{y\text{-}beam} \geq \frac{\phi_b R}{t_w(N + 2.5k)} \tag{5}$$

should be of no significance. However, if this equation is still enforced, as in the LRFD procedures, the resultant value of $N$ will become unrealistic, i.e. a very small or even negative value.

Considering the geometric relations:

$$\frac{N}{2} + \frac{3}{4} = t_a + \frac{3}{8} + e \text{ for Case I} \tag{6}$$

or

$$\frac{N + 2.5k}{4} + \frac{3}{4} = t_a + \frac{3}{8} + e \text{ for Case II} \tag{9}$$

The unrealistic bearing length $N$ would lead to a very small value of eccentricity $e$, which would in turn produce a very high design strength $\phi_b R$ when substituted into the equation for the limit state of flexure:

$$\phi_b R = \frac{0.225 F_{y\text{-}angle} L t_a^2}{e} \tag{4}$$

The inconsistency is an unsafe factor and will be illustrated more clearly in the following example.

**Example 2**

Rework Example 1 to illustrate the inconsistency of an unrealistic effective bearing length $N$ in the LRFD procedures.

*Solution*

Substitute $\phi_b R = 41.6$ kips, $F_{y\text{-}beam} = 36$ ksi, $t_w = 9/16$-in. and $k = 1.547$ in. into Equation 5:

$$36 \geq \frac{41.6}{(9/16)\,[N + (2.5)(1.547)]}$$

Solve for $N$:

$$N = \frac{41.6}{(9/16)(36)} - (2.5)(1.547) = -1.813 \text{ in.}$$

$N < 0$, **unrealistic!**

Because $N$ is less than zero there is no true bearing between the angle and steel beam. Regardless this fact, if we substitute $N = -1.813$ in., $k = 1.547$ in., $t_a = 1/2$-in. into Equation 9 and solve for $e$:

$$\frac{-1.813 + (2.5)(1.547)}{4} + \frac{3}{4} = \frac{1}{2} + \frac{3}{8} + e$$

$e = 0.3886$ in., **very small!**

Underestimating the eccentricity would lead to a very large design strength, therefore the LRFD procedures are unsafe in this aspect. (It should be pointed out that the overall LRFD procedures usually produce satisfactory and safe designs since, as will be noted later, the procedures are overly conservative in other aspects.) An examination of the total 140 cases listed in AISC Table 9-6 on unstiffened seated connections reveals that more than one third (38 percent) have unrealistic values of effective bearing length $N$, i.e. half (50 percent) of those cases controlled by Case II ($N < 2.5k$) have negative or zero values of beam bearing length $N$.

**Shear Force Effect on the Yielded Material**

Since the values of eccentricity $e$ (the distance from the toe of angle fillet to the location of reaction) are generally of the same order of magnitude when compared with the angle thickness, the shear force might have a significant reduction effect on the plastic moment capacity of the critical section. Ignoring the shear force effect, as in the LRFD procedures, may lead to an overestimation of the load-carrying capacity of the connection when the seat angle is thick. Drucker's yield criterion[41] for the combined plastic bending moment $M_{ps}$ and shear force $R_s$ at the critical section has the form:

$$\frac{M_{ps}}{M_0} + \left(\frac{R_s}{R_{os}}\right)^4 \le 1 \tag{13}$$

where

$M_0$ and $R_{os}$ = pure plastic bending moment capacity and shear force capacity, without coupling of seat-angle, respectively.

Using the Tresca's yield criterion, we have:

$$M_0 = \frac{F_{y\text{-}angle} L t_a^2}{4} \tag{14a}$$

$$R_{os} = \frac{L t_a F_{y\text{-}angle}}{2} \tag{14b}$$

Substitute Equations 14a and 14b into Equation 13 and rearranging:

$$M_{ps} = \frac{F_{y\text{-}angle} L t_a^2}{4}\left[1 - \frac{16 R_s^4}{(L t_a F_{y\text{-}angle})^4}\right] \tag{1a}$$

This is the plastic moment capacity of the critical section considering the reduction effect due to shear force. Using Equation 1a instead of Equation 1, Equation 4 will have the following modified form:

$$\phi_b R_s = \frac{0.225 F_{y\text{-}angle} L t_a^2}{e}\left[1 - \frac{16(\phi_b R_s)^4}{(L t_a F_{y\text{-}angle})^4}\right] \tag{4a}$$

where

$R_s$= load-carrying capacity of seat angle including the effect of shear force.

Also, Equations 7 and 10 will be modified to the following forms, respectively,

$$\frac{3.6 F_{y\text{-}angle} L t_a^2}{(L t_a F_{y\text{-}angle})^4}(\phi_b R_s)^4 + \frac{(\phi_b R)^2}{2 F_{y\text{-}beam} t_w} + \left(\frac{3}{8} - t_a - 1.25k\right)(\phi_b R) - 0.225 F_{y\text{-}angle} L t_a^2 = 0 \tag{7a}$$

$$\frac{3.6 F_{y\text{-}angle} L t_a^2}{(L t_a F_{y\text{-}angle})^4}(\phi_b R_s)^4 + \frac{(\phi_b R)^2}{4 F_{y\text{-}beam} t_w} + (0.375 - t_a)(\phi_b R) - 0.225 F_{y\text{-}angle} L t_a^2 = 0 \tag{10a}$$

The value of $\phi_b R_s$ can be determined by an iteration procedure.

**Example 3**

Using the LRFD procedures with the modified equations (7a and 10a) to compute the design strength of an unstiffened seated connection made with a thick seat angle, and assessing the maximum possible reduction effect of shear force on the load-carrying capacity of seat angle.

*Given*

$t_w$ = 9/16-in., $t_a$ = 1.0 in., $L$ = 6.0 in.

*Assume*

Beam set back = ¾-in., angle outstanding leg length = 4 in.

Use A36 steel for both seat angle and supported beam.

*Solution:*

Assume $2.5k \le N \le 3.25$ in., Case I—Basic Case

Substitute $t_w$ = 9/16-in., $t_a$ = 1.0 in., $L$ = 6.0 in., $k$ = 1.547 in. and $F_{y\text{-}beam} = F_{y\text{-}angle}$ = 36 ksi into Equation 7a;

$$\frac{(\phi_b R_s)^4}{2{,}799{,}360} + \frac{(\phi_b R)^2}{40.5} - 2.559(\phi_b R) - 48.6 = 0$$

$\phi_b R_s$ = 105.4 kips

Using Equation 8:

$$N = \frac{105.4}{(9/16)(36)} - (2.5)(1.547)$$

$N$ = 1.337 in. < $2.5k$ = 3.867 in.

Therefore, Case II applies.

Substitute $t_w$ = 9/16-in., $t_a$ = 1 in., $L$ = 6 in., $k$ = 1.547 in. and $F_{y\text{-}beam} = F_{y\text{-}angle}$ = 36 ksi into Equation 10a:

$$\frac{(\phi_b R_s)^4}{2{,}799{,}360} + \frac{(\phi_b R_s)^2}{81} - 0.625(\phi R_s) - 48.6 = 0$$

$\phi_b R_s$ = 82.3 kips

From LRFD Table 9-6:

$\phi_b R = 93$ kips

This example indicates that the overestimation of the load-carrying capacity is 13 percent by neglecting the effect of shear force in the LRFD procedure.

## THE BEHAVIOR OF SEATED BEAM CONNECTIONS

When designing a flexible angle, it is important to understand how the angle is being loaded, and how it reacts to this load. Figure 4(a) shows a simply supported beam placed on a seat angle. Due to loading on the beam, the beam deflects and its end rotates (Figure 4(b)). Consequently, the point of contact of the reaction $R$ tends to move outward. This increase in moment arm increases the bending moment on the seat, causing the leg of the angle to deflect downward. As the deflected leg takes the same slope as the loaded beam, the point of contact moves back, Figure 4(c). Two key issues involved in determining the load-carrying capacity of seated connections are the location of the critical sections (type of failure modes) and the point of reaction $R$ (bearing stress distribution).

### The Location of the Critical Sections

If a seated connection is used without attachment to the beam, Figure 5(a), it seems plausible to take the critical section as the net section through the upper bolt line,[37] i.e. b-b section in Figure 5(a), since this section has the smallest net area and largest eccentricity. However, if the second-order effect is considered, it can be shown that the section at the toe of the fillet on the leg which is bolted to the column, i.e. a-a section in Figure 5(a), is more critical than the section through the upper bolt line.

Figure 5(b) shows the resultant reaction $R$ acting on a deformed angle. Since the reaction force $R$ has to be perpendicular to the deformed surface of seat angle, it creates a horizontal component $R\sin\theta$ and a vertical component $R\cos\theta$. From the figure, it can be seen that:

$$M_{a\text{-}a} = e_1 R\cos\theta - kR\sin\theta \tag{15a}$$

$$M_{b\text{-}b} = e_2 R\cos\theta - lR\sin\theta \tag{15b}$$

where

$M_{a\text{-}a}$ and $M_{b\text{-}b}$ = bending moment values acting on the seat angle at Sections a-a and b-b, respectively.

By subtracting Equation 15b from Equation 15a and rearranging:

$$M_{a\text{-}a} - M_{b\text{-}b} = R[(e_1 - e_2)\cos\theta + (l - k)\sin\theta] \tag{16}$$

Considering the fact that $e_1 \approx e_2$ and $l >> k$, Equation 16 can be simplified as:

$$M_{a\text{-}a} - M_{b\text{-}b} \approx R(l - k)\sin\theta >> 0 \tag{16a}$$

That is, the bending moment on Section a-a will be much higher than that on Section b-b as the rotation angle $\theta$ and reaction $R$ increase. Therefore, the critical section should be taken at Section a-a. The corresponding failure mode is shown in Figure 5(c).

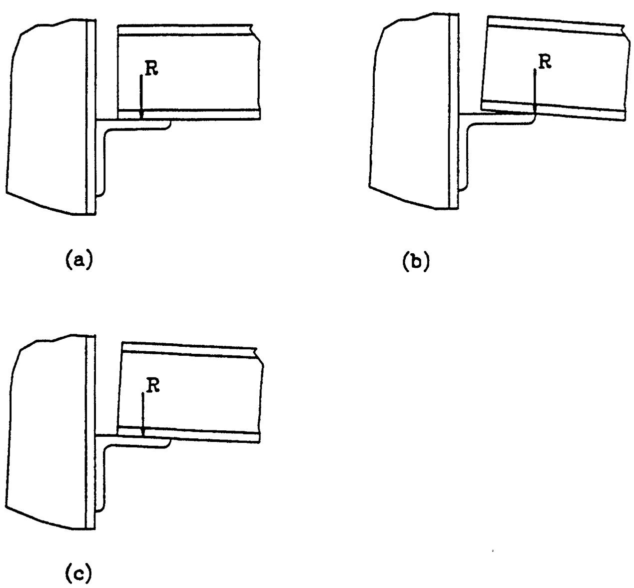



*Fig. 4. Reaction location for beam on seat angle.*

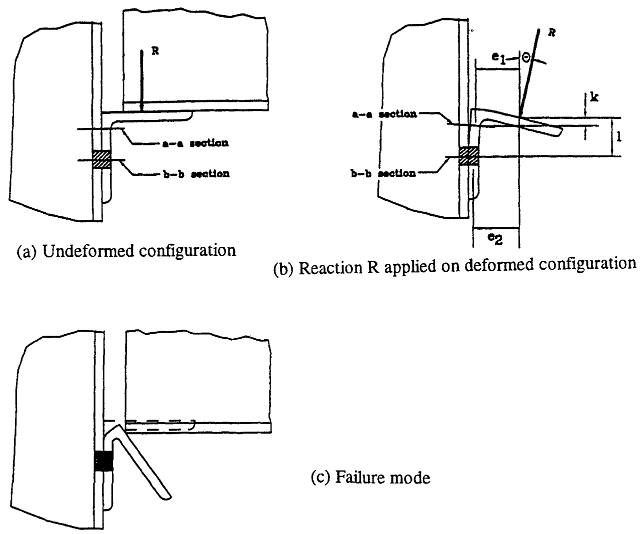



*Fig. 5. Location of critical section for no-bolt case.*

The beam should typically be attached to the seat as shown in Figure 6(a). Theoretically, the rotation of the beam at the end creates a horizontal force that tends to restrain the angle pull away from the column. The critical section for flexure is therefore taken at or near the base of the fillet on the outstanding leg, i.e. Section b-b in Figure 6(a). As a matter of fact, this is one of the basic assumptions used in the AISC LRFD procedures as reviewed in the preceding section. However, this assumption may not be valid when the entire length of the supported beam (not only one of the beam ends) is considered. As illustrated in Figure 7, the larger the beam end rotation ($\alpha$), the shorter the net distance between the beam ends (because the beam would be deflected and curved) and the larger the gap ($\Delta l$) between the beam end and column face. Depending on the span and stiffness of the supported beam, the beam end rotation may not necessarily generate a force that always restrains the angle pull away from the column.

In order to estimate the maximum capacity of the seat angle, the beam was assumed to be very stiff and the effect of beam end rotation was neglected. As a result, the correct failure mode would be the one shown in Figure 6(b), where the beam end is assumed to be a rigid body moving downward.

### The Point of Reaction *R*

Consider a simple beam of length $L_b$ supported by cantilever beams of length $l_a$ at both ends as shown in Figure 8. If we idealize the outstanding-seat-angle-legs as the cantilever beams, this simple model can be utilized to analyze the character of the shifting point of reaction $R$.

The assumptions embedded in the following derivations are: (1) the material is perfectly elastic; (2) the beams are initially straight and prismatic, with plane cross-sections remaining plane after deformation; and (3) the assumed point of contact for $R$ is located where the two deformed surfaces are tangent to each other.

Based on mechanics of material, the slopes of deflected chords of the cantilever and simple beams due to a concentrated load $P$ can be written in the following forms, respectively:

$$y_a' = \begin{cases} \dfrac{Px}{4E_aI_a}(2x_0 - x) & \text{for } 0 \le x \le x_0 \\ \dfrac{Px_0^2}{4E_aI_a} & \text{for } x_0 \le x < l_a \end{cases} \tag{17a}$$

$$y_b' = \begin{cases} \dfrac{P}{16E_bI_b}(L_b - 2x_0)^2 & \text{for } 0 \le x \le x_0 \\ \dfrac{P}{16E_bI_b}[(L_b - 2x_0)^2 - 4(x - x_0)^2] & \text{for } x_0 < x < L_b \end{cases} \tag{17b}$$

where

$y_a'$ and $y_b'$ = slopes of the deflected chords of the cantilever and simple beams, respectively

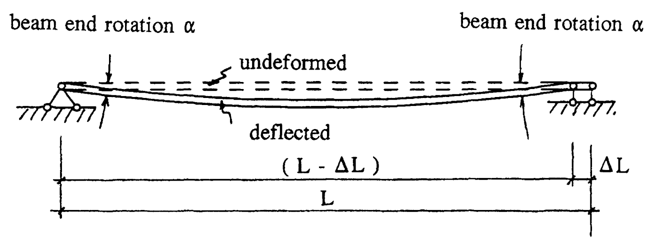
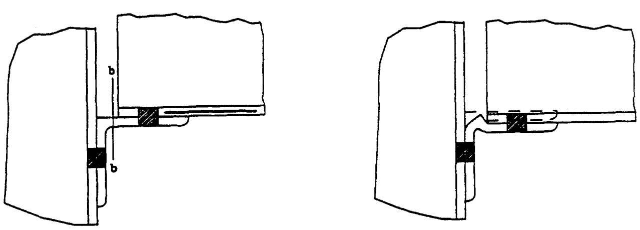



*Fig. 7. Correlation between beam end rotation and net span.*




(a) Undeformed configuration

(b) Failure mode

*Fig. 6. Beam attached to seat angle.*

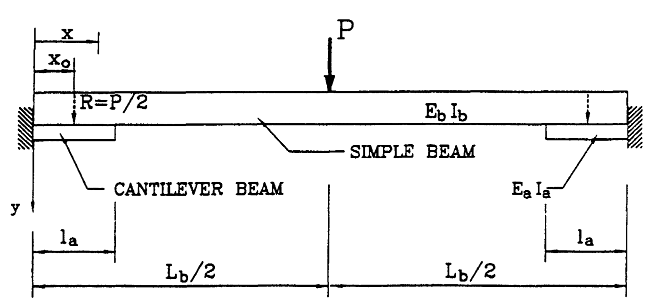



*Fig. 8. Idealized model for evaluating shift of reaction point R.*

$E_a$ and $E_b$ = elastic moduli of the cantilever and simple beam, respectively

$I_a$ and $I_b$ = moment of inertia (with respect to the bending axis) of the cantilever and simple beams, respectively

$x$, $x_0$, $l_a$, and $L_b$= dimensions as shown in Figure 8.

From Equation 17a and 17b, it is noted that the conditions for single contact point are satisfied automatically, i.e.

$$y_a' < y_b' \text{ for } x < x_0 \tag{18a}$$

$$y_a' = y_b' \text{ for } x = x_0 \tag{18b}$$

$$y_a' > y_b' \text{ for } x > x_0 \tag{18c}$$

By substituting Equations 17a and 17b into Equation 18b, we can write the following expression and solve for $x_0$:

$$\frac{Px_0^2}{4E_aI_a} = \frac{P}{16E_bI_b}(L_b - 2x_0)^2$$

$$0 \le x_0 = \frac{L_b/2}{1 + \sqrt{(E_bI_b)/(E_aI_a)}} \le l_a$$

Considering the fact that $I_b >> I_a$ and $E_a = E_b$ for practical seated connections made of steel, the above equation can be simplified as:

$$0 \le x_0 \approx \frac{1}{2}\sqrt{\frac{I_a}{I_b/L_b^2}} \le l_a \text{ for concentrated load} \tag{19a}$$

If the simple beam in Figure 8 is subjected to an evenly distributed load $w$ throughout its span $L_b$ instead of the concentrated load $P$, then the following expression for $x_0$ can be similarly derived:

$$0 \le x_0 \approx \sqrt{\frac{I_a}{6I_b/L_b^2}} \le l_a \text{ for distributed load} \tag{19b}$$

Equations 19a and 19b are plotted in Figure 9 for comparison. It can be seen that the point of reaction $R$ (or bearing stress distribution pattern) depends on load conditions and relative stiffness of seat-angle-leg and supported-beam. When all other conditions are identical, the $x_0$ value for concentrated load is 22.5 percent more than for the distributed load case. When the dimensions of the supported beam (i.e., $I_b$ and $L_b$) are given, then the bearing stress distribution will depend mainly on the thickness of the angle leg. If the leg of the angle is thin, it will deflect easily. Consequently, the point of contact will shift to the end of the beam (i.e. $x_0$ decreases), and the triangular distribution of Figure 10(a) is justified. If the leg of the angle is stiff, it will deflect less and the point of contact will extend farther out along the leg (i.e. $x_0$ increases), and thereby the reaction contact moves towards the outer edges as in Figure 10(b).

### The Effect Of Beam-Bottom-Flange To Seat-Angle Attaching Bolts

Considering the beam-angle-column subassemblage as shown in Figure 11(a), the overall effect of bolting the beam to the seat angle is to add redundancy to the connection subassemblage. The structural subassemblage, which is statically determinate before bolting, Figure 11(b), will become statically indeterminate when the bolts are installed and properly tightened, Figure 11(c). For the no-bolt case shown in Figure 11(b), a single plastic hinge formed at any location in the angle will lead to a sudden collapse of the beam. The corresponding load-carrying capacity can be determined using simple mechanics. For the bolted case shown in Figure 11(c), the formation of one plastic hinge in the seat angle does

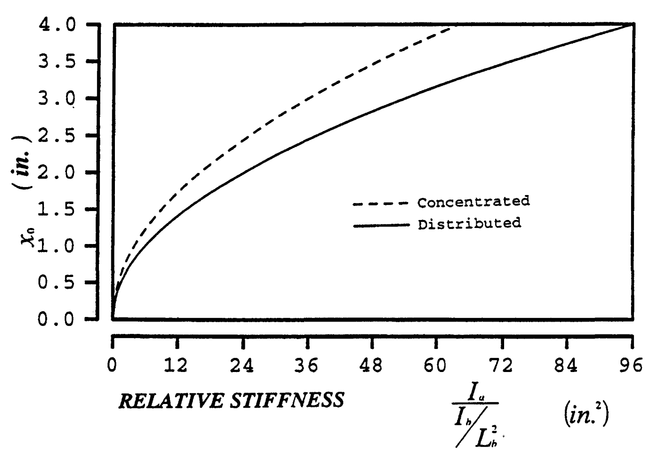
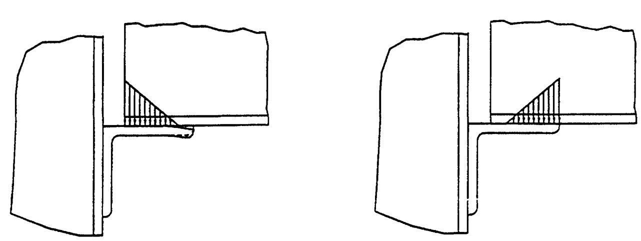



*Fig. 9. Relationship between relative stiffness and location of reaction point R.*



(a) Flexible angle

(b) Thick stiff angle

*Fig. 10. Bearing stress distribution patterns.*

not necessarily result in the failure of the connection. The load-capacity needs to be determined by plastic hinge analysis using assumed failure mechanisms.

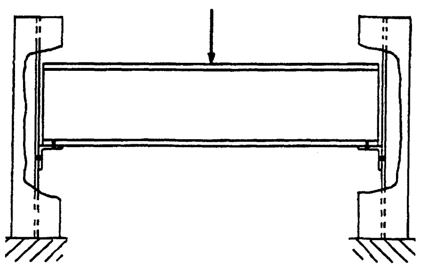
The local effects of the beam-bottom-flange to seat-angle bolts are different depending on the relative stiffness of the seat angle and the supported beam. Figure 12 illustrates the effect of the bolts for a connection consisting of a thick angle and a weak beam. Figure 12(c) indicates that tightening the bolts is equivalent to adding a clockwise moment $M_b$ to the no-bolt case, Figure 12(b). This clockwise moment $M_b$ is unfavorable to the performance of the connection since it increases the bending moment on the seat angle from $M_{ub}$ to $M_{ub} + M_b$. Also, it would increase the bearing reaction concentration at the toe of the outstanding angle leg and might overstress the beam web in bearing. The limit state of beam web crippling or local yield is, therefore, more likely to control in this case. Figure 13 is for connections involving a flexible angle and a strong beam. Tightening the bolts is essentially the same as applying a counter clockwise moment $M_b$ to the no-bolt case, Figure 13(b) and (c). This counter clockwise moment $M_b$ would improve the performance of the connection by reducing the bending moment on the seat angle ($M_{bt} = M_{ub} - M_b$) and increasing the length of contact for proper support of the beam web. Attaching the beam to the seat using properly tightened bolts can significantly increase the load carrying capacity of seated connections consisting of a flexible angle and a strong beam, since the strength of the seat angle itself would control the structural behavior of this kind of connection and the failure mode of seat angle would shift from the one shown in Figure 6(c) to that in Figure 7(b) due to bolting.

## THE LOAD-CARRYING CAPACITY OF SEATED-BEAM CONNECTIONS

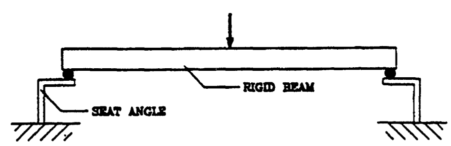
The preceding section shows that the beam-bottom-flange to seat-angle attachment bolts have a significant effect on the behavior of seated connections. When the bolts are not installed and tightened during construction, the load-carrying capacity of the connection should be calculated using the model shown in Figure 11(b). When the bolts have been installed and tightened during construction, then the model shown in Figure 11(c) should be used to compute the design strength of the connection. The following sections will demonstrate that the load-carrying capacity is dramatically different for the no-bolt case and the bolted cases.

### No-Bolt Case

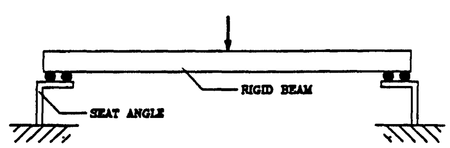
If the beam is not attached to the seat angle, the load-carrying capacity of the connection is directly related to the value of eccentricity, which is in turn a function of the relative stiffness



(a) Beam-angle-column subassembledge

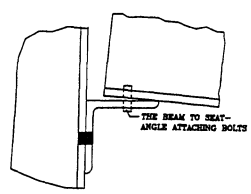



(b) Statically determinate model for no-bolt case

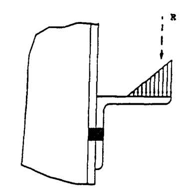



(c) Statically indeterminate model for bolted case

*Fig. 11. Overall effect of attaching beam to seat angles.*




(a) Configuration before tightening blots




(b) Bearing stress pattern before tightening bolts

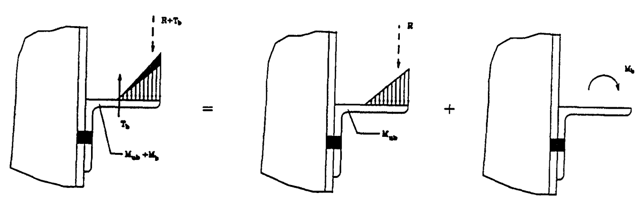



*Fig. 12. Local effect of tightening the bolts (thick angle and flexible beam case).*

parameter at the initial elastic stage as discussed earlier. The smaller the parameter

$$\left(\frac{I_a}{I_b / L_b^2}\right)$$

the smaller the eccentricity (see Equation 19 and Figure 9). For connections consisting of a flexible angle and a strong beam, both the triangular and parabolic bearing stress distributions shown in Figure 14 can be considered as rational models under service load conditions. Their eccentricity values can be estimated using the following equations:

$$e = \frac{N}{3} + b_s - \frac{t_a}{2} \text{ for triangular case} \tag{20a}$$

and

$$e = \frac{N}{4} + b_s - \frac{t_a}{2} \text{ for parabolic case} \tag{20b}$$

However, it should be noted that as the load increases, plastification would be developed and Equation 19 and Figure 9 no longer apply. As the outstanding leg of the angle deflects downward due to plastic and elastic deformations, the eccentricity would keep decreasing as the limit load is approached. At the same time, we note that the value of eccentricity cannot be physically less than that of beam setback minus one half of the angle thickness ($b_s - t_a/2$). Therefore, if we assume that $e = b_s - t_a/2$, then the maximum load-carrying capacity can be computed.

Since the eccentricity $e$ has the same order of magnitude when compared with angle thickness, we must therefore consider the effect of axial load on the yielding of material due to bending. The nondimensionalized expression for the reduced plastic moment capacity of a rectangular section can be written as:[40]

$$\frac{M_{pc}}{M_o} = 1 - \left(\frac{R}{R_{0c}}\right)^2 \tag{21}$$

where

$M_{pc}$ = reduced moment capacity due to the presence of the compressive force $R$ that acts on the critical section

$M_o$ and $R_{oc}$ = pure plastic bending moment capacity and axial load capacity, without coupling of seat angle, respectively.

Substitute $M_{pc} = eR$ into Equation 21 and rearrange:

$$\left(\frac{1}{R_{0c}}\right)^2 R^2 + \left(\frac{e}{M_0}\right) R - 1 = 0 \tag{22}$$

where

$$R_{0c} = F_{y\text{-}angle} L t_a \tag{23}$$

$$M_0 = \frac{F_{y\text{-}angle} L t_a^2}{4} \tag{24}$$

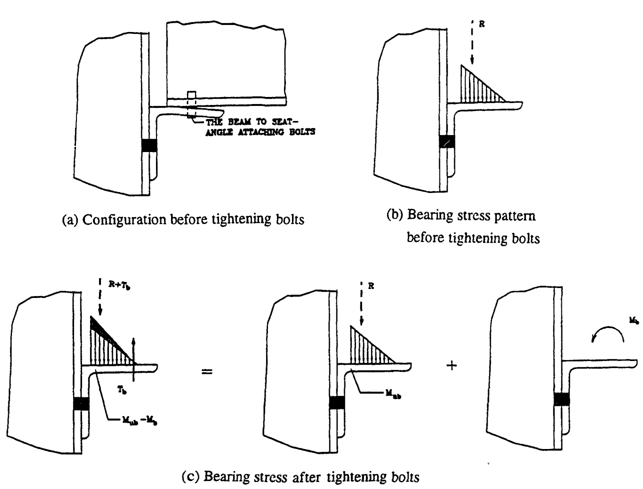
Substituting Equations 23 and 24 into Equation 22, and using $e = b_s - t_a/2$, then:

$$\frac{1}{(F_{y\text{-}angle} L t_a)^2} R^2 + \frac{4b_s - 2t_a}{F_{y\text{-}angle} L t_a^2} R - 1 = 0 \tag{22a}$$

where

$R$ = load-carrying capacity of seat angle including the effect of axial force.

Equation 22a can be solved for positive values of $R$, then the design strength can be obtained by multiplying $R$ by a resis-

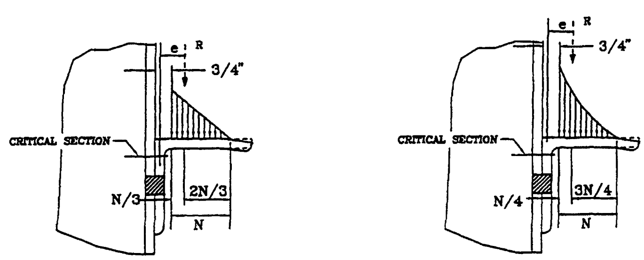


(a) Configuration before tightening bolts

(b) Bearing stress pattern before tightening bolts

(c) Bearing stress after tightening bolts

*Fig. 13. Local effect of tightening bolts (flexible angle and strong beam case).*



(a) Triangular distribution

(b) Parabolic distribution

*Fig. 14. Bearing stress pattern for flexible angle and stiff beam case.*

**Table 1.**
**Summary of Load-Carrying Capacity Calculated Using Various Theoretical Models***

| Angle Length (in.) | Angle Thickness (in.) | Rational Models (kips) | | LRFD** (kips) | |
|---|---|---|---|---|---|
| | | No-bolt Case | Bolted Case | $F_y = 36$ | $F_y = 50$ |
| 6 | 3/8 | 11.8 | 21.7 | 23.5 | 27.7 |
| | 1/2 | 22.9 | 37.0 | 36.8 | 44.7 |
| | 5/8 | 38.9 | 53.1 | 50.6 | 62.4 |
| | 3/4 | 60.4 | 69.4 | 64.6 | 80.4 |
| | 1 | N/A | 102.0 | 93.0 | 117.0 |
| 8 | 3/8 | 15.8 | 28.9 | 27.2 | 32.0 |
| | 1/2 | 30.6 | 49.3 | 41.6 | 50.3 |
| | 5/8 | 51.9 | 70.8 | 56.5 | 69.2 |
| | 3/4 | 80.5 | 92.5 | 71.6 | 88.5 |
| | 1 | N/A | 136.0 | 102.0 | 128.0 |

* Assume that beam web is thick enough so that the strength of seat angle controls; $F_{y\text{-}angle} = 36$ ksi;
** Values correspond to $t_w = 9/16$-in.

tance factor of 0.9. The results of selected examples from these procedures are listed in Table 1 for comparison purposes.

### Bolt-Tightened Case

The collapse mechanism for a seated connection consisting of a flexible angle and a strong beam is shown in Figure 15 when the beam bottom flange is properly attached to the seat angle. Since the distance between the two plastic hinges is the same order of magnitude when compared with the angle thickness, then the strength reduction effects due to shear force and axial load on the plastic moment capacity of the seat angle should be considered using Equations 13 and 21, respectively.

The work equation for the mechanism shown in Figure 15 is given by:

$$M_{ps}\theta + M_{pc}\theta = R\left(b_s - \frac{t_a}{2}\right)\theta \tag{25}$$

From Equations 13 and 21, respectively, the moment values are given as:

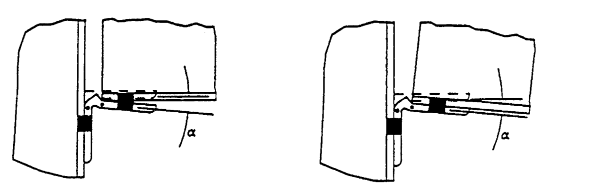
$$M_{ps} = M_0\left[1 - \left(\frac{R}{R_{0s}}\right)^4\right] \tag{26a}$$

$$M_{pc} = M_0\left[1 - \left(\frac{R}{R_{0c}}\right)^2\right] \tag{26b}$$

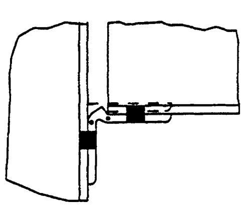
Substituting $M_{ps}$ and $M_{pc}$ from Equations 26a and 26b, and using Equations 14b, 23 and 24 for $R_{os}$, $R_{oc}$, and $M_o$, respectively, Equation 25 becomes:

$$\frac{16}{(F_{y\text{-}angle}Lt_a)^4}R^4 + \frac{1}{(F_{y\text{-}angle}Lt_a)^2}R^2 + \frac{4b_s - 2t_a}{F_{y\text{-}angle}Lt_a^2}R - 2 = 0 \tag{27}$$

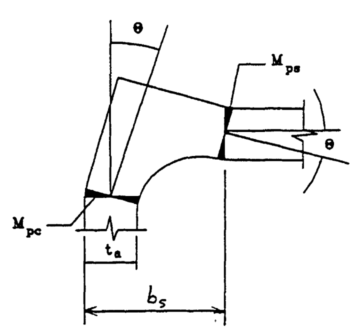
The value of $R$ can be determined by an iteration procedure. Then, the design strength can be obtained by multiplying $R$ by a resistance factor of 0.9. The results of selected examples from this method are listed in Table 1. It should be pointed out that these design strengths are determined solely from the limit state of plastic collapse of the seat angle. They represent



(a) Failure mode for bolt-untightened case (b) Failure mode for flexible beam case

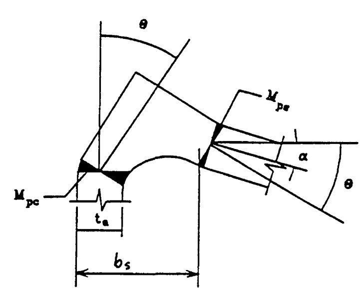


(a) Failure mode



(b) Analytical model

*Fig. 15. Failure mode for bolt-tightened case.*



(c) Analytical model

*Fig. 16. Failure mechanism for bolt-untightened or flexible beam case.*

an upper bound load carrying capacity of seated connection, since for the failure mode shown in Figure 15 the beam has to be rigid and the beam end rotation equals to zero. In reality, the beam can not be truly rigid and some finite beam end rotation will always exist. When the beam end rotation is significant and can not be neglected, then the model for the bolt-untightened case can be modified to consider the effect of beam end rotation on the load-carrying capacity of a seated-beam connection as discussed in the following section.

### Bolt-Untightened Case

A situation between the no-bolt and the bolt-tightened cases could occur if the bolts connecting the beam bottom flange to seat angle are installed but not tightened. For example, if the nut is threaded onto the shank of the bolt, but not fully turned until contact is achieved between the bottom of the nut and the connecting surface, then the failure mode should be like the one shown in Figure 16(a). A similar failure mode shown in Figure 16(b) applies for the case when the bolts are tightened but the beam is flexible and beam end rotation equals a finite value of $\alpha$. Analytically, both failure modes can share the same collapse mechanism for the seat angle as shown in Figure 16(c), except that for the bolt-untightened case the value of $\alpha$ depends on the position of nuts, while for the flexible beam case the beam end rotation $\alpha$ is a function of beam stiffness $EI$, beam span length $L$ and load distribution patterns. Again, the effects of shear force and axial load on the plastic moment capacity of seat angle must be considered.

The work equation for the mechanism shown in Figure 16(c) is given by:

$$M_{ps}(\theta - \alpha) + M_{pc}\theta = R\left(k - \frac{t_a}{2}\right)\theta \tag{28}$$

where

$M_{ps}$ and $M_{pc}$ are given by Equations 26a and 26b respectively.

In writing Equation 28, it is tactically assumed that the reaction $R$ is close to the beam end so that the effect of rotation $\alpha$ on the work done by reaction $R$ can be neglected.

Similar to the derivation from Equation 25 to Equation 27, Equation 28 can be simplified to the following fourth-order equation:

$$\frac{1 - \alpha/\theta}{(F_{y\text{-}angle}Lt_a)^4}R^4 + \frac{1}{(F_{y\text{-}angle}Lt_a)^2}R^2 + \frac{4k - 2t_a}{F_{y\text{-}angle}Lt_a^2}R - \left(2 - \frac{\alpha}{\theta}\right) = 0 \tag{29}$$

For the bolt-untightened case shown in Figure 16(a), the rotation angle $\alpha$ is a constant as far as the position of nuts are known. Therefore, the load-carrying capacity $R$ will be a function of only $\theta$ for given seat angle sizes. Given a specific value of $\theta$, the value of $R$ can be determined from Equation 29 by a simple iteration procedure. Thus, the $R$-$\theta$ relationship curve can be plotted using this method.

For the flexible beam case shown in Figure 16(b), the beam end rotation $\alpha$ is a linear function $R$, it can be expressed in the following general form:

$$\alpha = \Psi R \tag{30}$$

where

$\Psi = \dfrac{l^2}{12EI}$ for uniformly distributed load on a simple beam

$\Psi = \dfrac{l^2}{8EI}$ for a concentrated load at the mid-span of a simple beam.

Substituting $\alpha$ from Equation 30 into Equation 29 and rearranging we have:

$$\frac{\Psi/\theta}{(F_{y\text{-}angle}Lt_a)^4}R^5 - \frac{1}{(F_{y\text{-}angle}Lt_a)^4}R^4 - \frac{1}{(F_{y\text{-}angle}Lt_a)^2}R^2 - \left(\frac{4k - 2t_a}{F_{y\text{-}angle}Lt_a^2} + \frac{\Psi}{\theta}\right)R + 2 = 0 \tag{31}$$

Again, the load-carrying capacity $R$ is a function of $\theta$. Given a specific value of $\theta$, the corresponding $R$ can be determined from Equation 31 using an iteration procedure. Thus, the $R$-$\theta$ relation curve can be plotted.

**Table 2.**
**The Comparison of LRFD Design Strength Relative to the Results of Plastic Hinge Method***

| Angle length (in.) | 6 | | | | |
|---|---|---|---|---|---|
| Angle thickness (in.) | 3/8 | 1/2 | 5/8 | 3/4 | 1 |
| $F_y$ = 36 ksi | 8.3% | −0.5% | −4.7% | −6.9% | −8.6% |
| $F_y$ = 50 ksi | 27.6% | 20.8% | 17.5% | 15.9% | 14.9% |
| Angle length (in.) | 8 | | | | |
| Angle thickness (in.) | 3/8 | 1/2 | 5/8 | 3/4 | 1 |
| $F_y$ = 36 ksi | −5.9% | −15.6% | −20.2% | −22.6% | −24.8% |
| $F_y$ = 50 ksi | 10.7% | 2.0% | −2.3% | −4.3% | −5.7% |

* "+" = overestimate; "−" = underestimate.

## THE VALIDITY OF LRFD PROCEDURES AND PRACTICAL IMPLICATIONS

The load-carrying capacity of selected examples calculated using various analytical models described in the preceding sections are summarized and compared in Tables 1 and 2. All the values in the Table 1 include the appropriate resistance factors.

It can be observed from Table 2 that, based upon the strength given by the plastic hinge method, the LRFD method overestimates the load-carrying capacity by 15 percent to 28 percent when $L = 6$ in. and $F_{y\text{-}beam} = 50$ ksi. Moreover, the LRFD method underestimates the strength by 6 percent to 25 percent when $L = 8$ in. and $F_{y\text{-}beam} = 36$ ksi. Note, however, that the strength comparisons are more favorable (within $\pm 10$ percent) for the remaining cases listed in Table 2. Considering the inconsistencies and shortcomings of the LRFD procedures discussed previously, it is perhaps surprising that the design values generated from these procedures are not significantly off when compared with the results obtained using more rational and accurate models.

In the LRFD procedures, the unrealistic (very small or negative) value of effective bearing length $N$ or its resultant small eccentricity $\acute{e}$, together with neglect of the shear force effect on the plastic moment capacity will always result in overestimating the load-carrying capacity and lead to an unsafe design. However, since the LRFD procedures completely ignore both of the local and overall beneficial effects of beam-bottom-flange to seat-angle attaching bolts and employ an unrealistic and overly conservative failure mechanism, they tend to underestimate the load-carrying capacity and lead to an extremely conservative design. The combined effect of these opposite factors make the LRFD procedures reasonably safe, although accidently in many cases. However, the safety is not provided by the factors suggested in the design calculations.

The comparisons made in Tables 1 and 2 are within the range of variables listed in the LRFD Manual tables (i.e. ⅜-in. $\leq t_a \leq 1$ in. and $t_w \leq$ 9⁄16-in.). When the dimensions of the connection components exceed these ranges (although this may not happen very often in practice), the LRFD procedures will overestimate the load-carrying capacity more frequently. Therefore, it is reasonable to suggest that the design strength obtained using the plastic hinge analysis should be incorporated into the current LRFD procedures as an upper bound limiting strength. In other words, whenever the predictions of LRFD procedures for load-carrying capacity exceed those obtained using a simple plastic hinge analysis, the later should be used as design strength.

### Practical Implications

It is noted from Table 1 that the seated connections of unbolted cases generally have load-carrying capacities much lower than the design strengths of bolted connections. Moreover, the failure of an unbolted seated connection, Figure 5(c), would develop suddenly and without warning, which is generally not acceptable in engineering practice. Therefore, it is very important to install and tighten the beam-bottom-flange to seat-angle attaching bolts as soon as possible after the beam is placed on the seat-angle during construction. A seated connection should not carry any additional load from other structural members until the bolts are installed and securely tightened.

From the comparison made in Table 2, it is obvious that when $L = 6$ in. and $F_{y\text{-}beam} = 50$ ksi the LRFD results are not acceptable for safety consideration. This is because seat angles with a short length have less effective area to resist the load. In these cases the plastic moment capacity of seat angles become more sensitive to the reduction effect of axial load and shear force which are not accounted for in the LRFD formulations. Since the effects of axial load and shear force are, respectively, second and fourth order effects [see Equations 22a and 4a], their reduction effects would decrease rapidly as the angle length increases. This is why for $L = 8$ in. angle length the LRFD results are much more reasonable. According to the analysis made above, it is advisable to use seat angles with lengths greater than 8 in. whenever possible in practice. If a seat angle length less than 6 in. must be used, then care should be exercised in sizing the seat angle thickness in order to ensure a safe design.

## CONCLUSION

It is concluded that the current LRFD procedures for unstiffened seated connections are somewhat irrational, although they often produce acceptable results. The LRFD procedures employ a highly idealized and somewhat unrealistic distribution of forces within the connection, and they assume incorrect failure modes which ignore some critical behavioral factors. Since some of factors ignored in the LRFD procedures are unsafe while others are overly conservative, the combined effect of opposite factors may explain why seated connections have resulted in satisfactory designs for many years without significant problems. However, the current LRFD procedures are valid only within a particular limited range of connection parameters.

Improvements in the design procedures of seated connections may be desirable. It seems more reasonable to check the strength of the seat angle and beam web local yielding and crippling separately, rather than to account for them together as in the current LRFD procedures. Moreover, the level of safety can be more accurately controlled if the model that is used to predict the strength is rational and behaviorally correct.

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