Calculation of the Plastic Section Modulus Using the Computer

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ABSTRACT

A simple spreadsheet is presented which calculates the plastic section modulus of structural members. The method consists in dividing the cross section into rectangles and arranging all calculations conveniently into a spreadsheet program. The basic algorithm and the required spreadsheet formulas are given as well as a numerical example.

INTRODUCTION

With the increasing use of the limit states design of steel structures, engineers often have to calculate the plastic bending resistance, M_r, of structural members, which is a function of the plastic modulus, Z, of the cross section, that is,

$$M_r = \phi Z F_y \tag{1}$$

where

ϕ = performance factor
F_y = yield strength of steel.

Although the calculation of the plastic section modulus can be done easily by hand, it can also be done quickly and reliably using the computer. The following technical note presents a simple spreadsheet for the calculation of the plastic modulus. It is restricted to cross sections that can be approximated by a series of rectangles, which should cover most situations that structural engineers encounter in the design office.

SPREADSHEET ALGORITHM

The proposed algorithm is described below. The cross section to be analyzed must first be divided into N rectangles (Figure 1a). Each rectangle must comprise the entire width of the cross section at any particular height. Hence, the arrangement shown in Figure 1a is valid, while the one shown in Figure 1b is not valid.

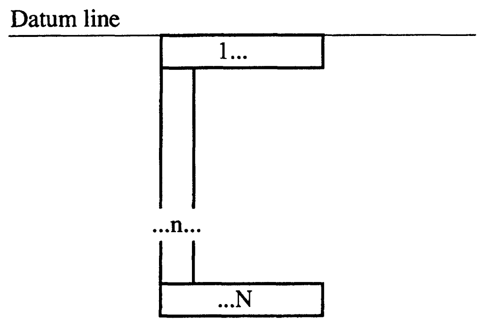
The width and the height of each rectangle will be entered into the spreadsheet, going consecutively from top to bottom of the cross section. These values are the only required input data. All the calculations presented below are arranged so that the equations can be expressed as spreadsheet formulas which will be evaluated automatically by the spreadsheet program.

With a datum line placed at the top of the cross section, the vertical distance from the datum line to the centroid, y_n, of the nth rectangle is equal to (Figure 2)

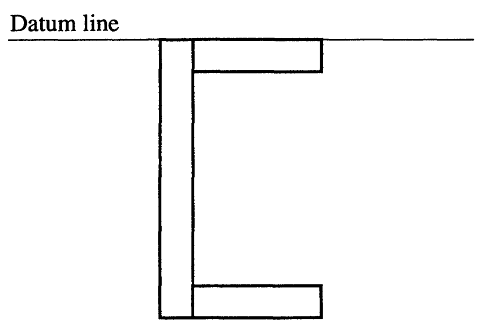
$$y_n = y_{n-1} + \frac{h_{n-1}}{2} + \frac{h_n}{2} \tag{2}$$

where

h_n = height of the nth rectangle



(a) valid arrangement



(b) invalid arrangement

Figure 1.

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$y_0 = h_o = 0.$

The cross-sectional area, A_n, of the nth rectangle is equal to

$$A_n = b_n h_n \tag{3}$$

where

b_n = width of the nth rectangle.

The total area, A_{total}, of the cross section is equal to

$$A_{total} = \sum_{n=1}^{N} A_n \tag{4}$$

The vertical distance, $\tilde{Y}$, from the datum line to the neutral axis, which divides the cross section into two portions of equal areas, is determined by noting that if the neutral axis passes through the nth rectangle, we must have

$$\frac{A_{total}}{2} = \sum_{i=1}^{n-1} A_i + b_n \tilde{h}_n \tag{5}$$

or

$$\tilde{h}_n = \frac{\frac{A_{total}}{2} - \sum_{i=1}^{n-1} A_i}{b_n} \tag{6}$$

where $\tilde{h}_n$ positions the neutral axis as shown is Figure 2. The rectangle through which the neutral axis passes is determined from the fact that it is the only one for which

$$\tilde{h}_n > 0 \text{ and } \tilde{h}_n < h_n \tag{7}$$

For all other rectangles, Equation 7 is not verified. Hence, the vertical distance from the datum line to the neutral axis, $\tilde{Y}$, is equal to

$$\tilde{Y} = \tilde{Y}_m = \sum_{i=1}^{m-1} h_1 + \tilde{h}_m \tag{8}$$

where the subscript m identifies the single rectangle for which Equation 7 is verified. For the other rectangles through which the neutral axis does not pass, the values

$$\tilde{Y}_n = \sum_{i=1}^{n-1} h_i + \tilde{h}_n \tag{9}$$

are meaningless and therefore discarded.

The contribution to the plastic modulus, Z_n, of each rectangle through which the neutral axis does *not* pass is equal to

$$Z_n = A_n \text{abs}(\tilde{d}_n) \tag{10}$$

where the distance from the neutral axis to the centroid of the nth rectangle, $\tilde{d}_n$, is equal to Figure 3a.

$$\tilde{d}_n = \tilde{Y} - y_n \tag{11}$$

The contribution to the section modulus, Z_m, of the rectangle through which the neutral axis *does* pass is equal to Figure 3b.

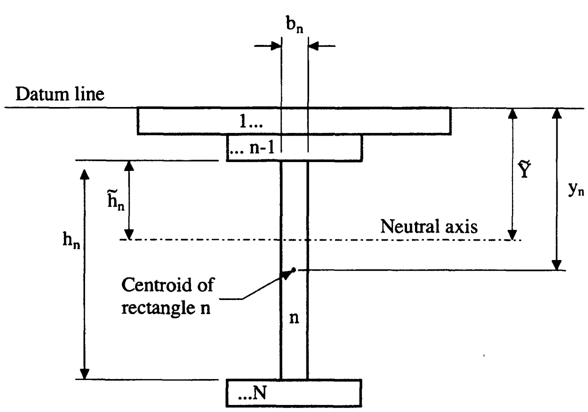


Figure 2.

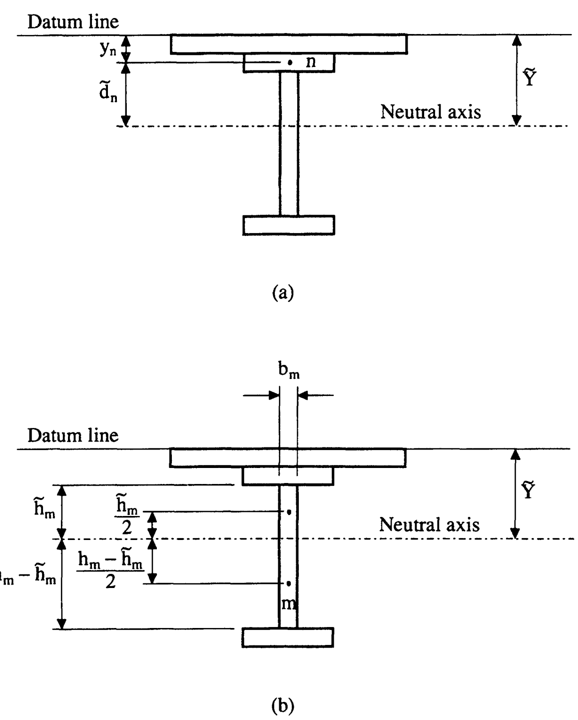


Figure 3.

$$Z_m = \frac{b_m \tilde{h}_m^2}{2} = \frac{b_m (h_m - \tilde{h}_m)^2}{2} \qquad (12)$$

Finally, the plastic modulus of the cross section, Z, is equal to

$$Z = Z_m + \sum_{n = 1 \text{ to } N \text{ except } m} Z_n \qquad (13)$$

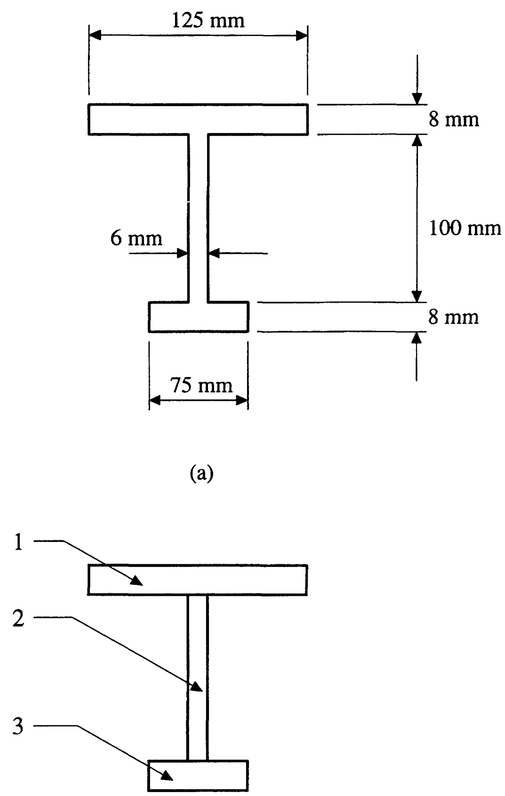
SPREADSHEET FORMULAS

The spreadsheet formulas required for the calculation of the plastic section modulus correspond to Equations 2 to 13 given above. There must be as many sets of formulas, arranged in rows in the spreadsheet, as there are rectangles into which the cross section is divided. Assuming that the cross section to be analyzed is composed of 3 rectangles (see the example below), there would be 3 sets of formulas, arranged in 3 rows, say rows 9 to 11 in the spreadsheet. The formulas for the first rectangle, in row 9, would be as shown in Table 1.

Note that the syntax used with the formulas given in Table 1 is that of Microsoft Excel. The formulas can be easily modified to meet the syntax rules of other spreadsheet programs such as Lotus 1-2-3, Quattro Pro, etc.

Table 1.
Spreadsheet Formulas

Expression	Equation No.	Cell	Formula
b_n		C9	input data
h_n		D9	input data
y_n	(2)	E9	D8/2+E8+D9/2
A_n	(3)	F9	C9*D9
$\sum_{i=1}^{n-1} h_i$		G9	D8+G8
$\sum_{i=1}^{n-1} A_i$		H9	F8+H8
$\tilde{h}_n$	(6)	I9	(F4-H9)/C9
$\tilde{Y}_n$	(7) and (9)	J9	IF(AND(I9>0;I9<=D9);G9+I9;0)
$\tilde{d}_n$	(11)	K9	IF(J9=0;J5-E9;'Neutral Axis')
Z_n or Z_m	(10) and (12)	L9	IF(J9=0;ABS(K9)*F9;C9*I9^2/2 +C9*(D9-I9)^2/2)
A_{total}	(4)	F5	SUM(F9:F11)
$A_{total}/2$		F4	F5/2
$\tilde{Y} \equiv \tilde{Y}_m$	(7) and (8)	J5	SUM(J9:J11)
Z	(13)	L5	SUM(L9:L11)



(b)

Figure 4.

EXAMPLE

Calculate the plastic modulus of the cross section shown in Figure 4a.

The cross section is divided into 3 rectangles as shown in Figure 4b. With the width and height of each rectangle entered as input data in cells C9, D9, C10, D10, C11 and D11, the value of the plastic modulus is calculated by the spreadsheet as $Z = 94{,}733$ mm^3 and displayed in cell L5 (Figure 5).

CONCLUSION

A simple spreadsheet is presented which can be used to calculate the plastic section modulus of structural members. It should be useful to design engineers, especially when there are many values to be calculated.

APPENDIX I. NOTATION

A_n = area of the nth rectangle
A_{total} = area of the entire cross section
b_n = width of the nth rectangle
F_y = yield strength of steel

h_n = height of the nth rectangle
$\tilde{h}_n$ = distance from the top of the nth rectangle to the neutral axis of the cross section
M_r = plastic bending moment resistance
$\tilde{Y}$ = distance from the datum line to the neutral axis of the cross section
$\tilde{Y}_m$ = correct value of $\tilde{Y}$ obtained with rectangle m ($\tilde{Y}_m \equiv \tilde{Y}$)
$\tilde{Y}_n$ = value of $\tilde{Y}$ obtained with the nth rectangle ($\tilde{Y}_n \neq \tilde{Y}$ for all rectangles except rectangle m)
y_n = distance from the datum line to the centroid of the nth rectangle
Z = plastic section modulus
Z_m = contribution of rectangle m to the plastic section modulus
Z_n = contribution of the nth rectangle to the plastic section modulus
ϕ = performance factor

Column —>		C	D	E	F	G	H	I	J	K	L
↓ Row											
3											
4				$A_{total}/2$ =	1100						
5				A_{total} =	2200			$\tilde{Y}$ =	24.667	Z =	94733.3
6											
7	n	b_n	h_n	y_n	A_n	$\sum_{i=1}^{n-1} h_i$	$\sum_{i=1}^{n-1} A_i$	$\tilde{h}_n$	$\tilde{Y}_n$	$\tilde{d}_n$	Z_n
8											
9	1	125	8	4	1000	0	0	8.8	0	20.667	20666.7
10	2	6	100	58	600	8	1000	16.667	24.667	N. Axis	21666.7
11	3	75	8	112	600	108	1600	-6.667	0	-87.33	52400

Note: (shaded cells) **Indicates input data.**

Figure 5.