Rational Basis for Increased Fillet Weld Strength

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BACKGROUND

The 1993 AISC LRFD Specification permits, for the first time, optional design use of nominal shear strength in excess of $0.6F_{exx}$ for fillet welds that are loaded in-plane, where F_{exx} is the electrode classification number, or its minimum specified strength. The traditional $0.6F_{exx}$ value, and its $0.3F_{exx}$ allowable counterpart in Allowable Stress Design (ASD) have their origins in the experimental data of Preece (Ref. 1) and in the theoretical von Mises shear yield stress of 0.577 times the tension yield stress. Upon rounding to a 0.6 coefficient for LRFD, or dividing this value by the factor of safety of two for ASD, this constant fillet weld design limit has been well established in design practice for more than two decades. The new LRFD Appendix J2.4 now contains the alternative provisions that permit designs with variable increased fillet weld strength, for which $0.6F_{exx}$ is actually the lower bound.

The fillet weld strength increase occurs because of sensitivity to the direction of loading. At a loading perpendicular to the longitudinal axis of the weld, the weld has been shown to be on the order of 50 percent stronger than the lower bound for parallel loading. A full range of strength variation exists for intermediate force angles of 0-90 degrees. (see Fig. 1) In the past, this effect had been conservatively and conveniently ignored for design but has now been recognized. While the detailed experimental justification reported in the literature is convincing by itself, the objective of this paper is to also demonstrate this increased strength with a simple rational model derived from first principles. The strength/ductility trade-off, i.e. lower weld ductility at higher strength, the shape of the weld load-deformation curve, and deformation compatibility for weld groups are not addressed here even though they are also important considerations covered in Appendix J2.4.

In addition to Ref. 1, the new weld design strength and ductility criteria are amply justified by empirical evidence on multiple strength curves originating from Butler, Pal, & Kulak (Ref. 2) through the most recent research by Lesik and Kennedy (Ref. 3). Previous editions of the AISC Manual of Steel Construction partially implemented this information in the development of the "C" ultimate strength values for the eccentrically loaded weld tables based upon the instantaneous center of rotation solution, in order to realize load redistribution benefits. However, because the previous AISC Specifications unequivocally restricted the weld strength to $0.6F_{exx}$ (or $0.3F_{exx}$ in ASD), these "C" Tables were always adjusted to comply with this limit. Application of Appendix J2.4 without imposing this shear yield limit may result in substantially higher capacity than permitted by the prior 1986 LRFD, from a maximum of 50 percent for some cases to a more common range of 10-30 percent increases. The weld design tables in the 1993 LRFD Manual of Steel Construction, Vol. II, reflect these higher values.

EQUILIBRIUM DERIVATION

An analytical first order derivation of the general fillet weld strength as a function of force angle, θ, can be based on the three-dimensional equilibrium of the effective throat, assum-

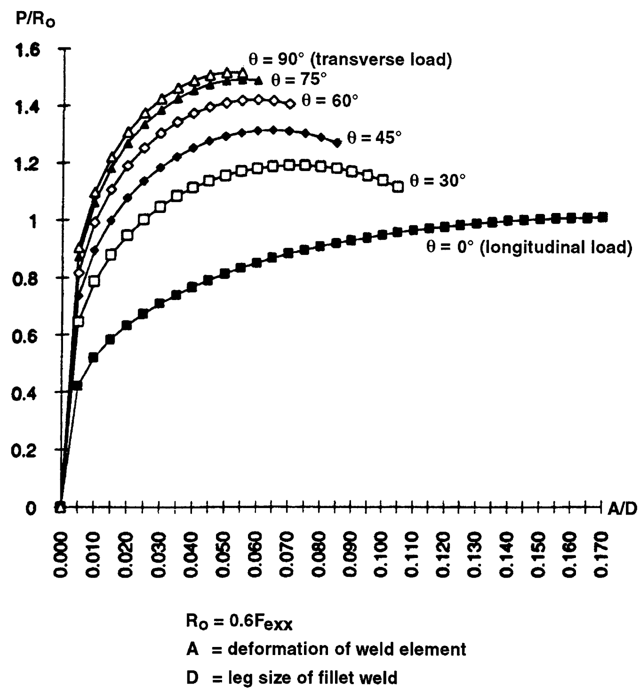

Fig. 1. Fillet weld strength curves (Ref. 3).

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ing equal weld leg sizes. Fig. 2 illustrates a free-body diagram of a weld loaded by P in the X-Z plane at an angle θ to its axis. The three force components of the weld resistance are identified as R_1 in the plane of the throat area, R_2 perpendicular to the plane of the throat area, and R_3 along the weld axis (Z direction). Both R_1 and R_2 are also parallel to the X-Y plane and perpendicular to R_3. It is assumed that the baseline weld strength limit $R_o = 0.6F_{exx}$ applies to the resultant shear in the plane of the effective throat at 45° from the leg:

$$R_o = \sqrt{R_1^2 + R_3^2} \tag{1}$$

Balance of forces in the Y-direction dictates that R_1 equals R_2:

$$R_1 \sin 45° = R_2 \sin 45$$

$$R_1 = R_2 = R \tag{2}$$

Equilibrium in the longitudinal weld Z-direction requires that

$$P_z = P\cos\theta = R_3$$

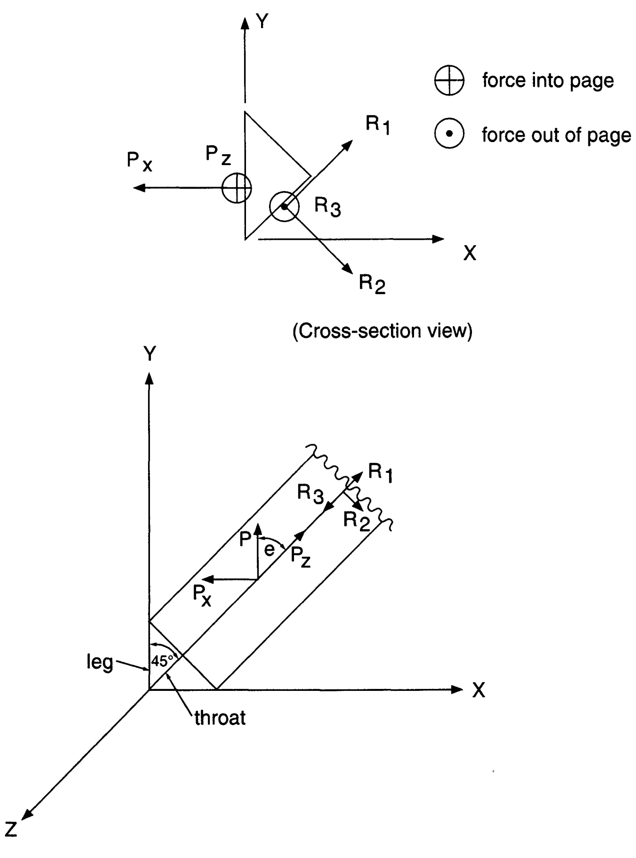

Fig. 2. Assumed fillet weld free-body diagram.

or, from (1) and (2),

$$P_z = P\cos\theta = \sqrt{R_o^2 - R^2} \tag{3}$$

Equilibrium in the X-direction requires that

$$P_x = P\sin\theta = R_1 \cos 45 + R_2 \cos 45$$

or

$$P_x = P\sin\theta = 1.414R \tag{4}$$

The geometry of the X-Z in-plane load P provides the last condition:

$$\tan\theta = \frac{P_x}{P_z} = \frac{1.414R}{\sqrt{R_o^2 - R^2}} \tag{5}$$

Eq. (5) can be algebraically re-arranged to obtain

$$R = R_o \sqrt{\frac{\sin^2\theta}{(cos^2\theta + 1)}} \tag{6}$$

Finally, the magnitude of the total load P as a function of R_o and θ can be determined as

$$P = \sqrt{P_x^2 + P_z^2}$$

$$= \sqrt{2R^2 + (R_o^2 - R^2)}$$

$$= \sqrt{R_o^2\left(1 + \frac{\sin^2\theta}{(cos^2\theta + 1)}\right)}$$

$$P = R_o \sqrt{\frac{2}{(cos^2\theta + 1)}} \tag{7}$$

or

$$P = R_o \sqrt{\frac{2}{(2 - \sin^2\theta)}}$$

Non-dimensionalizing Eq. (7) to express the ratio (P/R_o) gives the final result showing the dependence of the strength increase relative to R_o on the load angle θ. Table 1 illustrates this effect and compares it to the empirically based LRFD App. J2.4 strength factor (Ref. 3)

$$\left(1 + \frac{\sin^{1.5}\theta}{2}\right) \tag{8}$$

The theoretical Eq. (7) is within approximately 10 percent of the App. J2.4 factor, thereby providing additional justification for this behavior. In summary, for longitudinally loaded welds (θ = 0°), the traditional lower bound limit of $R_o = 0.6F_{exx}$ applies while for any other load angle, a higher nominal

Table 1.

θ degrees	(P/R_o) by Eq. (7)	$\left(1+\dfrac{\sin^{1.5}\theta}{2}\right)$
0	1.0	1.0
30	1.07	1.18
45	1.15	1.30
60	1.26	1.40
75	1.37	1.47
90	1.41	1.50

Table 2.

θ degrees	$\left(\dfrac{P}{R_o}\right)$ by Eq. (13)
0	1.0
30	1.05
45	1.10
60	1.16
75	1.21
90	1.23

strength up to a maximum of $0.9F_{exx}$ for transverse loading ($\theta = 90°$) can be realized.

OTHER MODELS

Two authors have previously reached identical conclusions to Eq. (7) from slightly different perspectives. In Ref. (4), Marsh assumed an elliptical interaction between the transverse and parallel load with their associated weld resistance of the form:

$$\left(\frac{P_z}{R_o}\right)^2 + \left(\frac{P_x}{R_o\sqrt{2}}\right)^2 = 1.0 \tag{9}$$

Kamtekar (Ref. 5) employed a three-dimensional principal stress analysis subject to von Mises yield criterion to determine the same result. These offer additional independent verification for Eq. (7).

In the draft Eurocode No. 3, Annex M, Alternative Method for Fillet Welds (Ref. 6), a similar increased weld strength formulation is given in terms of its resistance components and, the von Mises yield criterion. Using terminology consistent with this presentation, the proposed Eurocode limit may be expressed as:

$$\sqrt{R_2^2 + 3(R_1^2 + R_3^2)} \le F_{exx} \tag{10}$$

Eq. 10 results directly from the general von Mises strength of materials formula for the *m-n* plane

$$\sqrt{\sigma_m^2 + \sigma_n^3 - \sigma_m\sigma_n + 3\tau_{mn}} \le \sigma_y \tag{11}$$

where

σ_m = normal stress on *m* surface
σ_n = normal stress on n surface (orthogonal to *m* surface)
τ_{mn} = shear stress in *m-n* plane
σ_y = tensile yield stress

with one of the fillet normal stresses equals zero along the weld axis, R_2 is the remaining normal stress, and τ_{mn} is the resultant shear in the plane of the effective throat.

Equations 2-5 already provide the needed conversions to Eq. (10) in terms of the total force P and its angle θ:

$$\sqrt{\frac{P^2\sin^2\theta}{2} + 3\left(\frac{P^2\sin^2\theta}{2} + P^2\cos^2\theta\right)} \le F_{exx}$$

or

$$P \le \frac{F_{exx}}{\sqrt{2+\cos^2\theta}} \tag{12}$$

Solution of Eq. (12) for $\theta = 0$ results in the expected theoretical $0.577F_{exx}$ minimum for R_o. Therefore, non-dimensionalizing Eq. (12) by this R_o produces

$$\left(\frac{P}{R_o}\right) = \frac{1}{.577\sqrt{2+\cos^2\theta}} \tag{13}$$

which now is in a form that may be compared to Eq. (7). The strength increases thereby allowed by the proposed Eurocode are summarized in Table 2.

While these strength increases are somewhat smaller than those now permitted by the 1993 AISC-LRFD, they offer another persuasive analytical argument for the concept.

ASSUMPTIONS

One major simplifying assumption implicit in the previous derivations is that the weld failure plane remains constant at the location of the fillet throat, i.e. symmetrical at 45° to the legs. In reality, Refs. (7) and (8) indicate from recent test data that the average inclination of this weld fracture surface changes from about 49–58° for longitudinal loads ($\theta = 0°$) to a flatter 14–18° for transverse loads ($\theta = 90°$)

In addition, Ref. (7) addresses the presence of weld boundary condition effects in the form of lateral restraint at the root; this consideration adds another force component variable and