Load and Resistance Factor Design and Analysis of Stepped Crane Columns in Industrial Buildings

ROBERT A. MacCRIMMON and D. J. LAURIE KENNEDY

1.0 INTRODUCTION

Load and Resistance Factor Design (LRFD), generally known as Limit States Design (LSD) outside of the United States of America, is replacing working stress design (WSD) worldwide for good reason. Not only does it provide much more uniform reliability with load and resistance factors based on the statistical variation of the relevant variables but also, by implicitly eliminating unnecessary conservatism at the same time, it can effect economies as well.

A major advantage of LRFD over working stress design is inherent in the fact that the ultimate limit states, those associated with failure of all or part of the structure, are checked against factored load combinations, which are chosen to have the required low probability of being exceeded. This means that both the second-order geometric effects, resulting from the deformation of the structure and the non-linear effects due to material behaviour can be taken into account in a straightforward manner at the load levels associated with failure. The load deformation response can be determined to whatever load level is desired. Herein this level is taken as that approaching the formation of the first plastic hinge. In working stress design the response is assumed to be linearly elastic but is not established beyond the working load level. Second-order effects cannot be included directly. The differences in the two analyses are demonstrated by White and Hajjar[1] who show responses for various types of analyses.

Furthermore, using computers for frame analyses allows the calculation of second-order effects to be determined (within the limits of the basic assumptions) to any degree of accuracy. Different modes of failure that are likely influenced by the boundary conditions can be examined on a logical basis. It is noted that when second-order effects are taken into account directly, LRFD requires analyses to be made for the several load combinations and not for the separate loads which are then combined. As well, LRFD uses a slightly different approach from WSD in assessing end conditions to determine the equivalent lengths for stepped columns. Be that as it may, the inherent superiority of LRFD behooves us to apply this design philosophy to stepped crane columns as used in industrial buildings.

The objective of the paper is, having established member sizes based on experience, trial and error, or following the procedures given by Fisher,[2] to first present to the practitioner the various aspects a designer has to consider such as loads and load combinations and establishing what combinations are likely to be critical. Information is presented in sufficient detail so that the designer can confirm the calculations and comfortably apply the procedures in other circumstances. This is followed by comparing three different methods of analysis, all using LRFD principles, to determine the effect of loads for critical load combinations. Two code checks are then made to confirm the structural adequacy. These involve using AISE[3] concepts to determine equivalent lengths of the stepped columns. The analyses are: an amplified first order analysis; a second-order elastic analysis and a second-order elastic analysis embodying the notional load concept. Differences among the analyses are discussed.

Two different methods of analysis following the 1993 AISC LRFD Specification[4] are used to determine the axial forces and moments acting on the members. Both recognize that the second-order effects must be included. These methods are an amplified first-order elastic analysis and a second-order elastic analysis. The results of these analyses are in good agreement with each other. The AISC LRFD interaction equations are then used to check the adequacy of the columns. This check requires the use of the AISE[3] concepts, as modified for LRFD, to determine column strengths and P-δ amplification factors as a function of equivalent lengths of stepped columns.

A third analysis, also a second-order elastic analysis, is used to determine the axial forces and moments. In it the factored lateral loads are increased by the addition of a notional lateral load of 0.002 times the sum of the factored vertical loads acting on the structure for the load combination under consideration. The concept of the notional load is to replace the sidesway buckling mode by an equivalent column strength problem and allow the interaction equations to be used with column strengths based on pinned-pinned conditions. Thus, in using the AISE concepts, equivalent lengths based on the fixed-slider conditions are no longer required.

Robert A. MacCrimmon is project engineer, Acres International Ltd., Niagara Falls, Ont., Canada.

D. J. Laurie Kennedy is professor emeritus, dept. of civil engineering, University of Alberta, Edmonton, Alta., Canada.

This approach is akin to that given in CSA Standard S16.1[5] where, however, the notional load is considered to be a minimum value and is not used in conjunction with other loads. The AISC LRFD[4] Specifications with minor modifications are again used to check the adequacy of the columns.

A second order analysis is shown to be much superior to an amplified first order analysis while the second-order notional load analysis eliminates the need for calculating effective length factors, and equivalent length factors for other than the pinned-pinned condition. As presented, the notional load analysis also allows different failure modes to be assessed independently.

2.0 STRUCTURAL ANALYSIS

2.1 The Frame

The example presented is a typical industrial type steel building that contains an overhead traveling crane. A cross-section is shown in Figure 1. The roof trusses are connected to the columns at the top and bottom chords to achieve frame action. The columns are fabricated from I-shaped rolled sections in a "*stepped*" configuration and are braced in the direction of the weak axis at 16-ft, 32-ft, 40-ft and 45-ft from the fixed bases. The term "*segment*" is used to describe the upper and lower portions of the stepped column. The building is braced along its length on both column lines, and in the plane of the underside of the roof trusses. The tops of the crane girders are linked to the columns and therefore transmit crane side thrust to the columns. The crane girder bearing detail is such that no significant bending moments are imposed about the weak axis of the columns. The bents are spaced at 20 feet on centres.

2.2 Dead and Live Loads

In addition to the weight of the building, the loads included are: vertical loads and side thrust from an overhead crane of about 15 tons capacity, both uniform and unbalanced snow loads, and wind loads with internal pressure. Other possible loads such as those due to earthquake and ponding are not considered in this example. Crane loads were established in accordance with current AISC criteria and as given by Fisher.[2] In accordance with the LRFD Specification,[4] impact from crane loads is not included in the design of the columns.

Crane side thrust is shared with adjacent frames by means of the horizontal bracing at the underside of the roof trusses. A 3-dimensional model of the framing system showed that, in this case, about two thirds of the total side thrust would be transferred to adjacent frames. For this example, load sharing of the side thrust is allowed for by imposing loads at the bottom chord of the truss that are opposite in direction to the crane side thrust and two thirds its magnitude. Other more sophisticated approaches to this load sharing are beyond the scope of this example. See also Section 2.4.

In accordance with current practice and the LRFD Specification,[4] all the crane load was considered to be live load. If the total weight of the crane were to be considered as dead load, with the appropriate AISC dead load factor of 1.2, the total factored maximum wheel load would be only 85 percent of that when all the crane load is taken to be live load. Whether or not the load is moving or dynamic in nature, the dead weight of the crane is known with certainty, and therefore merits consideration as such.

The specified loads are as follows:

1. Crane loads, *C*:	
1.1 rated capacity	30 kips
1.2 maximum wheel load	30 kips, not including impact
1.3 total weight of crane	47.3 kips
1.4 weight of trolley	18 kips
1.5 total side thrust	9.6 kips
1.6 longitudinal force	6 kips at each rail
2. Snow loads, *S*:	
2.1 full	40 psf
2.2 partial	20 psf
3. Wind loads, *W*:	
3.1 windward wall pressure	6 psf
3.2 leeward wall suction	4 psf
3.3 windward roof suction	10 psf
3.4 leeward roof suction	6 psf
3.5 internal pressure	5 psf
4. Roofing and roof truss dead loads:	15 psf

2.3 Load Combinations

Separate load cases were assembled and then combined and factored in accordance with the 1993 AISC LRFD Specification.[4]

This example focuses on the right hand column and therefore the load cases and load combinations given in Tables 1

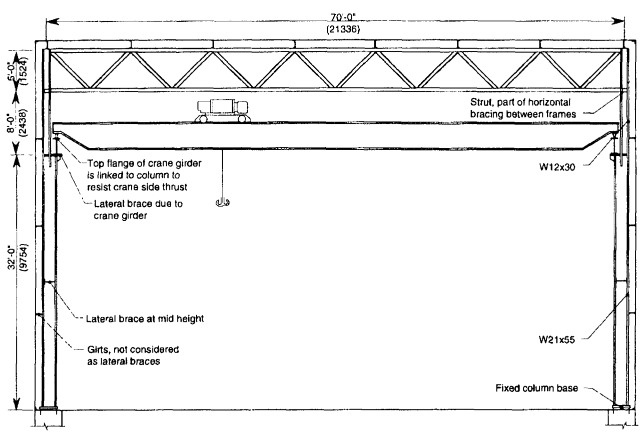


Fig. 1. Example frame.

Table 1. Load Cases		
Load Case	**Symbol**	**Description**
1	D	Dead load
2	S_f	Full snow load
3	S_r	Unbalanced snow load right
4	W	Wind, left to right, with pressure inside
5	C_r	Crane, maximum reaction to the right
6	C_l	Crane, maximum reaction to the left
7	C_{sr}	Crane side thrust to the right

Table 2. Load Combinations	
Load Combination	**Description**
1	$1.2D + 1.6(C_r + C_{sr}) + 0.5S_f$
2	$1.2D + 1.6(C_l + C_{sr}) + 0.5S_f$
3	$1.2D + 1.6(C_r + C_{sr}) + 0.5S_r$
4	$1.2D + 1.6(C_l + C_{sr}) + 0.5S_r$
5	$1.2D + 1.6S_f + 0.5(C_r + C_{sr})$
6	$1.2D + 1.6S_f + 0.5(C_l + C_{sr})$
7	$1.2D + 1.6S_r + 0.5(C_l + C_{sr})$
8	$1.2D + 1.6S_r + 0.5(C_l + C_{sr})$
9	$1.2D + 1.3W + 0.5(C_r + C_{sr} + S_r)$
10	$1.2D + 1.3W + 0.5(C_l + C_{sr} + S_r)$

and 2 respectively are those that have the most significance for it. This suggests that there are many load combinations for which the crane bay bent must be analyzed. It is realized, however, that the loadings and the structure under consideration require and deserve such detailed examination. Other load cases and load combinations are possible and may, in fact, govern the design of components of the roof truss.

2.4 Amplified First-Order Analysis

A first-order elastic analysis was performed using the computer program P-Frame 1.06 of Softek Services Ltd.[6] in order that the AISC LRFD[4] approximate procedure of Section C1 for considering second-order geometric effects could be followed. The computer model is described in Section 2.5. By Section C1 the required flexural strength is given by:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{1}$$

To determine the moments M_{nt} and M_{lt} where asymmetric loading causes sidesway, the frame must be propped laterally to find the non-translational moments, M_{nt}, due to the applied load. To find the moments M_{lt}, the props are released and are replaced by loads equal and opposite to the prop reactions

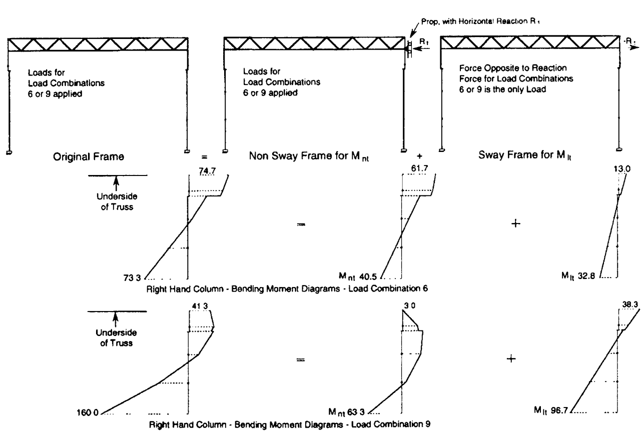


Fig. 2. Frame models for M_{nt} and M_{lt}.

with no other loads on the structure. This is illustrated in Figure 2 where the moments in kip-ft from a first-order analysis are shown for load combinations 6 and 9. To minimize the computational effort, first-order analyses without props were conducted for the 10 load combinations of Table 2. This revealed that combinations 6 and 9 would be the governing load combinations for the upper and lower column segments, respectively. For both load combinations all of the cladding loads have been conservatively taken to act at the top of the columns. As well, instead of carrying out a detailed analysis to determine the longitudinal distribution of the crane side thrust, two-thirds of the thrust has been assumed to be transferred to adjacent bents by the horizontal bracing at the level of the bottom chord of the truss. These factored load combinations are shown in Figure 3. The moments M_{nt} and M_{lt} were therefore determined only for these two load combinations.

Figure 2 shows that the prop to determine the moments M_{nt} for the non-sway was applied at the bottom chord level of the roof truss. It is argued that the depth of the truss in fact represents a braced storey. The portion of the column within the truss is also part of this braced storey and therefore the stepped column segments were considered to extend from the base plate to the underside of the truss.

Notwithstanding that not all crane supporting structures are framed exactly as in this example, Moore[7] and Vilas[8] approach the problem in a different manner and consider the stepped column to extend to an assumed pin midway between the two truss chords for the "no wind" case (little sway of bents) and to the bottom chord of the truss where a slider connection is provided for the "wind" case (in sway of bents). While the latter is equivalent to the approach given here, the distinction between "no wind" and "wind", as is evident from load combinations 6 and 9 ("no wind" and "wind" respectively) is not made here. Rather the depth of the truss is considered to be what it is in essence—a braced storey for all load cases. The distribution of lateral loads acting on a single frame due to crane side thrust could be determined by a 3-dimensional analysis, considering the lateral bracing at the level of the bottom chord of the truss. Here, for expediency, as noted in section 2.2, and based on a 3-dimensional model, about two thirds of such loads are transmitted to adjacent frames by the lateral bracing. The advantage to this approach is that the prop force, which is needed in *both* cases because the structure sways in both, is always applied at the level of the bottom chord of the truss. It would be difficult to imagine it being applied at the mid-truss depth but that is what the "no wind" approach would require. When second-order effects have to be considered, as they do in LRFD but not in WSD, some of these previously acceptable assumptions become less appropriate. When a second order analysis is performed as in sections 2.5 and 2.6, the length of the column as required to calculate P-δ effects, is irrelevant, because the analysis will yield the elastic P-δ *and* P-Δ effects. To calculate the axial strength of segments, an inherent part of the AISC LRFD[4] approach is a requirement for some judgement on the part of the designer in choosing the most appropriate end conditions, based on the deflected shape of the column.

The calculations to determine the amplification factor, B_2, for translational moments given by

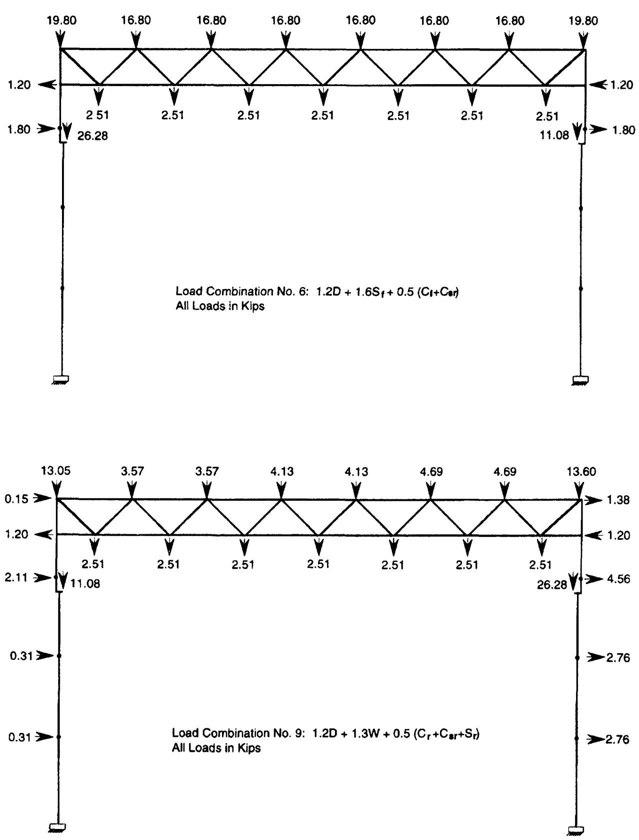


Fig. 3. Governing factored load combinations for stepped column.

$$B_2 = \frac{1}{1 - \dfrac{\Sigma P_u \Delta_{oh}}{\Sigma HL}} \qquad (2)$$

are summarized in Figure 4. In Equation 2 (AISC equation C1-4), P_u is, of course, the total factored vertical load on the storey and L, based on the previous arguments, is the height to the underside of the truss. The lateral interstorey deflection, Δ_{oh}, is the lateral deflection at the underside of the truss due to the force causing the drift. This is equal and opposite to the prop force. Thus for load combinations 6 and 9, the values of B_2 are respectively,

$$B_2 = \frac{1}{1 - \dfrac{195.3 \times 0.692}{2.26 \times 480}} = 1.14$$

and

$$B_2 = \frac{1}{1 - \dfrac{106.3 \times 2.038}{6.65 \times 480}} = 1.07$$

The application of this approximate method to find the second-order effects due to sway deflections for frames with two crane bays of different storey heights obviously becomes even more complicated. This suggests that a second-order analysis should be used from the onset.

The amplification factor to account for the P-δ effects is

$$B_1 = \frac{C_m}{\left(1 - \dfrac{P_u}{P_{el}}\right)} \geq 1 \qquad (3)$$

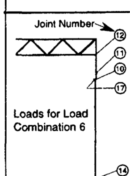
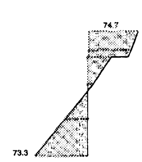
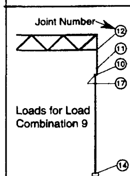
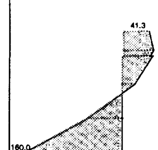
Loaded Structure	Bending Moment Diagrams First Order Analysis Unpropped Frame	Joint Bending Moments (kip-ft)	Prop at 12 Reaction (kips)	M_{nt} (kip-ft)	M_{lt} (kip-ft)	ΣP_u (kips)	ΣH (kips)	Δ_{oh} (in.)	L (in.)	B_2
		12 74.7	2.257	61.7	13.0			0 692		
		11 64.3		58.4	5.9					
		10 58.2		54.3	3.9					
		17 37.8		34.1	3.7					
						195.3	2 26		480	1.14
		14 73 3		40.5	32.8					
		12 41.3	6.645	2 9	38.4			2.038		
		11 47.1		29.7	17.4					
		10 40.7		29.2	11.5					
		17 49.1		38.1	11 0					
						106.3	6.65		480	1.07
		14 160 0		63.3	96.7					

Fig. 4. Determination of factors M_{nt}, M_{lt}, and B_2.

Table 3. Values of Parameters for Calculating B_1										
	Upper Segment					Lower Segment				
Load Combinations	K_s	P_u kips	P_{el} kips	C_m	B_1	K_s	P_u kips	P_{el} kips	C_m	B_1
6	2.63	79.1	1,069	1.0	1.08	1.36	90.1	1,207	0.39	1.00
9	2.96	36.2	844	1.0	1.05	1.24	62.5	1,452	0.48	1.00

where

C_m = equivalent moment factor, depends on the shape of the moment diagram and has a maximum value of 1.0 when the moments are uniform along the length

P_{el} = Euler buckling load

In the WSD Specification, for sway frames, the equivalent of P_{el} can be based on an effective length factor, K, greater than 1.0 whereas in the LRFD Specification it is based on a braced frame analysis and, therefore, K has a maximum value of 1.0.

Also, for stepped columns, as discussed subsequently, the equivalent length factor, K_s, is introduced in calculating the Euler buckling load P_{el}. The length of either column segment is multiplied by the equivalent length factor, K_s, related to it, to establish the length of a prismatic column of the same cross section as that of the segment and to have the same buckling characteristics as the stepped column.

For the upper segment as shown in Figure 5 the moment diagram in fact approaches a constant value and therefore

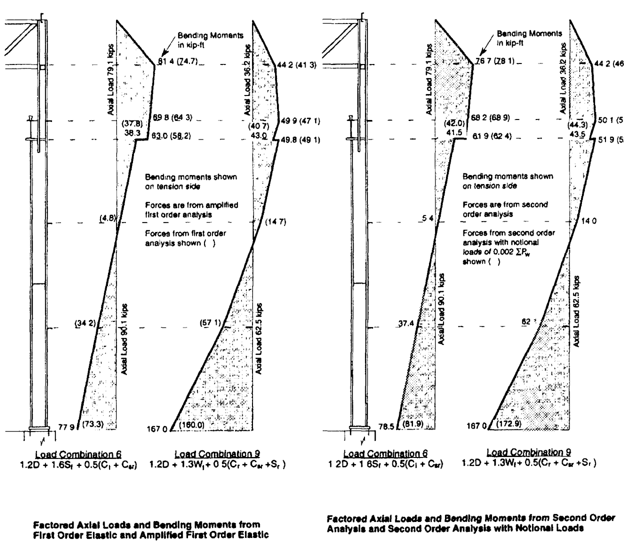


Fig. 5. Factored axial loads and bending moments for four methods of analysis.

C_m approaches 1. This is not consistent with the general statement in AISE Technical Report No. 13[3] that a C_m value of 0.85 is applicable for wind load. This latter value should be restricted in its use to the design of the lower segment.

Determination of the values of the equivalent length factors as used here are discussed subsequently. Calculations follow for finding values of B_1 based on the first-order analyses moments, as given in Figure 5, for the two load cases and for both the upper and lower segments. Relevant values of the parameters and a summary of the calculations are given in Table 3.

2.4.1 Load Combination 6—Upper Segment

$$P_{el} = \frac{A_g F_Y}{\lambda_c^2}$$

where

$$\lambda_c = \frac{K_s L}{r\pi}\sqrt{\frac{F_Y}{E}}$$

therefore

$$P_{el} = \frac{\pi^2 EI}{(K_s L)^2} = \frac{\pi^2 \times 29{,}000 \times 238}{(2.63 \times 96)^2} = 1{,}069 \text{ kips}$$

C_m = 1.00 (say) based on shape of moment diagram.

Therefore

$$B_1 = \frac{1.00}{1 - \dfrac{79.1}{1{,}069}} = 1.08$$

2.4.2 Load Combination 6—Lower Segment

$$P_{el} = \frac{\pi^2 \times 29{,}000 \times 1{,}150}{(1.36 \times 384)^2} = 1{,}207 \text{ kips}$$

$$C_m = 0.6 - 0.4M_1/M_2 = 0.6 - 0.4(37.8/73.3) = 0.39$$

$$B_1 = \frac{0.39}{\left(1 - \dfrac{90.1}{1{,}207}\right)} = 0.42, \geq 1.0$$

Use B_1 = 1.0

2.4.3 Load Combination 9—Upper Segment

$$P_{el} = \frac{\pi^2 \times 29{,}000 \times 238}{(2.96 \times 96)^2} = 844 \text{ kips}$$

$C_m = 1.0$ (say) based on shape of moment diagram

Hence,

$$B_1 = \frac{1.00}{\left(1 - \dfrac{36.2}{844}\right)} = 1.05$$

2.4.4 Load Combination 9—Lower Segment

$$P_{el} = \frac{\pi^2 \times 29{,}000 \times 1{,}150}{(1.24 \times 384)^2} = 1{,}452 \text{ kips}$$

$$C_m = 0.6 - 0.4(49.1 / 160) = 0.48$$

$$B_1 = \frac{0.48}{\left(1 - \dfrac{62.5}{1{,}452}\right)} = 0.50 \geq 1.0$$

Use $B_1 = 1.0$.

Having established values of B_1 and B_2 for use in Equation 1 the values of the amplified first-order moments are established for the two load combinations as given in Figure 5.

2.5 Second-Order Analysis

Second-order elastic analyses were performed for load combinations 6 and 9 using the stability analysis option of the computer program P-frame 1.06 of Softek Services Ltd.[6] Iterations are performed until the difference between the determinant values before and after the non-linear matrix force-displacement equation is solved is less than 0.1 percent, the default value. Convergence was obtained in three iterations indicating that the structure is relatively stiff. The computer model frame with a typical exaggerated deflected shape is shown in Figure 6.

It is noted that the tangent to the top end of right-hand column (which is critical) has a slope that is more favourable than the vertical slope for the fixed-slider condition for which the effective length factor is 1.0. Therefore the effective length factor of the critical column is less than 1.0. Were the roof truss infinitely stiff, then and only then, would the effective length factor equal 1.0 for this load case. All connections were taken to be fully fixed except the truss to column connections which were considered to be pinned. The eccentricity between the centre-lines of the two segments of the stepped column and that between the crane girder and the lower segment centre-lines were modeled using very stiff horizontal members. In addition, as shown in Figure 6, extra nodes were inserted at girt locations in both the upper and lower segments. The top flange of the crane girder is also laterally supported at the girt location in the upper segment as shown in Figure 1. The results of the second-order elastic analyses for the right-hand column are given in Figure 5. Comparison to the amplified first-order analyses moments of Figure 5 show the latter to be in substantial agreement with the second-order results. The computer program, Staad-III, Revision 16.0[9] was used to confirm the results of the P-frame analysis. The second-order elastic analyses results were used in the AISC LRFD design checks subsequently.

For this type of structure, a second-order elastic computer analysis is much simpler to carry out that an amplified first-order elastic analysis. Not only is it not necessary to analyze the frame when laterally propped and when the prop forces are reversed but, as well, all the intricate calculations to determine the amplification factors B_1 and B_2 are avoided. Furthermore the second-order analysis generates these member forces throughout the structure while when following the amplified first-order analysis, the additional step of adjusting the forces in the remainder of the structure must be undertaken. It is also easier to do second-order analyses for the various load combinations rather than to try to assess which ones may be critical on the basis of first-order analyses.

2.6 Second-Order Elastic Analysis with Notional Loads

The concept of the notional, imaginary or pseudo lateral load is simply this: by applying the notional load, expressed as a small portion of the vertical loads acting on the storey, the sway buckling or bifurcation problem is transformed into an in-plane strength problem without need for recourse to effective length factors greater than one. The use of sway buckling effective length factors is thus avoided. Kennedy[10] demonstrated that the notional load should be applied in conjunction with the actual factored loads on the structure and that its value should decrease from that for a flagpole column with a sway buckling effective length factor of 2.0 to that for a column with fixed ends with a sway effective length factor of

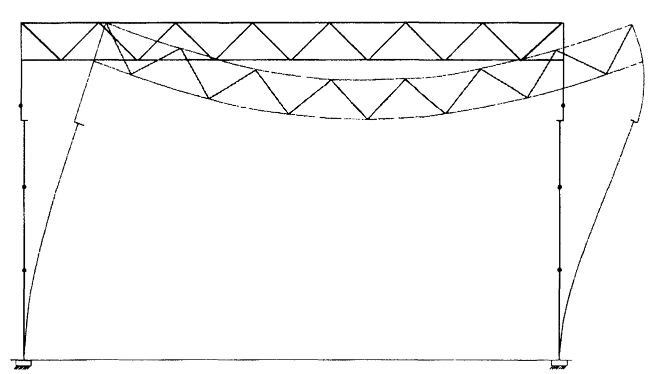


Fig. 6. Computer model frame showing typical exaggerated deflected shape.

1.0. In this example a notional load of 0.002 times the vertical loads, as used in Australian Standard AS4100,[11] is used.

The second-order elastic analyses previously described were therefore performed for load combinations 6 and 9 when in addition to the factored lateral loads, additional notional lateral loads of $0.002\Sigma P_u$ were applied to the frames at the level of the bottom chord of the truss, the location where the prop forces were applied and in the direction to increase the bending moments in the columns. The notional loads were therefore $0.002 \times 195.3 = 0.39$ kips for load combination 6 and $0.002 \times 106.3 = 0.21$ kips for load combination 9. Having determined the bending moments due to the lateral loads increased by the notional loads the equivalent length factors for the stepped column segments are logically based on the pinned-pinned condition.

The results of these analyses together with the amplified first-order analyses and the second-order analyses without notional loads are summarized in Figure 5.

3.0 EQUIVALENT LENGTHS

3.1 AISE Procedures

The AISE[3] gives a method to determine an equivalent length of a stepped column based on: the ratio of the length of the upper segment to the total length of the column, the ratio of the moment of inertia of the bottom segment to that of the top segment, the ratio of the axial force in the upper segment, P_1, to the additional axial force applied at the step, P_2, for a variety of end conditions. Anderson and Woodward[12] and Agrawal and Stafiej[13] give the derivation of the equivalent length factors. Equivalent length factors can also be derived from results of a computerized buckling analysis. This type of analysis is useful for more complex stepped columns where, for instance, more than one step is involved or where springs may be introduced as part of the support system. As a slight modification of the AISE procedure, the length of either segment is multiplied by its equivalent length factor to establish the length of a pin-ended prismatic column of the same cross section as that of the segment and, when loaded at the ends, has the same buckling load as the stepped column under its loading condition.

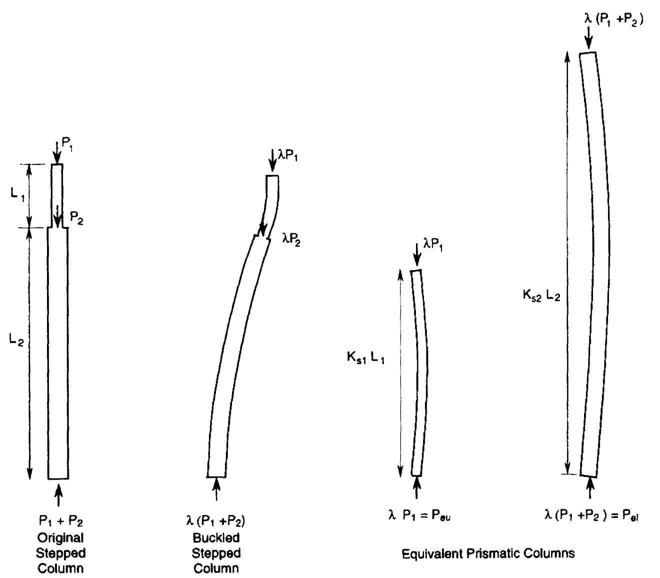


Fig. 7. Equivalent lengths for the two segments of a fixed-slider column.

This is illustrated in Figure 7 for a stepped column with fixed-slider end conditions. The set of three columns comprising the original stepped column (with the given load configuration and end conditions) and the equivalent prismatic columns for the two segments (loaded at their ends with their respective loads only and with pin-ends) buckle elastically at the same load factor λ. The following relationships exist, where P_{eu} and P_{el} are the Euler buckling loads for the equivalent prismatic columns.

$$\frac{P_1}{P_{eu}} = \frac{1}{\lambda} = \frac{P_1 + P_2}{P_{el}} \tag{4}$$

$$\frac{P_{eu}}{P_{el}} = \frac{P_1}{P_1 + P_2} \tag{5}$$

The equivalent prismatic columns with the given cross sections, lengths, and end conditions are used to establish both the Euler buckling loads, P_e and the nominal axial strengths, P_n.

In the AISC WSD Specification (not used herein), the assumed end conditions are used to determine both F'_{ex}, the Euler stress divided by the safety factor, and F_a, the allowable axial stress for buckling about the x-x axis. In the AISC LRFD Specification[4] for amplified first-order analyses, the Euler buckling load, P_{el}, used in determining the amplification factor, B_1, is based on the pinned-pinned condition while the nominal axial strength, P_n, is based on the assumed end conditions. In this specification, for second-order analyses there is no amplification factor B_1 and P_n is based on the assumed end conditions. When notional loads are introduced, as discussed in Section 2.6, the pinned-pinned condition is used for both the amplification factor and P_n calculations.

3.2 Equivalent Length Factors, K_s, for Load Combinations 6 and 9

An in-house program of Acres based on Anderson and Woodward[12] and on Agrawal and Stafiej[13] was used to calculate the equivalent length factors given in Table 4.

4.0 DESIGN CHECKS USING AISC LRFD 1993

The basic interaction equations for uniaxial bending, as exists here, are:

Table 4. Equivalent Length Factors, K_s

Load Combination	End Conditions: Pinned-Pinned Upper	Pinned-Pinned Lower	Fixed-Slider (as used to calculate B_1 in Section 2.4) Upper	Fixed-Slider Lower
6	2.63	1.36	3.28	1.69
9	2.96	1.24	3.96	1.65

(a) For

$$\frac{P_u}{\phi_c P_n} \geq 0.2 \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9}\frac{M_{ux}}{\phi_b M_{nx}} \leq 1.0 \tag{6}$$

(b) For

$$\frac{P_u}{\phi_c P_n} \leq 0.2 \quad \frac{P_u}{2\phi_c P_n} + \frac{M_{ux}}{\phi_b M_{nx}} \leq 1.0 \tag{7}$$

4.1 Factored Axial Loads and Bending Moments

The factored axial loads and bending moments from the second-order analyses for load combinations 6 and 9 are given in Figure 5. Load combination 6 is critical for the upper segment and 9 for the lower.

4.2 Upper Segment

Relevant properties of the W12×30 from the AISC handbook are:

$F_Y = 36$ ksi	$A = 8.79$ in.^{2}	$I_x = 238$ in.^{4}
$r_x = 5.20$ in.	$S_x = 38.6$ in.^{3}	$I_y = 20.3$ in.^{4}
$r_y = 1.52$ in.	$Z_x = 43.1$ in.^{3}	$J = 0.46$ in.^{4}
$C_w = 720$ in.^{6}		

Calculations show that the section is compact. From Figure 5 for load combination 6, $P_u = 79.1$ kips and $M_{ux} = 76.7$ kip-ft and the segment is bent in single curvature.

P_n is the lesser of the two values for weak axis and strong axis axial behaviour. For the weak axis, using an effective length factor of 1,

$$\frac{KL}{r_y} = \frac{1 \times 96}{1.52} = 63.2$$

$$\lambda_c = \frac{KL}{\pi r_y}\sqrt{\frac{F_Y}{E}} = \frac{63.2}{\pi}\sqrt{\frac{36}{29{,}000}} = 0.708$$

$$P_n = 0.658^{\lambda^2} A F_Y = 0.658^{0.708^2} \times 8.79 \times 36 = 256 \text{ kips}$$

and

$$\phi_c P_n = 0.85 \times 256 = 218 \text{ kips}$$

For the strong axis, the equivalent length factor for fixed-slider conditions (Table 4) is 3.28

therefore

$$\lambda_c = \frac{K_s L}{\pi r_x}\sqrt{\frac{F_y}{E}} = 0.678, \text{ and is not critical}$$

In calculating M_n the possibility of lateral torsional buckling must be taken into account.

(a) For yielding, the flexural design strength is

$$\phi_b M_n = 0.9 M_p = 0.9 F_Y Z = \frac{0.9 \times 36 \times 43.1}{12} = 0.9 \times 129$$

$$= 116 \text{ kip-ft}$$

(b) For lateral torsional buckling, L_b = distance between braces = 96 in. and from the LRFD Manual, page 4-20

$L_p = 6.3 \times 12 = 75.6$ in.,

$L_r = 19.1 \times 12 = 229$ in.,

$M_r = 75.3/0.9 = 83.7$ kip-ft

$$C_b = \frac{12.5 \times 76.7}{(2.5 \times 76.7) + (3 \times 65.6) + (4 \times 69.3) + (3 \times 73)} = 1.08$$

From the AISC Manual, page 4-134, for a W12×30, with an unbraced length of 8 ft, the beam design moment $\phi_b M_n$, for $C_b = 1$, is 111 kip-ft.

For $C_b = 1.08$, $\phi_b M_n = 1.08 \times 111 = 120$ kip-ft. This exceeds $\phi_c M_p = 116$ kip-ft, and therefore the flexural design strength, $\phi_b M_n = 116$ kip-ft.

From the values calculated

$$\frac{P_u}{\phi_c P_n} = \frac{79.1}{218} = 0.364 > 0.2$$

and Equation 6 (AISC interaction equation H1-1a) governs and gives

$$\frac{79.1}{218} + \frac{8 \times 76.7}{9 \times 116} = 0.363 + 0.588 = 0.951 < 1.0 \quad \textbf{o.k.}$$

Other design checks for shear, serviceability and the like would now normally be carried out. The top segment is **o.k.**

4.3 Lower Segment

Relevant properties of the W21×55 (a Canadian section) from the CISC Handbook are:

$F_Y = 44$ ksi	$A = 16.3$ in.^{2}	$I_x = 1{,}150$ in.^{4}
$r_x = 8.40$ in.	$S_x = 111$ in.^{3}	$I_y = 48.3$ in.^{4}

r_y = 1.72 in. Z_x = 126 in.[3] J = 1.27 in.[4]
C_w = 4,970 in.[6]

Calculations based on:

$b = 8.22/2 = 4.11$ in.

$t = 0.522$ in.

$h = 18.25$ in.

$t_w = 0.375$ in.

show the section to be compact.

P_n, for the weak axis, is based on the bottom half of the column segment below the lateral brace and $K = 0.8$ because of the base fixity. Thus,

$$\lambda_c = \frac{0.8 \times 192}{1.72 \times \pi}\sqrt{\frac{44}{29{,}000}} = 1.11 < 1.5$$

and the factored compressive strength

$$= \phi_c P_n = 0.85 \times 0.658^{1.11^2} \times 44 \times 16.3 = 364 \text{ kips}$$

For the strong axis, the equivalent length factor for fixed-slider (Table 4) equals 1.65 and therefore

$$\lambda_c = \frac{1.24 \times 384}{8.40 \times \pi}\sqrt{\frac{44}{29{,}000}} = 0.935, \text{ and is not critical.}$$

As for the upper segment, the nominal flexural strength, M_n, must be based on the possibility of lateral torsional buckling.

(a) For yielding, the flexural design strength is

$$\phi_b M_n = 0.9 M_p = F_Y Z = \frac{0.9 \times 44 \times 126}{12} = 0.9 \times 462$$

$$= 416 \text{ kip-ft}$$

(b) For lateral torsional buckling, the flexural design strength, from Section F1.2(a) or (b), depends on the unbraced length and the shape of the bending moment diagram.

For an unbraced length of 192 inches and for the bending moment diagram given in Figure 6, the AISC equations lead to a value of C_b of 1.50 and a lateral torsional buckling nominal strength greater than M_p. Therefore, the flexural design strength

$$\phi_b M_n = 0.9 \times 462 = 416 \text{ kip-ft.}$$

From the values calculated

$$\frac{P_u}{\phi_c P_n} = \frac{62.5}{364} = 0.172 < 0.2$$

and equation 7 (AISC equation H1-1b) governs.

$$\frac{62.5}{2 \times 364} + \frac{167}{416} = 0.086 + 0.401 = 0.487 < 1.0 \quad \textbf{o.k.}$$

The bottom segment is **o.k.** The low interaction value indicates that a smaller section could be considered. However, the W21×55 was selected to provide sufficient clearance between the end of the bridge girder and the upper segment and, as well, to provided sufficient lateral stiffness.

5.0 DESIGN CHECKS WITH NOTIONAL LOADS

The basic interaction equations of the AISC Specification[4] are used with some minor modifications because the cross-sectional strength, in-plane and out-of-plane checks are made independently. Both the Canadian Standard, S16.1-94,[5] and the Australian Standard, AS4100-1990,[11] and as recommended by Trahair,[14] treat these failure modes independently.

5.1 Factored Axial Loads and Bending Moments

The factored axial loads and bending moments from the second-order analyses with the addition of notional lateral loads for load combinations 6 and 9 are given in Figure 5. As for the AISC LRFD check, load combination 6 is critical for the upper segment and 9 for the lower.

5.2 Upper Segment

The relevant properties of the W12×30 are given in Section 4.2.

Cross-Sectional Strength

Adapting Equation 6 (AISC[4] equation H1- la) for a cross-sectional strength check the critical section has P_u = 79.1 kips and M_{ux} = 78.1 kip-ft, $\phi_c P_Y$ = 269 kips and $\phi_o M_p$ = 116 kip-ft, hence:

$$\frac{P_u}{\phi_c P_Y} + \frac{8}{9}\frac{M_{ux}}{M_p} = \frac{79.1}{269} + \frac{8 \times 78.1}{9 \times 116} = 0.294 + 0.598$$

$$= 0.892 \quad \textbf{o.k.}$$

In-Plane Behaviour

From Figure 56, for load combination 6, P_u equals 79.1 kips and M_{ux}, with notional load effects included, equals 78.1 kip-ft.

The factor C_m is computed to account for non-uniform moments but is not restricted to values greater than 1 as in the LRFD Specification[4] because only in-plane behaviour is being checked.

$$C_m = 0.6 + 0.4\left(\frac{62.4}{78.1}\right) = 0.920$$

P_{el}, using K_s = 2.63 for pinned-pinned conditions, is

$$P_{el} = \frac{\pi^2 EI}{(K_s L)^2} = \frac{\pi^2 \times 29{,}000 \times 238}{(2.63 \times 96)^2} = 1{,}069 \text{ kips}$$

P_n is calculated for the x-axis for in-plane behaviour with K_s

equal to 2.63 for the pinned-pinned condition. As calculated in Section 4.2, therefore,

$$\lambda_c = \frac{K_s L}{\pi r_x}\sqrt{\frac{F_Y}{E}} = 0.544$$

and therefore

$$\phi_c P_n = \phi_c(0.658^{\lambda^2} A F_Y) = 0.85 \times 0.658^{0.544^2} \times 8.79 \times 36$$

$$= 238 \text{ kips}$$

and, for in-plane bending

$$M_n = \phi_b M_p = 0.90 \times 129 = 116 \text{ kip-ft}$$

Therefore

$$\frac{P_u}{\phi_c P_n} = \frac{79.1}{238} = 0.332 > 0.2$$

and Equation 6 (AISC[4] equation H1-1a) governs, giving

$$\frac{79.1}{238} + \frac{8 \times 78.1 \times 0.920}{9 \times 116} = 0.332 + 0.551 = 0.883$$

$$< 1.0 \quad \textbf{o.k.}$$

Out-of-Plane Behaviour

The difference from the in-plane check is that the values of P_n and M_n in the interaction equation are now based on out-of-plane behaviour. Therefore P_n is calculated for weak axis buckling, and $\phi_c P_n$ is 218 kips as for the second-order AISC LRFD check. With $P_u / \phi_c P_n$ = 79.1 / 218 = 0.362, Equation 6 governs. In the AISC LRFD check, lateral-torsional buckling was found to not be critical and the plastic moment could be reached. Therefore the design moment, $\phi_b M_n = \phi_b M_p = 0.90 \times 129 = 116$ kip-ft as before. The value of C_m is taken to be not less than 1.0 because the effect of the non-uniform moment diagram has already been considered in establishing M_n. Therefore, using Equation 6 gives

$$\frac{79.1}{218} + \frac{8}{9}\left(\frac{78.1}{116}\right) = 0.363 + 0.598 = 0.961 < 1.0 \quad \textbf{o.k.}$$

With all interaction values less than 1.0 the upper segment is **o.k.** The second value for out-of-plane behaviour is virtually the same as that computed for the AISC LRFD[4] check in this case. The two interaction values for the upper segment are about equal and thus indicate that both failure modes are equally likely to occur.

5.3 Lower Segment

The relevant properties of the W21×55 are given in Section 4.3.

Cross-Sectional Strength

Adapting Equation 7 (AISC[4] H1-1b) for a cross-sectional strength check the critical section has P_u = 62.5 kips and M_{ux} = 172.9 kip-ft, $\phi_c P_Y$ = 610 kips and $\phi_b M_p$ = 416 kip-ft, hence:

$$\frac{P_u}{2\phi_c P_Y} + \frac{M_{ux}}{M_p} = \frac{62.5}{2 \times 610} + \frac{172.9}{416} = 0.051 + 0.416$$

$$= 0.467 \quad \textbf{o.k.}$$

In-Plane Behaviour

From Figure 5, for load combination 9, P_u equals 62.5 kips and M_{ux}, with notional load effects included, equals 172.9 kip-ft. The factor, C_m is computed to account for non-uniform moments but is not restricted to values greater than 1 because only in-plane behaviour is being checked.

$$C_m = 0.6 - 0.4\left(\frac{52.6}{172.9}\right) = 0.478$$

P_{el}, using K_s = 1.24 for pinned-pinned conditions, is

$$P_{el} = \frac{\pi^2 EI}{(K_s L)^2} = \frac{\pi^2 \times 29{,}000 \times 1{,}150}{(1.24 \times 384)^2} = 1{,}452 \text{ kips}$$

P_n is calculated for the x-axis for in-plane behaviour with K_s for the pinned-pinned behaviour condition equal to 1.24 as calculated in Section 4.3, therefore,

$$\lambda_c = \frac{K_s L}{\pi r_x}\sqrt{\frac{F_y}{E}} = 0.703$$

and therefore

$$\phi_c P_n = 0.85 \times 0.658^{0.703^2} \times 16.3 \times 44 = 496 \text{ kips}$$

and, for in-plane behaviour,

$$M_n = \phi_b M_p = 0.9 \times 462 = 416 \text{ kip-ft.}$$

Therefore

$$\frac{P_u}{\phi_c P_n} = \frac{62.5}{496} = 0.126 < 0.2$$

and Equation 7 (AISC Equation H1-1b) governs, giving

$$\frac{62.5}{2 \times 496} + \frac{0.478 \times 172.9}{416} = 0.063 + 0.199 = 0.262$$

Out-of-Plane Behaviour

The values of P_n and M_n in the interaction equation are now based on out-of-plane behaviour . Therefore P_n is calculated for weak axis buckling, and as in the AISC LRFD[4] check with a brace at mid-height and a K factor, with a fixed base, of 0.8, $\phi_c P_n$ = 364 kips. With $P_u / \phi_c P_n$ = 62.5 / 364 = 0.172, Equation 7 governs. In the AISC LRFD[4] check lateral-torsional buckling was found to not be critical and the plastic moment could be reached. Therefore the design moment, $\phi_b M_n = \phi_b M_p =$

$0.90 \times 462 = 416$ kip-ft as before. Therefore, using Equation H1-1b with C_m not less than 1.0, as before, gives

$$\frac{62.5}{2 \times 364} + \left(\frac{172.9}{416}\right) = 0.086 + 0.416 = 0.502 < 1.0 \quad \textbf{o.k.}$$

With both interaction values less than 1.0 the lower segment is **o.k.** The second value for out-of-plane behaviour is again about the same as that computed for the AISC LRFD[4] check. The out-of-plane interaction value is much greater than the in-plane strength value and indicates that this failure mode is more likely.

6.0 CONCLUSIONS AND RECOMMENDATIONS

1. The AISC LRFD Specification, coupled with the AISE equivalent length factors for stepped columns, provides an effective means for designing stepped columns for industrial buildings.
2. Second-order elastic analyses, using readily available computer software that runs even on 386s, are much easier to perform than amplified first-order analyses. As well, approximations inherent in the amplified analyses are avoided, fewer factors need be determined, the convergence of the solution is indicative of the stiffness of the structure, and many load combinations can be readily examined.
3. For crane column bents, with roof trusses participating in the frame action by virtue of the fact that the columns are connected to both the top and bottom chords of the truss, the stepped column can be considered to extend only from the base to the underside of the truss. The total depth of the truss acts as a braced storey. In an amplified first order analysis, this implies that the prop to prevent sidesway when determining the nonsway moments should be located at the underside of the truss.
4. For crane column bents with roof trusses participating in the frame action, a value of C_m of about 1.0 is suggested for the upper segment of stepped columns. The value of 0.85 given in AISE Technical Report No. 13 should be used for the lower segment only.
5. Examination of the deflected shape of the structure for the critical load combination shows that the effective length of the critical column is less than the theoretical value, i.e., less than 1.0 for the fixed-slider configuration.
6. The concept of equivalent lengths for stepped columns means this: the original stepped column and the equivalent prismatic columns for the two segments buckle elastically at the same load factor λ, i.e., at the same multiple of the applied loads.
7. The use of the notional load concept which allows equivalent lengths to be based on the pinned-pinned condition is simple to apply, requires no effective lengths to be computed and allows the in-plane and out-of-plane modes of failure to be assessed independently.

7.0 ACKNOWLEDGMENTS

The support of Acres International Limited with release time, resources, and finances to the first author and that of the Natural Sciences and Research Council of Canada to the second is gratefully acknowledged.

8.0 NOTATION

A (A_g)	Cross-sectional area, (gross)
B_1	Amplification factor to account for P-δ effects
B_2	Amplification factor to account for P-Δ effects
C	Crane load with subscripts r, l, and sr for maximum reaction to the right, to the left and with side thrust to the right respectively
C_b	Bending coefficient to determine lateral buckling strength depending on the moment gradient
C_m	Coefficient to determine equivalent uniform bending effect in beam columns
C_w	Warping constant
E	Modulus of elasticity
F_Y	Specified minimum yield strength
H	Horizontal force
I	Moment of inertia
J	Torsional constant
K	Effective length factor for prismatic member
K_s	Equivalent length factor for a segment of a stepped column
L	Storey height; length of member
M_1	Smaller moment at the end of the unbraced length
M_2	Larger moment at the end of the unbraced length
M_{lt}	Required flexural strength due to lateral frame translation
M_n	Nominal flexural strength
M_{nt}	Required flexural strength without frame translation
M_p	Plastic moment
M_r	Limiting buckling moment
M_u	Required flexural strength
P_1	Load on upper segment of stepped column
P_2	Load moment at step of stepped column
P-δ	Incremental moment due to force P acting on column displacement δ
P-Δ	Incremental moment due to force P acting on sway displacement Δ
P_{el}	Euler buckling load for a braced frame
P_{eu}	Euler buckling load for equivalent prismatic upper segment
P_{el}	Euler buckling load for equivalent prismatic lower segment
P_n	Nominal axial strength
P_u	Required axial strength in compression
P_Y	Yield load

r	Radius of gyration
S	Snow loads with subscripts f and r for full snow load and unbalanced snow load right, respectively; elastic section modulus
W	Wind load
x, y	Subscripts relating to the x and y axis
Z	Plastic section modulus
Δ_{oh}	Translation deflection of storey under consideration
ϕ_b	Resistance factor for flexure
ϕ_c	Resistance factor for compression
λ	Load factor
λ_c	Equivalent slenderness parameter
Σ	Sum

REFERENCES

1. White, D. W. and Hajjar, J. F., "Application of Second-Order Elastic Analyses in LRFD; research to practice." AISC *Engineering Journal*, 4th Qtr., 1991, pp. 133–148.
2. Fisher, J. M., "Industrial Buildings, Roofs to Column Anchorage." AISC *Steel Design Guide Series*, No. 7, Chicago, IL, Oct., 1993.
3. Association of Iron and Steel Engineers, "Guide for the Design and Construction of Mill Buildings," *Technical Report No.13*, Pittsburgh, PA, 1991.
4. American Institute of Steel Construction, *Load and Resistance Factor Design Specification for Structural Steel Buildings*, Chicago, IL, 1993.
5. Canadian Standards Association, *CAN/CSA-S16.1-94 Limit States Design of Steel Structures*, Rexdale, ON, M9W 1R3, Canada, 1994.
6. Softek Services Ltd., *P-Frame 1.06*, Vancouver, BC, Canada, 1991.
7. Moore, W. E., "A programmable solution for stepped crane columns," AISC *Engineering Journal*, 2nd Qtr., Vol. 23, 1986, pp. 55–58.
8. Vilas, H. K., "Mill building frame analysis: distribution of lateral loads," AISC *Engineering Journal*, 3rd Qtr., Vol. 24, 1987, pp. 101–112.
9. Research Engineers, Inc., *Structural Analysis and Design, Staad III, Revision 16.0a*, Marlton, NJ, 1992.
10. Kennedy, D. J. L., "Limit States Design of Beam-Columns in CSA S16.1-94," *Proceedings of the International Conference on Structural Stability and Design*, Sydney, Australia, Oct. 30–Nov. 1, 1995, pp. 461–465.
11. Standards Association of Australia, *1990 Australian Standard AS4100-1990 Steel Structure*, Sydney, Australia, 1990.
12. Anderson, J. P. and Woodward, J. H., "Calculation of Effective Lengths and Effective Slenderness Rations of Stepped Columns," AISC *Engineering Journal*, October, 1972, pp. 157–166.
13. Agrawal, K. M. and Stafiej, A. P., "Calculation of Effective Lengths of Stepped Columns," *Engineering Journal*, AISC, 4th Qtr, 1980, pp. 96–105.
14. Trahair, N. S., "Design Strengths of Steel Beam-Columns," *Canadian Journal of Civil Engineering*, 13(6), pp. 639–646.