# Improved Shear Strength of Webs Designed in Accordance with the LRFD Specification

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## ABSTRACT

The dependence of the LRFD design rules for shear resistance on the web elastic local buckling coefficient is demonstrated. A finite strip method of analysis, based on complex arithmetic and that incorporates shear, is then used to derive elastic local buckling coefficients that incorporate the restraint provided by the flanges of an I-section member. It is shown by an example how these enhanced buckling coefficients lead to greater calculated shear strengths, thereby producing economies in the use of steel.


## 1. INTRODUCTION

In the LRFD Specification,[1] the design shear strength of a web subjected to shearing stress in the plane of symmetry is governed by the elastic local buckling coefficient $k$. When the web is stocky and free from the influences of local buckling, it will fail by yielding in shear close to the theoretical shear yield stress $F_{yw}/\sqrt{3}$, where $F_{yw}$ is the uniaxial tensile yield strength of the web in ksi. On the other hand, when the web is sufficiently slender, its strength is governed by the elastic local buckling stress $F_{ol}$. Intermediate values of web slenderness are governed by the interaction of yielding and buckling in much the same way as the treatment of members subjected to bending or compression in the LRFD.

The local buckling stress in shear $F_{ol}$ may be written as[2]

$$F_{ol} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t_w}{h}\right)^2 \tag{1}$$

where

$E$ = the Young's modulus of the steel
$\nu$ = its Poisson's ratio
$t_w$ = the web thickness, and
$h$ = the web depth (Figure 1)

The local buckling coefficient for a web plate with simply supported longitudinal edges is usually taken in theory as[2]

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$$k = 5.35 + \frac{4}{(L/h)^2} \tag{2a}$$

when $L > h$, or as

$$k = \frac{5.35}{(L/h)^2} + 4 \tag{2b}$$

when $L < h$

where

$L$ = the local buckling half-wavelength.

This relationship is plotted in Figure 2, and it can be seen that the usual ensemble of garland shaped curves, typical for plates subjected to compression,[3] is absent for the shear case.

If $a$ represents the spacing between intermediate transverse stiffeners in a stiffened web, then the LRFD assumes that the buckling half-wavelength $L$ between the stiffeners will equal $a$, as indicated in Figure 2. The LRFD thus uses a simplified compromise for Equations 2, so that

$$k = 5 + \frac{5}{(a/h)^2} \tag{3}$$

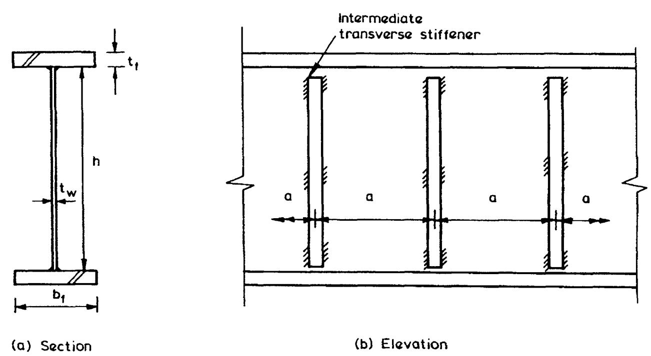


*Fig. 1. Stiffened web.*

This is also shown plotted in Figure 2, and it can be seen that Equation 3 is close to Equation 2. For an unstiffened web, $a$ approaches infinity, so that $k = 5$.

In the LRFD, when the modified web slenderness satisfies

$$\frac{h}{t_w}\sqrt{\frac{F_{yw}}{k}} < 187 \tag{4}$$

the web yields in shear before buckling, and its nominal resistance $V_n$ is governed by the shear yield stress $F_{yw}/\sqrt{3}$. Thus

$$V_n = 0.6A_wF_{yw} \tag{5}$$

where

$A_w$ = the net web area.

When the modified web slenderness satisfies

$$\frac{h}{t_w}\sqrt{\frac{F_{yw}}{k}} > 234 \tag{6}$$

the web buckles locally in the absence of yielding, and

$$V_n = \frac{26{,}400}{(h/t_w)^2}kA_w \tag{7}$$

If the modified web slenderness lies between the values of Equations 4 and 6, then there is an interaction between yielding and buckling, and the LRFD uses

$$V_n = 0.6F_{yw}\left\{\frac{187\sqrt{k/F_{yw}}}{(h/t_w)^2}\right\}A_w \tag{8}$$

A similar philosophy exists in the Australian limit states steel standard AS 4100[4] and in the British limit states BS 5950,[5] except that in the former code no provision is made for the interaction of buckling and yielding.

This paper presents a method, based on a theoretical study, which allows for the enhancement of the resistance $V_n$ of a web plate in a doubly symmetric I-beam, when the design is carried out by ignoring the favorable tension field[2] contribution $C_v$ in the LRFD. This design proposal is based on the use of accurately determined values of the elastic local buckling coefficient $k$. The theory, which has been well-documented elsewhere,[6,7] is described only very briefly, and design charts are presented for the elastic local buckling coefficient which incorporate the favorable effect which the flange restraint provides to the web. The proposed design method, which should lead to improved economies of steel, is then illustrated by an example.

## 2. THEORY

The local buckling coefficients for stiffened webs of I-beams, such as that shown in Figure 1, were obtained by means of the elastic complex finite strip method.[6,7] This is a form of the semi-analytical finite strip method presented by Cheung,[8] but it employs complex arithmetic to handle shear stresses which cannot easily be treated by Cheung's formulation.

The finite strip method differs from the finite element method by the way in which the I-section is subdivided. For the finite strip method, the member is subdivided into longi-

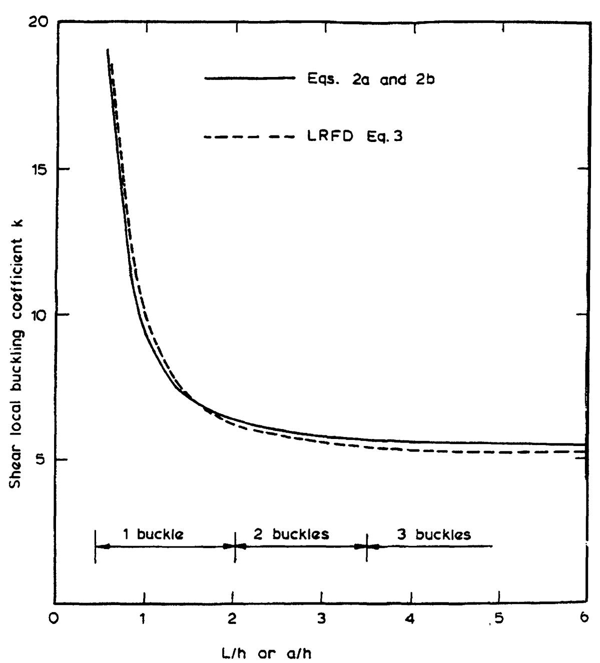


*Fig. 2. Buckling coefficient of web.*

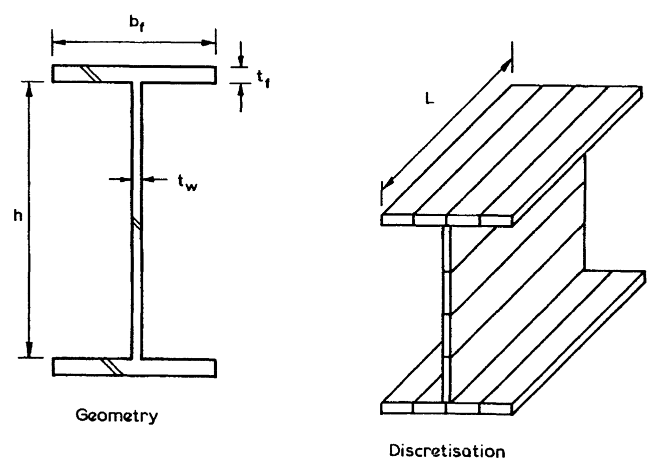


*Fig. 3. Discretized I-section.*

tudinal strips, as shown in Figure 3, and the longitudinal variation of displacements is represented by harmonic functions. In the finite element method, the strips would be further subdivided into rectangular elements whose displacements are represented by polynomials.

After assembly of the strip elastic stiffness and stability matrices, the global buckling equation, based on the principle of virtual work, becomes

$$([K] - \lambda[G]).\{\Delta\} = \{0\} \tag{9}$$

where

$[K]$ and $[G]$ = the generally complex global stiffness and stability matrices respectively
$\lambda$ = the buckling load factor by which the initial shear stress distribution is multiplied at buckling, and
$\{\Delta\}$ = the global buckling degrees of freedom.

The buckling condition for nontrivial $\{\Delta\}$ is that

$$|[K] - \lambda[G]| = 0 \tag{10}$$

the solution of which can be obtained for $\lambda$ by complex eigenvalue routines.[7] With each flange divided into four strips and the web divided into six equal width strips, the solution of Equation 10 was very rapid on a desktop workstation.

## 3. APPLICATION

The complex finite strip method outlined in the previous section was used to analyze doubly symmetric I-beams under pure shear for a wide range of geometries. Similar parameter studies were also undertaken by the author in Refs. 8 and 9. In all cases, a value of $h/t_w$ of 200 was adopted. It was found that this ratio does not affect the values of $k$, because of the non-dimensional way in which they are plotted. The rationale behind this simplification in plotting the results is explained more fully in Ref. 10.

Figures 4a through 4d plot the elastic local buckling coefficient $k$ as a function of the dimensionless half-wavelength $L/h$. The values of $k$ were obtained from Equation 1 with $\nu = 0.3$, and it is evident that the inclusion of flange restraint in these figures produces garland shaped curves. It can be seen that the minimum local buckling coefficient varies from 5.35 for a simply supported (S.S.) web to 8.95 for a web with built-in (B.I.) longitudinal edges. The value of 5.35 is close to the LRFD value of 5 for an unstiffened web. From these curves, increasing the stockiness of the flange relative to the web results in more web restraint, and hence a larger value of $k$.

For an unstiffened web, the limiting values of 5.35 (S.S.) and 8.95 (B.I.) are universally accepted,[3] and agree with elastic buckling tests of long plates. However, the experiments which have been undertaken on real transversely stiffened steel beams have their strengths enhanced by tension field action.[2] Because of this, experiments on steel beams are not of relevance in validating the elastic theoretical solutions plotted in Figures 4a through 4d. However, the finite strip theory used to derive these curves has been validated in Refs. 6 and 7 with independent theoretical treatments, so that confidence can be placed in using these elastic buckling solutions.

Because the buckling coefficients in Figures 4a through 4d are plotted as a function of the buckling half-wavelength $L$, the translation of each of the curves to the right by $nL$, where $n = 2, 3, 4, \ldots$, will produce an ensemble of curves whose lower bound represents the variation of $k$ with the length $a$ between

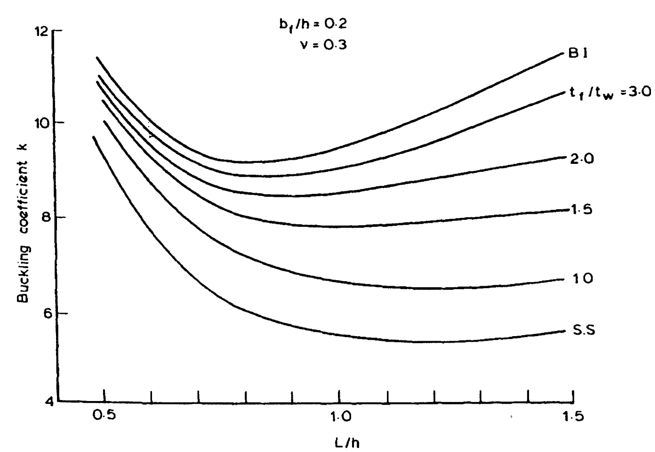


*Fig. 4a. Buckling curves.*

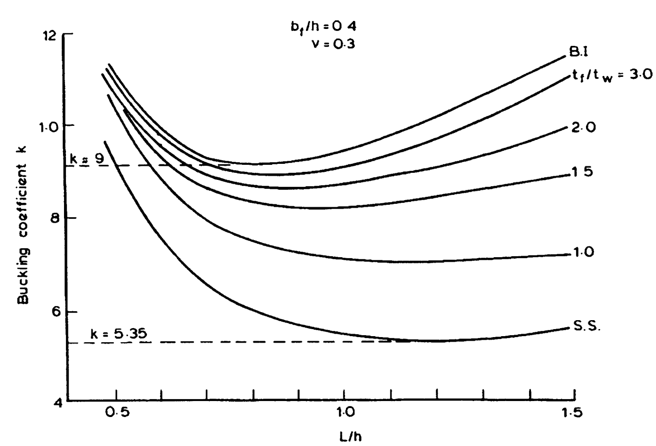


*Fig. 4b. Buckling curves.*

transverse stiffeners.[3] When this is done? the lower bound curve forms a basis for obtaining the local buckling coefficient for any value of the non-dimensional stiffener spacing $a / h$.

By fitting a quadratic interpolating function through three points close to the localized minima as depicted in Figure 4, a design curve for the minimum local buckling coefficient which is applicable for unstiffened webs is obtained. This was done in Figure 5, where the local buckling coefficient $k$ in pure shear is plotted against the flange to web width ratio $b_f / h$ for various values of $t_f / t_w$. It is clear from the graph that the values of $k$ remain relatively stationary for values of $b_f / h$ above about 0.5.

Figures 4a through 4d form a rational basis for obtaining the value of $k$ to be used in Equations 4, 6, 7 and 8 herein which are presented in the LRFD Specification. The calculated strengths are those obtained by ignoring tension field action, which is often done in design. Calibration of the Australian AS 4100[4] has been performed, in which tension field action was included,[11] and consistent values of the reliability index $\beta$ greater than 3 were obtained. The Australian standard is less conservative than the LRFD, so that deployment of the design method, even for webs where the tension field inelastic shear buckling coefficient $C_v$ is used, should result in a consistent and acceptable value for the reliability index $\beta$.

The formulation herein, however, was intended for transversely stiffened webs where tension field action is neglected. Because the values of $k$ in the design graphs are greater than those of Equation 3, which is based on a simply supported web, it is recommended that they be used in design to produce greater economy of steel.

## 4. DESIGN EXAMPLE

### 4.1. Problem

Calculate the design shear strength of a fabricated I-beam with a web of Grade 36 steel with $h$ = 36 in.; $b_f$ = 7.2 in.; $t_w$ = 0.4 in.; and $t_f$ = 1.2 in. The stiffener spacing is 60 in. Ignore the tension field contribution.

### 4.2. Solution

$$a / h = 60 / 36 = 1.67$$

$$b_f / h = 7.2 / 36 = 0.2$$

$$t_f / t_w = 1.2 / 0.4 = 3.0$$

From Figure 4a, $k$ = 10.8

$$\frac{h}{t_w}\sqrt{\frac{F_{yw}}{k}} = \frac{36}{0.4}\sqrt{\frac{36}{10.8}} = 164 < 187$$

The web therefore yields before buckling.

From Equation 5, using $\phi_v$ = 0.9,

$$\phi_v V_n = 0.9 \times 0.6 \times 36 \times 0.4 \times 36 \text{ kips}$$

$$= 279.9 \text{ kips}$$

If the present provisions of the LRFD are used, then from Equation 3

$$k = 5 + \frac{5}{(60 / 36)^2} = 6.8$$

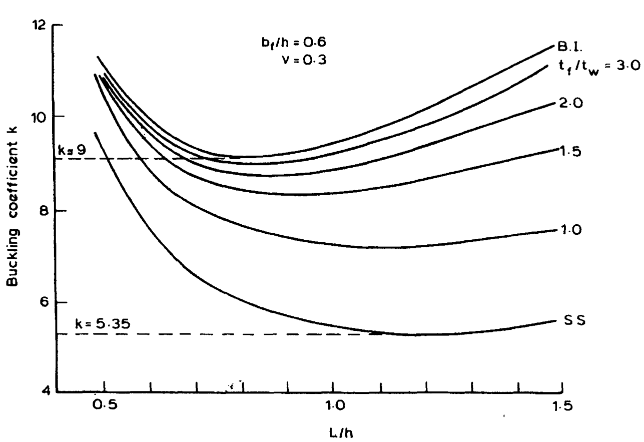


*Fig. 4c. Buckling curves.*

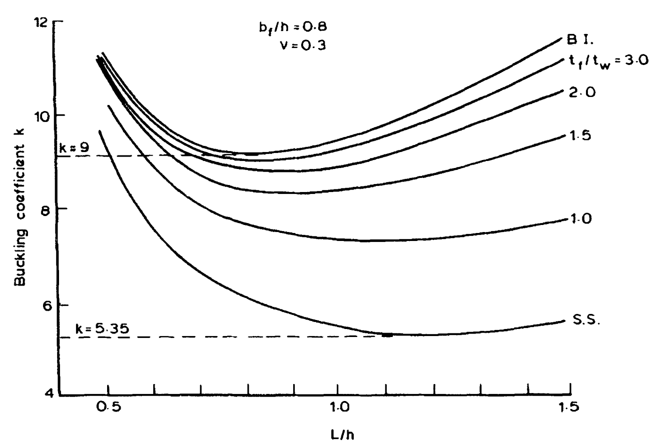


*Fig. 4d. Buckling curves.*

$$\frac{h}{t_w}\sqrt{\frac{F_{yw}}{k}} = \frac{36}{0.4}\sqrt{\frac{36}{6.8}} = 207 > 187$$

Hence Equation 8 governs, and

$$V_n = 0.6 \times 36 \left\{ \frac{187\sqrt{6.8/36}}{(36/0.4)^2} \right\} \times 0.4 \times 36 \text{ kips}$$

$$= 280.9 \text{ kips}$$

and

$$\phi_v V_n = 0.9 \times 280.9 = 252.8 \text{ kips}$$

The design proposal thus represents an increase in strength of 11 percent over the strength calculated according to the LRFD Specification.

## 5. CONCLUDING REMARKS

Based on the complex semi-analytical finite strip method of analysis, design graphs were obtained for the local buckling coefficient of the webs of I-beams under shear. The variation of this coefficient with the section geometry was demonstrated. Local buckling coefficients greater than that of the LRFD Specification, which are based on a web with simply supported edges, were obtained. It is proposed that these enhanced local buckling coefficients be used to calculate the shear resistance of I-beams. The savings which may be achieved were illustrated by an example.


## 6. ACKNOWLEDGEMENTS

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## 8. NOTATION

| | |
|---|---|
| $A_w$ | net area of web |
| $a$ | transverse stiffener spacing |
| $b_f$ | flange width |
| $E$ | Young's modulus of elasticity |
| $F_{ol}$ | elastic buckling stress in shear |
| $F_{yw}$ | yield stress of web |
| $h$ | web depth |
| $k$ | web local buckling coefficient in shear |
| $L$ | buckling half-wavelength |

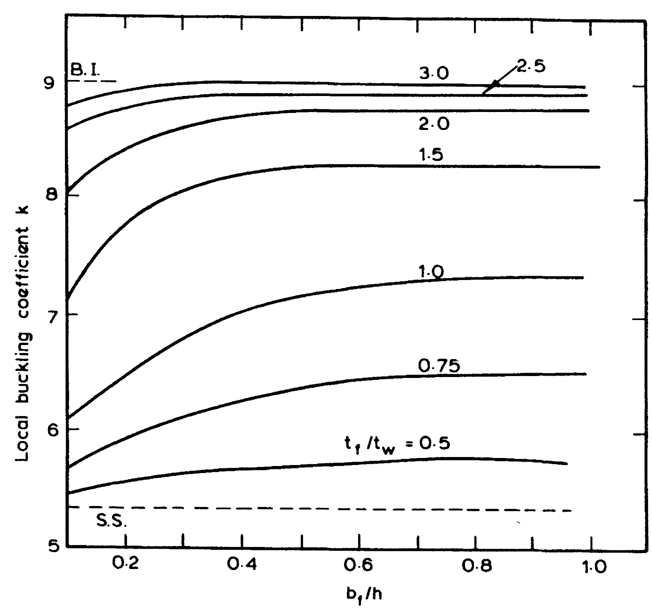


*Fig. 5. Local buckling design chart.*

$t_f$ thickness of flange
$t_w$ thickness of web
$V_n$ nominal shear resistance
$\beta$ reliability index
$\nu$ Poisson's ratio
$\phi_v$ shear resistance factor