# A Closer Examination of the Axial Capacity of Eccentrically Loaded Single Angle Struts

LeROY A. LUTZ

## INTRODUCTION

The purpose of this article is to address the axial capacity evaluation of eccentrically loaded single angles. The focus will be to examine and expand on the axial capacity evaluation made in Example 3-8 of the 1993 LRFD *Manual.*[1]

The Example as presented did not illustrate the less conservative approach permitted by the *Specification for Load and Resistance Factor Design of Single-Angle Members*. The evaluation of the axial capacity with due consideration of the sign of the stress is addressed. Also, since the example is supposed to represent the capacity obtained by load from a gusset plate attached to one leg, an examination of this situation is made.

## EXAMPLE 3-8 REVISITED

The L2×2×¼ used in Example 3-8 is shown in Figure 1. Added to the Example's Figure is the corrected $e_x$ value of 0.792 and the distance of the load point from the outside of the angle legs based on the $e_w$ and $e_z$ values shown.

The less conservative and more correct approach permitted by the single angle specification will be addressed first using the load eccentricities shown in Figure 1 and an effective length $KL = 4.0$ ft. Since stress from both $e_z$ and $e_w$ eccentricity should be added at some common location at the tips of the angle legs, the point at mid-thickness of angle leg is suggested as the best location. This point is very close to the actual $c_z$ on the radius.

The radius on the angle is not defined for a section and will vary with producer. This location is not exact for obtaining the extreme major axis stress, but the minor axis stress is more sensitive in the stress evaluation due to the smaller section modulus. These locations shown as points A and B in Figure 1, can be used for both unequal leg and equal leg angles.

For the L2×2×¼

$$I_z = Ar_z^2 = 0.938(0.391)^2 = 0.1434 \text{ in.}^4$$

$$I_w = 0.348 + 0.348 - .143 = 0.553 \text{ in.}^4$$

LeRoy A. Lutz is structural engineer and vice president, Computerized Structural Design, Inc., Milwaukee, WI, and chairman, AISC Task Committee 116 on single angles.

At the angle corner (Point C)

$$S_z = 0.1434 / 0.837 = 0.1713 \text{ in.}^3$$

At the angle tip (Points A & B)

$$S_z = 0.1434 / ((2 + .25/2)/\sqrt{2} - .837) = 0.2154 \text{ in.}^3$$

$$S_w 0.553 / ((2 - .25/2)/\sqrt{2}) = 0.417 \text{ in.}^3$$

**Determine $M_{nw}$:**

Per Single Angles Section 5.3.1

$$M_{ob} = C_b 0.46Eb^2t^2 / L$$

$$= 1.0(0.46)(29{,}000)2^2(.25)^2 / 48 = 69.5 \text{ k-in.}$$

$$M_y = F_y S_w = 50(0.417) = 20.85 \text{ k-in.}$$

Since $M_{ob} > M_y$ (Section 5.1.3),

$$M_{nw} = [1.58 - 0.83\sqrt{20.85/69.5}]M_y = 1.125M_y < 1.25M_y$$

based on lateral-torsional buckling

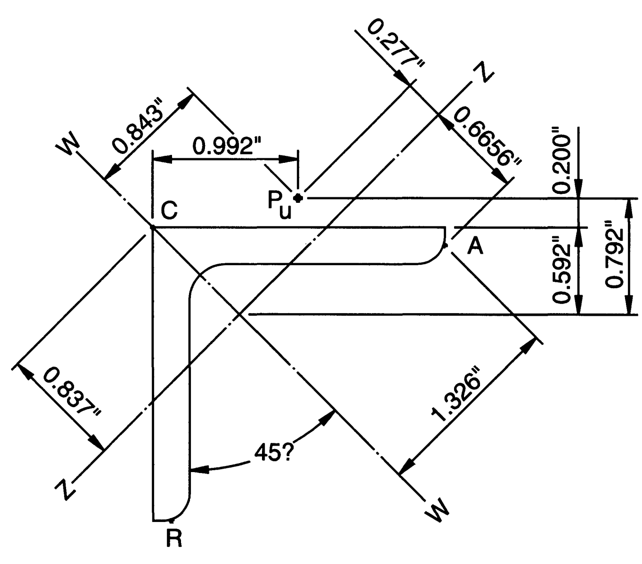


*Fig. 1. Example 3-8 angle.*

Check local buckling:

$$b/t = 2/0.25 = 8 < 0.382\sqrt{E/F_y} = 9.2$$

which permits an $M_n = 1.25M_y$

$$\therefore\ M_{nw} = 1.125(20.85) = 23.46 \text{ k-in.}$$

**Determine $M_{nz}$ at two locations:**

Per local buckling stress limits, $M_{nz} = 1.25M_y$. The $M_y$ used is the yield moment corresponding to the limit state at the point in question.

At angle corner,

$$M_{nzc} = 1.25(50)(0.1713) = 10.71 \text{ k-in.}$$

At angle tip,

$$M_{nzt} = 1.25(50)(0.2154) = 13.46 \text{ k-in.}$$

**Determine B1 factors for use in Equation 6-1a**

$$r_w = \sqrt{0.553/0.938} = 0.768 \text{ in.}$$

$$KL/r_w = 1.0(48)/0.768 = 62.5$$

From LRFD Spec. Table 8, $P_e/A_g = 73.3$

$$P_{ew} = 73.3(0.938) = 68.7 \text{ kips}$$

$$B_{1w} = 1/(1 - P_u/68.7)$$

$$KL/r_z = 1.0(48)/0.391 = 122.8$$

From Table 8, $P_e/A_g = 19.0$

$$P_{ez} = 19.0(0.938) = 17.8 \text{ kips}$$

$$B_{1z} = 1/(1 - P_u/17.8)$$

$C_m = 1$ is used above for both values of $B_1$ and should also have been 1 in the original Example 3-8.

**$P_u$ Evaluation with Due Regard to Sign**

Determine $P_u$ based on application of Equation 6-1a at the angle corner.

$\phi P_n = 14$ kips from Columns Table for L2×2×¼ with $KL = 4$ ft.

$$\frac{P_u}{14} + \frac{8}{9}\left(0 + \frac{0.277P_u(1)}{0.9(10.71)(1 - P_u/17.8)}\right) = 1 \qquad (1)$$

The w-axis flexural term is zero since the angle corner is on the w-axis. In the above expression, $P_u = 8.4$ kips.

Determine $P_u$ based on application of Equation 6-1a at point A in Figure 1.

$$\frac{P_u}{14} + \frac{8}{9}\left(\frac{0.834P_u(1)}{0.9(23.46)\left(1 - \frac{P_u}{68.7}\right)} + \frac{(-)0.277P_u(1)}{0.9(13.46)\left(1 - \frac{P_u}{17.8}\right)}\right) \qquad (2)$$

Based on the position of the load, it is clear that the moment $0.277P_u$ should be assigned a negative value since it produces tension at point A.

Substitute in the value of 8.4 kips obtained above for $P_u$

$$0.60 + \frac{8}{9}[0.382 - 0.364] < 1$$

Since the summation is less than 1 with $P_u = 8.4$ kips, a larger value of $P_u$ would be required for the sum to be 1. Therefore the 8.4 kips value represents the axial capacity of the cross section.

Had the negative sign not been used in the z-axis term interaction term of Equation 2, $P_u = 6.96$. Thus over a 20 percent increase in capacity is achieved by evaluating the interaction expression and considering the proper signs for the terms.

Evaluation of the interaction expression at point B is accomplished by using a negative sign for both flexural terms in Equation 2. With a value of $P_u = 8.4^k$, the interaction terms is –0.063. Therefore, $P_u = 8.4$ kips based on application of the interaction expression at point C (the angle corner).

## CONSIDERATIONS AS A WEB STRUT IN A TRUSS

The treatment of Example 3-8 to this point addresses the case of a single angle with an unbraced length of 48 inches and effective length factor $K = 1$ loaded at the prescribed location as shown in Figure 1. As a web strut in a truss, consideration must be given to both the *x* and *y* positions of the load and the effective length factor that is appropriate for the condition being considered.

### *X*-Location

The stresses produced in the angle are very sensitive to the *x*-location of the load as pointed out by McGuire.[2] The chord and/or gusset of the truss represents a significant rotational restraint to the single angle strut. The restraint about the y-axis (approaching a fixed-end condition) would tend to produce a uniform stress along the supported leg of the angle.

If the applied load would produce a uniform stress *at each end* of the member along the center of thickness of supported leg, then

$$\frac{(P_u e_z)(0.837 - 0.125\sqrt{2})}{0.1434} = \frac{P_u e_w}{0.417} - \frac{P_u e_z}{0.2154}$$

$$9.247e_z = 2.398e_w$$

Therefore $e_z = 0.2594e_w$ for this angle. As noted by Woolcock and Kitipornchai,[3] the 0.2594 represents the ratio of $I_z/I_w$, the eccentricity relationship that introduces x-axis bending to the web strut.

With $e_x$ remaining at 0.792 inches one can calculate that $e_w = 0.8894$ inches and $e_z = 0.0307$. The $e_y$ is now 0.466 inches as compared to 0.400 inches as illustrated in Figure 2.

Evaluating the interaction expression at the angle corner with the new $e_z$

$$\frac{P_u}{14}+\frac{8}{9}\left(0+\frac{0.2307}{0.9(10.71)}\left(\frac{P_u}{1-P_u/17.8}\right)\right)=1$$

$P_u = 8.81$ kips

This controls over the $P_u$ obtained from evaluation at point A, which indicates that due to moment magnification, the critical stress is at the angle corner (Point C). Even with uniformly applied stress along one leg at each end of the member, the stress at midspan is quite non-uniform. Undoubtedly with full restraint about the y-axis $e_y$ will be larger then 0.466 inches with the maximum compressive stress at each end occurring at point A, but with the maximum mid-length stress still occurring at the angle corner.

### *Y*-Location

If the tension and compression web struts are attached to the same side of the truss, the eccentricity $e_x$ can be considered less than that to the center of the web. Woolcock and Kitipornchai[3] recommended that an eccentricity of $c_x - t/2$ be used for this situation.

Thus $e_x = 0.592 - 0.25/2 = 0.467$ and if

$e_z = 0.2594e_w$ (as in previous calculation) and $e_z + e_w = e_x\sqrt{2}$ then $e_z = 0.136$ inches (See Figure 2)

Evaluating $P_u$ from interaction expression at the angle corner

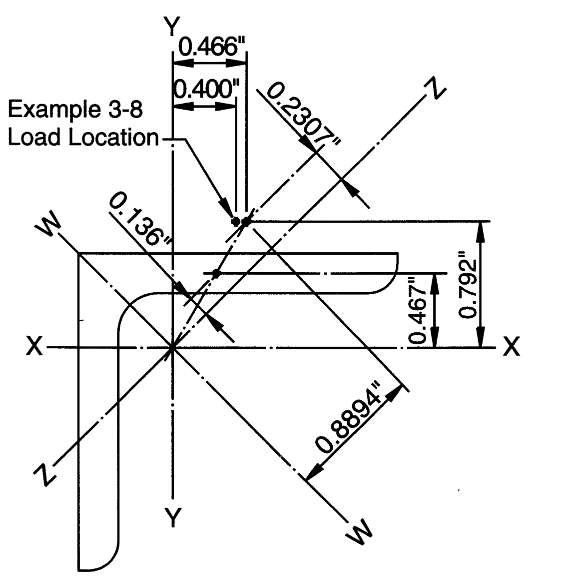


*Fig. 2. L2×2×¼ with new load locations.*

$$\frac{P_u}{14}+\frac{8}{9}\left(\frac{0.136P_u}{0.9(10.71)(1-P_u/17.8)}\right)=1$$

$P_u = 10.0$ kips

### Effective Length

End restraint not only reduces the applied end eccentricity to a single angle strut, but also reduces the effective length of the strut. Since the axis of the chord is not in line with the principal axes of the single angle strut, it is extremely difficult to evaluate effective length factors for single angle struts.

The ANSI/ASCE Standard 10-90, Design of Latticed Steel Transmission Structures[4] has a design procedure that considers the end restraint that occurs in single angle diagonal struts. Using this procedure the 48-in. long L2×2×¼ would have a $P_u$ value of 14.3 kips if each end is attached by welding or by two bolts.

If $e_z$ were 0.136 inches, an effective length considerably less than 1 would have to be used in order to find $P_u = 14.3$ kips using the AISC LRFD procedure for single angle web members in transmission structures. Since the axial capacity $\phi P_n$ is 14 kips for an effective length factor of 1, the effective length factor would have to be less than 1 if any amount of eccentricity exists in order to achieve a 14.3 kips axial capacity.

The intent here is not to suggest that the ANSI/ASCE 10-90 design value is correct, but that use of an effective length factor less than one may be appropriate. The effective length factor inherent in the ANSI/ASCE 10-90 procedure recognizes a considerable end restraint that may not be reached for the typical web strut in a truss.

## SUMMARY

Use of the AISC LRFD procedure for the design of single angle struts subjected to eccentric axial loads has been examined.

It has been demonstrated that consideration of the sign of the stress at a location when evaluating the interaction expression leads to a significant increase in axial capacity.

The selection of the eccentricity is discussed for the case where it is meant to represent the load application of a web strut attached at one leg to a chord web or gusset plate. The engineer must consider both the appropriate $e_y$ and appropriate $e_x$ based on the restraint conditions that exist. Use of effective length factor is also necessary to be able to determine the full capability of the single angle web strut.

It does appear that application of the less conservative AISC LRFD procedure for the web strut involves evaluation of the interaction expression at the angle corner only, regardless of what eccentricity and effective length factor are used. This means that only one flexural term which contains the $e_z$ value is involved in the evaluation of $P_u$.

## REFERENCES


1. *Load and Resistance Factor Design Specification for Structural Steel Buildings*, AISC, Chicago, IL, 1993.
2. McGuire, W., *Steel Structures*, Prentice-Hall Inc., Englewood Cliffs, NJ, 1968.
3. Woolcock, S. T. and Kitipornchai, S., "Design of Single Angle Web Struts in Trusses", *Journal of Structural Engineering*, ASCE, Vol. 112, No. 6, June, 1986.
4. ANSI/ASCE 10-90 *Design of Latticed Steel Transmission Structures*, ASCE, New York, NY, 1992.