

# Steel Rigid Frames With Leaning Columns — 1993 LRFD Example

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## ABSTRACT

Many buildings rely on rigid frames to provide lateral stability in one or both directions. Steel rigid frames often support “leaning” columns which hold their contributory gravity loads but rely on the frame for lateral stability. Rigid frames supporting leaning columns require special consideration whether a first-order or second-order analysis is undertaken. The design and analysis of rigid frames with leaning columns in accordance with the 1993 AISC LRFD is studied through use of an example eight story steel building which is supported laterally in each direction by isolated rigid frames. Results of first-order analyses with approximate moment magnification are compared with plane frame and space frame second-order elastic analyses.

## INTRODUCTION

Steel rigid frames have proven to be an effective and versatile structural system for resisting building loads. Due to the high ratio of strong axis to weak axis bending strength of typical wide flange shapes, and the desire to avoid complicated connections, steel buildings are often designed such that a single column is not integrated into perpendicular rigid frames. Columns which are not part of a rigid frame do not resist lateral loads since the beam to column connections provided in the plane of the considered lateral loads are not designed to resist bending moment. These columns are commonly referred to as “leaning” columns for that direction of load because they depend on, or “lean” on the rigid frames for lateral stability.

Special consideration is required in the design and analysis of rigid frames with leaning columns. Due to relatively high lateral deformations of rigid frames, second-order or “p-delta” effects must be considered. When a rigid frame supports leaning columns, it must be designed to resist the second-order effects of the gravity loads on the leaning columns as well as the second-order effects of the gravity loads applied directly to the frame. Furthermore, the effective

length factor for in-plane buckling of rigid frame columns must be increased to account for the destabilizing effects of the leaning column gravity loads.

In both first-order and second-order analyses there are uncertainties about the appropriate consideration of the leaning columns on the rigid frames. One common practice is to completely ignore these columns’ effects when designing a rigid frame. In this approach the rigid frame is analyzed as a plane frame subjected to the actual lateral and gravity loads applied to the frame. However, when a rigid frame sways from the lateral loads, shears develop in the leaning columns. These shears are opposite in direction to the shear forces in the frame, thereby increasing the shears (second-order) in the rigid frame. For multi-story frames, the inclusion of leaning columns in the structural model affects the rigid frame shears, even in a first-order analysis. This effect happens because the leaning columns, which are continuous through the floor levels, must bend to conform to nonuniform story drifts. Although it is generally negligible, one way to quantify the first-order effect is to run a space frame analysis, with the leaning columns being continuous through each floor level and connected to the structure with pin ended beams. First-order effects of the leaning columns can also be accounted for in a plane frame analysis by using a “dummy” column which is connected to the frame with pinned end beams and has a moment of inertia equal to the sum of the in-plane moments of inertia of all columns leaning on the frame being analyzed.

The exact or approximate computation of second-order effects for frames with leaning columns should account for the fact that the entire building weight contributes to the p-delta effect. First-order analyses, whether for a plane frame or space frame, do not include second-order effects which must be considered in design. The 1989 AISC *Allowable Stress Design (ASD) Specification and Commentary*<sup>1</sup> gives no guidance for treating leaning columns and there is no apparent means of accounting for them in the ASD moment magnifiers. A detailed presentation on an approximate manual analysis and design of steel rigid frames with leaning columns based on the AISC ASD manual<sup>1</sup> was given by LeMessurier<sup>2</sup> in 1977. Since that time it has become common to use numerical (computer) analyses of rigid frames, and the AISC *Load and Resistance Factor Design (LRFD) Specification*<sup>3</sup> has been introduced. The second edition of the LRFD

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manual<sup>4</sup> outlines considerations which must be given to rigid frames with leaning columns. These issues will be examined in detail with an example building in the next section.

The LRFD allows a second-order elastic analysis to be used in place of the approximate moment magnifiers. It is important that the leaning column effects be included in such analyses, and when performing a second-order planar analysis, the question of how to include the leaning columns again arises. At least two solutions are possible. The first is to place additional lateral loads on the frame.<sup>5,6</sup> A second approach is the dummy column method described earlier. In this case the gravity loads on all the leaning columns are summed and placed on the dummy column at each floor level. A second-order space frame analysis that models all of the columns will automatically account for the second-order effects of the leaning columns on the rigid frames.<sup>7</sup>

Several analyses were performed on an example building to quantify the leaning column effects on rigid frame design and analysis in accordance with the 1993 LRFD.<sup>4</sup> First-order and second-order plane frame computer analyses and first-order and second-order space frame analyses were performed on an eight story building with rigid frames and leaning columns.

### EXAMPLE BUILDING

Shown in Figures 1 to 3 are a framing plan and elevations of the rigid frames in an eight story structural steel building. The building is laterally supported in the NS direction by rigid frames C1-C2 and C7-C8 and in the EW direction by rigid frames C3-C5 and C4-C6. For lateral loading in the NS direction, columns C3 to C6 are leaning on rigid frames C1-C2 and C7-C8. Due to the symmetry of this structure, the discussion in this paper will focus only on the design of rigid frame C1-C2 for a NS loading. The lower section of leeward column C2 will be evaluated for compliance with the 1993 AISC LRFD<sup>4</sup> assuming: 50 ksi yield steel, earthquake loads not governing, and other loadings as stated below. For the

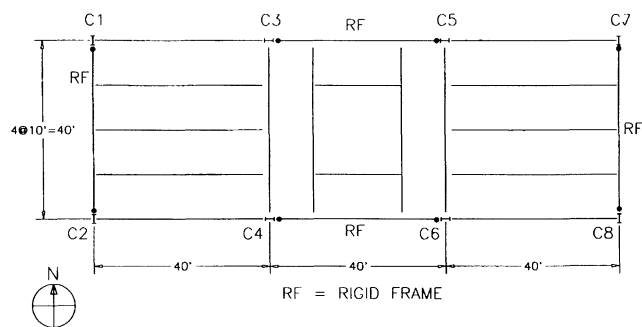


Fig. 1. Floor and roof framing plan.

rigid frame members the governing load combination is: 1.2 Dead (*D*) + 0.5 Live (*L*) + 1.3 Wind (*W*). Loads on the structure are: wind load = 30 psf from ground level to 48.5 ft. and 40 psf from 48.5 ft. to the top; floor dead load on the roof and floors = 90 psf; dead load of exterior walls and 3-ft. high parapet = 30 psf; live load on roof = 30 psf; and, live load on floors = 80 psf (reduced to 50 psf for 2nd through 5th floors). The exterior walls span vertically between floor levels and extend 1 ft. beyond the corner column centerlines. Loads for the factored gravity and wind load combination are shown in Figure 4. Column C2 from ground to the second floor, a W36x230 member bent about its strong axis, will be checked in accordance with the 1993 LRFD using several alternate treatments of the leaning columns.

### NUMERICAL ANALYSES

The numerical model for a plane frame computer analysis is shown in Figure 5. A “dummy” column is added to the model to represent all of the columns leaning on this frame. In this case the dummy column represents columns C3 and C4, and is loaded with the sum of their gravity loads. The dummy column is connected to the frame with pin ended beams and has a moment of inertia equal to the sum of the in-plane moments of inertia of the leaning columns. The following cases were run for the plane frame model:<sup>7</sup>

- a. first-order analysis for 1.2*D* + 0.5*L* + 1.3*W* [Case P1];
- b. first-order analysis for 1.2*D* + 0.5*L* [Case P1N];

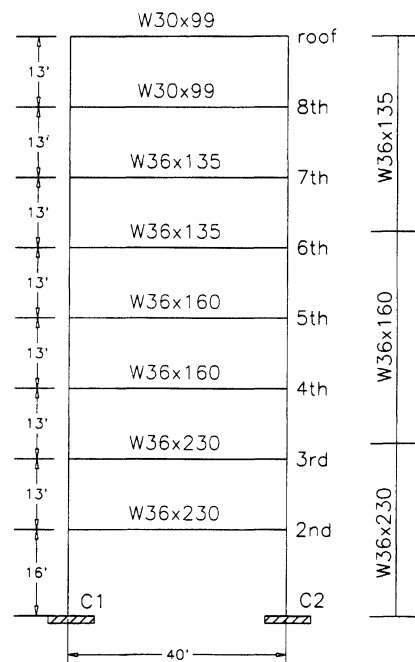


Fig. 2. Rigid frame C1-C2 (C7-C8 similar).

- c. second-order analysis for  $1.2D + 0.5L + 1.3W$  with the dummy loaded [Case P2D]; and,
- d. second-order analysis for  $1.2D + 0.5L + 1.3W$  with the dummy unloaded [Case P2].

The resulting column moments are shown in Figures 6(a) to 6(d), respectively. Column axial loads and lateral story deflections are given in Tables 1 and 2, respectively.

The inclusion of the dummy column has a small effect on the first-order (linear elastic) analysis results since the column has a relatively small moment of inertia and since the story drift levels are approximately equal (first-order base shear in the dummy column is  $-0.77$  kip). The dummy column does not significantly affect the second-order results by forcing the rigid frame to resist the p-delta effects from the leaning columns (second-order base shear in dummy column is  $-9.76$  kips).

First and second-order elastic space frame analyses were performed<sup>7</sup> for the case  $1.2D + 0.5L + 1.3W$ , and are designated as cases S1 and S2, respectively. Due to symmetry, only half of the building, including columns C1, C2, C3, and C4, was included in the space frame model. As a condition of this symmetry, columns C3 and C4 were restrained from moving out of the plane containing them at each floor level. Rigid frame column bending moment diagrams are shown in Figure 7. The resulting column axial loads and frame deflections are given in Tables 1 and 2, respectively.

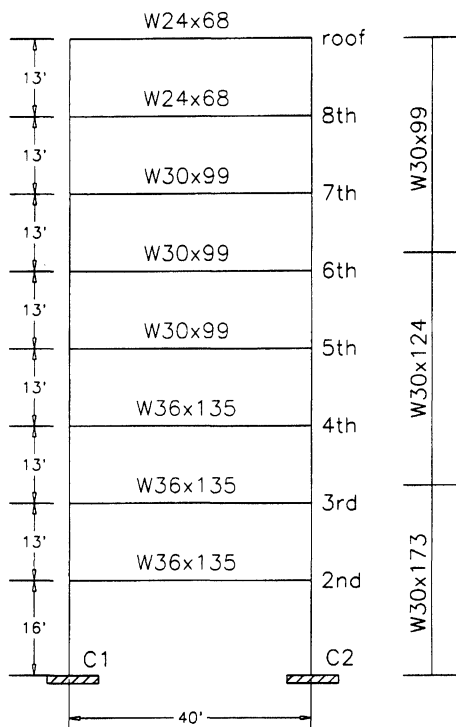


Fig. 3. Rigid frame C3-C5 (C4-C6 similar).

## COLUMN DESIGN

In all analyses for Column C2 from ground to the second floor; the effective length factor for weak axis buckling equals unity since the column does not resist any EW loads; and, the unbraced length for column buckling and lateral torsional buckling is 16 ft. The effective length factor for strong axis buckling,  $k_x$ , is determined from the nomograph in the AISC Manuals based on the fixity factors at each end of the column.<sup>1,3,4</sup> For C2 from the ground to the 2nd floor, the fixity factors are 1.0 at the base (fixed) and 5.58 at the 2nd floor joint, from which  $k_x$  is found to be 1.75 from the nomograph.

For rigid frames with leaning columns, Chapter C in the 1993 LRFD Commentary<sup>4</sup> states that when  $k_x$  is found from the nomograph it should be increased by multiplying it by  $N^{0.5}$ , where  $N$  is a destabilizing factor for leaning columns.  $N$  is equal to the sum of the axial forces in all columns of a story (from  $1.2D + 0.5L$ ) divided by the sum of the axial forces in the columns of rigid frames in the direction considered for same load case. In this case  $N$  equals the axial force ratio:  $(C1 + C2 + C3 + C4) / (C1 + C2) = (304 + 1060 + 1113 + 1113) / (304 + 1060) = 2.63$ , thus  $N^{0.5}$  equals 1.62. A quick approximation of  $N$  can be determined by assigning one part gravity load to the corner columns, two parts to the exterior non-corner columns, and four parts to the interior columns. The resulting  $N$  is the total number of parts divided by the number of parts assigned to the rigid frames in the direction considered. Here,  $N = (1 + 1 + 2 + 2) / (1 + 1) = 3.0$ . Multiplying

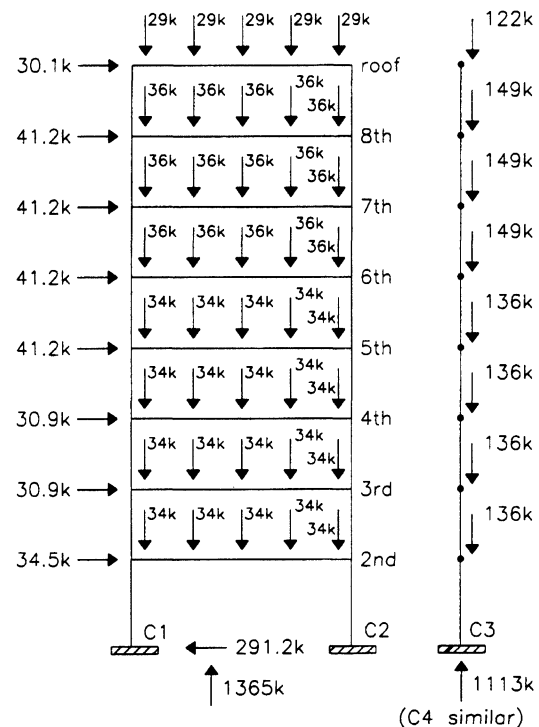


Fig. 4. Loads for  $1.2D + 0.5L + 1.3W$  (north-south wind).

Column	C1						C2						
	case	P1	P1N	P2D	P2	S1	S2	P1	P1N	P2D	P2	S1	S2
8-rf	63	73	63	63	63	63	82	73	82	82	82	82	82
7-8th	139	162	138	139	139	138	186	162	187	186	186	186	187
6-7th	197	252	194	196	197	194	308	252	311	309	308	308	311
5-6th	246	342	241	244	246	241	439	342	444	441	439	439	444
4-5th	275	427	266	272	274	266	580	427	589	583	581	581	589
3-4th	299	512	286	294	298	286	726	512	739	731	727	727	739
2-3rd	302	597	284	295	301	284	893	597	911	900	894	894	911
g-2nd	304	682	283	296	303	282	1060	682	1080	1070	1060	1060	1080

Key: P = Plane Frame; S = Space Frame; 1 = First-Order Analysis; 2 = Second-Order Analysis;  
N = no Wind; D = Dummy Column Loaded

$k_x$  from the nomograph (1.75) by the exact value of  $N^{0.5}$  (1.62) yields a modified value of  $k_x$  of 2.84.

### FIRST-ORDER ANALYSIS

Chapter H of the AISC LRFD code<sup>4</sup> gives the following equation for checking beam-columns, when the axial force is at least twenty percent of the member's compressive strength:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \geq 1.0 \quad (1)$$

The terms in Equation 1 are defined in the LRFD code. In order to account for second-order effects the 1993 LRFD code states:

"...In structures designed on the basis of elastic analysis,  $M_u$  for beam-columns, connections, and connected members shall be determined from a second-order elastic analysis or from the following second-order analysis procedure:"

The following equation is given:

$$M_U = B_1 M_{nt} + B_2 M_{lt} \quad (2)$$

where:

$M_{nt}$  = the required flexural strength in a member assuming there is no lateral translation of the frame and

$M_{lt}$  = the required flexural strength in member as a result of lateral translation of the frame.

(For biaxial bending there is a  $M_{ux}$  and a  $M_{uy}$  and all of the terms of Equation 2 require the appropriate  $x$  or  $y$  subscript.) Commonly,  $M_{nt}$  is the moment due to gravity loads and  $M_{lt}$  is the moment due to lateral wind loads. Separation of these moments often requires that two analyses be performed as done for this example [Cases P1 and P1N] with results shown in Figures 6a and 6b and Tables 1 and 2. Furthermore,  $M_{nt}$  and  $M_{lt}$  refer to the maximum "nt" and "lt" moments whether or not they occur at the same location. In fact, for columns in rigid frames such as the considered example, these maximum moments often occur at opposite ends.

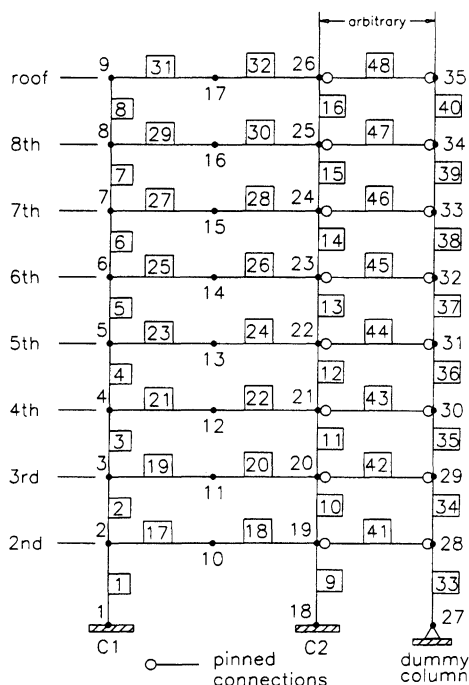


Fig. 5. Plane frame computer model.

case	P1	P1N	P2D	P2	S1	S2
roof	5.30	0	5.60	5.41	5.11	5.61
8th	4.89	0	5.18	5.00	4.91	5.19
7th	4.37	0	4.62	4.46	4.38	4.63
6th	3.73	0	3.95	3.81	3.74	3.96
5th	3.00	0	3.18	3.07	3.01	3.19
4th	2.21	0	2.34	2.26	2.22	2.34
3rd	1.43	0	1.51	1.46	1.43	1.51
2nd	0.74	0	0.78	0.75	0.74	0.78

Key: P = Plane Frame; 1 = First-Order Analysis; 2 = Second-Order Analysis; S = Space Frame; N = no Wind; D = Dummy Column Loaded

The calculation of  $B_1 = C_m / (1.0 - P_u/P_{el}) \geq 1.0$ , is intended to give the maximum moment, including second-order effects, due to the no-sway (gravity) loads. For beam-columns without transverse loads,  $B_1$  often calculates to less than unity and thus a value of 1.0 is used. This indicates that the maximum moment due to gravity loads, including second-order effects, is the same as the maximum linear elastic moment (i.e., no magnification of the maximum value) and occurs at the same location, namely one of the ends. If  $B_1$  calculates greater than unity when there are no transverse loads, then the maximum

second-order moment due to gravity loads occurs somewhere between the ends. This is uncommon since it requires either single curvature bending (negative  $M_1/M_2$  in the  $C_m$  equation) or an axial compression close to the in-plane no-sway elastic buckling strength  $P_{el}$ .

Section C1 in the 1993 LRFD<sup>4</sup> gives two alternate equations for  $B_2$ :

$$B_2 = \frac{1.0}{1.0 - \Sigma P_u \left( \frac{\Delta_{oh}}{\Sigma HL} \right)} \quad (3)$$

or

$$B_2 = \frac{1.0}{1.0 - \frac{\Sigma P_u}{\Sigma P_{e2}}} \quad (4)$$

where:

- $\Sigma P_u$  = required axial load strength of all columns in a story
- $\Delta_{oh}$  = the difference between the transverse deflections of the floors above and below the story considered, commonly referred to as the drift
- $\Sigma H$  = the sum of all story horizontal forces producing  $\Delta_{oh}$
- $L$  = the story height, and
- $\Sigma P_{e2}$  = the sum of the elastic in-plane sidesway buckling loads of all story columns resisting lateral loads in the direction considered.

When calculating  $\Sigma P_{e2} = \Sigma(\pi^2 EI) / (k_x L_x)^2$ , the appropriate

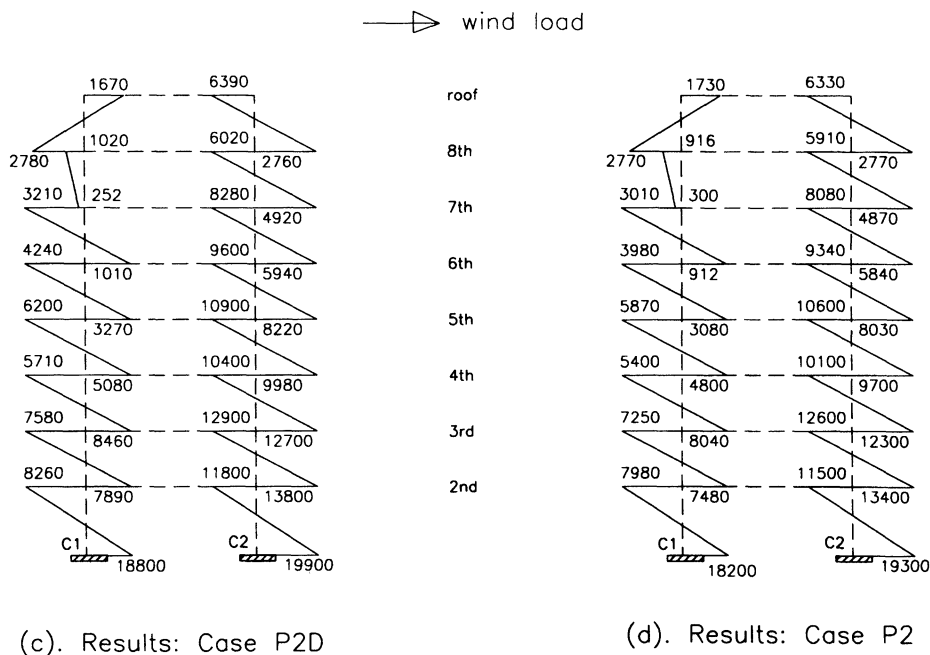


Fig. 6. Plane frame analysis results (cont.) column moments (in-kips).

$k_x$  value depends on how  $\Sigma P_u$  is calculated. If  $\Sigma P_u$  includes all story columns while the sum for  $P_{e2}$  only includes all lateral load resisting columns of the story, then  $k_x$  from the nomograph is not modified by the factor  $N^{0.5}$ . Alternately, if  $\Sigma P_u$  only includes loads directly applied to the rigid frame, then the sum for  $P_{e2}$  should only include the rigid frame columns and should be based on the modified  $k_x$  to account for the leaning column effects.

The intent of  $B_2 M_{nt}$  is to closely approximate the maximum moment, including second-order effects, due to the loads causing sway. For members without transverse loading, this maximum will virtually always occur at an end, the same end as the maximum linear elastic moment but often the opposite end from  $B_1 M_{nt}$ . For the most common case where there are no transverse loads, and the term  $B_1$  is unity (i.e., maximum  $B_1 M_{nt}$  occurring at an end), it is less conservative but more realistic to check the interaction equation separately at each end, not combining maximums that occur at different locations. For the rare case that  $B_1$  is greater than unity when there are no transverse loads, the maximum second order moment, although possibly not at an end, will still be less than the maximum of  $[B_1 M_{Jnt} + B_2 M_{Jlt}$  or  $B_1 M_{Knt} + B_2 M_{Klt}]$ , where  $J$  refers to one end of the member and  $K$  to the other. For members with transverse loads the interaction equation can be checked at several locations using the no-sway and sway moments for each location, or conservatively the  $B_2$  factor can be applied to the maximum total moment.

### EXAMPLE CALCULATION

Calculations of the interaction ratio for the lower section of C2 in the example structure are as follows. Using Figure 6b,

$C_m = 0.6 - 0.4 (605/1,830) = 0.468$ . The column axial force from Table 1 is  $P_u = 1,060$  kips.  $P_{el}$  is the elastic in-plane buckling load of the column based on an effective length factor of unity. Here  $P_{el} = 9.87EI_x/L^2 = 9.87(29,000)(15,000)/(192^2) = 116,000$  kips. The resulting  $B_1$  calculates as 0.47, and thus a value of 1.0 is used. This indicates that the maximum moment due to gravity loads, including second order effects, is the same as the maximum linear elastic moment (i.e., no magnification of the maximum value) and occurs at the same location, namely at the top end of this column.

For computing  $B_2$ :

$$\begin{aligned} \Sigma P_u &= 304 + 1,060 + 1,113 + 1,113 = 3,590 \text{ kips} \\ \Sigma H &= 291.2 \text{ kips, and} \\ \Delta_{oh} &= \text{horizontal deflection of second floor with respect to the ground floor} = 0.737 \text{ in. (from first-order analysis, see Table 2)} \\ L &= 16 \text{ ft.} = 192 \text{ in.} \\ \Sigma P_{e2} &= \text{sum of loads from the 2 rigid frame columns} \\ &= (9.87)(29,000)(15,000)/(1.75(192))^2 = 2(38,000) = 76,000 \text{ kips (note that since } \Sigma P_u \text{ includes the loads on the leaning columns, it is not necessary to multiply } k_x \text{ by } N^{0.5} \text{ when calculating } P_{e2}) \\ B_2 \text{ (Eq. 4)} &= 1/(1 - 3,590(0.737)/(291.2(192))) = 1.05, \text{ and} \\ B_2 \text{ (Eq. 5)} &= 1/(1 - 3,590/76,000) = 1.05. \end{aligned}$$

Using Figures 6a and 6b,  $M_{nt}$  for C2 between ground level and the second floor is 605 in-k at the bottom and 1,830 in-k at the top.

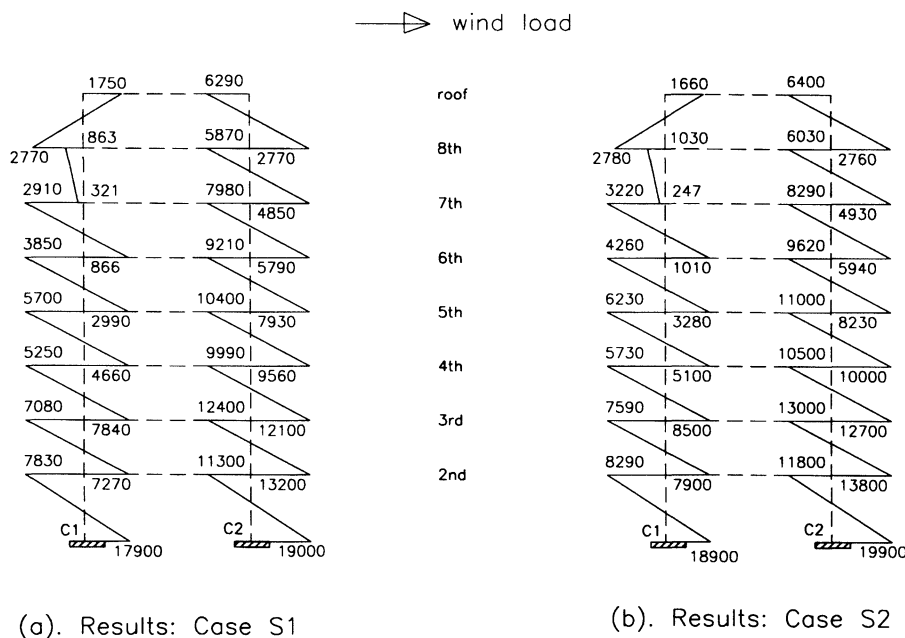


Fig. 7. Space frame analysis results column moments (in-kips).

$$M_{lt} = (1,130 - 1,830) = 9,470 \text{ in-k at top and} \\ (18,900 - 605) = 18,300 \text{ in-k at bottom.}$$

At the bottom of the column:

$$M_{ux} = 1.0(605) + 1.05(18,300) = 19,820 \text{ in-k (correct value)}$$

At the top of the column:

$$M_{ux} = 1.0(1,830) + 1.05(9,470) = 11,770 \text{ in-k.}$$

It is overconservative to combine the maximums:

$$M_{ux} = 1.0(1,830) + 1.05(18,300) = 21,050 \text{ in-k.}$$

Since the limiting laterally unbraced length for full plastic capacity,  $L_p$ , for a W36×230 Grade 50 is 158 inches, and is less than the actual unbraced length,  $L_b$ , of 192 inches, the strong axis moment capacity can be less than the plastic capacity of 42,480 in-k. However, in this case it equals the plastic capacity since  $C_b$  is considerably greater than unity and  $L_b$  is only a little greater than  $L_p$  (See 1993 LRFD Chapter F<sup>4</sup>).

The  $P_n$  is based on the maximum slenderness ratio. In this case

$$(kl/r)_y = 1.0(192)/3.73 = 51.5, \text{ and} \\ (kl/r)_x = 1.75(1.62)(192)/14.9 = 36.5$$

Thus, weak axis buckling governs and  $\phi_c F_{cr}$  for 50 ksi yield steel and a maximum slenderness ratio of 51.5 is 35.0 ksi;  $\phi_c P_n = 35 \text{ ksi} (67.6 \text{ in}^2) = 2,366 \text{ kips}$ . The axial force ratio is  $1,060/2,366 = 0.448$  which is greater than 0.2. It should be noted that in this case the leaning column factor  $N$  has no influence on  $P_n$  since out-of-plane buckling governs.

Using the first-order results, the interaction ratio is (Equation 1):

$$0.448 + 8/9 (19,820/42,480) = .448 + .415 \\ = 0.862 < 1.0 \quad \mathbf{o.k.}$$

Using the second-order results from Figure 6c or 7b:

$$1,080/2,366 + 8/9 (19,900/42,480) = .456 + .416 \\ = 0.872 < 1.0 \quad \mathbf{o.k.}$$

Omitting the dummy column from the second-order plane frame analysis would underestimate the interaction ratio:

$$1,070/2,366 + 8/9 (19,300/42,480) = .452 + .404 = 0.856.$$

The underestimation is small for this structure but will increase to significant levels for structures with high  $N$  values.

## RESULTS

The axial compressions found from all eight analyses were approximately the same in the example building studied. The

major differences were in the induced moments, depending on the type of analysis and interpretation of the AISC LRFD Code<sup>4</sup> requirements (Equation 1). With a literal interpretation of moment magnifier equation (combining maximum sway and non-sway moments from opposite ends) used in the first-order analysis, the calculated design moment was 21,050 in-k, which is about 6 percent higher when compared to the second-order analysis for the example studied. The use of a dummy column in a second-order plane frame analysis was shown to correctly model leaning columns with very little effort. The effect of the leaning column effect for this test case is moderate, but would be substantial for rigid frame buildings with many columns not participating in the lateral load resistance.

## CONCLUSIONS

Rigid frames with leaning columns require additional analysis considerations. There are several approaches for including the second-order effects of the leaning columns in a rigid frame design. For many practical structures, equivalent designs will result from any of the methods; the essential point is that they be accounted for by some method.

The analysis using a space frame to model the second-order effects and the effects of the leaning columns is the most accurate, but it is also the most time consuming. Results from this study indicate that the use of a second-order plane frame analysis with a dummy column results in an accurate and simpler design while taking very little extra analysis effort.

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