Discussion

Ponding of Two-way Roof Systems

Paper presented by FRANK J. MARINO (July, 1966, Issue)

Discussion by **DONALD A. SAWYER**

Mr. Marino has correctly pointed out that the problem of water ponding on flexible roof systems is more involved than some of the earlier papers implied and that the AISC Specification, Sect. 1.13, may give false information about the susceptibility of a given roof to ponding problems. In order to put the design of flat roofs on a more rational basis, he has derived certain relationships that are offered as possible design office procedures. His methods have been illustrated by several examples.

Up through Equation (4), his derivations are sound and there can be no argument with his use of certain acceptable approximations to arrive at the stability criterion for two-way roofs. The device used was that of assuming an arbitrary initial deflection in both the primary and the secondary beams, and proceeding to a final equilibrium position determined by the flexibility constants of the two types of members. For this part of the derivation, it does not matter how the initial deflections were caused, and Mr. Marino has stated that these may be due, in part, to initial crookedness. However, beginning with Equation (5), the derivation goes astray because the initial crookedness component has been omitted. The situation is similar to that for the case of initially crooked columns. It can be shown that initial crookedness in a column does not reduce its Euler buckling load but it does affect the load at which yielding first commences. For beams subject to ponding loads, the initial crookedness does not reduce the critical stability number but it does affect the equilibrium position and thereby the point at which yielding commences.

The major problem with Mr. Marino's derivation is that when initial crookedness is present, C_s/C_p is not a proper measure of δ_0/Δ_0 and Δ_w/Δ_0 is not a proper measure of f_w/f_0. These deficiencies can be remedied as follows. In general, the initial deflected shape of a beam at the onset of ponding is due to two components. One component is that due to the loads that are on the beam at the onset of ponding. This can be related to the stress f_0. The second component is that due to intentional or accidental crookedness. Its magnitude is independent of the initial stresses and would represent the shape of the beam in the unstressed condition. Assuming, with minor error, that the same constant would apply to both the primary and the secondary beams, their stress-related deflection at the onset of ponding can be expressed as

$$\Delta_0' = C_d f_{0p} \tag{9}$$

and

$$\delta_0' = C_d f_{0s} \tag{9a}$$

Assuming that the initial crookedness is known or can be estimated, its magnitude can be expressed as a multiple of the stress-related deflection. Thus,

$$\Delta_0'' = \beta_p \Delta_0' \tag{10}$$

$$\delta_0'' = \beta_s \delta_0' \tag{10a}$$

in which β_p and β_s are positive when the initial crookedness is such as to cause a sag. Again, with some error it may be assumed that

$$\Delta_w = C_d f_{wp} \tag{11}$$

$$\delta_w = C_d f_{ws} \tag{11a}$$

although the load distribution for the ponded water is somewhat different from the load distribution that exists at the onset of ponding. As Mr. Marino indicated, the deflections due to the loads can be expressed in terms of the flexibility constants also. Thus,

$$\Delta_0' = C_y C_p \tag{12}$$

$$\delta_0' = C_y C_s \tag{12a}$$

From Equations (10) and (12), the total deflections at the onset of ponding are

$$\Delta_0 = (1 + \beta_p) C_y C_p \tag{13}$$

$$\delta_0 = (1 + \beta_s) C_y C_s \tag{13a}$$

Then, from Equations (9), (10) and (11)

$$\frac{\Delta_w}{\Delta_0} = \frac{f_{wp}}{(1 + \beta_p) f_{0p}} \tag{14}$$

$$\frac{\delta_w}{\delta_0} = \frac{f_{ws}}{(1 + \beta_s) f_{0s}} \tag{14a}$$

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From Equations (13)

$$\frac{\delta_0}{\Delta_0} = \frac{(1 + \beta_s)C_s}{(1 + \beta_p)C_p} \tag{15}$$

When Equations (14) and (15) are substituted in the appropriate places in Mr. Marino's derivation of Equations (7) and (8), the following new relationships are found:

$$\left[\frac{1}{\text{F.S.}}\frac{F_y}{f_{0p}} - 1\right] \geq A_p(1 + \beta_p) + B_p(1 + \beta_s) \tag{7a}$$

$$\left[\frac{1}{\text{F.S.}}\frac{F_y}{f_{0s}} - 1\right] \geq A_s(1 + \beta_p) + B_s(1 + \beta_s) \tag{8a}$$

where,

$$A_p = \frac{\alpha_p\left(1 + \frac{\pi}{4}\alpha_s\right)}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

$$B_p = \frac{\frac{C_s}{1 - C_p}\frac{\pi}{4}(1 + \alpha_s)}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

$$A_s = \frac{\frac{C_p}{1 - C_s}\frac{\pi}{4}(1 + \alpha_p)}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

$$B_s = \frac{\alpha_s\left[1 + \frac{\pi^3}{32}\alpha_p + \left(\frac{\pi^3}{32} - \frac{\pi}{4}\right)\alpha_p\alpha_s\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s}$$

When β_p and β_s are both zero, the modified equations reduce to Mr. Marino's original Equations (7) and (8). Curves similar to Figs. 4 and 5 may be drawn for the four parameters A_p, B_p, A_s and B_s. A few computations will show that the initial crookedness can have a major influence on the results and that the effect of a given amount of crookedness is greatly different depending on whether the beam is erected with the crookedness adding to or subtracting from the deflection due to loads.

Some expected values of β_p and β_s can be estimated from the rolling tolerances allowed by practice. It is common to allow as much as ⅛-in. of crookedness for each 5 ft of beam length. In order to compare this with the dead load deflection of the example problems, it is convenient to express the deflection of a simple beam with its contributing area loaded with w psf as follows:

$$\Delta = \frac{w}{4.11} C_p \tag{16}$$

$$\delta = \frac{w}{4.11} C_s \tag{16a}$$

For Example 2 of the paper, the dead load is 15 psf and $C_p = 0.41$, so that the dead load deflection of the girders would be 1.50 in. The initial crookedness could be as much as $(0.125)(50/5) = 1.25$ in. Thus, β_p could lie somewhere between the limits of -0.84 and $+0.84$. Similarly, β_s could lie somewhere between -1.73 and $+1.73$. These values could affect the conclusions significantly. Obviously, even greater values of β_p and β_s can occur when intentional camber exists.

Note that when β_p and β_s are both equal to -1.00, Equations (7a) and (8a) would indicate that no allowance need be made for ponding regardless of the magnitudes of the flexibility constants. Of course this would be an ideal case where the roof is mathematically flat under full dead load and the rain could drain instantly over zero-height gravel stops. Clearly, one should not rely on such details to avoid ponding problems. However, considerations of favorable camber may help to explain why some apparently critical roofs have survived.

Because initial crookedness has not been taken into account in the numerical examples, it would be proper to rework them completely. However, it will be assumed that perfectly straight beams were used so that a few other comments will be pertinent. Mr. Marino has demonstrated that a designer may apply the procedure for any arbitrary amount of live load present at the onset of ponding. It was probably his intent to show the ease with which the computations could be made using Figs. 4 and 5. However, the impression is given that the designer should choose a proper fraction of live load to be present at the onset of ponding. But Example 3 shows that the procedure can be quite sensitive to this choice. That is, Example 3 demonstrates that the given roof satisfies the suggested criteria when one-fifth of the L.L. (4 psf) is on the roof at the onset of ponding. However, a check will show that the roof does not satisfy the criteria if one-fourth of the L.L. (5 psf) is on the roof at the onset. Thus, a difference of just 1 psf in that assumption would cause the roof to be either accepted or rejected. In southern regions where the true cause of roof live load is somewhat nebulous, it seems that the most logical source of the initial fraction must come from the rain storage caused by gravel stops or inadequate drainage. Some preliminary work by the writer indicates that much useful information for the selection of a factor somewhat akin to Mr. Marino's live load fraction will come from a study of roof hydraulics. In the meantime it appears that a designer must fall back on judgment to make a selection of the live load fraction if he chooses to apply Equations (7) and (8).

The application of the procedure to Example 1 shows that even an infinitely stiff joist would not be accepted. Clearly this is an extreme requirement. Actually, the example shows that the girder is so flexible that the stress requirements are not met even before the joist deflections are taken into account. Therefore, because this is a problem in the interaction of the two beam systems, it would be possible to make a redesign by changing only the girder rather than by changing both the girder and the joists as was done in Example 2. The correct course of action might be determined by a consideration of the weight required for various alternative designs. For the case of Example 1, it does seem best to modify both the girder and the joists to achieve a less heavy design. A check will show that 22H7 joists and a 24W68 girder would be acceptable according to Figs. 4 and 5 rather than the 24J8 joists chosen in Example 2. This modification is 516 lbs/bay lighter than the design of Example 2.

This writer has a similar paper in press[1] that takes into account several important parameters that were omitted from Mr. Marino's paper. That paper and one or two others due for publication within the next several months should do much to answer some of the remaining questions concerning this interesting roof ponding problem.

ADDITIONAL NOMENCLATURE

- C_d Deflection constant relating extreme fiber stress to deflection
- C_y Deflection constant relating deflection to the flexibility constant
- β_p Parameter relating initial crookedness to load deflection at onset of ponding for primary member
- β_s Parameter relating initial crookedness to load deflection at onset of ponding for secondary member
- Δ_0' Deflection of primary member due to load existing at the onset of ponding
- δ_0' Deflection of secondary member due to load existing at the onset of ponding
- Δ_0'' Initial crookedness of the unstressed primary member
- δ_0'' Initial crookedness of the unstressed secondary member

1. *Sawyer, Donald A.* Ponding of Rainwater on Flexible Roof Systems, *Journal of the Structural Division, ASCE (scheduled for publication in February, 1967).*

Discussion by **FRANK J. MARINO**

THE CONTENT and scope of Professor Sawyer's discussion indicate his keen understanding of the ponding phenomenon. He correctly points out, as the author was aware, that accidental initial crookedness might effect the equilibrium position of a member and hence the level of the maximum flexural stress. However, several factors led the author to ignore this effect in attempting to formulate a relatively simple analytical procedure, suitable for the design office.

The magnitude of the effect is relatively small. The allowable mill tolerance for camber of steel shapes, given in ASTM A6, is ⅛-in. per 10 ft of length, and not ⅛-in. per 5 ft as stated in Professor Sawyer's discussion. Moreover, the AISC Specification, Sect. 1.19.3, requires that, after erection, any accidental camber due to rolling or fabricating processes must be upward. Thus any accidental crookedness present in roof members should serve to lower the magnitude of ponding stresses. The effect, as previously pointed out, is small. Normal factor of safety requirements would cover the unlikely situation where, by some oversight, a roof member was fabricated and erected with a downward accidental camber. For instance, in the example cited by Professor Sawyer, referring to Example 2 of the author's paper, if the girder of that example had been erroneously erected with a downward camber, the amount of that accidental crookedness could have been, at most, 0.625 in. This would have the effect of increasing the final stress level in the girder by approximately 2 ksi, or about 7 percent.

Professor Sawyer points out that conformance to design criteria will depend on the amount of live load that is assumed on the roof at the onset of ponding. The fact that the amount of live load will affect the magnitude of final stress is self evident. Any structural member, designed against performance criteria, will satisfy, or not satisfy, those criteria depending on the amount of load assumed in the analysis. The determination of live load level for any aspect of design is a decision properly made by the designer for the specific conditions under consideration.

The author must confess that no attempt was made, in formulating the design examples provided with the paper, to reach an optimum or minimum weight design. The intent was merely to illustrate the application of the ponding analysis. Certainly, in attempting to modify an actual design that does not meet the ponding criteria, several avenues are open to the designer. He would normally choose that solution that optimized the design while still conforming to other nonstructural limitations.