Optimal Cost Design of Semi-Rigid, Low-Rise Industrial Frames

LEI XU, ARCHIE N. SHERBOURNE, and DONALD E. GRIERSON

ABSTRACT

The paper presents an efficient computer-automated method for the optimum design of steel frameworks accounting for the behaviour of semi-rigid connections. A continuous-discrete optimization algorithm is applied to minimize the "cost" of the connections and members for the structure subject to constraints on stresses and displacements under specified design loads. An example is presented to illustrate the features of the design method. The results suggest that the proposed semi-rigid design method realistically accounts for connection behaviour and produces more appropriate member sizes than does the traditional fully rigid design method.

INTRODUCTION

For conventional analysis and design of steel framed structures, the actual behaviour of beam-to-column connections is simplified to the two idealized extremes of either rigid-joint or pinned-joint behaviour. However, most connections used in steel frameworks actually exhibit semi-rigid deformation behaviour that can contribute substantially to overall nodal displacements and affect significantly the internal force distribution in the members. As such, steel frame connections should be treated as being "semi-rigid" for the purposes of proper analysis and design (Frye and Morris, 1975; Lui and Chen, 1986; Nethercot etc., 1988).

This paper presents a systematic method for the optimum design of steel frameworks accounting for the behaviour of semi-rigid connections. The design having the minimum combined "cost" of members and connections is sought, while ensuring that stresses and displacements are within acceptable limits. Members are sized using discrete standard steel sections, while connections are selected on the basis of their continuous-valued moment-rotation stiffnesses. A continuous-discrete optimization algorithm (Schmit and Fleury, 1979) is applied for the design of a typical three-bay, two-storey industrial steel frame under multiple load combinations and the results are compared with those for conventional rigid-joint design of the same structure.

Lei Xu is structural engineer, the Canam Manac Group Inc., Boucherville, Quebec, Canada J4B 5X9.

Archie N. Sherbourne is professor, department of civil engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.

Donald E. Grierson is professor, department of civil engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

NONLINEAR BEHAVIOUR OF SEMI-RIGID CONNECTION

Beam-to-column connections play an important role in the resistance to load of structural frameworks. The major function of these connections is to transfer the beam and floor loads to the columns. In general, the forces transmitted through the connections can be axial and shearing forces, and bending and torsional moments. The effect of torsion is neglected for the present study of planar frameworks as are the axial and shearing deformations since they are small compared to the rotational deformations of most connections.

The deformation behaviour of a connection is usually described by its moment-rotation (M-θ) relationship. Many tests have been conducted to study the flexural behaviour of different types of connections; typical responses for a variety of commonly used connections are shown in Figure 1. The curves for all connection types exhibit nonlinear behaviour over the entire range of loading between the two extreme cases of ideally pinned and fully rigid connections. Tests of prototype connections are usually done to obtain the actual moment-rotation behaviour which is then modelled approximately by mathematical expressions.

A four-parameter power model is adopted in this research to simulate the moment-rotation relationship of beam-to-column connections. It was first proposed Richard and Abbott (1975) who derived it from basic nonlinear elastic-plastic stress-strain relationships. Chen and Kishi (1987) used this equation to model top and seat-angle connections as well as single and double web-angle connections. Recently, Hsieh (1990) used this model to simulate nine types of beam-to-column connections (Kishi and Chen, 1986) and good curve-fitting was reported with test data.

The four-parameter power model of connection behaviour is of the form:

$$M = \frac{(R_e - R_p)\theta}{\left\{1 + \left[\frac{(R_e - R_p)\theta}{M_o}\right]^n\right\}^{1/n}} + R_p\theta \tag{1}$$

where,

R_e is the initial stiffness of the connection,
R_p is the strain-hardening stiffness of the connection,
M_o is a reference moment, and
n is the shape parameter of the M-θ curve.

A connection data base is established by adding and reorganizing the previous results (Kishi and Chen, 1986; Hsieh, 1990) for purposes of design. For illustration, the parameters in the nonlinear model and details of the Extended End Plate Connections are given in Tables 1 and 2. The analysis procedure is iterative and a secant stiffness approach is employed.

ANALYSIS

For the design of steel frameworks with semi-rigid connections it is necessary to have the capability to conduct corresponding analysis. To that end, the effects of connection flexibility are modelled by attaching rotational springs having stiffness moduli R_1 and R_2 to the ends 1 and 2 of a member. The stiffness matrix of a member i with semi-rigid restraint at the ends can be expressed as (Xu, 1994):

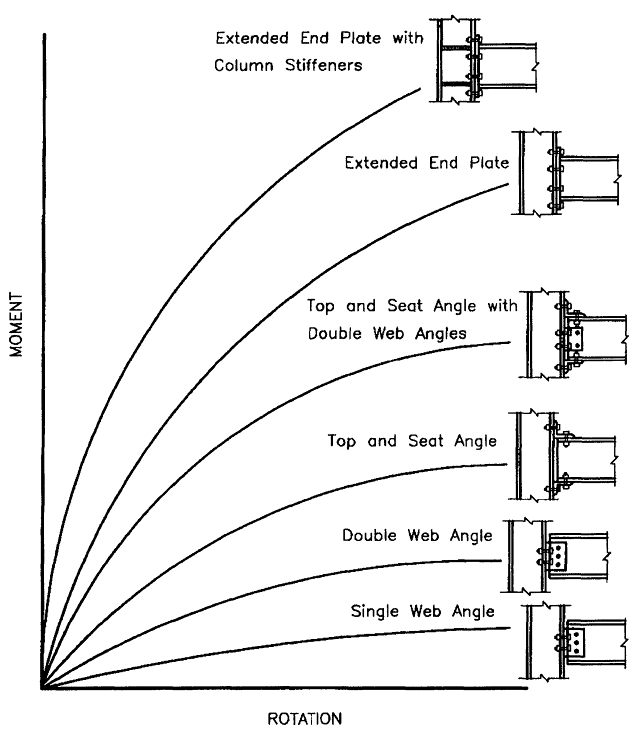

Fig. 1. Moment-rotation curves for various types of connections.

$$K_i = S_i - C_i + G_i \tag{2}$$

where

K_i is the stiffness matrix of member i accounting for semi-rigid end-connections
S_i is the elastic stiffness matrix of the member taken to have rigid ends
C_i is a correction matrix that is a function of the following *fixity factors* at the two member ends (Monforton and Wu, 1963)

$$r_1 = \frac{1}{1 + \frac{3EI}{R_1 L}};\ r_2 = \frac{1}{1 + \frac{3EI}{R_2 L}} \tag{3}$$

where

E is Young's modulus
L and I are the length and cross-section moment of inertia of the member, respectively, and
G_i is the geometrical stiffness matrix for the semi-rigid member, which is also a function of the fixity factors r_1 and r_2 (Xu, 1992).

The fixity factor r defines the stiffness of a connection relative to the attached member. For a pinned connection the value of the fixity factor is zero ($r = 0$), while for a fully rigid connection it is unity ($r = 1$). A semi-rigid connection has a fixity factor between zero and unity ($0 < r < 1$). Having the member stiffness matrices through Equation 1 for specified values of fixity factors, r, reflecting connection stiffness, the analysis of frameworks with semi-rigid connections is carried out using the conventional Displacement Method.

Having the stiffness matrix of a semi-rigid member, the iterative procedure using the secant stiffness approach for the second-order analysis of semi-rigid frames by considering non-linear connection behaviour, can be described as following:

Step 1.

Input the data and read the nonlinear parameters and reference stiffness R_f for the specified connections from the connection database. Set the iteration index $k = 0$, assign connection stiffnesses $R_{ij}^{(k)} = R_f$ and set member axial forces $N_i^{(k)} = 0$.

Step 2.

Update fixity factors r_{ij} through Equation 3, and then update member equivalent joint loads according to the updated r_{ij}. Generate each member stiffness matrix K_i through Equation 2 for the updated r_{ij} and the axial force $N_i^{(k-1)}$ from the previous iteration.

Step 3.

Assemble the structure stiffness matrix and solve for nodal

Table 1. Nonlinear Model Parameters for Extended End Plate Connections						
Id. no.	R_f kip-in./rad.	M_c kip-in	M_o kip-in	R_e kip-in/rad.	R_p kip-in/rad.	n
EEP_23	3.83e+04	5.02e+02	5.73e+02	6.66e+04	1.81e+03	1.36e+00
EEP_24	3.83e+04	5.02e+02	5.73e+02	6.66e+04	1.81e+03	1.36e+00
EEP_7	5.25e+04	7.61e+02	7.29e+02	5.98e+04	5.61e+03	2.61e+00
EEP_22	5.39e+04	6.88e+02	8.91e+02	1.36e+05	1.65e+03	9.64e-01
EEP_8	6.63e+04	7.81e+02	5.97e+02	9.81e+04	9.30e+03	3.97e+00
EEP_1	7.46e+04	8.97e+02	6.19e+02	2.18e+05	1.40e+04	2.46e+00
EEP_4	8.62e+04	1.06e+03	9.54e+02	4.16e+05	1.12e+04	9.49e-01
EEP_14	1.05e+05	1.15e+03	1.16e+03	1.94e+05	6.59e+02	2.42e+00
EEP_13	1.08e+05	1.22e+03	1.06e+03	1.75e+05	8.56e+03	3.50e+00
EEP_15	1.26e+05	1.40e+03	1.22e+03	1.68e+05	9.22e+03	5.11e+00
EEP_16	1.31e+05	1.42e+03	1.40e+03	2.07e+05	1.83e+03	3.22e+00
EEP_9	1.33e+05	1.49e+03	1.19e+03	7.58e+05	1.50e+04	1.97e+00
EEP_11	1.71e+05	2.14e+03	2.12e+32	2.06e+05	5.04e+03	3.16e+00
EEP_5	1.76e+05	2.19e+03	2.69e+03	7.00e+05	9.46e+03	8.02e-01
EEP_2	1.90e+05	2.22e+03	1.96e+03	5.59e+06	2.22e+04	6.61e-01
EEP_12	1.95e+05	2.17e+03	2.22e+03	3.99e+05	2.03e+03	1.98e+00
EEP_18	2.91e+05	3.31e+03	3.49e+03	6.08e+05	2.29e+03	1.70e+00
EEP_20	3.06e+05	3.52e+03	3.36e+03	3.84e+05	9.32e+03	3.96e+00
EEP_21	3.10e+05	3.56e+03	3.09e+03	6.57e+05	2.68e+04	2.19e+00
EEP_19	3.37e+05	3.48e+03	3.49e+03	6.71e+05	3.26e+02	3.25e+00

displacements, member axial forces $N_i^{(k)}$ and connection moments $M_{ij}^{(k)}$.

Step 4.

Calculate connection rotations $\theta_{ij}^{(k)} = M_{ij}^{(k)} / R_{ij}^{(k)}$ and obtain the corresponding moments $M(\theta_{ij}^{(k)})$ from Equation 1.

Step 5.

Check for convergence by comparing the connection moments $M_{ij}^{(k)}$ obtained through analysis with the moments $M(\theta_{ij}^{(k)})$ calculated in Step 4, and also by comparing the member axial forces $N_i^{(k)}$ with the forces $N_i^{(k-1)}$ from the previous iteration; if $|M_{ij}^{(k)} - M(\theta_{ij}^{(k)})| \le \varepsilon_1$ and $|N_{ij}^{(k)} - N_{ij}^{k-1)}| \le \varepsilon_2$, then stop; else go to Step 6.

Step 6.

Update connection secant stiffnesses $R_{ij}^{(k)} = M(\theta_{ij}^{(k)}) / \theta_{ij}^{(k)}$ and iteration index $k = k + 1$, and go to Step 2.

The subscript i (i = 1, ...total number of semi-rigid members) denotes the number of the semi-rigid member and subscript j (j = 1, 2) denotes one of the two ends of a semi-rigid member.

DESIGN

It is required to design a steel framework for minimum combined "cost" of members and connections while accounting for semi-rigid behaviour of the connections. Herein, the cost of each member, i, is represented by its weight, while the cost of each connection, j, is taken to be directly related to its rotational stiffness. Therefore, the total cost of a member i with two end-connections j = 1, 2 may be expressed as

$$Z_i = w_i a_i + \sum_{j=1}^{2} (\beta_{ij} R_{ij} + \beta_{ij}^o) \tag{4}$$

where

Table 2. Details of Extended End Plate Connections							
Id. No.	Beam	Column	lp(in)	tp(in)	g(in)	pt(in)	db
EEP_1	W14×22	W8×35	17.25	0.625	5.5	3.5	¾-in.
EEP_2	W18×35	W10×49	21.5	0.75	5.5	3.75	⅞-in.
EEP_4	W14×22	W8×35	17.25	0.875	5.5	3.5	¾-in.
EEP_5	W18×35	W10×49	21.5	1.25	5.5	3.75	⅞-in.
EEP_7	W14×22	W8×31	16.814	0.969	5.5	3.5	¾-in.
EEP_8	W14×22	W8×31	16.916	0.938	5.531	3.5	¾-in.
EEP_9	W16×26	W10×33	19.138	1.562	5.375	3.75	1 in.
EEP_11	12x5 RSJ32[a]	8x8 UC48[a]	—	1.0	4.0	5.5	⅞-in.
EEP_12	12x5 RSJ32[a]	8x8 UC48[a]	—	1.0	4.0	4.5	⅞-in.
EEP_13	10x4 UB17[a]	8x8 UC40[a]	—	0.75	4.0	4.375	⅝-in.
EEP_14	10x4 UB17[a]	8x8 UC40[a]	—	0.75	4.0	4.375	⅝-in.
EEP_15	10x4 UB19[a]	8x8 UC40[a]	—	0.75	4.0	4.375	¾-in.
EEP_16	10x4 UB19[a]	8x8 UC40[a]	—	0.75	4.0	4.375	¾-in.
EEP_18	15x6 UB40[a]	10x10 UC60[a]	—	0.75	3.5	0	¾-in.
EEP_19	15x6 UB40[a]	10x10 UC60[a]	—	0.75	3.5	0	1 in.
EEP_20	15x6 UB40[a]	10x10 UC60[a]	—	1.0	3.5	0	1 in.
EEP_21	15x6 UB40[a]	10x10 UC60[a]	—	1.0	3.5	0	1 in.
EEP_22	254×102 UB2[b]	152×152 UC3[b]	9.7638	0.5906	3.3858	3.8189	M16
EEP_23	254×102 UB2[b]	152×152 UC3[b]	9.7538	0.5906	3.3858	3.8089	M16
EEP_24	254×102 UB2[b]	152×152 UC2[b]	9.7638	0.5906	3.3858	3.8189	M16

[a]British section, "Steel Designers' Manual," 1966, The British Iron & Steel Federation.
[b]British section, "Steel Designers' Manual," 1992, The British Steel Construction Institute.

a_i and w_i are the member cross-section area and weight coefficient
R_{ij} and β_{ij} are the connection rotational stiffness and cost coefficient, and
β_{ij}^o is the cost of a pinned connection having zero rotational stiffness.

Each member i is to be sized using a commercial standard steel section and, as such, its cross-section area a_i is a *discrete* variable to the design. On the other hand, each connection j may be fabricated anywhere in the range from fully-pinned to fully-fixed and, as such, its rotational stiffness R_{ij} is a *continuous* variable to the design. That is, the design variables a_i and R_{ij} are restricted by the design as

$$a_i \in A_i;\ R_{ij}^L \leq R_{ij} \leq R_{ij}^U \tag{5a,b}$$

where Equation 5a requires the cross-section area a_i of member i to belong to the discrete set of areas $A_i \equiv \{a_1, a_2, ...\}$ prevailing for the standard section shape specified for the member (e.g., W-shape), while Equation 5b imposes specified lower and upper bounds R_{ij}^L and R_{ij}^U on the connection rotational stiffness R_{ij}.

In its general form, the optimal member-plus-connection design of a framework having semi-rigid connections involves stress and displacement constraints that are but implicit functions of the design variables. To facilitate computer solution of the design optimization problem it is first desirable to formulate each stress σ_k and displacement δ_l as an explicit function of the design variables. Since stresses and displacements generally vary inversely with member size a_i and connection stiffness R_{ij}, upon adopting the reciprocal variables

$$x_1 = \frac{1}{a_i}; f_{ij} = \frac{1}{R_{ij}} \tag{6a,b}$$

where

f_{ij} is connection *flexibility*, an explicit approximation of the design optimization problem for a semi-rigid framework with $i = 1, 2, ..., n$ members, is (Xu and Grierson 1993; Xu 1994):

$$\textit{Minimize: } Z = \sum_{i=1}^{n} \left(\frac{w_i}{x_i} + \sum_{j=1}^{2} \frac{\beta_{ij}}{f_{ij}} \right) \tag{7a}$$

Subject to:

$$\sigma_k^L \le \sum_{i=1}^{n} \left[\left(\frac{\partial \sigma_k}{\partial x_i} \right)^o x_i + \sum_{j=1}^{2} \left(\frac{\partial \sigma_k}{\partial f_{ij}} \right)^o f_{ij} \right] \le \sigma_k^U \; (k = 1, 2, \ldots, s) \tag{7b}$$

$$\delta_l^L \le \sum_{i=1}^{n} \left[\left(\frac{\partial \delta_l}{\partial x_i} \right)^o x_i + \sum_{j=1}^{2} \left(\frac{\partial \delta_l}{\partial f_{ij}} \right)^o f_{ij} \right] \le \delta_l^U \; (l = 1, 2, \ldots, d) \tag{7c}$$

$$f_{ij}^L \le f_{ij} \le f_{ij}^U \; (i = 1, 2, \ldots, n; j = 1, 2) \tag{7d}$$

$$x_i \in X_i \; (i = 1, 2, \ldots, n) \tag{7e}$$

where, from Equation 4 the objective function Equation 7a is a measure of the combined "cost" of the members and connections, Equation 7b and 7c define constraints on stresses σ_k and displacements δ_1 for the structure, and from Equation 5 the restrictions Equations 7d and 7e control the values of the continuous-valued connection stiffnesses, R_{ij}, and the discrete-valued member sizes, a_i, where, from Equation 7 the bounds $f_{ij}^L = 1/R_{ij}^U$ and $f_{ij}^U = 1/R_{ij}^L$, and the components of each discrete set $X_i = \{x_1, x_2, ...\} = \{1/a_1, 1/a_2, ...\}$. The stress and displacement gradients $\partial\sigma_k/\partial x_i$, $\partial\sigma_k/\partial f_{ij}$, $\partial\delta_l/\partial x_i$, and $\partial\delta_i/\partial f_{ij}$ include both the P-Δ effect and semi-rigid connection behaviour (Xu, 1994). The design optimization problem involves both continuous-valued variables, f_{ij}, and discrete-valued variables, x_i.

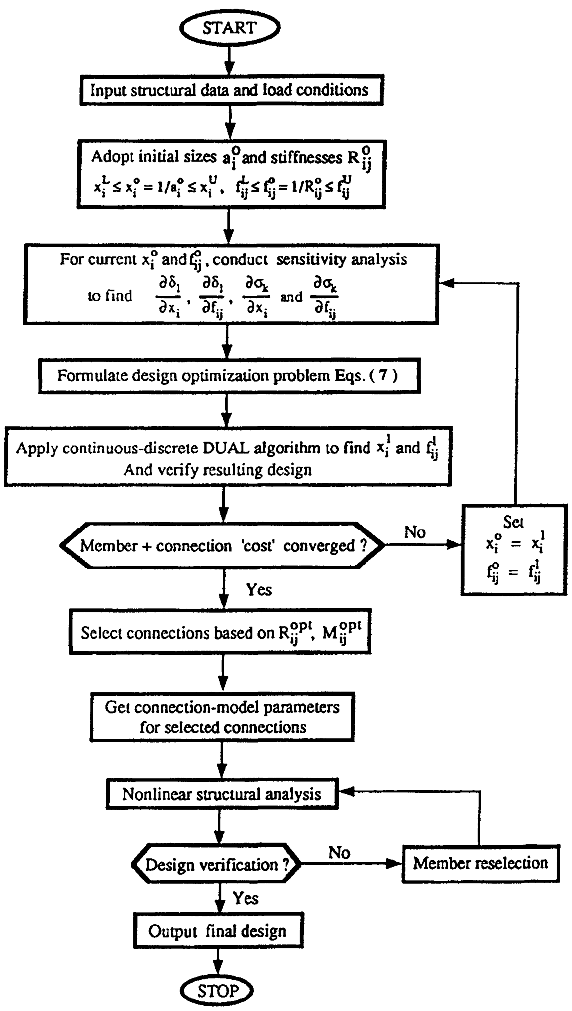

Fig. 2. Flowchart of overall design optimization procedure.

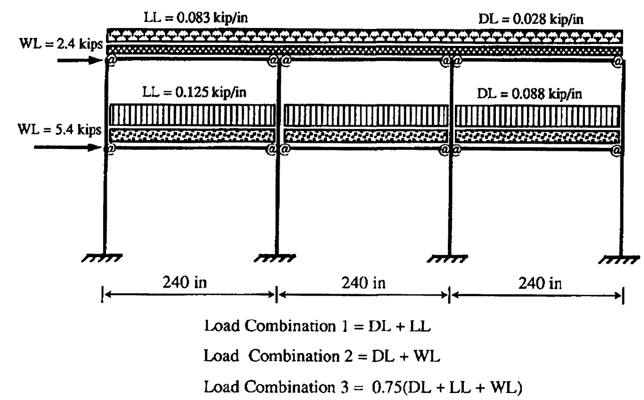

Fig. 3. Loading conditions of three-bay two-storey steel frame.

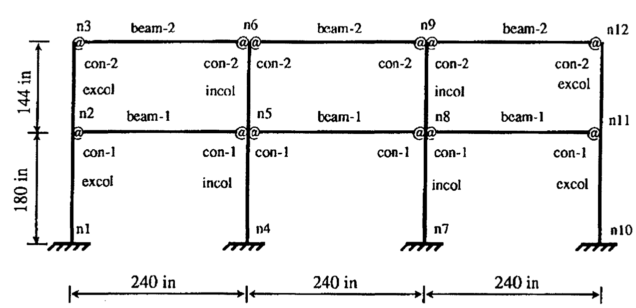

Fig. 4. Three-bay two-storey steel frame.

The design problem Equation 7 is solved at each design stage using a continuous-discrete optimization algorithm (Fleury and Sander, 1978; Schmit and Fleury, 1979) to find new values of x_i and f_{ij} for which the combined member-plus-connection "cost" of the structure is reduced relative to that for the previous design stage. The stress and displacement gradients are then updated for this new structure, and the design optimization Equation 7 are recalculated. This process is repeated until cost convergence occurs for successive design stages, at which point the minimum "cost" design has been found. The overall design procedure is illustrated by the flowchart in Figure 2.

APPLICATION AND DISCUSSION

The three-bay, two-storey steel framework is loaded as shown in Figure 3. The spacing of the frames is 20 ft. on center. Three different load combinations are examined and the framework has semi-rigid connections of equal rotational stiffness at each end of each beam as shown in Figure 4. Beams of standard W-section and identical beam-to-column connections are prescribed at each storey level. All columns have the same W-section over two stories. To investigate the effect of nonlinear semi-rigid connections on the design of low-rise steel frames, all column and beam members are restricted to W8 and W14-shapes, respectively, in order to select Extended End Plate Connections from the current connection database. The rotational stiffness, R_{ij}, of each semi-rigid connection is constrained to lie in the range $R_{ij}^L = 3.83\times10^4$ kips-in/rad to $R_{ij}^U = 3.37\times10^5$ kips-in/rad (Xu, 1994). The frame is to be designed in accordance with the strength/stability (stress) requirements of the AISC Allowable Stress Design (ASD) code (AISC 1978), while at the same time ensuring that beam deflections at mid-span do not exceed the limit of span/240 for Load Combination 1 (Dead Load + Live Load) and top-storey lateral sway does not exceed the limit of $h/400$ for the remaining load combinations. The following data are used to calculate the loads on the framework,

Floor Slab (4 in.)	50 lb/ft^2	Roof Dead Load	15 lb/ft^2
Roof Beam Weight	40 lb/ft^2	Floor Beam Weight	50 lb/ft^2
Snow Load	50 lb/ft^2	Wind Load	20 lb/ft^2
Floor Live Load	75 lb/ft^2		

Dead Loads:
Roof: $20 \times 15 + 40 = 340$ lb/ft = 0.028 kip/in
Floor: $20 \times 50 + 50 = 1050$ lb/ft = 0.088 kip/in

Live Loads:
Roof: $20 \times 50 = 1000$ lb/ft = 0.083 kip/in.
Floor: $20 \times 75 = 1500$ lb/ft = 0.125 kip/in.

Wind Loads:
Roof: $20 \times 20 \times 6 = 2400$ lbs = 2.4 kip
Floor: $20 \times 20 \times 0.5(15 + 12) = 5400$ lbs = 5.4 kip

The weight coefficient w_i in Equation 4 for each member is calculated as w_i = member length × material density. Regarding beam-to-column connection cost, CISC (CISC and CSCC 1983) suggested that if the unit beam weight is less than 51 kg/m (approximately 35 lb/ft), the cost of a steel member of W-section is increased by approximately 25 percent for pinned-jointed end-connections and by 70 percent if its end-connections are rigid jointed. For sections with unit weight between 51 kg/m (35 lb/ft) and 240 kg/m (161 lb/ft) the cost of a steel beam is increased by approximately 5 percent if it has pinned-jointed end-connections and by 43 percent if its end-connections are rigid-jointed. To avoid possible contradictions in directly applying CISC criteria for estimating connection costs, a modification is suggested that the cost of a steel beam is increased by 15 percent if it has pinned-jointed end-connections and 55 percent if it has rigid-jointed end-connections for W-sections with unit weight less than 240 kg/m (161 lb/ft) (Xu, 1994). The connection cost parameters β_{ij}^o and β_{ij} in Equation 4 for the beam members are evaluated from the above data. Therefore, the total member plus connection cost defined by Equation 4 is such that

$$1.15 w_i a_i \le w_i a_i + \sum_{j=1}^{2} (\beta_{ij} R_{ij} + \beta_{ij}^0) \le 1.55 w_i a_i \tag{8}$$

Recognizing that the cost of a member with pinned-jointed connections is

$$w_i a_i + \sum_{j=1}^{2} \beta_{ij}^0 = 1.15 w_i a_i \tag{9}$$

and the fact that $\beta_{i1}^o = \beta_{i2}^o$ it follows from Equation 9 that the cost of a pinned-jointed connection is $\beta_{ij}^o = 0.1 w_i a_i$. Moreover, since $\beta_{i1} R_{i1} = \beta_{i2} R_{i2}$ for each beam member of this design example, it follows from Equation 8 that each connection coefficient β_{ij} is restricted to

$$0 \le \beta_{ij} \le \frac{0.2 w_i a_i}{R_{ij}} \tag{10}$$

where the lower bound corresponds to a pinned connection while the upper bound corresponds to a rigid connection.

The upper bound term in Equation 10 provides the basis for establishing a reasonable estimate of the connection cost coefficient, β_{ij}. To this end, from Equation 3,

$$R_{ij} = \frac{3EI_i}{L_i} \cdot \frac{r_{ij}}{1 - r_{ij}} \tag{11}$$

Gerstle (1988), by reviewing typical building designs, indicated that the stiffness ratio EI/RL ranges from 0.02 to 0.1 for rigid frames. In terms of fixity factor, r, it ranges between 0.94 and 0.77. In this study, $r = 0.85$ is adopted from the average of 0.77 and 0.94 to represent the fixity factor for rigid connections. Substituting R_{ij} from Equation 11, with a fixity

Table 3.
Fully-Rigid Design and Semi-Rigid Design with Large Connection Stiffnesses

	Fully-Rigid			Semi-Rigid with Large Connection Stiffnesses			
Member Group	**Cross Section**	**Fixity Factor r**	**Connection Cost (lb)**	**Cross Section**	**Connection Stiffness (kip-in/rad)**	**Fixity Factor r**	**Connection Cost (lb)**
excol-1	W8×28	1		W8×28			
incol-1	W8×35	1		W8×35			
beam-1	W14×38	1	1.145×10^3	W14×38	1.265×10^6	0.9	1.145×10^3
beam-2	W14×22	1	6.635×10^2	W14×22	6.492×10^5	0.9	6.635×10^2
Weight	7.031×10^3		1.809×10^3	7.031×10^3		1.809×10^3 (lb)	
Total Cost	8.84×10^3 (lb)			8.84×10^3 (lb)			
Constraint	Limit	Actual	Ratio	Limit	Actual		Ratio
Top-Sway	0.81 (in.)	0.474 (in.)	0.585	0.81 (in.)	0.496 (in.)		0.613
Mid-Span Deflection	1.00 (in.)	0.310 (in.)	0.310	0.31 (in.)	0.345 (in.)		0.345
Stress Ratio	1.00	1.028	1.028	1.00	1.019		1.019
Governing Clause		Equation 1.6-2		Governing Clause		Equation 1.6-1a	
Governing Group		beam-1		Governing Group		excol-1	
Governing Load Combination: 2				Governing Load Combination: 2			

factor 0.85, into the upper bound term in Equation 10 results in

$$\beta = 0.01\frac{L_i}{EI_i} \cdot w_i a_i \qquad (12)$$

in which I_i and a_i are moment of inertia and section area of the beam, respectively.

The parameter β_{ij} has to be pre-determined in the optimization process. Many ways could be suggested to estimate I_i and a_i such as taking the section from a rigid design or selecting the section from beam tables by considering the gravity load only, etc. Numerical experiments have shown that the design is not sensitive to the value of β_{ij} (Xu and Grierson, 1993). In this example, floor and roof beams were assumed as W14×38 and W14×22. The resulting values of β_{ij} were 0.16×10^{-6} and 0.18×10^{-6} rad/in, respectively.

Investigations have been conducted for the optimum design of a three-bay, two-storey steel frame by assuming the frame is fully-rigid, semi-rigid with large connection stiffness, semi-rigid with connection stiffness between 2×10^5 to 50×10^5 kip-in/rad. (Gerstle, 1988) and semi-rigid with Extended End Plate Connections (Figure 5) having known nonlinear behaviour, respectively. All calculations included a second-order (P-Δ) analysis in the final designs. The final design results for the fully-rigid design and a semi-rigid design with large connection stiffnesses are presented in Table 3. Since fully-rigid connections are not attainable in practice, the semi-rigid design with large connection stiffnesses can be viewed as a realistic model of the fully-rigid case. The connection stiffnesses for this latter design case are assigned to be 1.25×10^6 kip-in/rad. and 6.492×10^5 kip-in/rad. for the connections of first and top storey, respectively, which correspond to the fixity factor for the beams in both the first and top storey being $r = 0.9$.

Two further semi-rigid design cases are presented in Table 4. The first case is the semi-rigid design represented by optimal standard sections and optimal connection stiffnesses from the design optimization process prior to selection of actual connections from the connection database. For the second case, Extended End Plate connections have been selected from the connection database consistent with the optimal connection stiffnesses and taking into account connection fabrication requirements.

The design optimization process commenced with an initial W8×67 column section and W14×120 beam section for all designs, and converged after 2 design cycles in each case. For the case of semi-rigid design with extended end-plate connections, the connection selection procedure was activated upon convergence to the optimal member sections and connection stiffnesses. For the specified type of connection,

Table 4.
Semi-Rigid Design Results

Semi-Rigid Design (no connection selection)					Semi-Rigid with Large EEP Connections		
Member Group	**Cross Section**	**Connection Stiffness (kip-in/rad)**	**Fixity Factor *r***	**Connection Cost (lb)**	**Cross Section**	**Connection or Fixity Factor *r***	**Connection Cost (lb)**
excol-1	W8×31		1		W8×31	1	
incol-1	W8×35		1		W8×35	1	
beam-1	W14×30	1.15×10^5	0.52	3.982×10^2	W14×38	EEP_4	3.620×10^2
beam-2	W14×22	1.88×10^4	0.21	2.256×10^2	W14×22	EEP_8	2.787×10^2
Weight	6.712×10^3			6.238×10^2	6.712×10^3		6.407×10^2
Total Cost	7.336×10^3 (lb)				7.353×10^3 (lb)		
Constraint	Limit	Actual	Ratio		Limit	Actual	Ratio
Top-Sway	0.81 (in.)	0.792 (in.)	0.977		0.81 (in.)	0.800 (in.)	0.987
Mid-Span Deflection	1.00 (in.)	0.707 (in.)	0.707		1.00 (in.)	0.595 (in.)	0.595
Stress Ratio	1.00	0.997	0.997		1.00	0.984	0.984
Governing Clause		Equation 1.6-2			Governing Clause		Equation 1.6-1a
Governing Group		beam-1			Governing Group		incol-1
Governing Load Combination: 2					Governing Load Combination: 2		

the connection selection procedure searched the connection database for a candidate having a reference stiffness that matched the optimal stiffness and which had an adequate moment capacity and appropriate dimensions compatible with the sizes of the column and beam to be connected. For this example, the fourth and eighth specimens of the Extended End Plate connections (EEP_4 and EEP_8) in the connection database Table 1 and 2 were selected for the first and top-storey beams, respectively. Once the connections were selected, the corresponding nonlinear connection behaviour was accounted for in the nonlinear analysis conducted during the subsequent design verification process. For the frame designed using Extended End Plate Connections it is noted, from Table 2, there are only a few connections available for W14-shapes. Therefore, connection EEP_8 and EEP_4 have been selected for this study. The arrangement and nonlinear moment-rotation curves of these connections are shown in Figures 5 and 6. From Tables 3 and 4 it is noted that all the

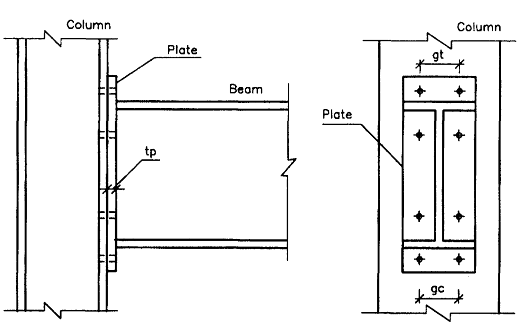

Fig. 5. Extended end plate connection (EEP).

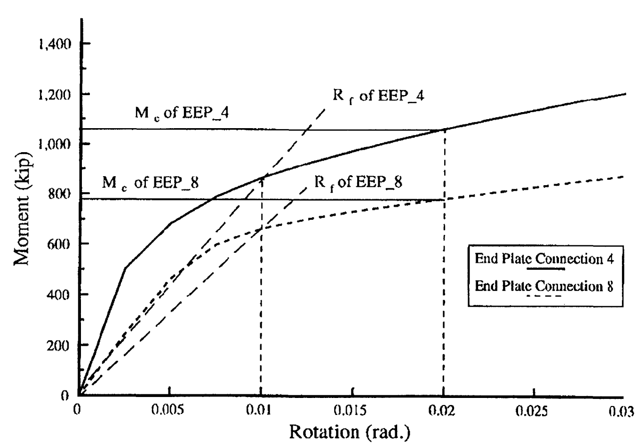

Fig. 6. Nonlinear M-θ curves of EEP_4 and EEP_8 connections.

optimal designs for this example are controlled by Load Combination 2 (Dead + Wind). To provide a better understanding of the influence of semi-rigid connection behaviour on the internal force distributions for such frameworks, the bending moments for all members and the axial forces for the columns are presented in Figures 7 to 10. The maximum rotations of the beam-to-column connections for the semi-rigid cases in Tables 3 and 4 are calculated by the secant formula $\theta^{max} = M/R$ and are as shown in Figures 7 to 10.

For fully-rigid frame design, the initial results generated by the computer-based optimization technique are:

excol: W8×28;
incol: W8×35;
beam-1: W14×43;
beam-2: W14×22

Since the flange width of the W14×43 exceeds that of the W8×28, a modification is made by adjusting the exterior column and floor beam to be W8×31 and W14×38, respectively. This adjustment causes a 2.8 percent violation of the stress response in the floor beam.

Upon comparing the traditional fully-rigid design with the semi-rigid design having large connection stiffnesses, Table 3 indicates that the top-sway and mid-span deflection respectively increase 5 percent and 10 percent for the latter case. The governing member group for the fully-rigid design is that for the beams in the first storey. However, the exterior column group controls the semi-rigid design with large connection stiffnesses. This perhaps indicates that traditional fully-rigid design may mislead the designer into having too much concern for the strength of the beams and not enough for the strength of columns.

When comparing the fully-rigid design in Table 3 with the semi-rigid design with extended-end plate connections in Table 4, it is found that the top sway and maximum mid-span deflection are respectively 69 percent and 92 percent greater for the latter case. This indicates that displacement-related serviceability conditions should be carefully examined for the design of steel frameworks involving semi-rigid connections. As well, since increased frame sway means increased P-Δ effects, it suggests that perhaps second-order effects should be considered in the design of low-rise steel frameworks with semi-rigid connections. It is noted from Tables 3 and 4 that any one of the semi-rigid designs for this low-rise framework has less member weight than the fully-rigid design, and thus is more economical from the point of view of member material consumption. By further considering connection costs, even more saving is possible through the use of semi-rigid connections since they are less complex than and, therefore, less costly to fabricate than so-called fully-rigid connections.

CONCLUDING REMARKS

The design of partially restrained, moment resisting frames is carried out using a mixed discrete optimization technique and a database of member sizes and connection properties developed in the United States. The data is, therefore, expressed in Imperial units. No comparable Canadian information is available as this problem is yet to be addressed in Canadian design codes.

The fixity factor, r, describing the end fixity of beam members varies, from 0 to 1, representing the pinned and fully fixed conditions corresponding to the extremes of restraint possible in a beam or column. It is understood, of course, that these extreme values are applicable only to idealised mathematical models suitable for analysis. Gerstle (1988) has proposed that field-bolted connections (Type 2) are in the range of rotational stiffness from 2×10^5 to 10×10^5 kip-in/rad, and that welded connections (Type 1) are in the range from 10×10^5 to 50×10^5 kip-in/rad. This classification ignores more recent developments, such as end plate connections employing prestressed bolts in tension, which can develop much larger

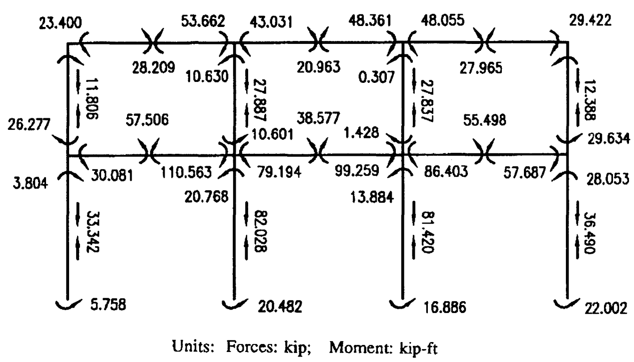

Fig. 7. Member forces for fully-rigid design.

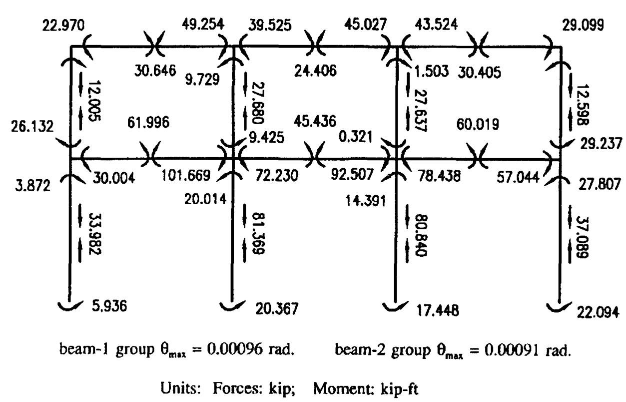

Fig. 8. Member forces and connection rotations for semi-rigid design with large connection stiffness (r = 0.9).

rotational stiffness. It might be more appropriate to suggest that bolted and welded connections can be used interchangeably over the entire range from 2×10^5 to 50×10^5 kip-in/rad.

Alternative designs involving the traditional ideal model (fully-rigid design, r = 1) and the partial restraint model (semi-rigid design) proposed herein are compared using equivalent costs. In this scenario, connection costs are expressed as incremental weights of the connected beams using data supplied by the CISC with recognizes heavier connections, and hence increasing cost, with increasing rotational stiffness. In the variants of the design example given, the total weight/cost is expressed as primary member weights plus equivalent connection weights and the following observations in order:

1. The proposed semi-rigid design method is valid and self consistent; with large values of connection stiffness, R, it is seen to converge to the traditional fully rigid design, r = 1, both as to weight/cost and performance.
2. Fully-rigid design is unrealistic. In fact, since fully-rigid connections are unattainable in practice and their realistic stiffness is often in the range of 80 to 90 percent of fully fixity, for design based on assuming fully-rigid connections the primary columns in the structure are over-stressed (being too light) and predictions of drift are unsafe (being too low). The connection costs for fully-rigid design are exorbitant and negate any comparison with those for semi-rigid design.
3. Semi-rigid design comes into its own for low and midrise structures, when gravity loads control the design and sway is relatively unimportant. Here, both member weights and connection costs are optimal and lower than those for comparable rigid frame design.
4. Nonlinear connection behaviour has a significant influence on the lateral sway of a steel frame; this can be seen in Table 3 in which all the designs were governed by the stress response except for the one with standard Extended End Plate Connections.
5. The current connection database does not meet the needs of practice being confined largely to connections of small capacity.

ACKNOWLEDGMENTS

This work was sponsored by the Natural Sciences and Engineering Research Council of Canada, and is based upon research conducted by the first author under the supervision of the second and third authors for the degree of Doctor of Philosophy in Civil Engineering at the University of Waterloo, Canada.

REFERENCES

American Institute of Steel Construction, *Manual of Steel Construction, Allowable Stress Design*, 8th Ed., Chicago, IL, 1978.

Chen, W. F. and Kishi, N., "Moment-Rotation Relation of Top- and Seat-Angle Connections," *Structural Engineering Report*, CE-STR-87-4, School of Civil Engineering, Purdue University, West Lafayette, IN., 1987.

Canadian Institute of Steel Construction and Canadian Steel Construction Council, *A Project Analysis Approach to Building Costs*, Ontario, Canada, 1983.

Frye, M. J. and Morris, G. A., "Analysis of Flexible Connected Steel Frames," *Canadian Journal of Civil Engineering*, Vol. 2, No. 3, 1975, pp. 280–291.

Gerstle, K. H., "Effects of Connections on Frames," *J. Construc. Steel Research*, 10, 1988, pp. 241–267.

Hsieh, S. H., *Analysis of Three-Dimensional Steel Frames with Semi-Rigid Connections*, Cornell University, Ithaca, NY, 1990.

Kishi, N. and Chen W. F., "Steel Connection Data Bank

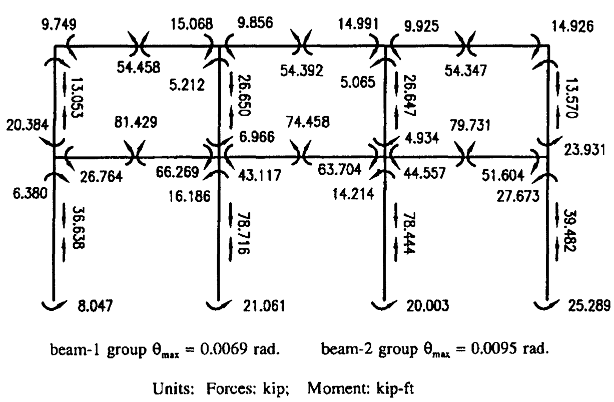

Fig. 9. Member forces and connection rotations for semi-rigid design (open connection selection).

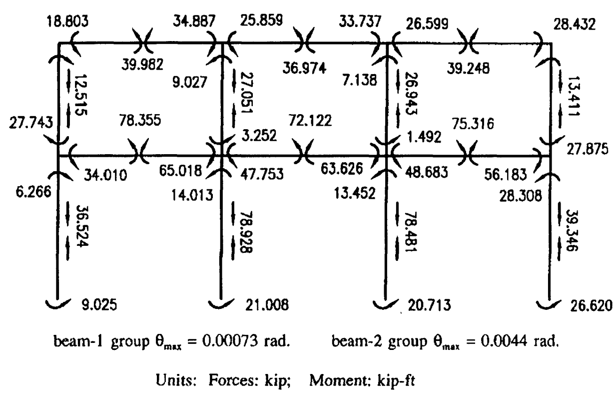

Fig. 10. Member forces and connection rotations for semi-rigid design with extended end plate connections.

Program", *Structural Engineering Report*, CE-STR-86-18, School of Civil Engineering, Purdue University, West Lafayette, IN, 1986.

Lui, E. M. and Chen, W. F., "Analysis and Behaviour of Flexibly-Joint Frames," *Engineering Structures*, No. 8, 1986 pp. 107–118.

Monforton, G. R. and Wu, T. S., "Matrix Analysis of Semi-Rigidly Connected Steel Frames," ASCE, *Structural Division*, Vol. 89, No. ST6, 1963, pp. 13–42.

Nethercot, D. A., Davison, J. B. and Kirby, P. A., "Connection Flexibility and Beam Design in Non-sway Frames," *Engineering Journal*, AISC, Vol. 25, No. 3, 1988, pp. 99–108.

Richard, R. M. and Abbott, B. J., "Versatile Elastic-Plastic Stress-Strain Formulation," ASCE, *J. of Engineering Mechanics Division*, 101, EM4, 1975, pp. 511–515.

Schmit, L. A. and Fleury, C., "An Improved Analysis/Synthesis Capability based on Dual Methods-ACCESS 3," AIAA Paper No. 79-0721, *Struct. Vol., AIAA/ASME/ASCE/AHS 20th Structures, Structural Dynamics and Materials Conference*, St. Louis, MO, 1979.

Xu, L., "Geometrical Stiffness and Sensitivity Matrices for Optimization of Semi-Rigid Steel Frameworks," *Journal of Structural Optimization*, Vol. 5, 1992, pp. 95–99.

Xu, L. and Grierson, D. E., "Computer-Automated Design of Semi-Rigid Steel Frameworks," *J. of Structural Engineering*, ASCE, Vol.119, No. 6, 1993, pp. 1740–1760.

Xu, L., "Optimal Design of Steel Frameworks with Semi-Rigid Connections," *PhD. Thesis*, University of Waterloo, Canada, 1994.

List of Notations

a	cross-section area
a_i	cross-section area of member i
A_i	a set of cross-section areas of the available discrete standard sections for member i
C	correction matrix for semi-rigid member
E	Young's material modulus
$f_{ij}, f_{ij}^L, f_{ij}^U$	connection flexibility for semi-rigid member i at end j and its associated lower and upper bounds
G_i	geometric stiffness matrix for semi-rigid member i
I	moment of inertia
K_i	structural stiffness matrix
i	index of member
j	index of member-end
L	member length
M	connection bending moment
M_c	connection moment capacity
n	curve-fitting shape parameter for connection modelling
r	fixity factor for semi-rigid member
r_{ij}	fixity factor for semi-rigid member i at end j
R	connection stiffness
R_f	connection reference stiffness
$R_{ij}, R_{ij}^L, R_{ij}^U$	connection stiffness for semi-rigid member i at end j and its associated lower and upper bounds
R_p	connection strain-hardening stiffness
S_i	elastic stiffness matrix for fully-rigid member
w_i	weight coefficient for member i
X_i	a set of reciprocal cross-section areas of the available discrete standard sections for member i
x_i	reciprocal section area for member i
Z	total member-plus-connection 'cost' for structure
Z_i	member-plus-connection 'cost' for member i
β_{ij}, β_{ij}^o	cost coefficients for connection at end j of semi-rigid member i
$\delta_1, \delta_1^L, \delta_1^U$	specified joint displacement and its associated lower and upper bounds
θ	connection rotation
$\sigma_k, \sigma_k^L, \sigma_k^U$	normal stress at section k and its associated lower and upper bounds