Effective Lengths of Uniform and Stepped Crane Columns

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INTRODUCTION

Crane columns are used extensively in warehouses and mill buildings to support overhead cranes. In addition to carrying the dead and live loads of the moving cranes, crane columns are required to support the roof structure of the buildings. The cross-section of a crane column can be uniform or stepped. In the former, a bracket bolted or welded to the face of the column is used to support the runway girder as shown in Figure 1a. In the latter, the runway girder rests directly on the enlarged segment of the column as shown in Figure 1b. In earlier years, it was common practice to erect the crane column independent of the frame column. The two were then connected by batten plates or laces to act as a composite unit as shown in Figure 1c. In any case, the base of the column may be pinned or fixed. If the upper shaft of the column is rigidly connected to the roof of the building, resistance to in-plane translation is provided by rigid frame action and the base of the column can be designed as pinned or fixed. However, if the upper shaft of the column is not rigidly connected to the roof as for the case when prefabricated roof trusses are used, the base of the crane column must be designed as rigid so that resistance to sway can be provided by cantilever action of the column.

Crane columns differ from ordinary frame columns in that heavy axial loads are present at some intermediate points of the columns. These loads are denoted as P_L in Figure 1. The design of crane columns is complicated by the fact that a simple procedure which gives reasonable accurate effective length factors (and hence the critical loads) for the columns under a variety of geometric, loading and boundary conditions is not readily available in design codes. For instance, no guidelines are provided in the current AISC LRFD Specification[1] for evaluating crane column effective lengths even though their design, which is often based on interaction equations, requires the use of the effective length factor K to determine their nominal axial strength P_n.

A simple but crude approach to evaluate effective lengths of crane columns for the frame shown in Figure 2a is to ignore the upper shafts of the crane columns and assume that the roof beam is connected directly to the crane load points B and F at the level of the crane beam (Figure 2b). This approach often gives erroneous results especially for cases in which the upper shafts of the crane columns are subjected to high axial loads. Another approach is to assume that the full lengths, $L_U + L_L$, of the crane columns are subjected to the combined axial loads of $P_U + P_L$ (Figure 2c). Although this approach often gives conservative results for uniform crane columns, the degree of conservatism may be quite high for cases in which the upper shafts of the crane columns are only lightly loaded. In addition, the application of the approach for stepped crane columns is somewhat awkward since the moments of inertia are not constant along the lengths of the columns.

In this paper, a simple yet reasonably accurate procedure for calculating effective length factors for crane columns of uniform and nonuniform (stepped) cross-sections with any

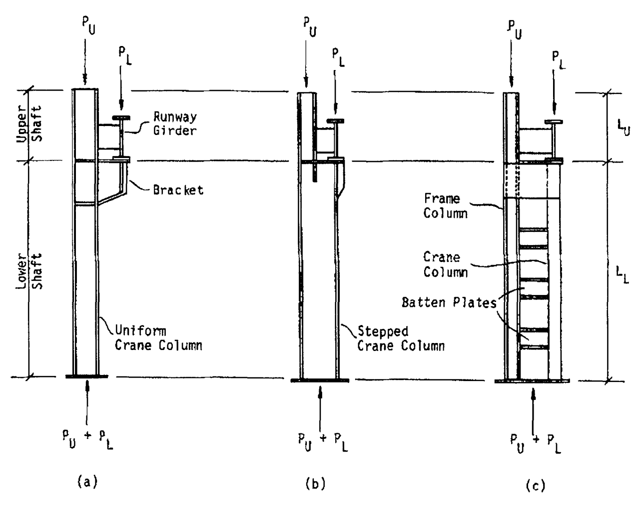

Fig. 1. Uniform and stepped crane columns.

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values of relative shaft lengths, moments of inertia, loading and boundary conditions is presented. The procedure is based on the story stiffness concept and it takes into account both member and frame instability effects in the formulation. The validity of the proposed methodology will be demonstrated by numerical examples.

BACKGROUND

In theory, the critical load of a column with any boundary conditions subjected to any types of axial loads applied anywhere on the column can be obtained from an eigenvalue analysis.[2,3] In an eigenvalue analysis the critical load is obtained as the eigenvalue of the characteristic or transcendental equation written with respect to the deformed geometry of the column. Depending on the complexity of the problem, recourse to numerical methods is often necessary to obtain solutions. For columns that are parts of a frame, the buckling load of the frame can be obtained from a stability analysis in which the stiffness equations of the frame are formulated using stability functions or finite elements. The critical load is obtained as the load when the stiffness of the frame vanishes. Mathematically, this critical load is the load that renders the determinant of the structure stiffness matrix zero. Once the critical load of the frame is obtained, the effective length factor of any column in the frame can be calculated from the equation

$$K = \sqrt{\frac{\pi^2 EI}{P_{cr} L^2}} \tag{1}$$

where

- K is the effective length factor
- P_{cr} is the column load at buckling
- E is the modulus of elasticity
- I and L are the moment of inertia and length of the column, respectively.

Because of the arduousness involved in the solution process, the use of a stability or numerical analysis for calculating critical loads is seldom undertaken by practitioners in design offices. For design purposes, simplified procedures which do not require the use of laborious computational techniques but capable of giving reasonably accurate results are often preferred. Over the past two decades, a number of simplified procedures have been proposed for evaluating effective length factors for crane columns. Anderson and Woodward[4] presented transcendental equations for a series of isolated stepped crane columns using a combination of pinned, fixed, and guided end conditions. The effective length of the crane column to be designed is obtained as an approximation of one of the idealized cases presented. To facilitate design, Agarwal and Stafiej[5] developed tables of effective length factors based on the Anderson-Woodward transcendental equations. Moore[6] also presented solutions for effective length factors of a special class of stepped crane columns in which the top of the column is pinned. The AISE Technical Report No. 13[7] provides tables for effective length factors of crane columns with fixed bases and pinned tops in terms of three parameters: (1) the ratio of the length of the upper shaft to the total length of the column; (2) the ratio of the moment of inertia of the lower shaft to that of the upper shaft; and (3) the ratio of axial force in the upper shaft to the additional axial force applied on the lower shaft. Using the same parameters but with a somewhat different range, Huang[8] presented values for the effective length factors of cantilever crane columns in the form of graphical charts. Based on a parametric study using an in-house stability analysis program, Fraser[9,10] proposed a

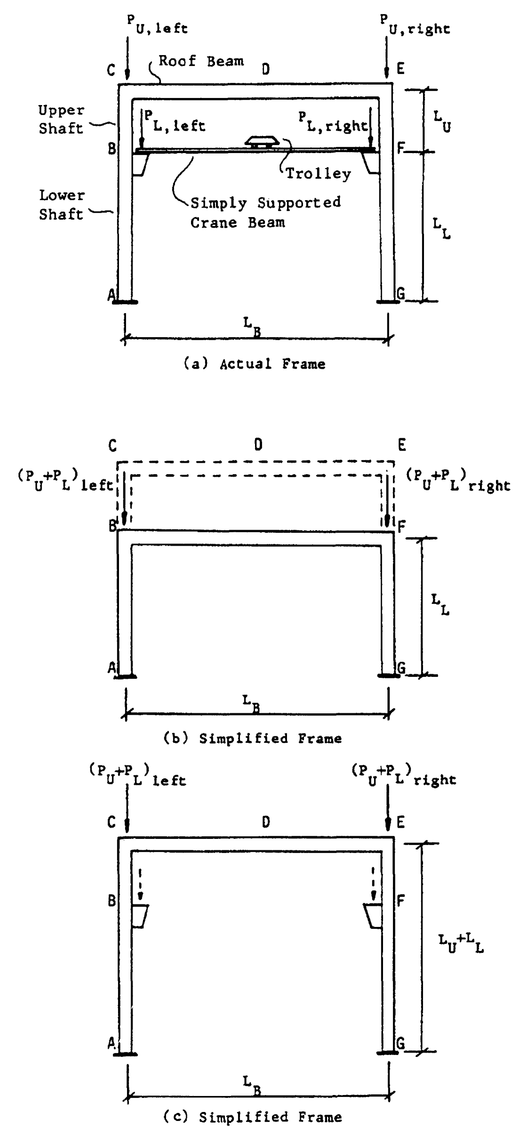


Fig. 2. Simplified frame models.

procedure by which the effective length factors of pin-based uniform and stepped crane columns can be calculated. The approach proposed by Fraser requires the use of the AISC K-factor alignment chart in conjunction with special factors and charts presented in References 9 and 10 for solutions. Recently, Bendapudi[11] presented a rather comprehensive approach for the design of heavily loaded laced mill building columns with fixed bases and pinned tops.

All the procedures mentioned in the preceding paragraph have limitations in terms of column geometry (relative lengths of the upper shaft and lower shaft, relative values of moments of inertia of the upper shaft and the lower shaft), loading conditions (relative magnitudes of axial loads on the upper shaft and on the lower shaft) and boundary conditions. More importantly, most of the studies reported were based on isolated member analysis which ignores the interaction effect between the left and right columns of the frame. It is a well-known fact that a lightly loaded column braces a heavily loaded column at buckling.[12] This member interaction effect should be incorporated in the design of crane columns since the overhead crane is in constant motion and thus tends to impart different live axial loads on the supporting columns. In what follows, a simple methodology which does not require the use of any special nomographs or charts to determine effective length factors for uniform and stepped crane columns with any values of relative shaft lengths, moments of inertia, loading and boundary conditions will be presented. The validity of the proposed method will be demonstrated by comparison with theoretical and numerical solutions.

In applying the proposed method, the following limitations must be observed:

1. At incipient instability, the columns must buckle in the sway uninhibited mode. This is an inherent assumption used in the story stiffness concept on which the proposed method is based.
2. The columns are single shafted columns (i.e., the cross-section of the column behaves as a single unit). The approach is not applicable for multiple shafted columns in which individual shafts do not act in a fully composite manner.
3. The lateral displacements at the immediate load points (i.e., points B and F in Figure 2a) of the columns at incipient instability should not differ by more than 25 percent. This value is well within practical limits. Because of the presence of the crane beam, the difference in lateral displacements at points B and F will not be appreciable.

K-FACTOR EQUATION

In Reference 13, a K-factor equation capable of accounting for the member interaction effect mentioned in the foregoing was derived based on the story stiffness concept. It is applicable not only for rigidly connected columns, but for columns with partial end restraints and leaning columns as well. The equation has the form

$$K_i = \sqrt{\left(\frac{\pi^2 EI_i}{P_i L_i^2}\right)\left[\left(\Sigma\frac{P}{L}\cdot\frac{1}{5\Sigma\eta}\right)+\left(\Sigma\frac{P}{L}\cdot\frac{\Delta_I}{\Sigma H}\right)\right]} \tag{2}$$

where

K_i	is the effective length factor of the i-th column
EI_i and L_i	are the flexural rigidity and length of the column, respectively
P_i	is the axial force in the column
$\Sigma(P/L)$	is the sum of the axial force to length ratios of *all* columns in the story
ΣH	is the sum of all fictitious lateral loads producing Δ_I; it is obtained by adding all the fictitious lateral loads applied at and above the story under consideration
Δ_i	is the first-order *interstory* deflection caused by ΣH, and η is a stiffness index given by

$$\eta = \left[3 + 4.8\left(\frac{M_S}{M_L}\right) + 4.2\left(\frac{M_S}{M_L}\right)^2\right]\frac{EI}{L^3} \tag{3}$$

in which

M_S and M_L are the numerically smaller and larger end moments of the column, respectively.

These moments are obtained from a first-order analysis when the frame is subjected to the fictitious lateral loads of ΣH. The ratio M_S/M_L is taken as positive if the column bent in reverse curvature and it is taken as negative if the column bent in single curvature.

It is important to note that ΣH is not the actual lateral loads that act on the frame. Rather, it is a set of fictitious lateral loads obtained as a fraction of the gravity loads applied at the joints of the frame. The reason behind subjecting the frame to this set of fictitious lateral loads is to create a deflected configuration for the frame which closely approximates the actual buckled shape of the frame. The magnitude of these fictitious loads to be used for the analysis is not important as long as they are proportional to the applied gravity loads. This is because in a first-order analysis the ratios M_S/M_L in Equation 3 and $\Delta_I/\Sigma H$ in Equation 2 will remain constant regardless of the load magnitude. While the magnitudes of the fictitious lateral loads are quite arbitrary, they must be applied in a direction consistent with the anticipated buckled geometry of the frame. For frames which exhibit no preferred direction for buckling (e.g., symmetrical frames subjected to symmetrical loadings), the direction of these fictitious lateral loads is inconsequential.

In Reference 13 a number of examples were given to explicate the use of Equation 2. The accuracy of the equation in determining column effective length factors was demonstrated for a variety of frames including cases in which the distribution of gravity loads was not constant throughout a story and cases in which the flexural rigidities of the columns vary significantly across a story as well as cases involving leaning columns. The validity of Equation 2 was further demonstrated in Reference 14 in which a number of comparative studies were performed using multistory multibay frames. In the following section, the procedure for using Equation 2 to calculate effective length factors for crane columns is given. This will be followed by numerical examples to demonstrate the accuracy of the proposed approach.

PROCEDURE

In applying the proposed procedure to frames with crane columns, the crane columns are considered to be consisted of two segments: the lower shafts and the upper shafts. The lower shafts will be parts of a "story" of the frame while the upper shafts will constitute another story. The fictitious lateral loads to be applied to create an approximate buckled configuration for the frame are calculated based on the gravity loads acting on the frame. A first-order analysis is then performed. From the results of this analysis, the moment ratios M_S / M_L of the lower shafts of the crane columns as well as $\Delta_I / \Sigma H$ are calculated. The effective length factors for the lower shafts of the columns are to be evaluated from Equation 2 and the effective length factors for the upper shafts of the columns are to be calculated using Equation 5. The step-by-step procedure for applying the proposed approach is outlined below:

1. For the frame given in Figure 3a, replace the vertical gravity loads by fictitious lateral loads as shown in Figure 3b. The fictitious lateral load factor α to be used is quite arbitrary. In Reference 13, α was taken as 0.001 but any convenient values of α can be used. The fictitious lateral loads should be applied in such a direction as to create a deflected geometry for the frame which closely approximates its true buckled configuration.
2. With the frame subjected to the fictitious lateral loads, perform a first-order analysis on the frame. Using results from this analysis, calculate the moment ratios M_S / M_L for the lower shafts (i.e., segments AB and FG) of the crane columns AC and EG. Also calculate $\Delta_I / \Sigma H$ where Δ_I is the average lateral deflections at the intermediate load points (i.e., points B and F) of the columns and ΣH is the sum of all fictitious lateral loads that act at and above the intermediate load points.
3. Calculate the stiffness index η according to Equation 3. Sum the stiffness indices of the two crane columns.
4. Calculate K factors for the lower shafts of the crane columns using Equation 2.
5. Once the K factors for the lower shafts of the columns are known, the K factors for the upper shafts of the columns can be calculated using the formula

$$\frac{P_U}{P_U + P_L} = \frac{(P_U)_{cr}}{(P_U + P_L)_{cr}} = \frac{\pi^2 EI_U / (K_U L_U)^2}{\pi^2 EI_L / (K_L L_L)^2} \tag{4}$$

which, upon rearrangement gives

$$K_U = K_L \left(\frac{L_L}{L_U}\right) \sqrt{\left(\frac{P_L + P_U}{P_U}\right)\left(\frac{I_U}{I_L}\right)} \tag{5}$$

where

K_U is the effective length factor of the upper shaft

K_L is the effective length factor of the lower shaft

L_U, L_L, I_U, I_L are the lengths and moments of inertia of the upper and lower shafts, respectively

P_U is the gravity load applied on the upper shaft and

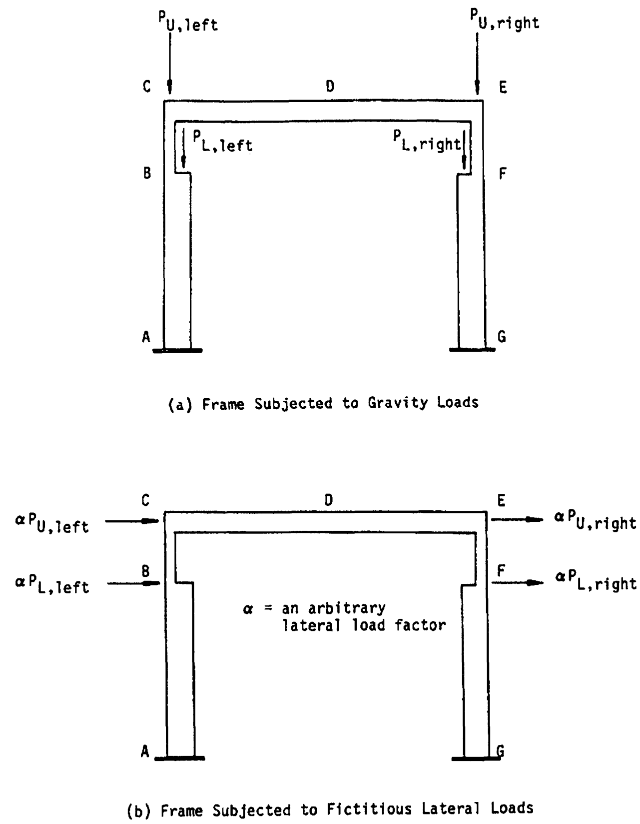


Fig. 3. Substitute frame for effective length factor computation.

P_L	is the gravity load applied on the lower shaft.

The sum $P_L + P_U$ is thus the total gravity load supported by the lower shaft of the crane column.

NUMERICAL EXAMPLES

In this section, three examples will be used to illustrate the procedure and to demonstrate the validity of the proposed approach for calculating effective length factors for uniform and stepped crane columns with different loading and boundary conditions.

Example 1

The effective lengths of the uniform crane columns shown in Figure 4a are to be calculated. The frame is an example frame used in Reference 9. The upper shafts of the crane columns are supporting a roof load of 234 kN each while the lower shafts of the crane columns are supporting crane loads of 700 kN and 200 kN, respectively, on the left and right columns in addition to the roof loads. The modulus of elasticity, E; the moment of inertia of the beam, I_B; and of the upper and lower shafts of the crane columns I_U and I_L are given in the figure. In reality, moments are induced at points B and F because the crane loads are applied at an eccentricity with respect to the column axes. Nevertheless, these moments are ignored in evaluating effective length factors for the columns.

To begin the analysis, we shall replace the gravity loads by a set of fictitious lateral loads. This is shown in Figure 4b. The fictitious lateral load factor used was 0.001. The magnitudes of the fictitious lateral loads were thus obtained as the product of 0.001 and the gravity loads. The fictitious lateral loads are to be applied in the anticipated buckled direction of the frame at locations where the gravity loads were. The results of a first-order analysis performed on the frame of Figure 4b are given as follows:

$$(M_S / M_L)_{AB} = 0,\ (M_S / M_L)_{FG} = 0$$

$$(\Delta_I)_B = 1.414 \times 10^{-3}\ \text{m},\ (\Delta_I)_F = 1.404 \times 10^{-3}\ \text{m}$$

so,

$$\Delta_I / \Sigma H = [(1.414 \times 10^{-3} + 1.404 \times 10^{-3}) / 2] / 1.368 = 0.00103\ \text{m/kN}$$

Using Equation 3, we obtain

$$\eta_{AB} = 1230\ \text{kN/m},\ \eta_{FG} = 1230\ \text{kN/m, so}\ \Sigma\eta = 2460\ \text{kN/m}$$

Also, we have

$$\Sigma(P / L) = (934/10) + (434/10) = 136.8\ \text{kN/m}$$

Now, using Equation 2 the effective length factors for the lower shafts of the crane columns are calculated as:

$$K_{AB} = \sqrt{\frac{\pi^2 E I_{AB}}{P_{AB} L_{AB}^2}\left[\left(\Sigma\frac{P}{L}\cdot\frac{1}{5\Sigma\eta}\right)+\left(\Sigma\frac{P}{L}\cdot\frac{\Delta_I}{\Sigma H}\right)\right]}$$

$$= \sqrt{\frac{\pi^2(200 \times 10^6)(2050 \times 10^{-6})}{(934)(10)^2}\left[\frac{136.8}{5(2460)} + (136.8)(0.00103)\right]}$$

$$= 2.57$$

$$K_{FG} = \sqrt{\frac{\pi^2 E I_{FG}}{P_{FG} L_{FG}^2}\left[\left(\Sigma\frac{P}{L}\cdot\frac{1}{5\Sigma\eta}\right)+\left(\Sigma\frac{P}{L}\cdot\frac{\Delta_I}{\Sigma H}\right)\right]}$$

$$= \sqrt{\frac{\pi^2(200 \times 10^6)(2050 \times 10^{-6})}{(434)(10)^2}\left[\frac{136.8}{5(2460)} + (136.8)(0.00103)\right]}$$

$$= 3.76$$

Once the effective length factors for the lower shafts have

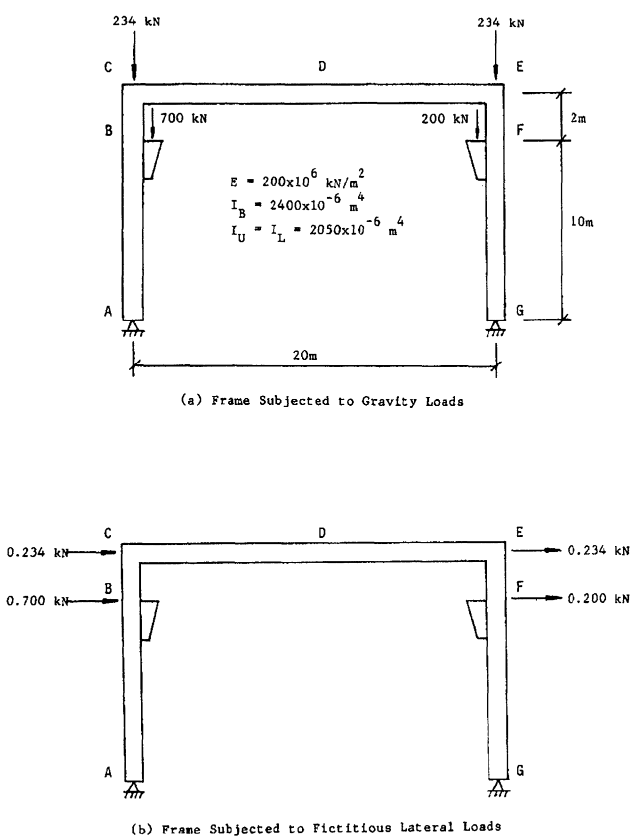


Fig. 4. Pin-based uniform crane columns.

been calculated, the effective length factors for the upper shafts can be calculated from Equation 5 as follows:

$$K_{BC} = K_{AB}\left(\frac{L_{AB}}{L_{BC}}\right)\sqrt{\left(\frac{P_{AB}}{P_{BC}}\right)\left(\frac{I_{BC}}{I_{AB}}\right)}$$

$$= 2.57\left(\frac{10}{2}\right)\sqrt{\left(\frac{934}{234}\right)\left(\frac{2050\times10^{-6}}{2050\times10^{-6}}\right)}$$

$$= 25.6$$

$$K_{EF} = K_{FG}\left(\frac{L_{FG}}{L_{EF}}\right)\sqrt{\left(\frac{P_{FG}}{P_{EF}}\right)\left(\frac{I_{EF}}{I_{FG}}\right)}$$

$$= 3.76\left(\frac{10}{2}\right)\sqrt{\left(\frac{434}{234}\right)\left(\frac{2050\times10^{-6}}{2050\times10^{-6}}\right)}$$

$$= 25.6$$

For purpose of comparison, the theoretical effective length factors calculated using an eigenvalue analysis and those reported in Reference 9 are given in Table 1.

As can be seen, the proposed method gives reasonably accurate results. For design, both the lower and upper shafts of the crane columns should be checked for compliance with the beam-column interaction equation. At first glance, it seems that the effective length factors for the upper shafts are extraordinarily high. However, it should be noted that the design strength of a column is dependent on its effective length KL, not just the effective length factor K. The upper shafts, being much shorter than the lower shafts, will naturally have larger K values according to Equation 4. Nevertheless, even if the design strengths $\phi_c P_n$ of the upper shafts are smaller than those of the lower shafts, the upper shafts are subjected to much smaller axial loads P_u. The result is that the ratio $P_u / \phi P_n$ will be well within reasonable limits. Very often, the design of uniform crane columns is controlled by the capacity of the lower shafts because of the presence of high axial loads in these portions of the columns.

Example 2

In this example, the effective length factors of the stepped crane columns shown in Figure 5a are to be determined. The frame is the same frame used in Reference 10 by Fraser. The upper shafts of the crane columns are supporting a roof load of 53 kips each. The lower shafts are carrying crane loads of 300 kips and 140 kips, respectively, on the left and right columns in addition to the roof loads. The modulus of elasticity, E; moment of inertia, I; and cross-sectional area, A; are given in the figure. The subscripts B, U and L denote the beam, the upper shaft and the lower shaft of the crane column, respectively. The fictitious lateral loads shown in Figure 5b were obtained by multiplying the gravity loads by a fictitious lateral load factor α of 0.001. A first-order analysis of the frame shown in Figure 5b gives the following results:

$$(M_S / M_L)_{AB} = 0,\ (M_S / M_L)_{FG} = 0$$

$$(\Delta_I)_B = 0.1086 \text{ in.},\ (\Delta_I)_F = 0.1077 \text{ in.}$$

so,

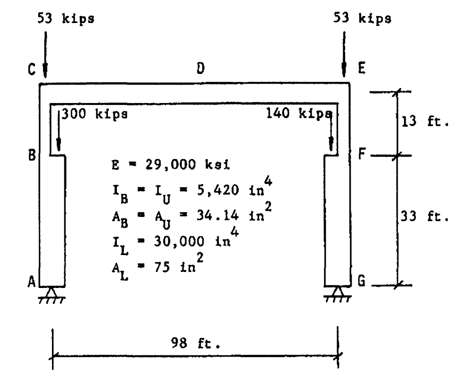
$$\Delta_I / \Sigma H = [(0.1086 + 0.1077)/2]/0.546 \text{ in/kips}$$

Using Equation 3, we obtain

$$\eta_{AB} = 42.03 \text{ kips/in.}$$
$$\eta_{FG} = 42.03 \text{ kips/in.}$$

so

$$\Sigma\eta = 84.06 \text{ kips/in.}$$



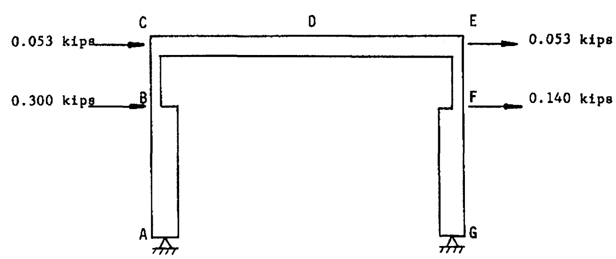


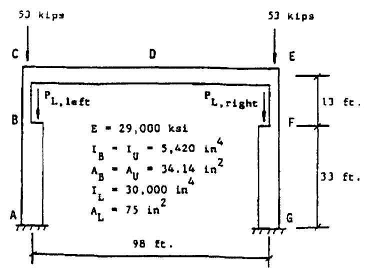
Fig. 5. Pin-based stepped crane columns (1 kip = 4.448 kN, 1 ft. = 0.305 m, 1 in. = 25.4 mm, 1 ksi = 6.895 MPa).

Table 1.			
Column Segment	Present Method	Theoretical K	Reference 9
AB	2.57	2.50	2.55
FG	3.76	3.66	3.74
BC	25.6	24.9	25.5
EF	25.6	24.9	25.5

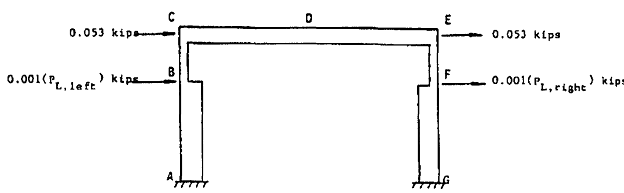
Also, we have

$$\Sigma(P/L) = (353/396) + (193/396) = 1.379 \text{ kips/in.}$$

Using Equation 2 we obtain



(a) Frame Subjected to Gravity Loads



(b) Frame Subjected to Fictitious Lateral Loads

Load Case	$P_{L,left}$ (kips)	$P_{L,right}$ (kips)
I	440	0
II	330	110
III	220	220
IV	110	330
V	0	440

Fig. 6. Fix-based stepped crane columns (1 kip = 4.448 kN, 1 ft. = 0.305 m, 1 in. = 25.4 mm, 1 ksi = 6.895 MPa).

Table 2.			
Column Segment	Present Method	Theoretical K	Reference 10
AB	6.55	6.46	7.00
FG	8.85	8.74	9.47
BC	18.2	18.0	19.5
EF	18.2	18.0	19.5

$$K_{AB} = \sqrt{\frac{\pi^2(29{,}000)(30{,}000)}{(353)(396)^2}\left[\frac{1.379}{5(84.06)} + (1.379)(0.198)\right]}$$

$$= 6.55$$

$$K_{FG} = \sqrt{\frac{\pi^2(29{,}000)(30{,}000)}{(193)(396)^2}\left[\frac{1.379}{5(84.06)} + (1.379)(0.198)\right]}$$

$$= 8.85$$

and, using Equation 5, we obtain

$$K_{BC} = 6.55\left(\frac{396}{156}\right)\sqrt{\left(\frac{353}{53}\right)\left(\frac{5420}{30{,}000}\right)}$$

$$= 18.2$$

$$K_{EF} = 8.85\left(\frac{396}{156}\right)\sqrt{\left(\frac{193}{53}\right)\left(\frac{5420}{30{,}000}\right)}$$

$$= 18.2$$

The effective length factors calculated above are compared with their theoretical values and those obtained in Reference 10 in Table 2. Good agreement is observed.

Example 3

As a third example, the effective length factors of the fix-based stepped crane columns shown in Figure 6a will be determined. The frame is the same as that used in Example 2 except for the boundary conditions at the bases of the crane columns. Also, to simulate the actual operation of the crane, five load cases are considered. In Load Case I, all the crane loads are assumed to act on the lower shaft of the left crane column whereas in Load Case V, all the crane loads are assumed to act on the lower shaft of the right crane column. Load Cases II, III and IV will represent different crane load distributions on the two crane columns when the crane is in the quarter, middle, and three-quarter point of the crane beam, respectively. Using the lateral load model shown in Figure 6b, a first-order analysis was performed on the frame for each load case. The results of the analyses together with the calcu-

Table 3.									
Load Case	Column Shaft	M_S/M_L	η (k/in)	$\Sigma\eta$ (k/in)	P (kips)	P/L (k/in)	$\Sigma(P/L)$ (k/in)	$\Delta_I/\Sigma H$ (in/k)	K
I	AB FG	0.167 –0.163	54.90 32.63	87.53	493 53	1.245 0.134	1.379	.0113	1.44 4.4
II	AB FG	0.108 –0.052	49.98 38.69	88.67	383 163	0.967 0.412	1.379	.0113	1.63 2.51
III	AB FG	0.036 0.036	44.53 44.53	89.06	273 273	0.689 0.689	1.379	.0113	1.93 1.93
IV	AB FG	–0.052 0.108	38.69 49.98	88.67	163 383	0.412 0.967	1.379	.0113	2.51 1.63
V	AB FG	–0.163 0.167	32.63 54.90	87.53	53 493	0.134 1.245	1.379	.0113	4.4 1.44

lated K factors for the lower shafts of the two crane columns are summarized in Table 3. Because of structural symmetry, the K factors for column shaft AB in Load Cases I and II should be the same as those for column shaft FG in Load Cases IV and V. This is indeed the case as depicted in the table. From the table, it can also be seen that the proposed approach successfully captures the interaction effect between the two crane columns: the lightly loaded column is bracing the heavily loaded column at buckling. The result is an increase in effective length factor for the lightly loaded column and a decrease in the effective length factor for the heavily loaded column.[13] For instance, in Load Case III when both columns are subjected to the same axial loads, the effective length factors are equal to 1.93 for both column shafts AB and FG. However, in Load Case I when all the crane loads are acting on column shaft AB, its effective length factor decreases to 1.44 while the effective length factor of column shaft FG increases to 4.4. This interaction effect must be considered in design. The column should be proportioned to withstand the most severe load case. For this problem, the most severe load case is the one which gives the largest value of $P_u / \phi_c P_n$.

Once the effective length factors of the lower shafts have been calculated, the effective length factors of the upper shafts can be determined from Equation 5. The results are summarized in Table 4.

In Table 5, the K factors calculated using the present approach are compared with the theoretical values obtained using an eigenvalue analysis. Because of structural symmetry, only Load Cases I, II, and III are shown. As can be seen, the effective length factors calculated using the present approach compare well with the theoretical values.

SUMMARY AND CONCLUSIONS

A simple methodology for calculating effective length factors for uniform and stepped crane columns with any values of relative shaft lengths, moments of inertia, loading, and boundary conditions was presented. The proposed approach can be applied easily without resort to using any special nomographs and charts for solutions. The only analysis required is a simple first-order analysis from which effective length factors of the lower and upper shafts of the crane column can be calculated using Equation 2 and Equation 5, respectively. Once the effective length factors have been calculated, the lower and upper shafts of the column can be designed using beam-column interaction equation following the usual procedure. Since the approach takes into consideration the destabilizing effect of the frame due to both member and frame instabilities as well as the interbracing effect due to interaction between the strong and weak columns in a story, rather accurate results can be obtained. Numerical examples were given to demonstrate the validity of the approach. Although this paper addresses only the calculation of elastic effective length factors, extension of the approach to inelastic K factor evaluation is rather straightforward using the tangent modulus concept.[15] Because of the ease with which effective length factors can be determined, the proposed approach can greatly facilitate the design of crane columns.

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Table 4.

Load Case	Column Shaft	*K*
I	BC EF	4.74 4.74
II	BC EF	4.73 4.75
III	BC EF	4.72 4.72
IV	BC EF	4.75 4.73
V	BC EF	4.74 4.74

Table 5.

Load Case	Column Shaft	K	$K_{theoretical}$
I	AB FG BC EF	1.44 4.40 4.74 4.74	1.51 4.61 4.97 4.97
II	AB FG BC EF	1.63 2.51 4.73 4.75	1.65 2.52 4.77 4.77
III	AB FG BC EF	1.93 1.93 4.72 4.72	1.95 1.95 4.77 4.77

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