Buckling of Webs in Unsymmetric Plate Girders

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ABSTRACT

The elastic buckling behavior of unsymmetric plate girder webs with and without a longitudinal stiffener is presented. The results of a finite element analysis were used to develop a buckling coefficient for longitudinally stiffened webs which takes into account the depth of the web in compression and the vertical location of the stiffener. The results are compared with the AASHTO Bridge Specifications and a design example is presented.

INTRODUCTION

Slender web unsymmetrical plate girders are often used for composite bridge construction. An unsymmetric girder is defined as a girder with a single plane of symmetry about the vertical axis of the web. The neutral axis of an unsymmetrical girder will not be at mid-height. Consequently the depth of the web in compression will be greater or less than half the web depth. The top flange of the girder in the positive bending moment region, where the composite concrete slab is in compression, is normally smaller than the bottom tension flange. The top flange is very close to the neutral axis of the composite section after the concrete slab has hardened. The top flange contributes very little to the load carrying capacity of the composite section. In the negative moment region of girders where the reinforcing steel is included in the composite section properties, a smaller top flange is also often used. In negative bending the difference in the size of the top and bottom flange is less because the reinforcing steel area is much smaller than the transformed slab area used in positive bending. The depth of the web in compression is largest in positive bending during the construction phase before the slab has hardened. In negative bending, the depth of the web in compression is largest after the slab has hardened.

In order to prevent the buckling of the web due to the bending stress in the web, longitudinal stiffeners are often required in bridge construction. Previous work by Rockey and Dubas has shown that the optimum vertical location of the longitudinal stiffener is 0.2 the web depth, D, from the compression flange for symmetrical girders with simply supported boundary conditions at the flanges.[1,2] The buckling coefficient for bend buckling of the web with a stiffener at this location is 129.3. The optimum location varies in actual girders with the rotational restraint offered by the flanges. The optimum location for webs in unsymmetrical girders is assumed in the AASHTO Bridge Design Specifications to be 0.4 the depth of the web in compression, D_c.[3] The size of the flanges is often changed along the length of the girder to reduce weight. The changes in the flange sizes causes the depth of the web in compression to vary along the length of the girder. If the longitudinal stiffener is located a fixed distance from the compression flange, the stiffener cannot be at the optimum location all along the girder. The position of the longitudinal stiffener relative to the web depth in compression also changes due to the shift in the neutral axis location after hardening of the concrete slab. A means of estimating the buckling capacity of a girder web which includes the position of the longitudinal stiffener relative to the depth of the web in compression is required to design these composite girders.

The buckling capacity of webs in unsymmetrical girders without longitudinal stiffeners must also be estimated for the various stages of construction of composite girders. A web in an unsymmetrical girder is treated in the AASHTO Design Specifications as equivalent to a symmetrical girder with a web depth equal to twice the web depth in compression. This approach is examined in this paper by comparison with more complex approaches taken in other specifications.

The results of a finite element analysis of girder webs with longitudinal stiffeners are presented. The results are used to develop a simple design method to estimate the buckling capacity of longitudinally stiffened girder webs as a function of stiffener location relative to the depth of the web in compression.

UNSTIFFENED GIRDER WEBS

The buckling of webs in unsymmetrical girders from in-plane bending stresses without longitudinal stiffeners can be conveniently calculated replacing the total web depth by twice the web depth in compression in the plate buckling equation. The traditional plate buckling equation is:

$$F_{cr} = \frac{0.904Ek}{\left(\frac{D}{t_w}\right)^2} \quad (1)$$

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where:

- E = Youngs Modulus of the web taken as 29,000 ksi in this paper,
- k = buckling coefficient, dependent upon plate geometry and boundary conditions
- D = web depth
- t_w = web thickness

Equation 1 can be used to express the buckling of webs in unsymmetrical girders by substituting $2D_c$ for the total web depth in the denominator. A more convenient method is to use a buckling coefficient k which can be expressed as:

$$k_{usym} = \frac{k_{sym}}{4}\left(\frac{D}{D_c}\right)^2 \tag{2}$$

Where k_{sym} and k_{usym} are the buckling coefficients for the symmetrical and unsymmetrical web respectively. The minimum buckling coefficient for a plate with all edges free to rotate, simply supported, subjected to a symmetrical bending stress is 23.9. The AISC[6] and AASHTO specifications recognize the partial restraint offered by the flanges by using a value of 36 which is equal to the value for a simply supported web, 23.9, plus 80% of the difference between the simply supported buckling coefficient and the value of 39.6 for fixed rotational boundaries.[4] The corresponding values for an unsymmetrical web are:

$$k_{ss} = 6\left(\frac{D}{D_c}\right)^2 \tag{3}$$

and

$$k_{partial\ fixity} = 9\left(\frac{D}{D_c}\right)^2 \tag{4}$$

The Eurocode No. 3 Part 1:19[5] gives the following equations for the buckling coefficient for girder web as a function of

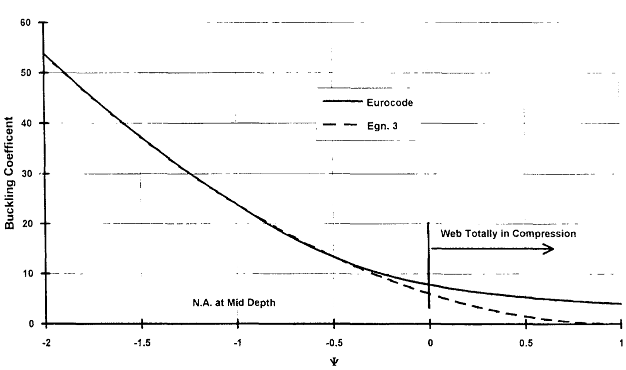

Fig. 1. Comparison of Equation 3 with Eurocode buckling coefficients.

$\psi = \sigma / \sigma_c$ where σ_c is the maximum compressive stress and σ is the maximum stress at the other edge of the web. Compression stress is positive and tension is negative. The buckling coefficient equations in the Eurocode for webs simply supported at the flanges are:

$$\begin{aligned} &\text{for } 1 \geq \Psi \geq 0 \\ &k = \frac{8.2}{1.05 + \Psi} \\ &\text{for } 0 > \Psi > -1 \\ &k = 7.81 - 6.29\Psi + 9.78\Psi^2 \\ &\text{for } -1 \geq \Psi \geq -2 \\ &k = 5.98(1 - \Psi)^2 \end{aligned} \tag{5}$$

Figure 1 compares the buckling coefficients from Equation 3 and the Eurocode equations. The simpler formulation in Equation 3 agrees with the Eurocode values over the range of ψ from –2 to –0.6. This range of ψ is the normal range of stresses found in plate girders. ψ equal to –1 is a symmetrical girder and ψ equal 0 would correspond to a girder with the neutral axis at the tension flange. The simpler formulation given by Equations 3 and 4 are recommended for design. Equation 4 should be used with the AISC or AASHTO Specifications since it matches the assumption of partial flange fixity used in these specifications.

COMPUTIONAL MODEL

The analytical study consisted of an elastic finite element analysis of web panels. The finite element program ANSYS was used for all the analysis. The web and the longitudinal and transverse stiffeners were modeled using 8 node shell elements. A 15 by 15 element mesh subdivided the 102 in. square web panel. The transverse stiffeners at the ends of the panel and the longitudinal stiffener were divided into 15 elements with a width equal to the full stiffener width. The transverse stiffener size in all the analysis was 7/16-in. thick and 5 in. wide. The number of rows of elements above the longitudinal stiffener varied with the stiffener location from 3 to 6. An aspect ratio for the web elements was maintained close to one for all the analysis.

The web was restrained from out-of-plane displacement but free to rotate along its vertical boundaries at the transverse stiffeners. The horizontal boundaries, where the flanges would be attached, were restrained against out-of-plane displacements for all the analyses. The web was allowed to rotate along the horizontal boundary for the majority of the analyses performed. The solutions corresponding to this boundary condition are referred to as simply supported solutions. The horizontal boundaries were restrained from rotation in selected analyses in order to determine the influence of the rotational restraint offered by the flange upon the solutions.

The solutions with these boundary conditions are referred to as fixed support solutions.

The loading consisted of a linear pressure gradient along the vertical web boundaries with compression in the top of the web and tension in the bottom. The value of the compression pressure at the top of the web was maintained at 1 ksi. The buckling stress was therefore equal to the eigenvalue. The location of the neutral axis was altered by adjusting the value of the maximum tensile pressure at the bottom of the web.

Prior to performing the parametric analysis of longitudinally stiffened webs, the analysis procedure was checked by comparing the solutions of webs with and without longitudinal stiffeners to published values. The results were within 2 percent of the published values.

The major variables in this study consisted of the depth of the web in compression (D_c), the vertical location of the stiffener (d_s) and the web thickness (t_w). The geometric parameters are shown in Figure 2. The location of the stiffener relative to the girder depth is denoted by the parameter $x = d_s / D$. The variable $\alpha = d_s / D_c$ defines the location of the stiffener from the top flange relative to the depth of the web in compression. Web slenderness is defined as the ratio of the web depth to the web thickness.

A matrix of values were considered for the depth of web in compression, longitudinal stiffener location, and web slenderness. The values of D_c / D considered were 0.5, 0.54, 0.61, 0.64, 0.69, 0.74, and 0.78. Four locations of the longitudinal stiffener were evaluated for each value of D_c / D. Three of the stiffener positions were fixed with respect to the web depth; x = 0.20, 0.25, and 0.33. The fourth location had a variable location depending upon the depth of the web in compression since it was positioned at α = 0.4. This last stiffener location was used to evaluate the stiffener location recommended in the AASHTO Specification. The values of web slenderness included were 204, 272, and 408.

In addition to the variables listed above, the size of the longitudinal stiffener was varied to determine the adequacy of the present AASHTO Specification. Analyses was also performed without the longitudinal stiffener but with the nodes along the stiffener locations fixed from out-of-plane displacement.

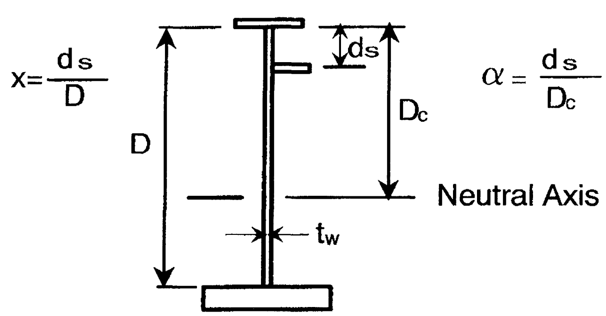

Fig. 2. Girder geometry.

COMPUTATIONAL RESULTS

The results from one series of analysis are shown in Figure 3. The results shown are for a ½-in. web with the longitudinal stiffener modeled by fixing nodes of the web against out-of-plane displacement at the stiffener location. The stiffener was not included in these models. The results shown are for four different stiffener locations. Three fixed locations of the stiffener with respect to the web depth; x = 0.20, 0.25, and 0.33. The fourth stiffener location is a fixed ratio of the depth of the web in compression, α, of 0.40. The depth of the web in compression was varied for each stiffener location to produce the variation in the compression web slenderness. The location of the stiffener is seen to have a significant effect upon the buckling stress. The results for α equal to 0.40 give the larger buckling stress. The fixed stiffener location of x = 0.20 gives identical values to that for α equal to 0.40 for the smallest web slenderness since this is the result from a symmetrical web loading and the stiffener location is identical. The results for a stiffener located at x = 0.33 show almost a constant value. The results for x = 0.25 are also almost independent of the slenderness ratio for smaller slenderness ratios.

The flattening of the results for larger values of x and smaller web slenderness is due to a change in buckling mode exhibited by the web. Figure 4 shows the buckled shape of web with stiffener at x = 0.25 and a compression web slenderness of 160. The web below the stiffener has buckled. When buckling occurs in the panel below the stiffener, changes in the depth of the web in compression cause a significant change in the buckling stress. An increase in the depth of web in compression increases the slenderness ratio of the panel below the stiffener which reduces the buckling stress. Figure 5 shows the result for a compression web

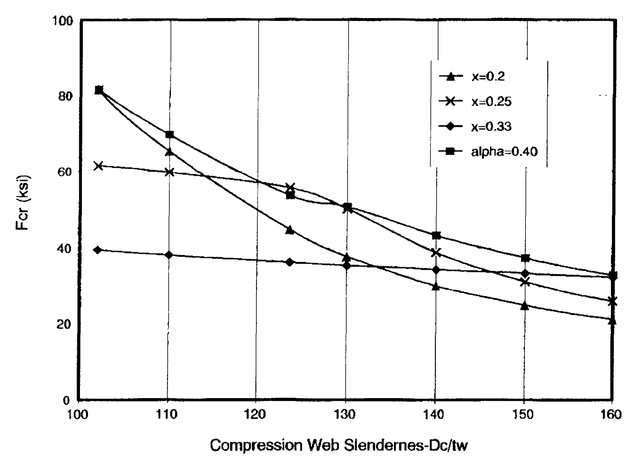

Fig. 3. Typical finite element analysis results.

slenderness of 102 and the same stiffener location. The buckle occurs in the panel above the stiffener. The change in the buckling mode is a result in the change in the value of α. The α value for the geometry in Figure 4 is 0.32. The value in Figure 5 is 0.50, the stiffener is located at the middle of the web depth in compression. All of the values of α for the web with x = 0.33 shown in Figure 3 have an α value greater than 0.4. The very small change in buckling stress with compression web slenderness for these webs with large α values occurs because the slenderness of the controlling top panel is independent of the depth of the web in compression. The depth of the web in compression changes the stress gradient in the panel not the slenderness ratio of the controlling top panel. The change in stress gradient caused by the shift in the neutral axis is responsible for the slight increase in buckling stress with small slenderness ratios.

The influence of the stiffener location upon the buckling capacity of the web is shown for two different web boundary conditions in Figure 6. The vertical axis is the buckling stress for a given compression web slenderness and stiffener location divided by the buckling stress for the identical web with the stiffener located at α = 0.4. The results for simply supported boundaries show a maximum at 0.4 with a ratio equal to one. This indicates that the optimum stiffener location is a value of α equal to 0.4. The maximum ratio occurs at approximately 0.43 for fixed boundary conditions. The rotational restraint at the top edge of the flange increases the buckling capacity of the top web panel. Therefore, a stiffener further from the flange produces a higher buckling capacity. Figure 7 shows the results for both boundary conditions for a fixed stiffener location of x = 0.25. The almost identical behavior of the webs with both boundary conditions when α is less than 0.4 occurs because the web below the stiffener is buckling. The edge boundary conditions primarily influence the results when the top panel controls. The top panel buckles when α is larger than the optimum value. The buckling stress for the web with fixed boundary conditions is larger when the top panel buckling occurs. At the optimum value of α, buckling occurs in both the panel above and below the stiffener.

The influence of stiffener size was also studied for webs with simply supported boundary conditions. The AASHTO LFD specifications have three requirements for the geometry of the longitudinal stiffener. The stiffener must meet the following requirements:

Slenderness Ratio:

$$\frac{b'}{t} \leq \frac{2,600}{\sqrt{F_y}}$$

Moment of Inertia:

$$I \geq Dt_w^3\left[2.4\left(\frac{d_o}{D}\right)^2 - 0.13\right]$$

Radius of gyration:

$$r \geq d_o\frac{\sqrt{F_y}}{23,000}$$

The equations for slenderness ratio and moment of inertia can be combined, if the web is neglected in the calculation, to produce the stiffener size with a slenderness ratio and moment of inertia equal to the requirements. The resulting stiffener size is the most slender size stiffener meeting the required moment of inertia. The radius of gyration requirement will always be met when this size stiffener is used. The influence of stiffener geometry upon the buckling stress is shown in Figure 8. The results are for a ½-in. web with the stiffener placed at x = 0.20, 0.25, and 0.33, and α = 0.40 with various web depths in compression. All webs were simply supported along the top and bottom. The vertical axis is the ratio of the buckling stress divided by the buckling stress calculated with the nodes at the stiffener location fixed against out-of-plane displacement. No stiffener was used in the fixed node case.

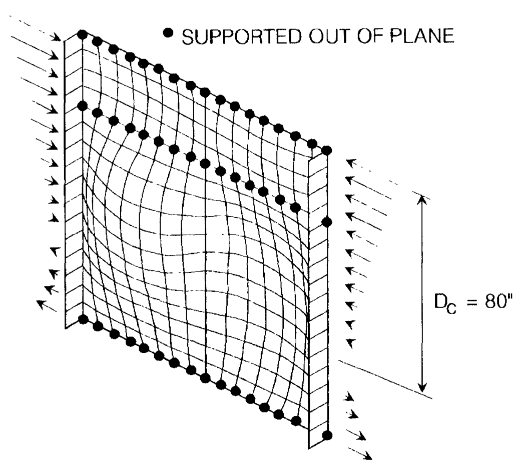

Fig. 4. Bottom panel buckling (x = 0.25, D_c / t_w = 160).

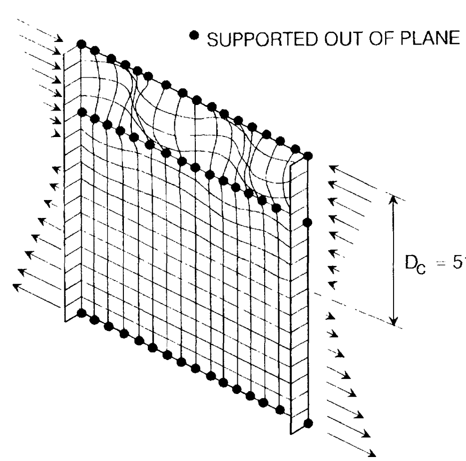

Fig. 5. Top panel buckling (x = 0.25, D_c / t_w = 102).

The fixed node case corresponds to a stiffener with infinite stiffness and no area. The small stiffener had a moment of inertia equal to 63 percent of the required value with a slenderness ratio of 98 percent of the maximum. The other points had a stiffener size which just met the moment of inertia requirement and had a slenderness ratio equal to the maximum. The stiffener size had little effect upon the buckling stress when α exceeded 0.40. When α was less than 0.40, the small stiffener produced buckling stresses below the fixed node case indicating the stiffener was not sufficient to restrain the web. The minimum stiffener which just met the AASHTO requirements produced higher buckling stresses than the fixed node case. This is due to the reduction in web stress in the middle of the panel as the stiffener stress increased. The stiffener participates in carrying the bending stress in the web. Since the buckling stress has been defined as the stress applied to the edges of the panel, the actual stress causing the buckling in the center of the web panel is less than the defined buckling stress. The buckling stress for small α values with the minimum stiffener are less than the fixed node cases due to local buckling of the stiffener. The buckling stress in these panels exceeded 75 ksi. The value of F_y used in the stiffener size equations was 50 ksi. A stiffener with a smaller slenderness ratio was required for these high buckling stresses.

When α is greater than 0.40 both stiffener sizes produce buckling stresses above the fixed node case. The top panel of the web buckles when α exceeds 0.40. The stiffener size is not as critical when the top panel buckles since the stiffener is located in a lower stress region of the web.

The results of the analysis indicate that the buckling stress of a web panel with a longitudinal stiffener is a function of the compression web slenderness and the location of the stiffener relative to the compression depth of the web. The optimum stiffener location for simply supported web boundary conditions is 0.40 times the web depth in compression from the compression flange. The buckling mode of webs with the stiffener above the optimum location is buckling of the web below the stiffener. The mode for webs with the stiffener below the optimum location is buckling of the top panel above the stiffener. The buckling strength of webs with the stiffener placed at a fixed location below the optimum location is not significantly influenced by the compression web depth. Stiffeners meeting the AASHTO requirements are sufficient to restrain the web from out-of-plane movement at the buckling stress.

BUCKLING COEFFICIENT FOR UNSYMMETRICAL WEBS WITH LONGITUDINAL STIFFENERS

The results of the finite element study were used to develop buckling coefficients for longitudinally stiffened webs with simply supported boundary conditions. The buckling coefficients were developed for two cases. The first case is when the stiffener is located below the optimum location of $\alpha = 0.4$. Buckling occurs in the panel between the stiffener and the compression flange. The buckling stress was found to be almost independent of the total depth of the web in compression for this case. This was shown in Figure 3 for small values of D_c / t_w with $x = 0.20$ and 0.25. The buckling coefficient that best fit the finite element results and also agreed with the published solution of 129 for a symmetric girder with a stiffener at $\alpha = 0.4$ was:

$$k = 5.17\left(\frac{D}{d_s}\right)^2 \quad \text{for } \alpha = \frac{d_s}{D_c} \geq 0.4 \tag{6}$$

Substituting this buckling coefficient into Equation 1 yields:

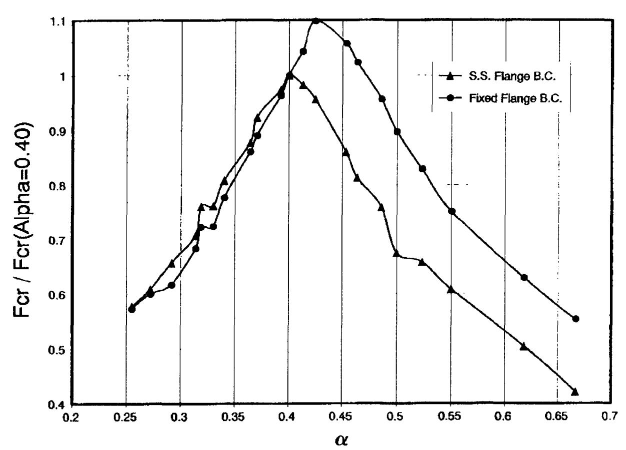

Fig. 6. Influence of web boundary conditions upon optimum stiffener location.

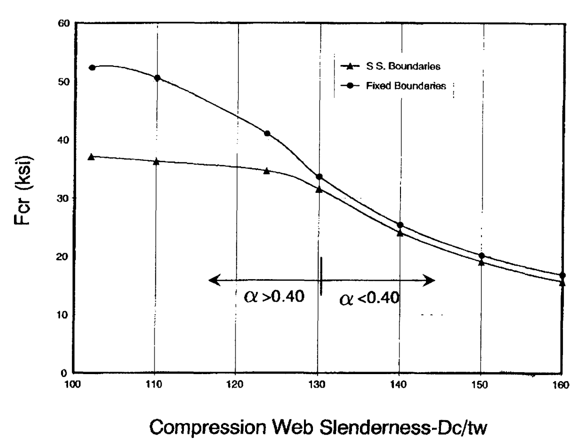

Fig. 7. Influence of boundary conditions for $x = 0.25$.

$$F_{cr} = \frac{0.904\,E\,5.17}{\left(\frac{d_s}{t_w}\right)^2} \tag{7}$$

The buckling is seen to be solely a function of the slenderness ratio of the panel above the stiffener, d_s / t_w. The buckling coefficient from the Eurocode formulation for a simply supported plate with a compression stress at the bottom equal to 0.8 of the top is 5.12 which is within 1 percent of the 5.17. At the limit of $\alpha = d_s / D_c$=0.4 and D_c=D / 2 (a symmetric girder with the stiffener located at the optimum location), d_s is equal to 0.2D. Substituting this value for d_s into Equation 6 yields a buckling coefficient of 129. Therefore the buckling coefficient matches at the limit of α = 0.4 both for the sub panel treated alone and it also matches the buckling coefficient value of 129.3 for a symmetrical girder. If α is greater than 0.4 the compression stress at the stiffener location is less than 0.8 of the stress at the top of the panel. This will increase the buckling coefficient. The resulting increase in F_{cr} is not appreciable as shown in Figure 3. This increase is not considered in the equation presented above to simplify the equation. Neglecting this slight increase in the buckling stress is conservative and simplifies the formulation.

The second case is when the stiffener is located above the optimum location and buckling occurs in the lower panel. The buckling coefficient for this case was found to be:

$$K = 11.64\left(\frac{D}{D_c - d_s}\right)^2 \tag{8}$$

$$\text{for } \alpha = \frac{d_s}{D_c} < 0.4$$

The formulation of this equation was developed by setting the lower panel compression depth, $D_c - d_s$, equal to $0.6D_c^*$. D_c^* is the compressive web depth if the stiffener is located at the optimum location of α = 0.4. The total web depth for a symmetrical girder is twice D_c^* or $10/3(D_c - d_s)$. Substituting this depth as D in Equation 1 and with a buckling coefficient of 129.3 yields the constant 11.64 given in the buckling coefficient equation.

The agreement between the buckling stress from the finite element study and that predicted using the two buckling coefficients given above in Equation 1 is shown in Figure 9. Only finite element results with stiffeners meeting the AASHTO design requirements are shown in the figure. The calculated buckling stress provides a reasonable lower bound to the finite element results. The finite element results with actual stiffeners are slightly greater than the predicted value since they include the beneficial participation of the stiffener with the web in carrying the applied web stress. The results with the nodes supported out of plane model a stiffener with no axial stiffness and infinite bending stiffness. The individual results that are aligned vertically are when the top panel controls. The buckling coefficient equation ignores this small increase in buckling stress due to the difference in the stress gradient in the top panel with changes in D_c.

DESIGN EQUATIONS

The present AISC[6] and AASHTO Specifications limit the web slenderness based upon elastic buckling limits, no inelastic reduction due to residual stress is considered. The AASHTO LFD specification has two web slenderness limits, upper limits for webs with and without longitudinal stiffeners. The AASHTO ASD specifies the web thickness requirements for webs in terms of the bending stress in the web along with upper limits. Upper limits in ASD on web slenderness are 170 for webs without longitudinal stiffeners and 340 for webs with longitudinal stiffeners. A design approach similar to the AASHTO ASD Specification is presented in this section. The design equations are written in terms of the bending stress in the web and are presented in a LFD format. The use of the

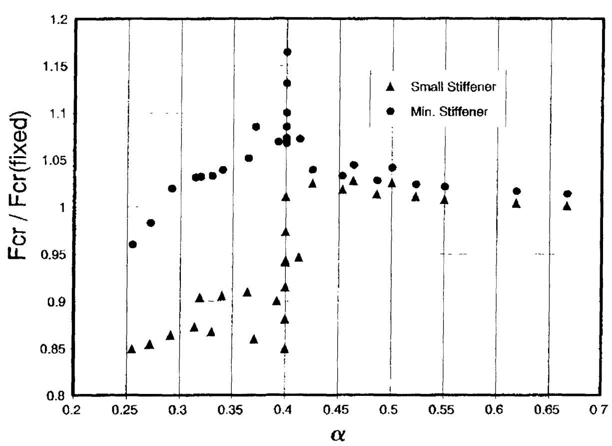

Fig. 8. Influence of stiffener on buckling strength.

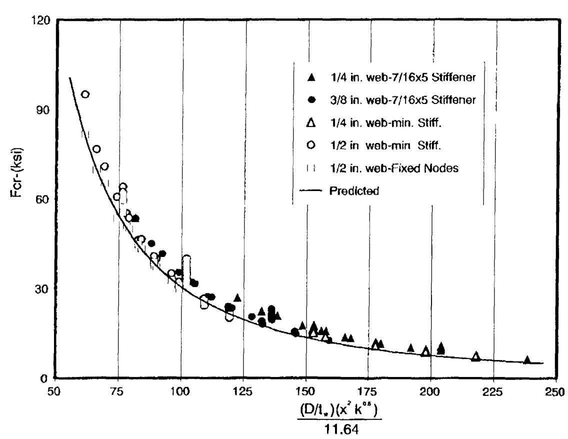

Fig. 9. Comparison of finite element results with predictions using derived buckling coefficients.

Table 1. Web Slenderness Limits			
Category	**AASHTO ASD[a]**	**AASHTO LFD**	**Proposed LFD[b]**
Compact	NA	$19{,}230 / (F_y)^{.5}$	$19{,}230 / (F_y)^{.5}$
Without a Longitudinal Stiffener	$31{,}000 / (F_y)^{.5}$	$30{,}800 / (F_y)^{.5}$	$5{,}120(k / f_b)^{.5}$
With a Longitudinal Stiffener	$62{,}000 / (F_y)^{.5}$	$73{,}000 / (F_y)^{.5}$	$5{,}120(k / f_b)^{.5}$

[a]Calculated assuming the bending stress f_b is equal to $0.55F_y$ (units for stress in the AASHTO Specification are psi).
[b]$k = 9(D / D_c)^2$ for webs without a longitudinal stiffener $K = 5.17(D / d_s)^2$ if $d_s / D_c \geq 0.4$ else $k = 11.64[D / (D_c - d_s)]^2$ for webs with longitudinal stiffeners.

design equations is illustrated by applying the equations to a typical composite plate girder bridge.

The present specification limits on web slenderness, D / t_w or $2D_c / t_w$, in the AASHTO Specification for allowable stress design (ASD) and load factor design (LFD) along with the proposed are presented in Table 1. The proposed specification values for non compact webs are not written as limiting web slenderness. They are similar to the format used in the ASD where the limit is based upon the compressive bending stress in the web. They are shown in a LFD format. The constants in Table 1 for the AASHTO ASD Specification were calculated by setting f_b equal to $0.55F_y$. An upper limit on web slenderness is needed to provide stiffness for fabrication and to limit wrinkling from distortions due to welding. The upper limits on web slenderness should not be a function of the web yield strength since they are not based upon strength requirements. A reasonable approach would be to limit the maximum web slenderness, D / t_w, to the values presently allowed in the ASD. These limits have served well in the past. The limits would be 170 for girders without longitudinal stiffeners and 340 for girders with longitudinal stiffeners.

In order to compare the proposed design equations and to provide the values for the design evaluation below, the limits for Grade 50 material, $F_y = 50$ ksi, are given in Table 2. The values given in the table are the maximum values of web slenderness to achieve a calculated bending stress resistance of F_y for LFD or $0.55F_y$ for ASD. The maximum web slenderness values are given for three cases. The first case is for a symmetrical girder with a longitudinal stiffener placed at the optimum location of $0.4D_c$ which is also $0.2D$, the AASHTO Specified location. The second case is an unsymmetrical web with the depth of the web in compression equal to 70% of the web depth and a stiffener located at $0.2D(0.29Dc)$ which is above the optimum location. The third case is the same depth of web in compression as case 2 but with the stiffener located at the optimum location. The present AASHTO Specification limits for unsymmetrical webs was calculated using $2D_c / t_w$ as the web slenderness. The limits for the unstiffened web are the same for all three specifications and cases within the round off error of the calculations. The symmetrical Case 1 of the proposed specification agrees reasonably well with the ASD requirements for longitudinally stiffened web. The LFD web slenderness is far above the buckling limit. Neither the ASD nor the LFD specification agrees with the proposed specification for an unsymmetrical girder. The present AASHTO Specification does not consider the effect of the location of the stiffener upon the buckling stress of the web. Consequently, both Case 2 and 3 have the same limits. The AASHTO Specification requirements are unconservative for Case 2 with the stiffener located at the AASHTO specified location of $0.2D$. The ASD specification agrees reasonably well with the proposed specification when the stiffener is located at the optimum location. However, the LFD design specification remains unconservative. If the limits in AASHTO are interpreted as simply D / t_w rather than $2D_c / t_w$, the limits for the unsymmetrical girders, Cases 1 and 2, would be the same as the symmetrical girder, Case 1. This interpretation would result in an extremely unconservative design.

EXAMPLE

A typical longitudinally stiffened composite plate girder bridge was selected to show the application of the proposed design formulas. The bridge was designed in accordance with the early LFD design specification which did not include consideration of the depth of the web in compression. The bridge is a three span continuous bridge with side spans of 215 feet and a center span of 290 feet. The flanges and web are 50 ksi yield strength material and the girder is composite in the spans (positive moment region) and at the piers (negative moment region). The ⅜-in. web is 105 in. deep, $D / t_w = 280$. The buckling capacity of the web for the positive moment sections of the side and center spans, and the negative moment section at the piers will be evaluated for the following three strength loading cases and sections:

1. Construction loading (non-composite steel section carries all the load),
2. Overload-Elastic composite section, and
3. Maximum Load-Inelastic or Elastic Composite Section.

The positive moment sections of the three spans consisted of two sections for each span. The top flange for all sections was a ¾ × 18-in. plate. The bottom flange width was 20 inches for all sections with a thicker flange used in the central portion of each span. Table 3 summarizes the steel section web depth in compression and the buckling stresses for the four positive moment sections in the bridge. The last two columns give the critical buckling stress for the web from the proposed design

Table 2. Web Slenderness Limits for 50 Grade Material						
	Case 1 $D_c/D = 0.5$ $d_s/D_c = 0.4$		Case 2 $D_c/D = 0.7$ $d_s/D_c = 0.29$		Case 3 $D_c/D = 0.7$ $d_s/D_c = 0.4$	
Specification	Unstiffened	Stiffened	Unstiffened	Stiffened	Unstiffened	Stiffened
AASHTO ASD	140	280	100	200	100	200
AASHTO LFD	138	326	99	233	99	233
Proposed	137	260	98	157	98	186

Table 3. Positive Moment Steel Sections Construction Loading					
Span	Section	Bottom Flange Thickness (in.)	D_c (in.)	F_{cr} (ksi) $0.2D$	F_{cr} (ksi) $0.4D_c$
Side	Ends	0.75	53.7	40.2	41.4
Side	Center	1.125	58.9	29.9	30.7
Central	Ends	1	57.3	32.7	29.3
Central	Center	1.5625	63.8	23.4	29.3

equations with the longitudinal stiffener at $0.2D$ and at $0.4D_c$ of the center section of each span. The AASHTO Specifications would require the stiffener be placed at $0.2D$ and limit the stress to the yield stress of 50 ksi. Placing the stiffener at the optimum location for the center section increase the buckling stress for both sections used in the side spans. The buckling stress of the end sections of the central span is decreased and the center section increased when the stiffener is located at the optimum location for the center section. It is impossible to increase the web buckling stress to the yield strength as implied in the AASHTO Specifications with only one longitudinal stiffener for all the sections.

A summary of the web depth in compression for the same sections for the overload elastic composite section and the maximum load plastic composite section is shown in the Table 4. The buckling stress of an unstiffened web is greater than the yield strength for the side span sections and the ends of the center span for the elastic composite section. No longitudinal stiffener is required for strength in these panels if the stress is limited to the yield strength of the web. The web of the center section of the center span has a buckling stress of 44.5 ksi neglecting the longitudinal stiffener. The buckling strength can be increased with a properly placed stiffener. However, if the stiffener is placed at the optimum location for the construction loading of $0.4D_c$, $0.4 \times 63.8 = 25.5$ in.; the stiffener would be almost at the elastic composite neutral axis and provide very little benefit.

The last column in Table 4 shows that the compression web slenderness at the plastic moment is less than the value given in Table 2. Therefore the webs are compact. The sections also satisfy the ductility requirements of the AASHTO Specifications. The maximum load capacity of the sections is the plastic moment, M_p.

The results for the pier section, negative moment section, are shown in Table 5. Only the elastic sections were considered since the web does not satisfy the compactness requirements. Two composite elastic sections were used in the calculations. The first is the elastic uncracked slab section which considers the effective width of the slab participating with the steel. The second elastic composite section is for a cracked section which considers only the longitudinal reinforcement acting with the steel girder. The buckling stresses given in the last three columns are for three different positions of the longitudinal stiffener. The first position is at $0.2D$, the second at $0.4D_c$ of the steel section, and the third at $0.4D_c$ of the composite section including the slab. The buckling stress for the stiffener located at 0.2D is less than the yield strength for the three cross sections analyzed. The AASHTO Specifications would allow a web stress of 50 ksi for all stages of loading with the stiffener located at $0.2D$. The AASHTO Specifications are obviously unconservative. The variation in buckling stress for the different sections is due to the change in the depth of the web in compression. The lowest buckling stress is for the elastic composite section with an uncracked slab when 69 percent of the web is in compression. When the stiffener is located at the optimum depth for the steel girder

Table 4. Positive Moment Composite Sections					
Span	**Section**	**Elastic D_c (in.)**	**Elastic F_{cr} (unstiffened) (ksi)**	**Plastic D_{cp} (in.)**	**Plastic $2D_{cp}/t_w$**
Side	Ends	19.5	> 50	8.4	45
Side	Center	23.2	> 50	8.6	46
Center	Ends	22.0	> 50	8.6	46
Center	Center	27.3	44.5	15.3	81

Table 5. Negative Moment Section				
Cross Section	**D_c (in.)**	**F_{cr} (ksi) Stiffener at 0.2D**	**F_{cr} (ksi) Stiffener at 0.4D_c (Steel)**	**F_{cr} (ksi) Stiffener at 0.4D_c (Slab)**
Steel Girder	48.7	43.2	50.2	22.9
Steel Girder and Slab	72.2	16.4	15.5	22.9
Steel Girder and Reinforcement	51.6	43.2	44.8	22.9

alone the buckling strength is increased to 50.2 ksi. However, the buckling stress is reduced for the composite slab cross section. Locating the stiffener at the optimum location for the composite slab case, the last column, yields the same buckling stress for all cross sections since the stiffener is below the optimum location for the steel alone and the composite reinforcement cross sections. The panel between the stiffener and compression flange controls the buckling strength.

The pier section requires a thicker web in order to have a capacity equal to M_y at maximum load. The cracked slab cross section should be used in calculation of the maximum load capacity. The elastic composite slab cross section should be used to evaluate the overload condition. The ratio of the design overload moment to the maximum load moment in the AASHTO LFD Specification is $1/1.3 = 0.77$. The corresponding required web buckling capacity is $0.77F_y$ equal to 38.5 ksi for 50 grade material. The AASHTO requirement of a maximum stress of $0.95F_y$ at overload does not control. The design constraints are a web with a buckling stress equal to or greater than 38.5 ksi for the composite section with the slab and a buckling stress greater than or equal to 50 ksi for the composite section with the reinforcing steel. The required web thickness can be calculated for a stiffener located at the optimum location, d_s=0.4D_c, by calculating the buckling coefficient as:

$$k = 5.17\left(\frac{D}{d_s}\right)^2 = 5.17\left(\frac{D}{0.4D_c}\right)^2 \tag{9}$$

The required web thickness can be found by rewriting the equation in Table 1 and solving for the web thickness.

$$t_w = \frac{D}{5{,}120}\left(\frac{f_b}{k}\right)^{0.5} \tag{10}$$

Since the required capacity is largest for the maximum load case, the web thickness for the composite section with the reinforcement with a bending stress of 50,000 psi will be calculated and then the overload capacity checked. The calculation for the maximum load case is shown below:

$$D_c = 51.6$$

$$k = 5.17\left(\frac{105}{51.6 \times 0.4}\right)^2 = 133.8$$

$$t_w = \frac{105}{5{,}120}\left(\frac{50{,}000}{133.8}\right)^{0.5} = 0.396$$

The required web thickness is 7⁄16-in. The overload capacity must now be checked for the stiffener located at d_s = 20.6 in. and a compression web depth of 72.2. The buckling stress can be calculated directly from Equation 1 with the buckling coefficient of:

$$\frac{d_s}{D_c} = \frac{20.6}{72.2} = 0.283$$

$$k = 11.64\left(\frac{105}{72.2 - 20.6}\right)^2 = 48.2$$

The buckling stress is:

$$F_{cr} = \frac{0.904 \times 29{,}000{,}000 \times 48.2}{\left(\dfrac{105}{\dfrac{7}{16}}\right)^2}$$

$$F_{cr} = 21{,}940$$

The calculated stress is less than the required value of 38.5 for the overload condition consequently a different stiffener location or larger web is required. If the stiffener is left at the optimum location for the composite section with the reinforcement, the required web thickness can be calculated by substituting f_b = 38,500 psi and a buckling coefficient of 48.2 into the Equation 10. The required web thickness is 0.579 in. or 9⁄16-in. Since the overload condition controls the design, an optimum solution for web thickness can be found by solving for the optimum buckling coefficient by substituting $0.4D_c = 0.4 \times 72.2$ into the buckling coefficient equation and then using the buckling coefficient in the web thickness equation. The optimum buckling coefficient is 68.3 and the resulting required web thickness is 0.487. Therefore a ½-in. web with a stiffener located at 0.4 × 72.2 = 28.8 in. will provide a buckling capacity of 38.5 ksi for the elastic composite section with the slab. A check of the steel section alone and the composite section with the reinforcement with a ½-in. web with a longitudinal stiffener 28.8 in. above the bottom flanges yields a buckling stress above 50 ksi. Therefore increasing the web thickness to ½-in. with longitudinal stiffener at the optimum location for the composite section with the slab, d_s = 28.8 in., yields the optimum solution. The increase in web thickness also increases the positive moment capacity for the steel section alone, construction loading, to M_y.

The web thickness in the design example had to be increased from ⅜ to ½-in. to provide adequate web buckling capacity in the negative moment section at the pier. The controlling loading case was the overload section consisting of the steel section acting compositely with the concrete slab. Cracking of the slab due to service stresses or shrinkage would reduce the web depth in compression. Cracking was ignored in the example to provide a conservative solution. The results of full size tests of composite sections in negative bending have shown that the effective slab stiffness prior to yielding of the steel section is closer to the full slab stiffness than the reduced stiffness considering only the reinforcement.[7] The optimum stiffener location for the positive moment section is controlled by the construction loading on the steel section. In both cases, both positive and negative bending, the longitudinal stiffener is not located at $0.2D$ but rather at the optimum location for each section of the girder.

The example above did not consider the superposition of the elastic stresses for the various stages of loading. The depth of the web in compression for the various cross sections analyzed was calculated assuming all the load was applied to the cross section. The depth of the web in compression should be calculated by superimposing the elastic web stresses for all cases except for the inelastic capacity M_p. The depth of the web in compression can be calculated as:

$$D_{c_n} = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} \dfrac{f_i}{D_{c_i}}} \tag{11}$$

where f_i normal is the value of the maximum compressive stress in the web at load stage "i" and D_{ci} is the distance from the underside of the compression flange to the neutral axis for the cross section "i". For example, the web depth in compression for the negative moment section at overload using the standard AASHTO load factor is:

$$D_{c_{OL}} = \frac{f_{dl1} + f_{dl2} + \tfrac{5}{3} f_{LL+I}}{\dfrac{f_{dl1}}{D_{c_{steel}}} + \dfrac{f_{dl2} + \tfrac{5}{3} f_{LL+I}}{D_{c_{slab}}}} \tag{12}$$

and for maximum load:

$$D_{C_{Max}} = \frac{1.3(f_{dl1} + f_{dl2} + \tfrac{5}{3} f_{LL+I})}{\dfrac{1.3 f_{dl1}}{D_{C_{steel}}} + \dfrac{1.3(f_{dl2} + \tfrac{5}{3} f_{LL+I})}{D_{C_{reinf.}}}} \tag{13}$$

The load factor 1.3 cancels out in Equation 13 but is shown for clarity. The composite section stresses, f_{dl2} and f_{LL+I}, should be calculated using the slab in the composite section for overload and only the reinforcement for maximum load. The web depth in compression in the equations are cross sectional properties and should correspond to the composite section used in calculating the stresses.

SUMMARY AND CONCLUSIONS

The buckling coefficients for singly symmetric girder webs with and without a single longitudinal stiffener have been developed. The buckling equations provided agree with the finite element analysis and also the published solutions available for symmetric stiffened webs and for the normal range of the depth of the web in compression for unstiffened webs. The AASHTO requirements for longitudinal stiffener moment of inertia and slenderness were found to be adequate when the limit load of the web is taken as the buckling load.

The design equations proposed provide a rational means to determine the buckling capacity of webs when the location of the neutral axis varies during the various stages of loading and along the length of girder. The positioning of the longitudinal stiffener cannot be at the optimum location for all stages of loading. The best position for the positive moment section in the design example presented was the optimum position for the steel section alone at mid span. The best position at the pier section was the optimum position for the composite overload section.

The AASHTO LFD design maximum web slenderness ratio for webs with longitudinal stiffeners is unconservative. The elastic buckling stress of the maximum slenderness webs is less than the yield strength of the web. The ASD design specifications agree with the proposed for symmetric girders. Both the LFD and ASD design specification are unconservative for unsymmetric girders if the stiffener is placed at $0.2D$.

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