A Practical Approach to the "Leaning" Column

LOUIS F. GESCHWINDNER

INTRODUCTION

The "leaning" column has existed as an element in steel structures from the time steel frames were first constructed. Although they provide the title for this paper, they themselves are not the problem which needs to be addressed. It is their impact on the other parts of the structure which must be a consideration in any design. A "leaning" column is a pin ended column which has no lateral stability of its own; thus, it must rely on other parts of the structure to provide for its lateral stability and the lateral resistance for the entire frame.

This paper will discuss the situations in which "leaning" columns will occur and look at practical ways to account for them in both LRFD[1] and ASD[2] design.

STRUCTURAL FRAMING

The framing plan for a low rise building is shown in Figure 1. In the long direction, lateral load is resisted by the rigid frames along column lines A and D, from line 2 to line 5. In the short direction, lateral load is resisted by the braced frames along column lines 1 and 6, between lines B and C. If the rigid frames are taken without consideration for the remainder of the structure, each would be designed to resist the gravity load applied to its columns plus one half of the total lateral load. Columns which are not a part of the two rigid frames would be designed to carry their part of the gravity load. These gravity only columns are not part of the rigid frames and are the ones referred to as "leaning" columns.

If these "leaning" columns were considered part of an unbraced framing system, they would have an infinitely long effective length, $K = \alpha$, and thus be impossible to design. If, on the other hand, they are assumed to be laterally supported at their ends, they could be considered simple pin ended columns and be designed with an effective length equal to their actual length, K = 1. For this situation to be at all reasonable, something must provide the required lateral restraint.

The question that must be answered is; "Where does the lateral restraint for 'leaning' columns come from?" If one were to examine all of the possible contributors to the needed lateral restraint, a number of possible answers would come to mind. First, there are the walls of the building, both interior and exterior. Depending on their construction, they may provide lateral restraint even though they are not counted upon to participate in resisting the applied lateral load. Another source might be the moment capacity of the connections between the "leaning" columns and the beams which frame into them. A source of restraint which may not have been counted upon in the calculations to resist applied lateral load but which may actually be available to provide lateral stiffness. A third source for lateral resistance comes from the rigid frames along column lines A and D which, in addition to being required to carry the lateral load, may provide the required restraint for the remainder of the columns which are not part of these frames. The required stiffness may already exist in a frame as a result of members being designed for different loading conditions or may result from the reserve stiffness of members which have been designed for buckling out of the plane of the frame. Any of these are acceptable sources for the lateral support assumed at the ends of the "leaning" columns when they are designed with K = 1.

The decision as to where stability comes from in a particular design is critically important. It is the designers responsibility to be sure that the assumed stability is actually provided. As more and more factors are accounted for in an analysis, it becomes more and more important to include those same factors in the corresponding design. An ASCE committee has found that there are many approaches being taken and assumptions being made in the analysis of steel building frames[3] but that these assumptions may not be fully incorporated

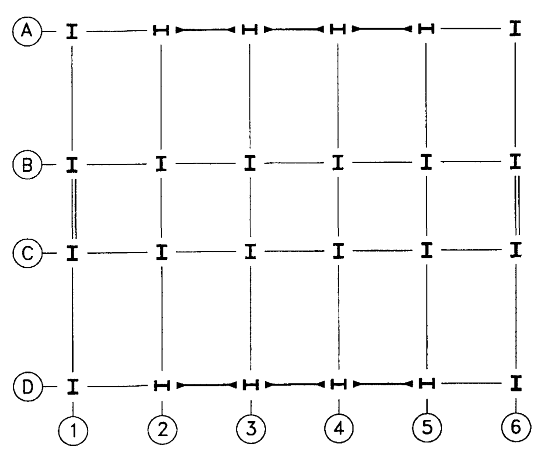

Fig. 1. Low rise building framing plan.

Louis F. Geschwindner is professor of architectural engineering, The Pennsylvania State University.

throughout all phases of a design. Heavy masonry walls may have easily provided lateral stiffness for structures built at one time but today's light curtain walls normally should not be counted upon for that purpose. Beam connections which may provide lateral resistance should be designed for that purpose from the start in order to insure that all framing components are adequate to meet the strength and stiffness needs. Partially restrained connections are permitted in both LRFD and ASD Specifications and may reasonably be counted on to provide the needed stiffness. If the rigid frame is to provide the lateral stability, then it too must be properly sized to provide all of the restraint that it will be called upon to give.

METHODS OF ANALYSIS

Although the LRFD and ASD Specifications permit a first order analysis for design purposes, they do require that the second order effects be accounted for. A first order analysis is formulated around the undeflected geometry of the structure. This is the normal approach taken to structural analysis through such well known methods as moment distribution, slope deflection, or the stiffness method. A second order analysis is formulated around the deflected geometry of the structure and will normally include an iterative or approximate solution. There are many references available for both first order and second order analyses.[4,5]

Two different second order effects will impact on the design of a single column. The first, illustrated in Figure 2a for a column in which the ends are prevented from displacing, is the result of the deflection along the length of the column. It can be seen that the moments along the column will be increased due to the column deflection, by an amount $P\delta$. This increase in moment due to member deflection is referred to as the member effect.

The column in Figure 2b is part of a structure which is permitted to sway laterally an amount Δ. As a result, the moment required in the end of the column to maintain equilibrium in the displaced configuration is given as $P\Delta$. This member moment is referred to as the structure effect, since the lateral displacement of the column ends is a function of the properties of all of the members of the structure participating in sway resistance.

The deflections, δ and Δ, shown in Figure 2 are second order deflections, resulting from the applied loads plus the second order forces. Although requiring a second order analysis, both of these effects appear to be straightforward for the individual column of Figure 2. However, when columns are combined to form frames, the interaction of all of the members of the frame significantly increases the complexity of the problem. The addition of columns which do not participate in lateral resistance but which do carry gravity load brings further complexity to the problem.

This can most readily be seen with reference to the structures shown in Figure 3. The frame in Figure 3a is composed of two identical columns carrying the same load, P_a. If the columns behaved elastically, their capacity would be determined using Euler's equation with $K = 2$, thus $P_a = \pi^2 EI / 4L^2$. The addition of the lateral load, H, in the frame of Figure 3b, means that the ability of the columns to carry axial load will be reduced to P_b, since now each column must also resist an applied moment, $HL/2$. This reduced capacity is normally accounted for through the use of an interaction equation provided by the design specification, LRFD H1-1a and H1-1b or ASD H1-1, H1-2 and H1-3. Now an additional column, EF, which carries a load Q, is added to the previous frame as shown in Figure 3c. With column EF pinned at each end, it is seen that the original columns AB and CD are still called upon to resist all of the lateral load, H. If the column EF has no impact on columns AB and CD, as would appear from a first order analysis, they would continue to carry the gravity load, P_b. However, if the addition of column EF, with its load Q, does have an impact on the ability of other columns to carry load, then their load must be appropriately reduced to P_c. Column EF is a "leaning" column. It would be designed with an effective length factor $K = 1$ and would rely upon

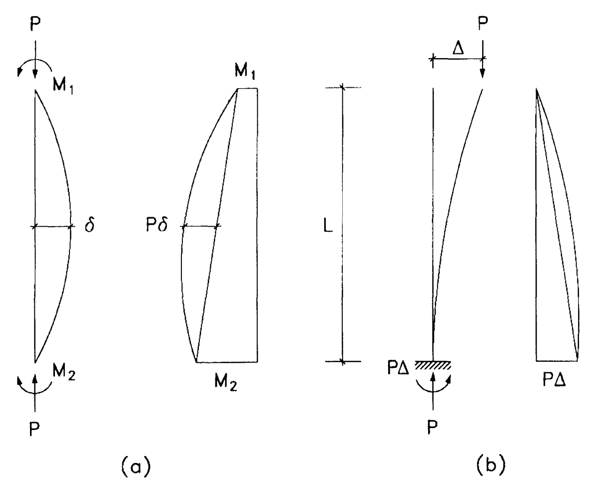

Fig. 2. Second Order Effects.

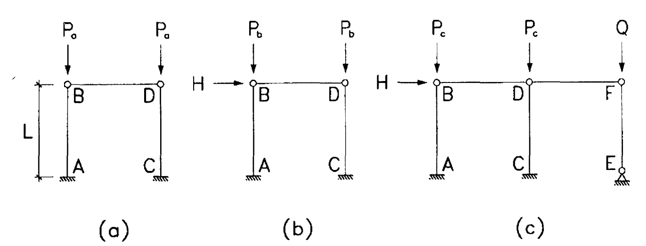

Fig.3. Unbraced frames.

columns AB and CD to keep its upper end from moving laterally.

EFFECTIVE LENGTH

The normal approach to finding the axial load capacity of a column involves the determination of the effective length factor, K, which relates the actual critical buckling load of the column to that predicted by the Euler buckling equation so that

$$P_{cr} = \frac{\pi^2 EI}{K^2 L^2} \quad (1)$$

Perhaps the most commonly used approach to the determination of K factors is the nomograph found in the Commentary to the Specifications,[1,2] and shown in Figure 4 for unbraced frames. The equation upon which the nomograph is based is given here as Equation 2.

$$\frac{G_A G_B (\pi / K)^2 - 36}{6(G_A + G_B)} = \frac{\pi / K}{\tan(\pi / K)} \quad (2)$$

The assumptions used in the development of the nomograph are detailed in the Commentaries. Two of the most important assumptions are 1) that all members behave elastically and 2) that all columns in a story buckle simultaneously. This latter assumption is critical since it eliminates the possibility that any column in an unbraced frame might contribute to the lateral sway resistance of any other column. If the nomograph is used for column EF of Figure 3c, the pinned ends yield $G_A = G_B = \alpha$ which, in turn, yields $K = \alpha$. By this approach, the critical load would be undefined, a particular problem for the designer.

Since the behavior of columns in actual structures would seem to show that columns like column EF do, in fact, have some axial capacity, it is important to find a model that will reasonably predict the capacity of a frame including these "leaning" columns. Numerous approaches intended to account for the effect of "leaning" columns have been presented in the literature. These approaches offer a wide range of mathematical complexity and practical usefulness. Three approaches that have been presented in the literature for including the "leaning" column will be discussed along with some simplified equations that are included in the Commentary of the second edition of the LRFD Specification. In addition, an equation similar to that used to develop the nomograph will be developed to include the leaning column. As always, the designer is called upon to decide on the appropriate approach to use in a particular design situation.

Modified Nomograph Equation: The derivation of Equation 2 is readily found in numerous references.[5] Following the same procedures and assumptions, with the addition of the "leaning" column as shown in Figure 5, a new equation may be developed. Viewing the structure in its displaced equilibrium configuration, the "leaning" column and the restraining column are separated as shown in Figures 5b and c. The load Q on the leaning column EF must be balanced by the horizontal force, $Q\Delta / L$, at F, for equilibrium of the "leaning" column. This force must then be applied as a load at B on the restraining column AB.

Equations of equilibrium at the joints of column AB and the sway equilibrium equation can be written for the structure in the displaced configuration. Member end moment equations are then written using the slope deflection method, incorporating the stability functions necessary to account for the influence of axial load on column AB.[5] Combining these

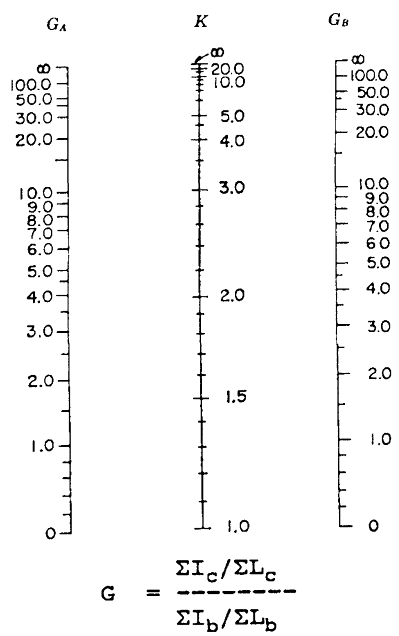

Fig. 4. Unbraced frame nomograph (1).

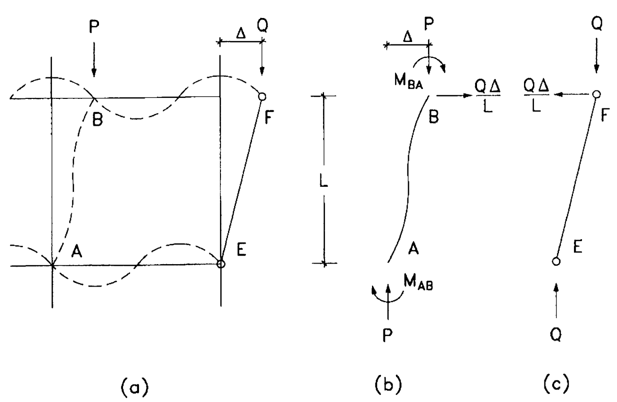

Fig. 5. Frame model for buckling with "leaning" column.

equations and setting the determinate of the coefficients equal to 0.0 will yield the buckling condition equation.

$$\frac{G_A G_B(\pi/K)^2 - 36}{6(G_A + G_B)}\left(1 + \frac{Q}{P}\right) - \frac{\pi/K}{\tan(\pi/K)}\left(1 + \frac{Q}{P}\right) + \frac{6\tan(\pi/2K)Q}{(G_A + G_B)(\pi/2K)P} + \frac{Q}{P} = 0 \quad (3)$$

If the "leaning" column load is 0.0, Equation 3 reduces to Equation 2. Since neither of these equations can be solved explicitly, an iterative approach may be used or, in the case of the frame without "leaning" columns, the nomograph already discussed may be used.

The Yura Approach (6): This is perhaps the easiest approach to develop since it relies on a straightforward interpretation of the physical problem. For the unbraced frame shown in Figure 6, equilibrium will be established for the structure in the undeflected configuration and again in the deflected configuration. The first order, undeflected equilibrium configuration forces are shown in Figure 6a. If the frame is permitted to displace an amount Δ, equilibrium in this displaced configuration will be as shown in Figure 6b. In order for column EF to be in equilibrium, a lateral force, $Q\Delta / L$ as shown at F is required. This force must be equilibrated by an equal and opposite force shown at B. Thus, when column AB buckles, it buckles with a moment of $(P\Delta + Q\Delta)$ at its base. It is observed that this is the same moment that would result if the individual column AB were to buckle under the axial load of $(P + Q)$. The assumption that the buckling load is $(P + Q)$ is only slightly conservative for the individual column AB, since the deflected shape due to an axial load and a lateral load differ only slightly. In order to insure sufficient lateral resistance to buckling for column EF, column AB must be designed to carry a fictitious load $(P + Q)$.

In order to compare this approach to others presented in the literature, it is helpful to convert it to an effective length approach. If column AB is to be designed to carry the load P but have the capacity $(P + Q)$, a modified effective length factor will be required. K_o is defined as the effective length factor that would be determined from the nomograph of Figure 4, which does not account for the leaning column. In this case $K_o = 2$. K_n is defined as the effective length factor which will account for the "leaning" column. Thus, based on the buckling load being $(P + Q)$

$$(P{+}Q) = \frac{\pi^2 EI}{K_o^2 L^2} \quad (4)$$

If the column is to be designed to carry the actual applied load, P, with the leaning column accounted for through K_n, then

$$P = \frac{\pi^2 EI}{K_n^2 L^2} \quad (5)$$

Solving Equations 4 and 5 for their corresponding K's and taking the ratio K_n^2 / K_o^2 yields

$$\frac{K_n^2}{K_o^2} = \frac{P{+}Q}{P} \quad (6)$$

which may be solved for K_n as

$$K_n = K_o\sqrt{\frac{P+Q}{P}} \quad (7)$$

If column AB from Figure 6a were designed to carry the load P using the effective length factor K_n it would provide sufficient lateral restraint to permit column EF to be designed to carry the load Q using $K = 1.0$.

For frames with more than one "leaning" column and more

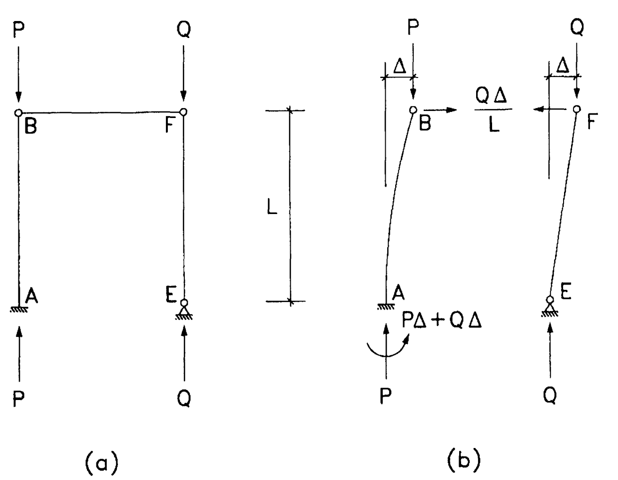

Fig. 6. Equilibrium forces for Yura derivation.

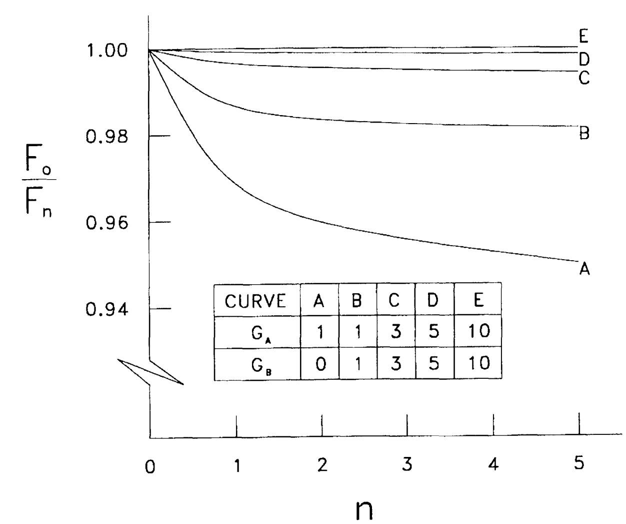

Fig. 7. F_o / F_n vs. n for Lim & McNamara Approach (7).

than one restraining column, P and Q will be replaced by ΣP and ΣQ. It should also be noted that this approach maintains the assumption that all restraining columns in a story buckle in a sidesway mode simultaneously.

Lim & McNamara Approach (7): Another approach that will account for the "leaning" column was proposed by Lim and McNamara for columns of unbraced tube buildings. Their development is also based on the assumption that all columns in the restraining frame buckle in a sidesway mode simultaneously; however, they developed the sway buckling equation through the use of stability functions and an eigenvalue solution.

The resulting effective length factor, accounting for the "leaning" column is given in their paper as

$$K_n = K_o\sqrt{1+n}\,F_o / F_n \qquad (8)$$

where K_n and K_o are as defined earlier, $n = \Sigma Q / \Sigma P$ and F_o and F_n are the eigenvalue solutions for a frame without "leaning" columns, F_o, and with "leaning" columns, F_n. Figure 7 shows the relationship between F_o/F_n and n as a function of end conditions for five different columns. Since normal column end conditions fall somewhere between fixed and pinned, it can be seen that taking $F_o/F_n = 1.0$ should provide a K-factor on the conservative side by at most 2 percent. Substituting for n and using $F_o/F_n = 1.0$, the Lim & McNamara approach gives the same K-factor as the modified Yura approach where

$$K_n = K_o\sqrt{1+\frac{\Sigma Q}{\Sigma P}} = K_o\sqrt{\frac{\Sigma P + \Sigma Q}{\Sigma P}} \qquad (9)$$

LeMessurier Approach (8): A more complex, yet still very practical approach was presented by LeMessurier for frames with and without "leaning" columns. The basic equations were developed for a single cantilever column and then extended to the general frame. Where the two previous approaches determined a constant value for a story by which the nomograph value of K_o was modified, this approach determines a constant value for a story which then multiplies the individual column moment of inertia divided by the column load, I_i / P_i, for each column, i. Thus, the contribution of each column to the lateral resistance is accounted for individually. The effective length factor for each column that participates in resisting sidesway buckling is given by

$$K_i^2 = \frac{I_i}{P_i}\pi^2\frac{\Sigma P + \Sigma Q + \Sigma(C_L P)}{\Sigma(\beta I)} \qquad (10)$$

where

$$\beta = \frac{6(G_A' + G_B) + 36}{2(G_A + G_B) + G_A G_B + 3} \qquad (11)$$

$$C_L = \frac{\beta K_o^2}{\pi^2} - 1 \qquad (12)$$

C_L = 0 for leaning columns.
K_o = the column effective length based on the nomograph.
G_A, G_B = column end conditions as defined for use with the nomograph.
P_i = load on column, i.
I_i = moment of inertia for column, i.
ΣP = load on the restraining columns in a story.
ΣQ = load on the leaning columns in a story
$\Sigma(C_L P)$ = sum of $(C_L P)$ for each column in the story.
$\Sigma(\beta I)$ = sum of (βI) for each column participating in lateral sway resistance.

Commentary Equations: Although use of Equation 10 is not particularly complex, there have been some suggested simplifications which may be useful to the designer. Two modified LeMessurier equations are presented in the second edition of the Commentary to the LRFD Specification. The first simplification is to assume that there is no reduction in column stiffness due to the presence of axial load. This is accomplished by taking $C_L = 0$ for all columns. This leads to $\beta = \pi^2 / K_o^2$. Substitution of these values into Equation 10 yields:

$$K_i^2 = \frac{I_i}{P_i}\pi^2\frac{\Sigma P + \Sigma Q}{\Sigma(\pi^2 I / K_o^2)} \qquad (13)$$

which reduces to

$$K_i = \sqrt{\frac{I_i}{P_i}\frac{\Sigma P + \Sigma Q}{\Sigma(I / K_o^2)}} \qquad (14)$$

For a structure in which only one column can be considered to provide lateral stability, the summation in the denominator is unnecessary and Equation 14 reduces to

$$K_i = K_o\sqrt{\frac{\Sigma P + \Sigma Q}{P_i}}$$

which is the same equation that resulted from the modified Yura and Lim & McNamara approaches, Equations 7 and 9 respectively.

For the development of the second simplified equation, stiffness reduction due to axial load is included as though all columns were cantilevers with a buckled shape in the form of a half sine curve as shown in Figure 2b, thus $C_L = 0.216$. Since the "leaning" columns have no lateral stability of their own, $C_L = 0.0$ for all "leaning" columns. The equation given in this paper as Equation 10 is just one form of the effective length factor equation given by LeMessurier. Another form that uses the ratio of lateral displacement of a story to the lateral load as a measure of lateral stiffness is also available through the same derivation.[8] This equation is given as

$$K_i^2 = \frac{I_i}{P_i}\frac{\pi^2 E}{L^3}\frac{\Delta_{oh}}{\Sigma H}(\Sigma P_T + \Sigma C_L P_T) \quad (15)$$

where

ΣH = the total lateral load supported by the level under consideration,

Δ_{oh} = is the corresponding lateral displacement of the level and

ΣP_T = is the total load on the given story.

In order to account for $C_L L = 0$ on the "leaning" columns, the load on these "leaning" columns must be subtracted from the total load on the story so that $(\Sigma P_T + \Sigma C_L P_T) = (\Sigma P_T + 0.216(\Sigma P_T - \Sigma Q)$. Making this substitution and factoring out ΣP_T yields

$$K_i^2 = \frac{I_i}{P_i}\frac{\pi^2 E}{L^3}\Sigma P_T \frac{\Delta_{oh}}{\Sigma H}\left(1 + \frac{0.216(\Sigma P_T - \Sigma Q)}{\Sigma P_T}\right) \quad (16)$$

The bracketed term in Equation 16 is simplified in the commentary equation as

$$K_i^2 = \frac{I_i}{P_i}\frac{\pi^2 E}{L^3}\Sigma P_T \frac{\Delta_{oh}}{\Sigma H}\left(\frac{1}{0.85 + 0.15\Sigma Q / \Sigma P_T}\right) \quad (17)$$

This simplification is not really necessary since, in the original form, the equation is no more complex. A plot of the bracketed terms and their square roots is shown in Figure 8. It can easily be seen that the simplified terms vary from the original terms as derived from the LeMessurier equations, particularly when the "leaning" column loads are relatively small.

EXAMPLES

The following three examples will show how these approaches may be used successfully to evaluate columns in unbraced frames, including "leaning" columns.

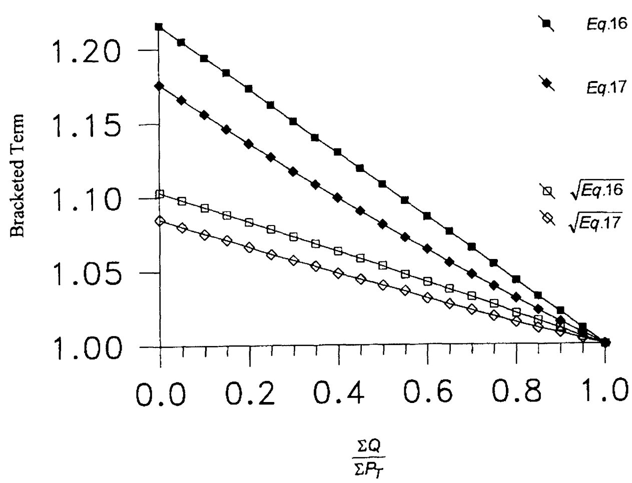

Fig. 8. Comparison of Equation 16 and Equation 17.

Example 1

The unbraced frame with "leaning" columns as shown in Figure 9 is to be checked for strength and stability. $F_y = 36$ ksi. This is the same frame as used in Yura's presentation,[6] modified with a load factor of 1.43 to permit a check by LRFD. The frame is assumed to be braced out of the plane.

Yura Approach: Since sway will likely control the design of column AB, try a W12×30. Due to symmetry, one half of the load on the "leaning" columns will be assigned to a single column, AB.

$\Sigma P = 57$ kips, $\Sigma Q = 150$ kips,

$(\Sigma P + \Sigma Q) = (57 + 150) = 207$ kips.

$$G_{top} = \frac{238/16}{586/(35(2))} = 1.78$$

Note: to account for the pin ended beam, the length is modified by the factor 2.

G_{bot} = 10

K = 2.1 from the nomograph of Figure 4

K = 1.0 for out of plane buckling

KL/r_x = 2.1(16) (12)/5.21 = 77.4

KL/r_y = 1.0(16) (12)/ 1.52 = 126.3

For strength in the y-axis, $\lambda_c = 1.42 < 1.5$ thus, use LRFD E2-2. $F_{cr} = 15.54$ ksi, $P_u = 116$ kips > 57 kips thus, the strength is adequate.

For stability about the x-axis, $\lambda_c = 0.868 < 1.5$ therefore from LRFD E2-2, $F_{cr} = 26.26$ ksi, $P_u = 196.2$ kips < 207 kips, thus the column will not be sufficient to provide stability.

Lim & McNamara Approach: Again, a W12×30 will be considered for column AB. Using the ratio of $\Sigma Q / \Sigma P = n$ = 150/57 = 2.63. As before, $K_o = 2.1$ so $k_n = 2.1\sqrt{1 + 2.63} = 4.0$. As already shown, the column will be adequate for strength. Now, checking for stability, KL/r_x = 4.0(16) (12)/5.21 = 147.4, $\lambda_c = 1.65$ and $F_{cr} = 11.55$ ksi. Thus, $P_u = 86.3$ kips > 57

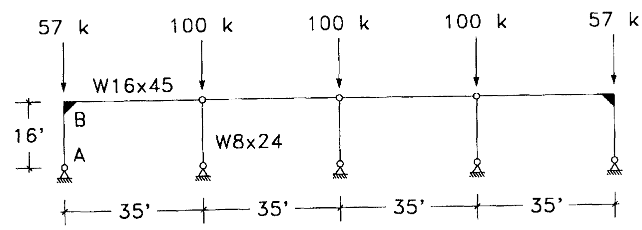

Fig. 9. Frame for Example 1.

kips applied load which shows that the column is also adequate for stability.

LeMessurier Approach: Since the W12×30 column was shown to be adequate, it will again be checked. Using the values obtained above, for column AB,

$$K_o = 2.1$$

$$\beta = \frac{6(1.78 + 10) + 36}{2(1.78 + 10) + (1.78)(10) + 3} = 2.40$$

$$C_L = \frac{2.40(2.1)^2}{\pi^2} - 1 = 0.0724$$

$$K_n^2 = \frac{I_i}{57}\pi^2 \frac{207 + 0.0724(57)}{2.40 I_i} = 15.23$$

$$K_n = 3.9$$

Thus, with $K_n = 3.9 < 4.0$ from above, the W12×30 will be sufficient to provide lateral restraint.

The difference between the Yura and Lim & McNamara approaches has to do with the use of the effective load or the effective length. With the effective load approach, the column is designed with the shorter effective length given through use of K_o and the larger load given by $(P + Q)$. With the effective length approach, the load is maintained as P_i; however, the effective length given through K_n is larger. Figure 10 shows a plot of the elastic and inelastic strength equations from the LRFD Specification. In addition, the slenderness parameters for the two design approaches are shown. If column design were controlled by the upper, elastic, equation in the figure, LRFD E2-3, Yura's approach would have shown that the W12×30 was adequate.

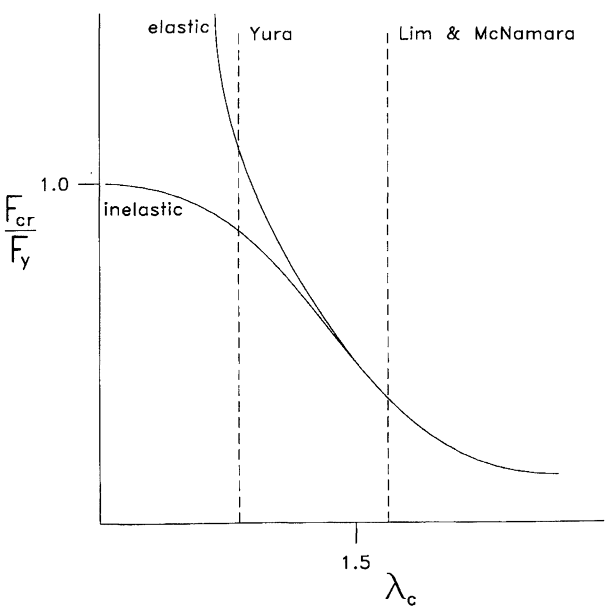

Fig. 10. LRFD column strength.

Modified Nomograph Equation: An iterative solution of Equation 3 with G_A, G_B, Q, and P as given above yields $K_n = 3.879$. As can be seen, this value compares quite well with the values already obtained and the column will be adequate.

Commentary Equations: For the first simplified equation, Equation 14, $K_n = 4.0$, as would be expected from the derivation shown above, since there is only one restraining column.

The use of the second simplified equation, Equation 16, requires an analysis of the structure to determine the ratio of lateral displacement to load. Since the analysis carried out for the previous approaches assumed that the pin connection at the base was not a true pin but one which resulted in $G_B = 10$, this must be included in the analysis. With this provision accounted for as shown in Figure 11, an arbitrary lateral load of 5 kips results in a deflection $\Delta_{oh} = 2.3715$-in. Thus Equation 16 results in:

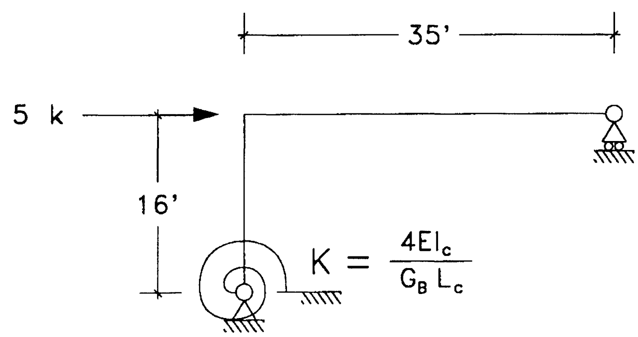

Fig. 11. Model for lateral displacement computation.

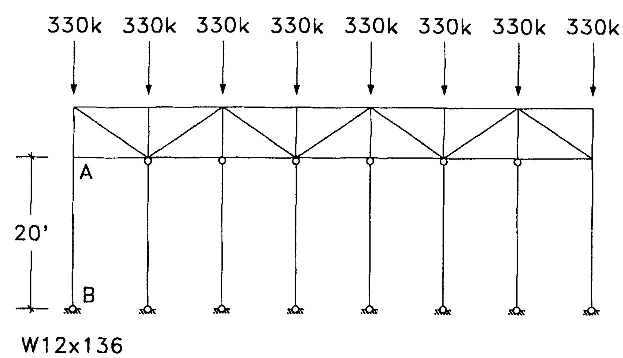

Fig. 12. Frame for Example 2.

$$k_i = \pi\sqrt{\frac{29000(238)}{(16(12))^3}\frac{207}{57}\frac{2.3715}{5}\left(1+\frac{0.216(207-57)}{207}\right)}$$

$= 4.379$

and using Equation 17, $K_i = 4.313$. Even with these somewhat larger K values, the W12×30 will provide sufficient strength and stiffness. Thus, all five approaches yield similar results.

Example 2

The frame shown in Figure 12 was used by Cheong-Siat-Moy[9] to show that Yura's approach would not work and by de Buen[10] to present his new approach. The truss is assumed to provide sufficient rotational restraint at the top to permit that end of column AB to be treated as a fixed end while the bottom of the column is pinned, thus from the nomograph, $K_o = 2.0$. Recognizing that Yura's approach will yield a larger column than required and that LeMessurier's approach will give the correct results, as shown by the two previously cited references, the Lim & McNamara approach will be compared to the LeMessurier approach. For this example, $F_y = 36$ ksi will be used.

Lim & McNamara: With one half of the leaning column load resisted by the single restraining column, $n = 990/330 = 3.0$. Thus,

$$K_n = 2.0\sqrt{1+3.0} = 2.0\sqrt{4.0} = 4.0$$

Using $K_n = 4.0$, $KL/r_x = 4.0(20)(12)/5.58 = 172.0$, $\lambda_c = 1.93 > 1.5$, thus from LRFD E2-3, $F_{cr} = 8.48$ ksi, thus $P_u = 287$ kips < 330 kips.

LeMessurier: For a fixed-pinned column, $\beta = 3.0$ and $C_L = 0.216$ so that Equation 10 yields:

$$K_n^2 = \frac{I_i}{330}\pi^2\frac{4(330)+0.216(330)}{3I_i} = 13.87$$

$K_n = 3.72$ as shown in References 9 and 10.

Using $K_n = 3.72$, $KL/r_x = 3.72(20)(12)/5.58 = 160.0$, $\lambda_c = 1.79 > 1.5$, thus from LRFD E2-3, $F_{cr} = 9.85$ ksi, thus $P_u = 334$ kips > 330 kips.

Although the Lim & McNamara approach does not show the W12×136 to be adequate, it requires a column only one size larger than the other method.

Modified Nomograph Equation: Using G values consistent with a fixed end and a pinned end along with $Q/P = 3.0$, an iterative solution of Equation 3 results in $K_n = 3.718$. Again, the result is similar to the previously obtained values, particularly that from the LeMessurier approach.

Commentary Equations: As was shown for Example 1, where only one column provides lateral support the results from Lim & McNamara and the simplified equation, Equation 14, are identical, $K_n = 4.0$. The LeMessurier analysis, Equation 10, assumed that the columns were fixed at the upper end and had true pins at the lower end. This results in $C_L = 0.216$ which is the assumption for the second simplified equation, Equation 16. With the lateral displacement calculated for a cantilever beam with a 5 kip load, $\Delta_{oh} = 0.6407$-in. This yields $K_n = 3.724$ from Equation 16 and $K_n = 3.698$ from Equation 17. The results from Equation 16 are identical to the previously calculated value using the complete LeMessurier approach as would be expected.

Example 3

An interesting problem was originally proposed by Zweig[11] and later discussed by Yura. The structure, a portion of which is shown in Figure 13, is a large, unbraced one story industrial building. Deep roof trusses with infinitely large stiffness compared to the columns, frame in each direction. In order to equalize sway restraint in each direction, alternate columns have their strong axes turned 90 degrees. The nomograph approach with $G_{top} = 0$ and $G_{bot} = 10$ yields $K_o = 1.65$. Without considering "leaning" columns, $KL = 1.65(20) = 33$-ft. and a W12×65 will be required to carry $P_u = 234$ kips, using steel with $F_y = 36$ ksi. Each column in the building would be a W12×65. If the strong axis column is used to brace the weak axis column, there should be some savings available. Each of the approaches previously discussed will be used to check a W12×53 column to determine whether this smaller column would be adequate. Throughout this example, the columns will be taken as pairs, one strong axis and one weak axis column.

Yura Approach: Using $K_o = 1.65$, the capacity of the W12×53 for sway buckling about the x-axis, column 1, and the y-axis, column 2, will be added. The combined capacity must be equal to or greater than the total load 2(234) = 468 kips. For $KL/r_x = 1.65(20)(12)/5.23 = 75.7$, $\lambda_c = 0.849 < 1.5$, thus $F_{cr} = 26.6$ ksi and $P_u = 353$ kips. For $KL/r_y = 1.65(20)(12)/2.48 = 159.7$, $\lambda_c = 1.79 > 1.5$, thus $F_{cr} = 9.40$ ksi and $P_u = 125$ kips.

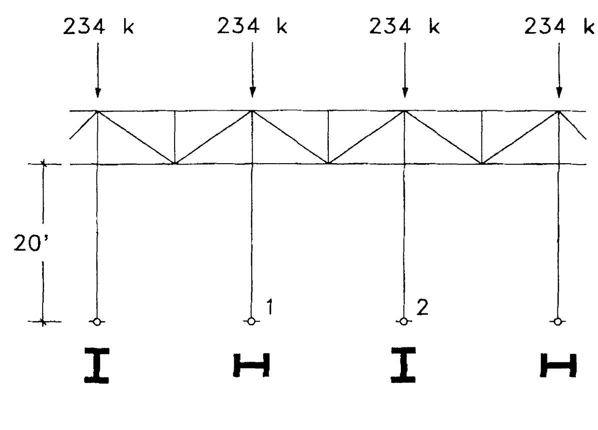

Fig. 13. Frame for Example 3.

The combined capacity is 353 + 125 = 478 kips which is greater than the 468 kips combined load. Thus, the W12×53 will be acceptable for the columns.

Lim & McNamara Approach: If column 1 is taken as the restraining column and column 2 as the "leaning" column, n = 234/234 = 1.0. Thus, for column 2, K = 1.0 and KL = 1.0(20) = 20 ft. For the W12×53 buckling about the y-axis, P_u = 292 kips > 234 kips. For column 1, $K_n = 1.65\sqrt{1+1}$ = 2.33 and KL = 2.33(20) = 46.6-ft. For buckling about the x-axis, P_u = 261 kips > 234 kips. Thus, the W12×53 is adequate for both columns in each direction.

LeMessurier Approach: With $G_{top} = 0$ and $G_{bot} = 10$ for both columns 1 and 2, β = 4.17 and K_o = 1.65. $C_L = [4.17(1.65)^2/\pi^2 - 1] = 0.15$. For the W12×53, $I_1 = 425\text{-}in^4$ and $I_2 = 95.8\text{-}in^4$. Thus, from Equation 10

$$K_i^2 = \frac{I_i}{234}\pi^2 \frac{468 + 2(0.15)(234)}{4.17(425 + 95.8)} = 0.01045 I_i$$

which yields K_1 = 2.11 and K_2 = 1.0. Thus, for column 2, KL/r_y = 1.0(20)(12) / 2.48 = 96.8, λ_c = 1.09 < 1.5, thus F_{cr} = 21.9 ksi and P_u = 292 kips > 234 kips. For column 1, KL/r_x = 2.11(20)(12) / 5.23 = 96.8, λ_c = 1.09 which is the same as for column 2 so P_u = 292 kips. This shows that the W12×53 column is adequate for both carrying the load and providing lateral restraint.

Modified Nomograph Equation: In this case, with end stiffness and load ratio as for the other approaches, Equation 3 yields K_n = 2.267. It is clear that this approach does not account for the difference in columns as found with the LeMessurier approach; however, it is satisfactory when compared to the Lim & McNamara approach.

Commentary Equations: For the first simplified equation, Equation 14, K_o = 1.65 for both columns. Thus

$$K_1 = 2.108 \text{ and } K_2 = 1.001$$

As was the case in Example 1, the application of second simplified equation requires the analysis of the structure with a connection stiffness at the base that will result in G_B = 10. With a 5 kip lateral load, the resulting lateral deflection, Δ_{oh} = 1.1986-in. Thus, from Equation 16,

$$K_1 = 2.162 \text{ and } K_2 = 1.026$$

while Equation 17 yields

$$K_1 = 2.136 \text{ and } K_2 = 1.014$$

A check of the W12×53 column with these larger effective length factors shows that the column still provides the required strength and stiffness.

CONCLUSIONS

It should be clear that the "leaning" column is a consideration that results from the structural framing, not from the use of ASD or LRFD. It is also clear, from Example 3, that accounting for the total lateral resistance in a frame may lead to reduced column sizes with their accompanying savings.

Although the Yura approach does, for some conditions, give overly conservative results, it can readily be modified to yield the Lim & McNamara approach, Equations 7 and 9, which provides sufficiently accurate results for design. This was shown, particularly in Example 2, even for those cases reported in the literature for which the Yura approach is overly conservative.

The equations proposed by LeMessurier are generally recognized as the most accurate of those presented. In addition, the LeMessurier equations are not difficult to use nor do they require the graphic analysis for C_L or β that was presented in the original work. The determination of K_o may be accomplished through the nomograph, as is normally done, or by an iterative solution of the controlling equation, Equation 2. Thus, it is not unrealistic to use the LeMessurier equations for effective length factors in normal engineering practice.

It was also shown that through an iterative solution of Equation 3, a more accurate value of K_n may be obtained than that from Equation 2. In this case, the leaning column loads are accounted for; however, the other limitations of the nomograph solution are still present.

It is suggested that different stages of design might benefit from the use of different levels of accuracy in determination of effective length. Although the LeMessurier approach is not overly complicated to use, designers wishing to use an even simpler approach may find that the Lim & McNamara equation for K_n provides a sufficiently accurate way to account for "leaning" columns, particularly in preliminary stages of design. Lim & McNamara could be quite appropriate for preliminary design while LeMessurier would be more appropriate for final design. Although simplified equations are presented in the Commentary to the second edition of the LRFD Specification, there is really no need to use them. Once the effective length factors have been determined, design by either ASD or LRFD may proceed.

REFERENCES

1. *Load and Resistance Factor Design Specification for Steel Buildings,* AISC, 1986.
2. *Specification for Structural Steel Buildings—Allowable Stress Design and Plastic Design,* AISC, 1989.
3. ASCE Task Committee on Drift Control of Steel Building Structures, "Wind Drift Design of Steel-Framed Buildings: State-of-the-art Report," *Journal of Structural Engineering*, ASCE, Vol. 114, No. 9, 1988, pp. 2085–2108.
4. West, Harry H., *Analysis of Structures,* John Wiley and Sons, New York, 1989.
5. Galambos, T. V., *Structural Members and Frames.* Prentice-Hall, Englewood Cliffs, NJ, 1968.
6. Yura, J. A., "The Effective Length of Columns in Unbraced Frames," *Engineering Journal*, AISC, Vol. 8, No. 2, 1971, pp. 37–42.

7. Lim, L. C. and McNamara, R. J., "Stability of Novel Building System," *Structural Design of Tall Steel Buildings. Vol. II-16, Proceedings, ASCE-IABSE International Conference on the Planning and Design of Tall Buildings,* Bethlehem, PA, 1972, pp. 499–524.
8. LeMessurier, Wm. J., "A Practical Method of Second Order Analysis," *Engineering Journal*, AISC, Vol. 14, No. 2, 1977, pp. 49–67.
9. Cheong-Siat-Moy, F., "Column Design in Gravity-Loaded Frame," *Journal of Structural Engineering*, ASCE, Vol. 117, No. 5, 1991, pp. 1448–1461.
10. de Buen, Oscar, "Column Design in Steel Frames Under Gravity Loads," *Journal of Structural Engineering*, ASCE, Vol. 118, No. 10, 1992, pp. 2928–2935.
11. Zweig, A., "Discussion of 'Column Stability Under Elastic Support' by T. R. Higgins," *Engineering Journal*, AISC, Vol. 2, No. 3, 1965, pp. 105–106.