Design of Braced Multi-Story Frames by the Plastic Method

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SINCE 1956 an extensive research program has been carried out at Lehigh University to study the inelastic behavior and strength of multi-story frames subjected to various types of applied loads and to develop design methods utilizing the reserved strength in the inelastic range. In this program, because of the difference in load-carrying behavior, multi-story frames were divided into two types: braced and unbraced frames, and a separate investigation was made for each type. The results obtained from this research together with the recommended design procedures were presented in a comprehensive set of lecture notes and a design aids booklet,[1, 2] which were prepared for a special Summer Conference on "Plastic Design of Multi-Story Frames" held in August, 1965, at Lehigh University. This paper summarizes the procedure described in the lecture notes for the design of braced frames.

A braced frame is defined as a frame in which all resistance to lateral force, sway and frame instability is provided by a specially designed bracing system. The required member sizes for the beams and columns in such a frame are often governed by gravity load. It is therefore possible in the initial stage of design to treat the frame and the bracing as two separate load-carrying systems and to perform independent designs for them. The interaction between the frame and the bracing system is then checked after the approximate member sizes are selected. The following are the general conditions assumed in the development of the design procedure.

1. All members in the frame lie in a single plane and all loads are applied in the same plane. Biaxial bending of beams and columns is not permitted.
2. Lateral bracing with simple connections are provided to prevent out-of-plane deformations. All deflections (lateral and vertical) are therefore confined in the plane of the frame.
3. Fully welded rigid connections are used throughout the frame.
4. Floor loads, including both dead and live loads, are considered as uniformly distributed loads. Wind loads or other types of lateral loads are assumed to act as concentrated loads at floor levels.

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PRELIMINARY DESIGN

The purpose of a preliminary design is to select approximate member sizes which can later be checked for other design requirements. Before the preliminary design can actually proceed, several preparatory steps must first be completed. These include: determination of dimensional layout based on functional requirements of the building, selection or assignment of design loads, distribution of loads to plane frames and tabulation of girder and column loads. These steps are the same in plastic design as they are in allowable-stress design. An additional step which is done only in plastic design is to multiply the design loads by the load factors. The recommended load factors are 1.70 for gravity loads and 1.30 for gravity-plus-wind loads. The use of these load factors results in two separate sets of factored loads for which the frame must be designed:

1. Factored gravity (including both dead and live) loads equal to 1.70 times the working values.
2. Factored gravity and wind loads equal to 1.30 times the working values.

These loads are also called the "design ultimate loads" and can be directly used in the subsequent calculations.

In the preliminary design, all girders and columns are proportioned to resist the factored gravity loads. The girders are assumed to fail by forming beam mechanisms in the clear spans and the required plastic moment can be calculated from the equation

$$M_p = \frac{1}{16} w_T L_g^2 \qquad (1)$$

where w_T is the factored total load and L_g the clear span length. With the required M_p known, the girder size can be selected by entering the plastic modulus tables contained in the AISC Manual.

Columns in a braced frame must have sufficient strength to resist the axial loads from the floors above and the bending moments transmitted to the joints by the girders. The maximum moment that can be transmitted by the girder is equal to the plastic moment of the girder plus an increment of moment caused by the vertical shear acting at the column face, that is,

$$M = M_p + V\frac{d_c}{2} \qquad (2)$$

in which V represents the shear force from the girder and d_c the column depth. At an exterior joint the two columns immediately above and below the joint are selected so that the total resisting moment is equal to or greater than the applied moment. At interior joints the unbalanced moment that must be resisted by the columns is equal to the difference between the two moments transmitted from the adjacent girders.

In selecting preliminary column sections, the maximum resisting moment provided by a column may be assumed to be equal to the reduced plastic moment, M_{pc}. The actual moment capacity is likely to be less than this value due to column instability effect. The difference will be taken into account in the final design. Tables giving M_{pc} values of column sections for various axial force ratios can be found in Reference 2.

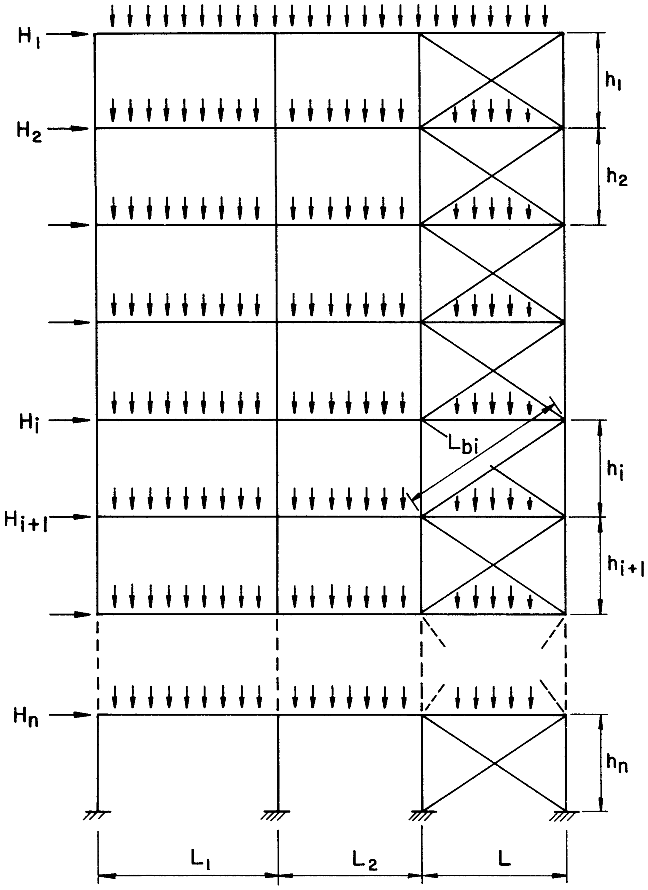


Fig. 1. A three-bay, n-story braced frame

SELECTION OF BRACING SYSTEM AND SIZES

The bracing system in a multi-story frame is designed to serve the following important functions: (1) resisting lateral loads, (2) counteracting the overturning moment ($P\Delta$ moment) due to gravity loads, (3) preventing frame buckling and (4) improving sway behavior. Many types of bracing are used in building frames, such as diagonal bracing, K-bracing, knee braces, and shear walls. The type of bracing used and its location are usually dictated by architectural, functional and structural considerations. The discussion presented herein will be concerned only with diagonal bracing, although the same principles can be applied to other types of bracing as well.

Figure 1 shows a three-bay, n-story frame braced by diagonal braces spanning between panel points in the right bay. The bracing system is assumed to behave as a pin-connected Pratt truss in resisting the lateral loads. It is further assumed that all braces can resist only tension forces with a limiting capacity equal to the yield force. The frame is loaded either by gravity loads alone, or by gravity loads in combination with lateral loads. When gravity loads alone act on the frame, the primary function of the bracing is to prevent over-all frame buckling. The required cross-sectional area for the brace in the ith story from the roof is given by

$$A_{bi} = \frac{L_{bi}^3}{EL^2h_i}\sum_1^i W_i \qquad (3)$$

where

L_{bi} = length of brace
L = span of braced bay
h_i = story height
E = modulus of elasticity, and
$\sum_1^i W_i$ = sum of all gravity loads above and including ith level, load factor = 1.70

For the case of combined gravity and lateral loading, the bracing area required to resist the lateral loads and to counteract the overturning effects can be determined from the equation

$$A_{bi} = \frac{L_{bi}}{\sigma_y L}\sum_1^i H_i + \frac{L_{bi}^3}{EL^2h_i}\sum_1^i W_i \qquad (4)$$

where σ_y represents the yield stress of the bracing and $\sum_1^i H_i$ is the sum of all the lateral loads at the ith level. A

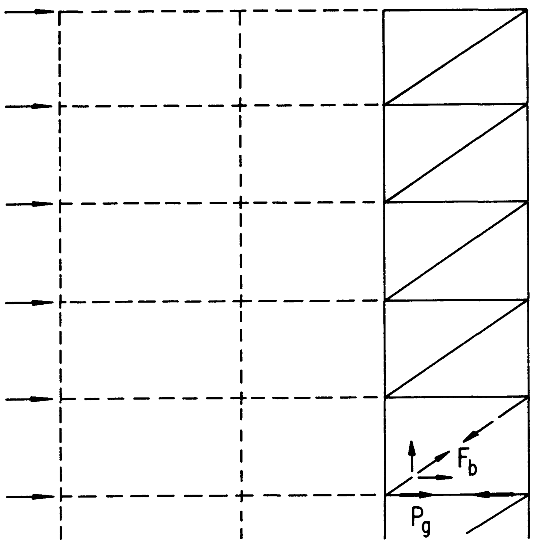


Fig. 2. Axial force in girder in braced bay

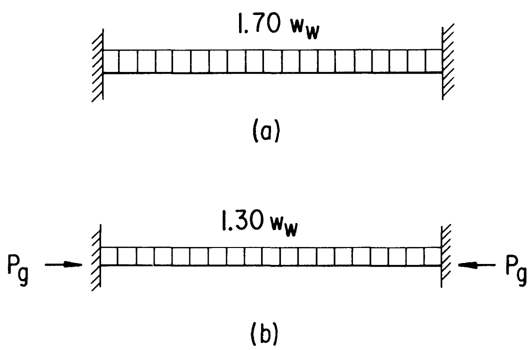


Fig. 3. Forces applied to girder in braced bay

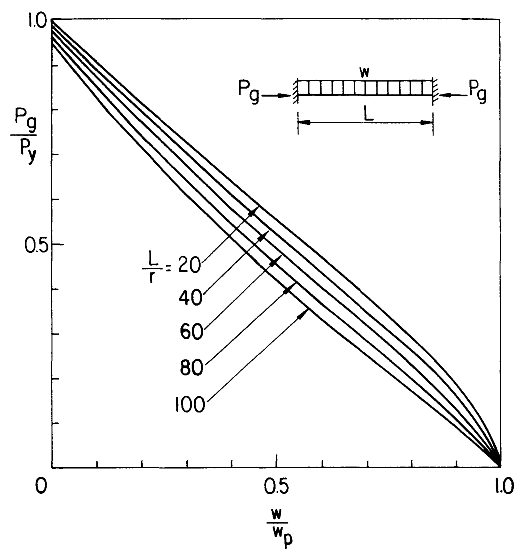


Fig. 4. Ultimate strength interaction curves for combined axial force and gravity load

load factor of 1.30 is applied to both H_i and W_i. The development of Equations (3) and (4) is described in Reference 1.

It is usually also required that the slenderness ratio of the brace be less than a certain maximum allowable value. The AISC Specification gives a maximum slenderness ratio for bracing members of 300. For a brace of length L_{bi} the minimum radius of gyration should meet the requirement

$$r_{bi} \geq \frac{L_{bi}}{300} \tag{5}$$

Experience has shown that the area computed from Equation (3) is often smaller than that required by Equation (4) and that the slenderness requirement of Equation (5) is likely to govern the bracing sizes in the top stories. A convenient design procedure would be to compute first the required areas from Equation (4) and to select tentative bracing sizes. These sizes can then be checked for the requirements of Equations (3) and (5).

FINAL DESIGN OF GIRDERS

The girders selected in the preliminary design for the full gravity loading with a load factor of 1.70 will usually have sufficient strength to resist the combined loads. This is because of the fact that a lower load factor of 1.30 is specified for the case of combined loading and that all the lateral loads are assumed to be resisted by the bracing system. Additional considerations, however, should be given to the girders in the braced bay. These girders often act as the web members of the bracing truss and are thus subjected to compressive forces when lateral loads are applied to the frame (Fig. 2). The force in the girder at the ith story level may be conservatively computed from the formula

$$P_{gi} = \sum_{1}^{i} H_i + \frac{\Delta_i}{h_i} \sum_{1}^{i} W_i \tag{6}$$

Where Δ_i is the relative sway between floor levels i and $i + 1$ and can be estimated from a knowledge of the elongation of the bracing in that story. A value of Δ_i/h_i = 0.004 has been suggested for practical computations. Since both ΣH_i and ΣW_i increase from the top down, the force P_g may become significantly large in the lower story girders.

The girder which was previously designed to resist gravity load only (Fig. 3a) should now be checked for combined axial and gravity loads (Fig. 3b). The presence of the axial thrust causes the girder to fail by instability at a load less than the beam mechanism load. The latter was used as the basis of design for gravity load. The reduced load-carrying capacity corresponding to a given axial thrust can be found from the interaction curves given in Fig. 4. In this figure the axial thrust, P_g, is nondimensionalized with respect to the axial yield load, P_y, and the maximum girder load, w, is expressed in terms of the beam mechanism load, w_p. The girder size selected previously is adequate if the maximum load from the curves is greater than the factored gravity load (1.30 times working value).

The maximum strength can also be computed approximately from the following formula[3]:

$$w = \frac{1 - \dfrac{P_g}{P_e}}{1 - 0.4\dfrac{P_g}{P_e}}\left(1 - \frac{P_g}{P_0}\right) w_p \tag{7}$$

in which P_e is the elastic buckling load in the plane of bending and P_0 the critical load if axial thrust alone existed.

The above discussion was for frames which are braced only in one bay. The force P_g at a given story level is assumed to be applied entirely to the girder in that bay. In frames with multiple braced bays, this force is usually distributed to more than one girder and, consequently, would be less critical in affecting the selection of girder sizes.

FINAL DESIGN OF COLUMNS

As pointed out in the discussion on preliminary design, the resisting moment of a column is likely to be reduced by the effect of column instability. This reduction must now be taken into account in the final design. In the following, only the basic concepts of the design methods will be presented; the detailed development and some special considerations are given in Reference 1.

There are two possible loading conditions which are often encountered in column design:

1. Full dead and live loads on all girders, and
2. Checkerboard (or partial) live load on alternate girders.

Whether the columns in a given frame should be designed for full loading or checkerboard loading depends on the judgment of the designer and/or the code requirement.* In general, the checkerboard loading would cause more severe bending moment distribution and is likely to govern the selection of column sizes.

Design for Full Loading—Figure 5 shows two tiers of a braced frame loaded by the factored total load, w_T, on its girders. Beam mechanisms are assumed to form in the girders at this load, and a moment equal to that given by Equation (2) is applied by each girder to its supporting

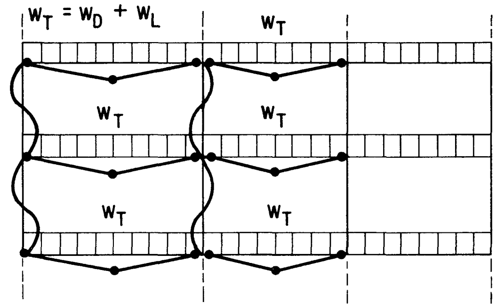


Fig. 5. Full dead and live loads on all girders

joints. These moments cause both the exterior and the interior columns to deform in double curvature (S-shapes). It is appropriate in design calculations to assume that each column is subjected to two end moments of equal magnitude and acting in the same sense. The end moment ratio, q, of the columns is therefore taken as 1.0. This assumption is adequate for most building columns (whose slenderness ratio is usually less than 40), because the moment carrying capacity of low slenderness ratio columns is not significantly affected by a small variation in the end moment ratio. This point is illustrated in Fig. 6. The column selected for this illustration has a slenderness ratio of 40 and is subjected to axial forces equal to 0.6 P_y and 0.8 P_y. In the first case the strength of the column is not affected by a change in q, while in the second case a small reduction occurs for $q = 0.6$. However, the reduction is usually so small that its effect can be neglected in design calculations.

The foregoing discussion justifies the design assumption that all the columns may be assumed to be bent in symmetrical double curvature. Analytical studies have shown that the moment carrying capacity of double curvature columns is either equal to M_{pc} or slightly below it. Figure 7 shows the relationships between the maximum moment and the axial force ratio for three columns with slenderness ratios equal to 30, 40 and 50. It is seen that the maximum moment may be taken as M_{pc} for P/P_y equal to or less than 0.6. For higher P/P_y ratio (up to 0.9) the maximum moment will be somewhat less than M_{pc}.

Another important property of double curvature columns is that the moment-rotation curves usually have a horizontal plateau. The maximum moment can be maintained at the ends while the column rotates through a certain angle. Therefore, in designing the columns for the case of full loading, it is only necessary to satisfy a statical condition for each joint

$$M_{a_{\max}} + M_{b_{\max}} \geq M_j \tag{8}$$

in which $M_{a_{\max}}$ and $M_{b_{\max}}$ represent, respectively, the maximum moments that can be resisted by the columns above and below the joints. For the exterior columns, the joint moment M_j is the maximum moment applied by

* *For example, the American Standard Building Code (1955 edition) requires that the effect of partial loading be considered when the live load exceeds 150 lbs per sq ft or is twice the dead load.*

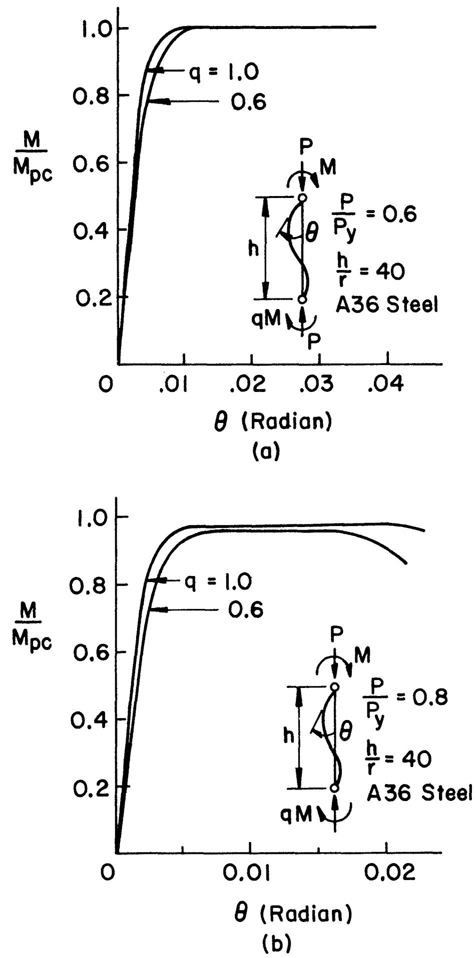


Fig. 6. Moment-rotation relationships of double curvature columns

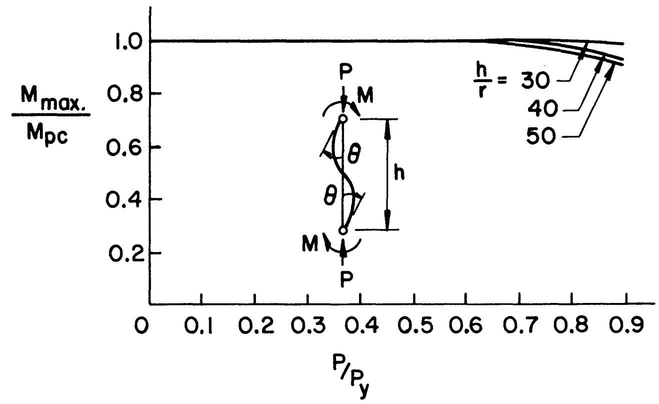


Fig. 7. Moment-carrying capacity of symmetrical double curvature columns

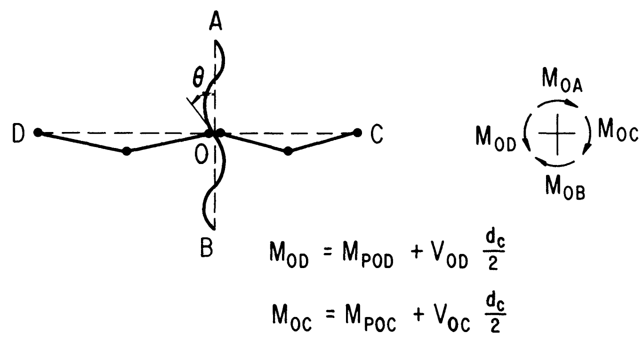


Fig. 8. Girder-and-column system assumed in design of columns for full loading

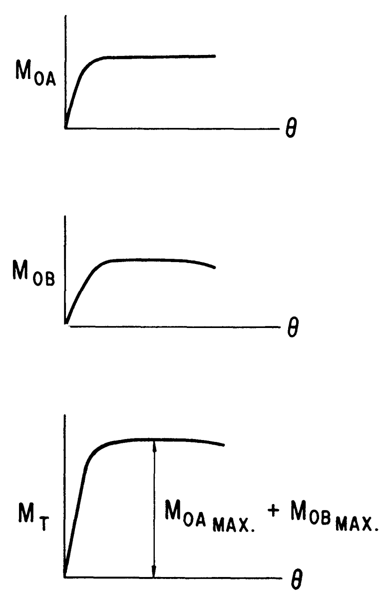


Fig. 9. Determination of total resisting moment of two double curvature columns

the exterior girders. For the interior columns, M_j is equal to the difference between the two maximum moments transmitted from the adjacent girders.

The above procedure is further explained in Figs. 8 and 9. It is required to check the adequacy of some trial sections selected for the two interior columns of the frame shown in Fig. 5. The two columns shown as **OA** and **OB** in Fig. 8 are considered adequate if the total resisting moment provided by the columns is equal to or greater than the net joint moment $M_{OD} - M_{OC}$. The total resisting moment can be determined by constructing the individual moment-rotation curves for the trial columns and by combining the curves in the manner illustrated in Fig. 9. It is obvious that the maximum resisting moment of the two columns is equal to the sum of the individual maximum moments.

Design for Checkerboard Loading—Consider now the case where the same columns are to be designed for checkerboard loading. Figure 10 shows a typical loading pattern which may be assumed for the purpose of design. In the exterior bay the live loads on the two alternate girders are now removed, thus reducing the

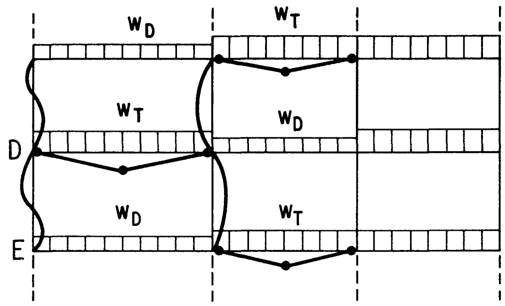


Fig. 10. Checkerboard live load on alternate girders

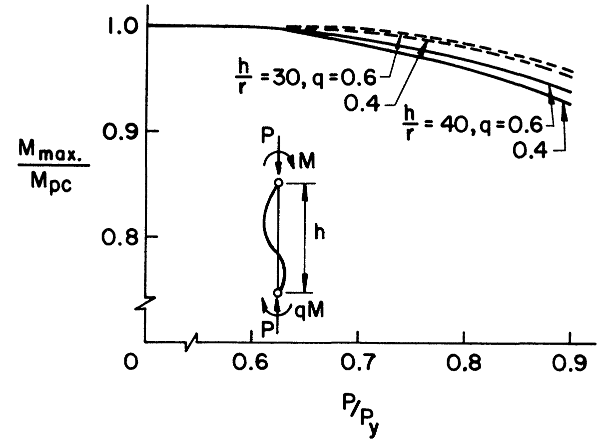


Fig. 11. Influence of axial force, slenderness ratio and end moment ratio on strength of columns

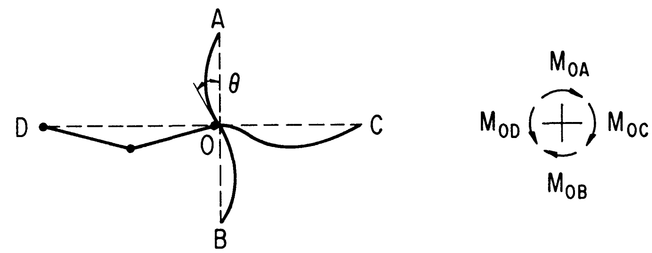


Fig. 12. Girder-and-column system assumed in design of columns for checkerboard loading

bending moments applied to the exterior joints. The reduction in joint moment causes a change in the end moment ratio, q, of the two exterior columns and may, in turn, decrease the moment-carrying capacity. The removal of the live loads may therefore reduce the total resisting moment provided by the columns at point **D**.

It has been found that the strength of beam-columns with slenderness ratios smaller than 40 and subjected to axial forces less than 0.6 P_y is not significantly affected by a change in q from 1.0 to 0 (one end free of moment).[2] The maximum resisting moment is usually equal to M_{pc}. Therefore checkerboard loading does not require an increase in the column sizes.

For columns subjected to axial forces higher than 0.6 P_y, a change in q usually results in a corresponding change in the moment-carrying capacity. An estimate of q is therefore needed in the design. To estimate the value of q for column **DE** in Fig. 10, it is first necessary to compute the moments applied at the two joints by the girders. The moment at the upper joint is $M_D = M_p + (w_T L_g/2)\ d_c/2$, and that at the lower joint is $M_E = (w_D L_g^2/12) + (w_D L_g/2)\ d_c/2$ (considering the lower girder to be fixed at its two ends and assuming elastic behavior). For practical calculations, the q value of the column may be assumed to be equal to the ratio of M_E to M_D. With q known, the maximum moment that can be applied to the upper end of the column may be estimated from Fig. 11. The same procedure can also be used to determine the maximum resisting moment at the lower end of the upper column. The condition to be satisfied in the design is that the sum of the two resisting moments should be equal to or greater than M_D.

Next, consider the design of the two interior columns for the loading pattern shown in Fig. 10. The removal of the live loads on the alternate girders reduces the maximum moments that may be transmitted to the three interior joints from those girders. This reduction causes a change (in magnitude and direction) of the net unbalanced moments acting at these joints. Because of this change, both columns may now be bent in single curvature by end moments acting in the opposite sense. It is known from beam-column studies that the behavior and strength of single curvature columns are significantly different from those of double curvature columns and that special considerations are often necessary in the design.

The procedure for designing these columns can be explained with reference to the girder-and-column system shown in Fig. 12. The left girder is loaded by the factored total load, w_T, and the right girder by the factored dead load, w_D. A maximum moment $M_{OD} = M_{POD} + (w_T L_g/2)\ d_c/2$ is applied to the joint by the left girder. This moment is resisted by the two columns and the right girder which can be considered as a restraining member for the columns. The adequacy of the trial column sizes can be checked by determining the moment-rotation curves of the three members. Before the moment-rotation curves of the columns can be constructed, it is first necessary to know (or to assume) the end moment ratio, q. Figure 13 has been prepared to permit rapid selection of q values. The subscripts l and r are used to refer to the quantities associated with the left and right girders, respectively. After the q value is selected, the moment-rotation curves can be found by entering the charts given in Reference 2. It should be pointed out that the moment-rotation curves of a majority of single curvature columns do not have a horizontal plateau. Unloading usually takes place after reaching the maximum moment. The moment-rotation curves of the two columns are depicted in Figs. 14a and 14b.

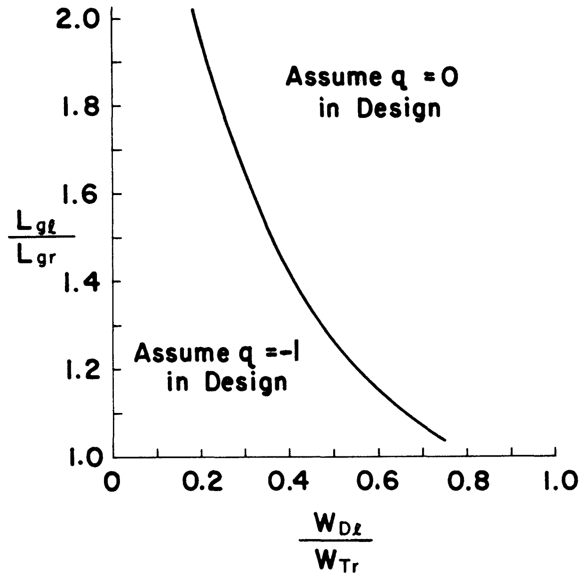


Fig. 13. Selection of end moment ratio for interior columns

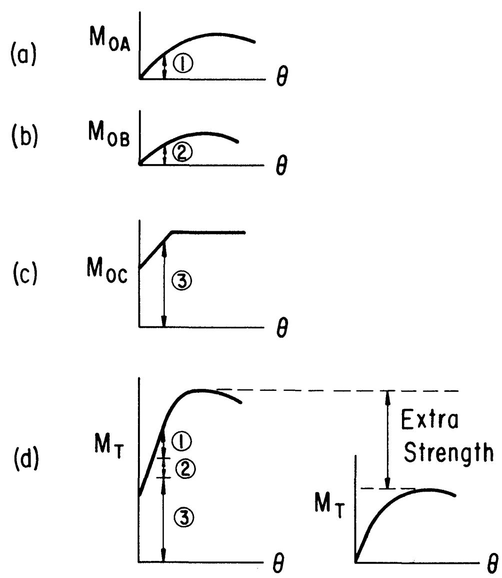


Fig. 14. Determination of total resisting moment of two single curvature columns and a restraining girder

The moment-rotation curve of the right girder can be determined by simple analytical calculations. Under the factored dead load, the moment at the left end of the girder is assumed to be equal to the fixed end moment $(w_D L_g^2/12) + (w_D L_g/2)\ d_c/2$. As the applied moment M_{OD} increases, this moment also increases, but will reach a limit when a plastic hinge forms at this end. The detailed procedure for computing the rotation corresponding to plastic hinge formation can be found in Reference 1. The resulting moment-rotation curve of the girder is shown in Fig. 4c.

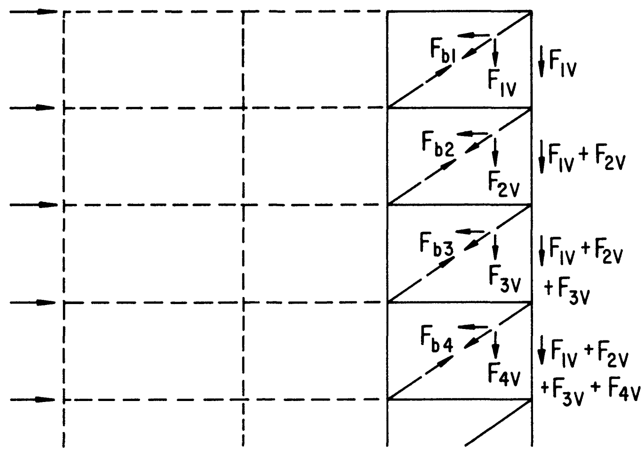


Fig. 15. Axial forces applied to columns in braced bay by lateral loads

Combination of the three moment-rotation curves gives the M_T-θ curve shown in Fig. 14d. The peak of this curve determines the maximum moment that may be resisted by the three members. The trial column sizes are satisfactory if the total resisting moment is greater than the moment M_{OD}. It is noted that in this design procedure the extra strength provided by the girder carrying only the dead load is fully utilized.

Additional Check for Columns in Braced Bay— The application of lateral loads causes tension forces to develop in all the braces. In each story the vertical component of the bracing force introduces an additional compressive force to the leeward column of the braced bay. These forces accumulate from the top story down in the manner shown in Fig. 15. The columns according to the above procedure for gravity loading (L.F. = 1.70) must be checked for their capacities to resist the axial forces under combined gravity and lateral loading (L.F. = 1.30).

The cumulated compressive force in the leeward column of the ith story can be computed from the following formula (see Fig. 1 and Equation (4)):

$$F_{ci} = \frac{1}{L}\left\{h_1\left[H_1 + \frac{\Delta_1}{h_1}W_1\right] + h_2\left[(H_1 + H_2) + \frac{\Delta_2}{h_2}(W_1 + W_2)\right] + h_i\left[\sum_1^i H_i + \frac{\Delta_i}{h_i}\sum_1^i W_i\right]\right\} \quad (9)$$

The relative story deflections, $\Delta_1, \Delta_2, \ldots, \Delta_i$, in this equation can be estimated by the procedure described in the next section or may be conservatively taken as 0.004 times the story heights. To this force F_{ci} must be added the axial force due to gravity load. The total axial force may exceed the axial force present in the column when gravity load alone is applied to the frame. If this occurs, the column size selected previously for the case of gravity loading is no longer satisfactory, and a new column must be tried.

ESTIMATE OF SWAY DEFLECTIONS

The sway deflections of a braced frame can be conveniently estimated by assuming that: (1) braced bay alone resists all the lateral loads and (2) deflections are caused only by the lateral loads. The secondary deflections due to unsymmetrical gravity load and sway moment are often neglected in the estimation. Because of these assumptions, the deflections of the entire frame may be determined by computing only the deflections of the braced bay.

There are three effects which contribute to the deflections of the braced bay: (1) brace elongation, (2) girder shortening, and (3) column elongation and shortening. The following equations will be given for computing the story deflections caused by lateral loads only:

Brace Elongation:

$$\frac{\Delta_i}{h_i} = \frac{L_{bi}^3}{A_{bi}Eh_iL^2}\sum_1^i H_i \tag{10}$$

Girder Shortening:

$$\frac{\Delta_i}{h_i} = \frac{L}{A_{gi}Eh_i}\sum_1^i H_i \tag{11}$$

where A_{gi} is the area of the girder.

Column Shortening and Elongation:

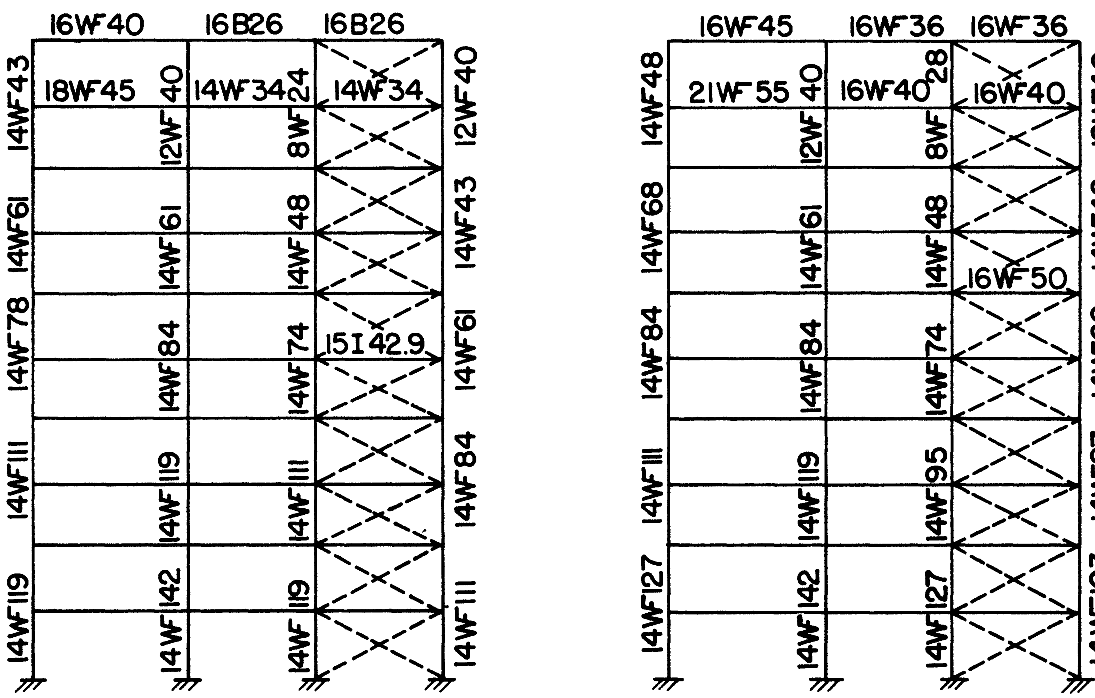
$$\frac{\Delta_i}{h_i} = \frac{2F_{ci}h_i}{A_{ci}EL} \tag{12}$$

where A_{ci} is the area of the column and F_{ci} the axial force. F_{ci} can be computed from Equation (10) by neglecting the terms involving ΣW_i (which represents the effect of sway moment). The ratio Δ_i/h_i computed from Equation (12) can also be interpreted as the story rotation caused by the axial deformations of the columns within the ith story. The lowest story will have no rotation, and the rotation of the higher stories will consist of the sum of the rotations in the stories below.

These equations show that the deflections are linearly proportional to the lateral loads. They can be used to estimate both the working load and factored load deflections.

DESIGN EXAMPLES

The method presented in this paper has been applied to the design of three example frames. The frames are a three-story two-bay, a ten-story three-bay, and a twenty-four story three-bay frame. The dimensions and the loads assumed in the design are described in detail in Reference 1. Figure 16 shows the members selected for the ten-story frame by both the plastic method and the allowable-stress method. A comparison of the two designs shows that the column sizes selected by the two methods are almost the same and that most of the saving comes from the girders. Figure 7 gives comparisons of the girder weight, column weight and total weight of the two designs. A saving of 3.2 tons (8 percent) is achieved by the plastic design.



PLASTIC DESIGN ALLOWABLE-STRESS DESIGN

Fig. 16. Member sizes of a ten-story frame designed by plastic and allowable-stress methods

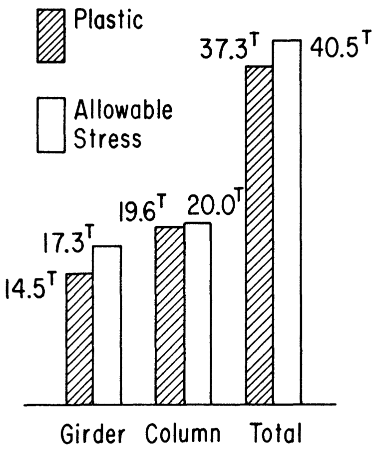


Fig. 17. Weight comparison for a ten-story frame

SUMMARY

1. As a result of extensive research on steel frames, a method is now available for designing braced multi-story frames on the basis of their plastic strength.

2. Limited experience has indicated that the method leads to more economical and rational designs for routine multi-story frames.

3. The validity of the method has been confirmed by laboratory tests on large scale frames.[4]

4. The basic concepts presented in the paper can be applied to designs using both structural carbon steels and high strength steels. Special considerations with regard to the use of high strength steels in plastic design are described in Reference 5.

A plastic method has also been developed for unbraced multi-story frames using a subassemblage concept.[6] Further work on unbraced frames is currently in progress.

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