An Assessment of *K* Factor Formulas

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INTRODUCTION

Current structural design practices recognize that the maximum strength of frames and the maximum strength of component members are interdependent, but it is not practical to take this interdependence into account rigorously.[1] This reasoning lends support to the Structural Stability Research Council (SSRC) technical memorandum which states that *"in design practice, the two aspects, stability of individual members and elements of the structure and stability of the frame system as a whole, be considered independently."* However, it is admitted that *"difficulties are encountered in complex frameworks when attempting to compensate automatically in column design for the instability of the entire frame."* Considerable attention has been paid in the literature to the study of different types of frame elements such as compression members, beams, bracing system and connections and also their effects on the stability of the frame and as a result, several methods have been proposed for evaluating the frame strength. However, the effective length concept for evaluating the frame strength is the most popular method for estimating the interaction effects of a framed member on the total frame stability, and it is recommended by almost all the current specifications.[2–4]

Multiple column curves which are based on results obtained from theoretical and experimental studies of several practical columns have been used traditionally by designers for the design of isolated columns. These curves account for the influence of residual stresses, cross-sectional shapes and imperfections on column strength. The effective length concept can be considered to relate these curves to framed columns for which the amount of rotational and translational restraint provided at the ends by other members of the frame cannot be assessed accurately by simple means. According to this concept, the strength of a framed compression member of length L is equated to an equivalent pin-ended member of length KL, subject to axial load only, by means of K factors. The effective length (K) concept is considered to be an essential part of many analysis procedures and it can handle several cases which can occur practically in all structures. The concept is valid for ideal structures, but its implementation involves several assumptions.

There are a number of methods[5–8] suggested for the calculation of K factors and they are based on different assumptions and types of modeling the frame behavior. Alignment charts, for example, have been prepared assuming the buckling of an idealized subassemblage in a frame and that all columns in a story buckle simultaneously (Figure 1). Where the assumptions are violated, the use of alignment charts results in erroneous values for K factors. LeMessurier's formula,[7] on the other hand, offers some correction to the alignment chart assuming the stronger columns in a story brace the weaker ones in that story. A method proposed recently by Lui[9] appears to be simple and more effective in evaluating the K factor for framed columns in sway frames. K factor values obtained by using the overall system buckling analysis are, however, considered to be the exact solution.[8]

It should be appreciated that the effective length factor is not a constant and it varies depending upon several factors such as structural shape, member geometry and relative dimensions, framing members and load distribution. Prismatic members may have different values of K factors when compared to members with varying cross-section having same

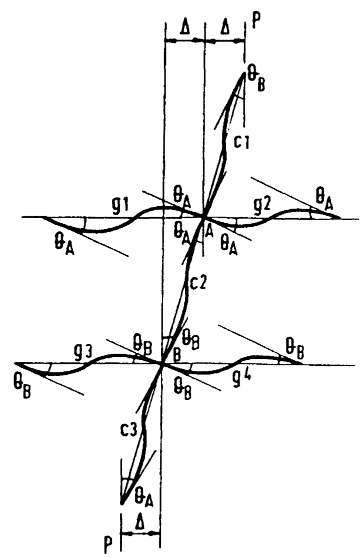

Fig. 1. Subassemblage of an unbraced frame used in the development of AISC alignment chart.

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length and load distribution. Weaker columns or columns with larger loads may exhibit larger values of effective length factors and vice versa. This paper is concerned with a brief review and assessment of some of the methods for K factor determination and, results obtained from the analysis of a number of frames of different parameters by using these methods are presented and compared. Also, the merits in using the Lui's method is highlighted and a brief outline of the method is presented. The general validity and simplicity of the Lui's method is demonstrated. Against the background of this information, the Lui's method is recommended for general use and code adoption.

METHODS TO CALCULATE *K* FACTORS

The development and implementation of effective length factors have undergone several stages. A number of methods have been proposed and Liew et al.[10] have, recently, shown that AISC-LRFD beam-column design approach without using a K factor could lead to inaccurate results. The methods proposed to date predict K values which, when used in frame design, produce results of varying degrees of accuracy depending upon the geometry, size, support conditions and applied loading. This is due to the assumptions and simplifications made in different methods. A brief summary of these methods are given below:

Alignment Chart Method

The AISC-LRFD Specification Commentary recommends the use of the alignment charts to compute K factors. The charts are based on the buckling of the subassemblage shown in Figure 1 and involve several assumptions. The buckling solution for the unbraced assembly results in a transcendental equation of the form

$$\frac{G_A G_B (\pi / K)^2 - 36}{6(G_A + G_B)} - \frac{\pi / K}{\tan(\pi / K)} = 0 \qquad (1)$$

in which G_A and G_B are the column to beam stiffness ratios at the two column ends as

$$G_A = \frac{\sum_A (EI/L)_{column}}{\sum_A (EI/L)_{beam}}$$

$$G_B = \frac{\sum_B (EI/L)_{column}}{\sum_B (EI/L)_{beam}}$$

The solution of Equation 1 is expressed in the form of the sidesway permitted alignment chart, and another chart for the subassemblage corresponding to frames in which sidesway is prevented is also given in the AISC LRFD manual. Modifications to calculate K factors accounting for inelasticity in the columns are also suggested.[6] This method is approximate in the sense that it does not account for the bracing effect that may be provided by stronger columns on weaker columns in a particular story. Duan and Chen[11,12] proposed a procedure in which the far ends of the columns above and below the column being considered are not necessarily continuous but can either be hinged or fixed.

LeMessurier's Method

A more accurate method to compute K factors was given by LeMessurier,[7] who proposed an approach in which the lateral restraining effect between columns can be accounted for. This approach accounts for the fact that all columns in a story buckle simultaneously, that a strong column or a column with low axial force will brace a weak column or a column carrying high axial load, or that some columns lean on others in the same story. The effective length factor for column 'i' of a story in accordance with LeMessurier, can be obtained by using the expression

$$K_i^2 = \frac{\pi^2 EI_i}{P_{ui} L_i^2}\left[\frac{\Sigma P_u + \Sigma C_L P_u}{\Sigma P_L}\right] \qquad (2)$$

where

EI_i = flexural rigidity of column 'i'
L_i = actual height of column 'i'
P_{ui} = required axial compressive strength for i-th rigid column
ΣP_u = required axial compressive strength of all columns in a story
$P_L = \dfrac{\beta EI}{L^2}$
$\beta = \dfrac{6(G_A + G_B) + 36}{2(G_A + G_B) + G_A G_B + 3}$
$C_L = \left(\beta \dfrac{K^2}{\pi^2} - 1\right)$
K = K factor obtained from the sidesway permitted alignment chart.

Equation 2 accounts directly for leaner columns sized for gravity loads only. A conservative and simple design approximation using a modified elastic effective length factor K given by

$$K_i^2 = \frac{I_i}{P_{ui}} \frac{\Sigma P_u}{I / \Sigma K^2}, \; P_{ui} > 0 \qquad (3)$$

is suggested in the revised AISC LRFD Manual.

Lui's Method

A simple and elegant method which accounts for both member instability and frame instability in the calculation of effective length factors was proposed recently by Lui.[9] Member instability, referred to as the P-δ effect is considered in terms of stability functions[13] which are simplified to a great extent by using a Taylor series expansion. Frame instability, referred to as the P-Δ effect, is accounted for by the use of a story stiffness concept. The two effects are explicitly combined into one formula, for which K_i factor for a member i in a frame can be determined as

$$K_i^2 = \frac{\pi^2 EI_i}{P_i L_i^2} \Sigma \frac{P}{L} \left(\frac{1}{5\,\Sigma\,\eta} + \frac{\Delta_{oh}}{\Sigma\,H} \right) \tag{4}$$

where

P_i = compressive axial force in member i

$\Sigma \frac{P}{L}$ = sum of the axial force to length ratio of all members in a story

ΣH = sum of the story lateral forces at and above the story under consideration

Δ_{oh} = inter-story deflection i.e. relative displacement between adjacent stories

η = $\frac{(3 + 4.8m + 4.2m^2)EI}{L^3}$

m = M_A / M_B

M_A, M_B = member end moments with $M_A < M_B$

$\Sigma\eta$ = sum of η of all members in the story being considered.

System Buckling Method

The most accurate of all the methods of calculating effective length is to use a system buckling analysis. In this method, the K factor is found by equating the axial force in a member at the incipient buckling of the frame, P_{cr}, to the buckling load associated with an effective length KL. K factor can be obtained accurately for non-rectangular and irregular frames and structures with different types of elements by using this method. Even inelasticity can be accounted for in this method. For detailed discussion on this, reference may be made to Liew et al.[10]

ASSESSMENT OF THE METHODS

The concept of effective length has been proposed to simplify the process of incorporating the frame action into the design interaction formulas. The method used to obtain such K factors should be simple for design office use and free from elaborate mathematical computations. It should be explicit and versatile, accounting for the most possible variations that may occur in any framed structure, and applicable to a variety of conditions. Values of K factors calculated must be sufficiently accurate for application to design calculations. The methods discussed in the previous section will be analyzed now in view of these considerations.

Alignment Chart Method

The alignment chart method is the most widely used since it provides a direct means to obtain K factors. It should, however, be appreciated that it involves a number of simplifications or assumptions which are not realistic and results obtained are, therefore, inaccurate when these assumptions are violated. For example, the assumption that all columns in a story buckle simultaneously cannot be satisfied because in any practical frame it is unlikely to achieve the same stiffness parameter $L\sqrt{P/EI}$ for all columns in a particular story. Columns may vary in geometry or in dimensions and, if they are the same, axial loading may be distributed unevenly between columns. More active research on flexible connections is in progress and as a result, increased use of PR construction with semi-rigid connections could violate the condition that all joints are rigid. K factors obtained by this method are not influenced by members in the adjacent bay. Moreover, the accuracy of the alignment charts depends essentially on the size of the charts and on the readers' judgement.

LeMessurier's method

This method accounts accurately for the fact that all the rigid columns in a story participate in any sidesway buckling mode, and that the stronger column (smaller $L\sqrt{P/EI}$ ratio) braces the weaker column (larger $L\sqrt{P/EI}$) until sidesway buckling occurs. The method accounts directly for the presence of gravity or leaner columns in the story. LeMessurier has also outlined an iterative procedure for calculating inelastic K factors.

There are several deficiencies in the alignment chart solutions and they have been adequately addressed by several researcher[9] Even the methods such as LeMessurier's method which were proposed to rectify certain inadequacies require the use of alignment charts that may not be convenient for computer implementation.

Lui's Method

The formula given by Lui's method is simple and needs only a first-order frame analysis. Also, no special charts or iterative procedures are required. It is more amenable for computer based design and it takes into account explicitly the member instability (P-δ) effects and frame instability (P-Δ) effects. Also, it has been shown that the K factor can be predicted by this formula with sufficient accuracy for columns in unbraced frames with unequal distribution of lateral stiffness and gravity loads, and for frames with leaner columns.[9]

The first term within the bracket on the right side of Lui's equation (Equation 4), ($\frac{1}{5}\Sigma\eta$), represents member instability effect, while the second term ($\Delta_{oh} / \Sigma H$) accounts for the frame instability effect. The first step in the procedure is to carry out a first-order elastic analysis of the frame in question to determine the horizontal deflection at each story level. The only loads applied on the frame are the lateral loads calculated as a fraction, say 0.1 percent, of the applied story gravity load for each of the stories. In practice, any value can be chosen since the quantities $\Delta_{oh} / \Sigma H$ and M_A / M_B required in Equation

4 are not affected by the value of lateral load used. In the first-order elastic analysis, all quantities vary linearly with the applied lateral load and so the ratio of the quantities remains unaffected.

The second step in the procedure is to calculate the values $\Delta_{oh} / \Sigma H$ for each of the stories. In the third step, the values of m, η, P/L for each of the columns in a particular story are evaluated. Equation 4 is then used to obtain the value of K for each rigid column in that story. The final step is repeated for each of the stories. All calculations can be carried out conveniently in a tabular form for each story.

An Illustrative Example

The following example of a frame with a leaner column illustrates the computation of K factors by using the four methods described above. The frame, which was considered by Lui,[9] is shown in Figure 2(a). The loading case (A) in which a gravity load of P is applied on each column is taken for illustration. The K factor for the right column AB is evaluated as follows:

Alignment Chart Method

$$G_A = \infty;\ G_B = 2\,\frac{\Sigma\,(EI/L)_{column}}{\Sigma\,(EI/L)_{beam}} = 2.0$$

From the alignment chart with

$$G_A = \infty;\ G_B = 2.0$$

We have $K = 2.6$.

Le Messurier Method

For this frame, since only Column AB provides stability to the system, LeMessurier's Equation 2 reduces to[7]

$$K_{AB}^2 = \frac{\pi^2}{P_{AB}}\left[\frac{\Sigma\,P + (C_L P)_{AB}}{\beta_{AB}}\right] \tag{5}$$

From the expressions for $(C_L P)_{AB}$ and β_{AB} as given in Equation 2 with $G_A = \infty$, $G_B = 2.0$, and $K = 2.6$, one obtains these values, respectively, as

$$\beta_{AB} = \frac{6(G_A + G_B) + 36}{2(G_A + G_B) + G_A G_B + 3} = \frac{6}{2+2} = 1.5$$

$$(C_L P)_{AB} = \left(\beta_{AB}\frac{K^2}{\pi^2} - 1\right)P = \left[1.5\left(\frac{2.6}{\pi}\right)^2 - 1\right]P = 0.0275P$$

$$P_{AB} = P,\quad \Sigma P = 2P$$

Substituting in Equation 5, we get

$$K_{AB}^2 = \frac{\pi^2}{P}\left[\frac{2P + 0.0275P}{1.5}\right] = 13.34$$

For the right column AB, we have

Column	I (in.^{4})	L (in.)	P (kips)	P/L	m	η
AB	100	144	50	0.347	0	$\frac{3EI}{L^3}$
CD	100	144	50	0.347	0	$\frac{2.4EI}{L^3}$
Σ			100	0.694		$\frac{5.4EI}{L^3}$

$K_{AB} = 3.65$

Lui's Method

In the first step, a small lateral load equal to 1.0 percent of the gravity loads ($2P = 100$ kips) viz. 1.0 kip is applied as shown in Figure 2(b) in order to initiate sway. In the second step, the corresponding sway deflection, Δ_{oh} from the first-order analysis is found to be 0.681 in. In the third step, the remaining calculations to determine the K factor for the right column AB can be carried out as shown in the table above. It should be noted that η for the leaner column CD is $2.4EI/L^3$.[3] This is because the column CD buckles in a single curvature mode and m for that column is equal to -1. The value η for the column AB is, however, given by $3EI/L^3$ corresponding to $m = 0$ and using Equation 4 and the above table, we have

$$K^2 = \frac{\pi^2 EI_i}{P_i L_i^2}\,\Sigma\,\frac{P}{L}\left(\frac{1}{5\,\Sigma\,\eta} + \frac{\Delta_{oh}}{\Sigma\,H}\right)$$

$$= \frac{\pi^2(29000)(100)}{50(144)^2}\left[0.694\left(\frac{1}{5\times 5.4} + \frac{0.681}{1}\right)\right] = 13.75$$

From which we obtain $K_{AB} = 3.71$.

System Buckling Method

There are computer program[14] available to carry out elastic buckling analysis of frames. These programs can be used to determine the K factor of compression members in the frame. For this particular example the K value for the right column AB was obtained by Lui[9] as 3.69.

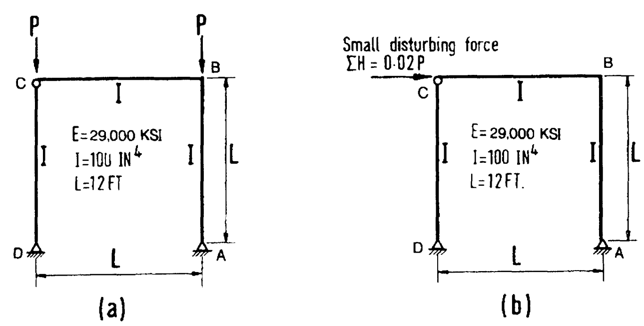

Fig. 2. An illustrative example of a leaned-column frame.

Some General Remarks on the Use of *K*-Factor

The *K* factor concept as used in the design of framed columns has been questioned by Cheong-Siat-Moy,[15] who has demonstrated that the concept can give rise to peculiarities that are in direct conflict with the common sense approach to design. He recommends that anomalies due to the *K* factor can be eliminated by employing column interaction equations that do not make use of such factors. Different schools of thought exist regarding the use of *K* factors. A recent study by Liew et al,[10] however, has shown that the LRFD beam-column interaction equations without the use of a *K* factor will lead to unconservative results. Several examples of frame design have been tested with and without the use of *K* factors and the results compared with those obtained from "exact" design. It has been shown that the *K* factor is probably necessary to obtain an accurate fit to results from refined analyses of simple structural systems for all possible combinations of axial force and end moment.

Easy accessibility to computers of reasonably large size by designers has enabled the use of computer based designs in a direct manner. The AISC-LRFD Specification and similar codes such as the Canadian Limit State Design Specification and the Australian Specification permit the use of second-order elastic analysis of steel frames. Even though there are several approximate methods that are available to calculate effective length factors and amplification factors, they are rather complex and fraught with several simplifying assumptions. There is increasing awareness of the capabilities of computer-based analysis and design amongst designers and more computer programs are being developed for a comprehensive second-order analyses that account for both P-δ and P-Δ effects.[16,17] Also, basic theory for second-order inelastic analysis is well established and documented.[18–21] Efforts in the area of providing the technology and the base of understanding for direct use of second-order inelastic analysis in frame design have already been initiated by at least two groups: AISC—Technical Committee 17 and SSRC—Task Group 29.[22] Good progress has been made, but much more remains to be done.[23]

NUMERICAL RESULTS FROM FRAME ANALYSES

Examples of unbraced frames having uneven distribution of geometry such as length, flexural rigidity and loading were considered for the analyses in order to study the validity of various methods to calculate *K* factors. The alignment chart method, LeMessurier's method, Lui's method and the system buckling method[14] were used in the analyses of simple to large frames having different parameters. Values of *K* factors thus obtained are presented and the validity of the methods assessed.

Single Story Frames

These frames were considered by Liew et al.[10] who computed *K* factors by using the alignment chart method, LeMessurier's method and the system buckling method. Results obtained by Liew et al. are reproduced for comparison in Figures 3–5. The frame shown in Figure 3 is fixed at the column base and symmetrical in geometry, but subjected to an unequal distribution of loading, P and αP, on the columns. The unevenness of loading is varied by changing the value of α from 0.0 to 4.0. *K* factors for the right column, K_{AB}, have been obtained for different values of G at joint B for each of the α values. The variation of K_{AB} with G at joint B is plotted for these values of α. Similar results are plotted in Figure 4 for the same frame given in Figure 3 but with different support conditions at the column base, which is pinned in this case. In Figures 3 and 4, values of K_{AB} obtained in the current investigation by using the Lui's method are superimposed.

Figure 5 shows the results for a frame with unequal length which is represented by γ, the ratio of column lengths,

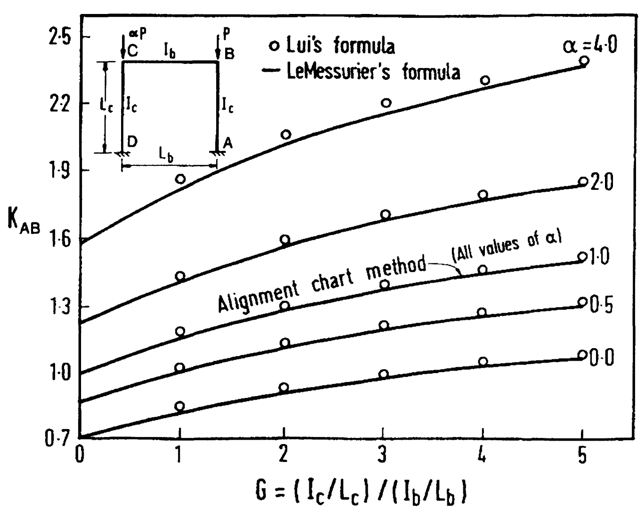

Fig. 3. Variation of K factors with different axial load distribution for portal frame with fixed base.

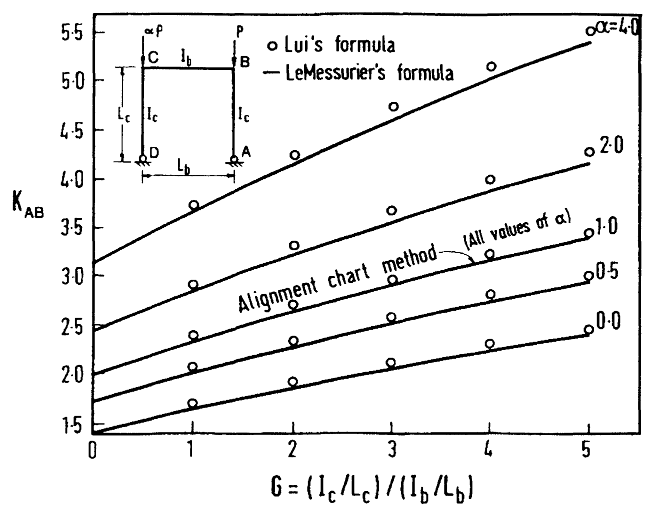

Fig. 4. Variation of K factors with different axial load distribution for portal frames with hinged base.

L_{CD} / L_{AB}. Effective length factors for the left column and right column are plotted as a function of γ, the results obtained by Liew[16] using LeMessurier's method and the system buckling method are reproduced in the figure. Values of K factors obtained in the present study by using Lui's method are also presented in Figure 5 for comparison.

Liew et al.[10] have reported that LeMessurier's method and the system buckling method give nearly identical results for K factors as shown in Figures 3–5. Also, when both columns in the frame are of the same length and carry the same load, the K factors obtained from the alignment chart coincide with those calculated by using the other methods. However, the alignment chart method is not applicable when α is not equal to 1.0. This is due to the fact that the alignment chart is based on the explicit assumption that all columns in the structural system have the same stiffness parameter ($L\sqrt{P/EI}$) at incipient buckling. As shown in Figures 3 and 4, the K factor values predicted by Lui's method lie very close to those obtained by other methods in all cases. Considering the simplicity of Lui's method in dealing with columns with unequal distribution of load over a wide range of G values, one can appreciate the efficiency and applicability of the method. It is also clear from the results shown in Figure 5 that Lui's method predicts the K values with good accuracy for frames having unequal length with γ not exceeding 2.0. When γ exceeds 2, the method underestimates the K values to the extent of about 20 percent; this γ value, however, lies outside the practical range.

The results for the frame in Figure 2 which were calculated in the previous section show the accuracy of Lui's method compared to LeMessurier's method and the system buckling method. All three methods are found to yield identical results for K factor. The alignment chart, however, underestimates the value and it shows the inability of the method in handling such conditions in structures. This example also demonstrates

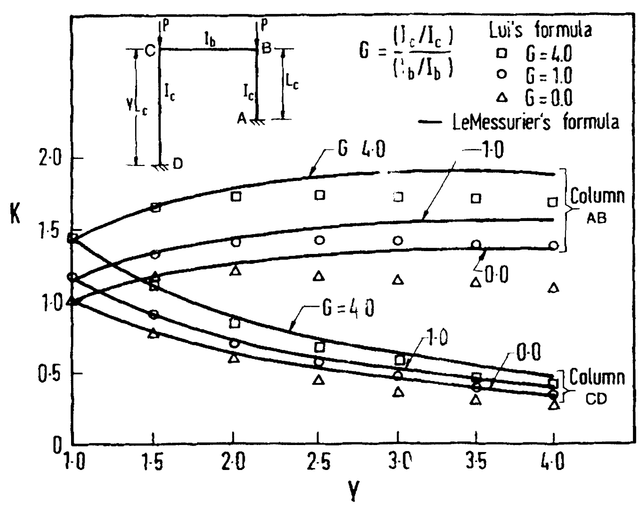

Fig. 5. Variation of K factors for portal frame with unequal column lengths.

the simplicity in calculation and independence of any charts of Lui's method.

Results for a single story, and two-bay frame which was first analyzed by LeMessurier[7] are given in Figure 6. The frame has both unequal distribution of load and column stiffness. Values of K factors for all the three columns calculated using the system buckling method, alignment chart method and Lui's method are listed in the figure along with the K factor values given by LeMessurier.[7] Close observation of these values show that Lui's method of prediction is very close to the system buckling and LeMessurier's methods, which are considered to be accurate. The predictions by the alignment chart in the case of exterior columns underestimate the K values to the extent of 34 percent compared to the exact solution, i.e., the system buckling analysis. The predictions by Lui's method, on the other hand, are exact and the advantage of using this method is apparent. The method is simple and yet accurate values for K could be obtained despite the unequal distribution of load and column stiffness.

Large Frames

Results presented so far, with respect of single story frames, show that most of the available methods could predict identical values for K factors except for frames with large values of γ, the column length ratio. Analyses were therefore carried out on larger frames in order to investigate the validity of these methods to predict K factors of columns in multi-story frames. One single-bay, three-story frame and three two-bay, three story frames were considered for the analyses. All the frames were analyzed using the four methods, i.e., system buckling method, Lui's method, the alignment chart method and LeMessurier's method. Details of the frames and their respective results are presented in Figures 7–10. Finally, a ten-story frame was also investigated to obtain the K factors of column members. The details of this frame and the corresponding results are shown in Figure 11. In all these cases, K factors of columns determined by using the four methods are presented for comparison.

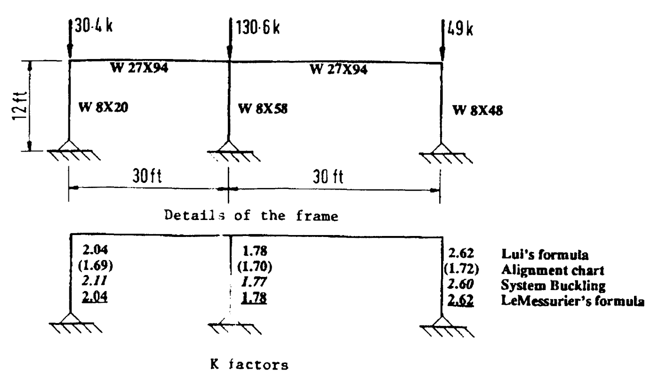

Fig. 6. Comparison of K factors for a two-bay single-story frame with uneven column loads.

All the frames considered in Figures 7–11 are typical of those that occur in practice and they are symmetrical in both geometry and loading. The three frames shown in Figures 8–10 differ in loading type and distribution of column sizes. Close observation of these results presented for all the frames show reasonable agreement between each of the K factor values predicted by the four methods considered, in most cases. Generally, the variation between the values is less than 10 percent. K factor values obtained by using Lui's formula are almost identical to those predicted by LeMessurier's formula. Values predicted by the system buckling method, however, differ by a larger margin in some cases. This is because of the fact that the criteria of buckling adopted in this case is different from that of the other methods. Buckling of the entire structural system is considered in this case while the LeMessurier and Lui formulas are based on story buckling concept. Buckling of adjacent members is taken into account in the formulation of alignment charts.

Results of frame analyses reported by Lui[9] and those shown in Figures 3–11 clearly establish the accuracy of the formula in Equation 4. The examples considered include extreme cases of unequal distribution of lateral stiffness, gravity loads, and leaner columns that may occur in practice. In all these cases, we are able to get close agreement between K factor values obtained by using LeMessurier's method, Lui's method and the system buckling method. Where its assumptions are violated, alignment charts could not predict K values accurately. Among the methods considered, Lui's formula appears to fulfil the requirements of a design office use since it is simple and direct without the requirement of any special charts or procedures. It does not involve any elaborate computer analyses and most of the calculations can be carried out using a pocket calculator. Even though K factor values predicted by Lui's formula and LeMessurier's formula are identical in most cases, the simplicity and independence of any chart in the case of Lui's formula make it more desirable for design office use.

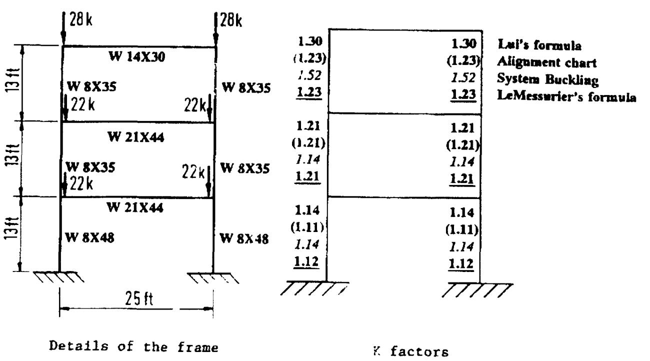

Fig. 7. Comparison of K factors for a single-bay three-story frame with even column loads.

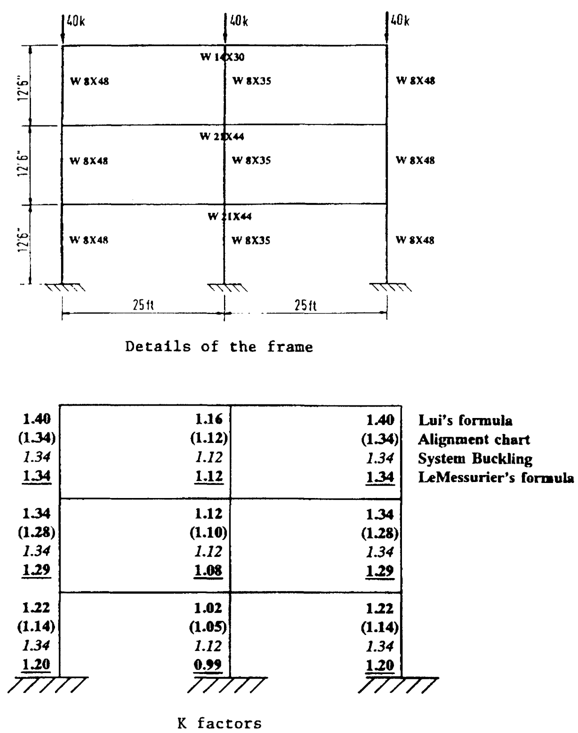

Fig. 8. Comparison of K factors for a two-bay three-story frame with even column loads.

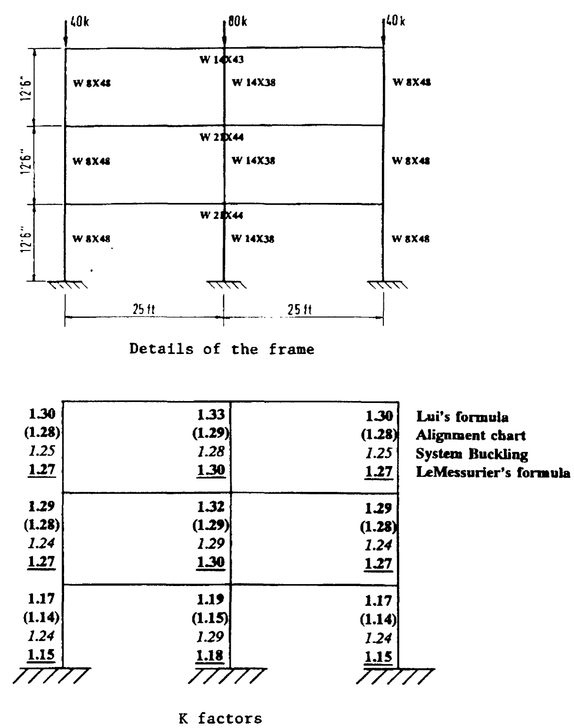

Fig. 9. Comparison of K factors for a two-bay three-story frame with uneven column loads.

CONCLUSIONS

Four different approaches, including the alignment chart, LeMessurier's formula, Lui's formula, and the system buckling method were considered to compute *K* factors of columns in frames of different proportions. All methods except the alignment chart were found to predict nearly identical values of *K* factors for columns in single story frames irrespective of the unequal distribution of column stiffness. In the case of multi-story frames, the predicted values of *K* factors by all four methods were found to agree within a reasonable accuracy. In view of the simplicity in dealing with columns with unequal distribution of gravity loads and lateral stiffness and the independence of any special charts or iterative procedures, it can be concluded that Lui's formula is the most appropriate for use in a design office and it is recommended for general use.

The present methods of allowance for second-order effects in frame design are generally tedious and approximate. Practical advanced inelastic analysis of frames enables engineers to predict accurately all possible failure modes of a system in a direct manner, when it is subjected to a given load combination. Such advanced analysis methods can be expected to gradually replace the conventional methods of frame design, which use the *K*-factor, in the near future.[22]

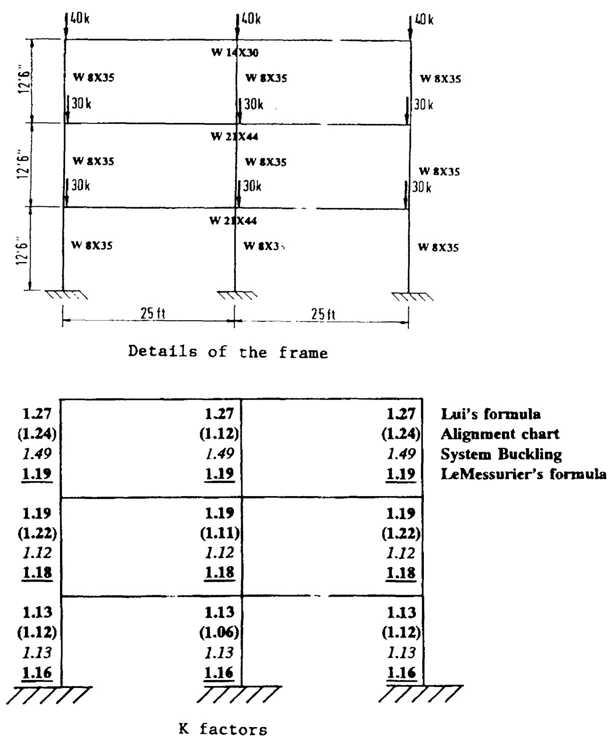

Fig. 10. Comparison of K factors for a two-bay three-story frame with even column loads.

ACKNOWLEDGMENTS

The first author acknowledges the support from the National University of Singapore during his sabbatical leave at Purdue University from May 1992 to October 1992. Useful suggestions at various stages from Professor E. M. Lui and Dr. J. Y. R. Liew are gratefully acknowledged.

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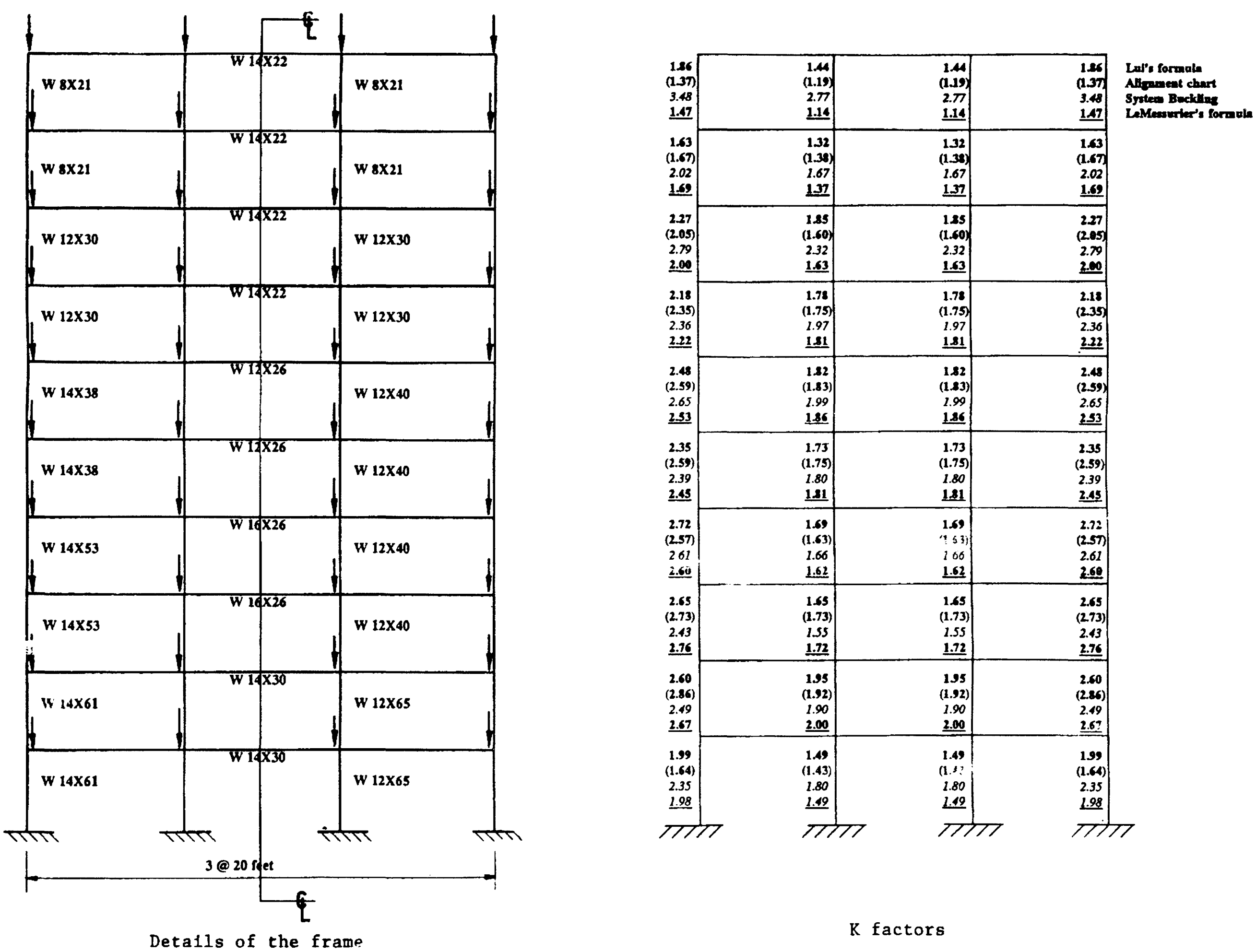

Fig. 11. Comparison of K factors for a ten-story frame with even column loads.