Steel Frame Stability Design

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INTRODUCTION

Many stability evaluation methods have been proposed over the last thirty years. The SSRC[9] lists five specific methods of analyzing steel frames for stability: the effective-length concept; second-order, inelastic analysis; the $P\Delta$ method; the Merchant-Rankine formula; and the moment amplification method. The SSRC recommends evaluating frame stability separately from individual member analysis, while the AISC[1] includes stability evaluation in the individual member selection process.

The AISC[1] requires that second-order effects be considered in the design of frames and that in unbraced frames the effective length factor, K, be not less than unity. The frame stability provisions of the AISC Specifications are based on the effective length concept, using the moment amplification method to account for second-order effects. It should be noted that the AISC does not prohibit the use of methods other than the effective length concept.

The effective length concept typically is implemented through the determination of column effective length factors, K, using an alignment chart. The sidesway uninhibited alignment chart[1] provides K factors for use in evaluating the lateral stability of the frame at the individual member level. The alignment chart is based on the assumption all columns in a story are equally critical and equally add to the lateral resistance of the frame.

A method for evaluating frames in which all members do not equally contribute to a frame's lateral resistance has been proposed by Yura.[10] However, it has been shown[3,4] Yura's method is overconservative in frames in which the stability is provided by columns carrying a small portion of the story gravity load. One reason for this conservatism is the application of inelastic buckling allowable stresses where elastic buckling is clearly applicable.

A method of stability design using a frame stiffness analysis has been proposed by Schilling[7,8] and de Buen.[4] In the frame stiffness approach, stability is verified through a comparison of the gravity loads to the frame's lateral stiffness. The approach has many advantages: it is easy to understand, easy to implement and uses information the designer already should have at hand. It also eliminates the potential overconservatisms of Yura's[10] method.

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The purpose of this paper is to assemble information presented in earlier research in order to propose a complete and practical method of implementing the frame stiffness approach; in addition, a new method of directly calculating correction factors is proposed. Included are discussions on second-order effects, inelastic behavior, and individual member analysis.

ELASTIC BUCKLING OF FRAMES

The frame stiffness approach to stability design is based in the recognition that the first order lateral stiffness of a frame is reduced by two second-order geometric effects,[4,7] designated as $P\Delta$ and $P\delta$. These second-order effects are due to vertical load interaction with lateral story displacement, Δ, as shown in Figures 1a and 1b, and with the displacements due to column curvature between the column's ends, δ, as shown in Figure 1b.

As shown in Figure 1a, leaner columns (those that provide no lateral stiffness to a frame) reduce frame stiffness through the application of an additional lateral force to the frame: the additional force is a function of the vertical load, P, and lateral displacement, Δ. As shown in Figure 1b, the effective stiffnesses of stability columns (those that provide lateral stiffness to a frame) are reduced by the introduction of P-delta generated flexure.

Consider the individual stability column shown in Figure 2. Ignoring shear and axial deformations, the interstory deflection due to the applied lateral load V_c can be calculated as shown by Le Messurier:[6]

$$\theta_a = M_a G_a h / 6EI \tag{1}$$

$$\theta_b = M_b G_b h / 6EI \tag{2}$$

$$V_c h = M_a + M_b \tag{3}$$

$$\theta_a = \frac{1}{EI}\int_{x=0}^{h} M(x)dx = \frac{V_c h^2}{2EI} - \frac{M_a h}{EI} + \theta_b \tag{4}$$

G_a and G_b are as defined in the AISC[1], SSRC[9] and elsewhere. Combining Equations (1), (2), (3) and (4):

$$\frac{M_a}{M_b} = \frac{G_b + 3}{G_a + 3} \tag{5}$$

$$M_b = \alpha V_c h \tag{6}$$

where:

$$\alpha = \frac{G_a + 3}{G_a + G_b + 6} \tag{7}$$

The equations for moment, curvature and deflection are:

$$M(x) = V_c(\alpha h - x) \tag{8}$$

$$\theta_b = \frac{\alpha V_c G_b h^2}{6EI} \tag{9}$$

$$\theta(x) = \left(\frac{1}{EI}\right)\int M(x)dx = \left(\frac{V_c}{EI}\right)\int(\alpha h - x)dx \tag{10}$$

$$\theta(x) = \left(\frac{V_c}{EI}\right)\left(\alpha h x - \frac{x^2}{2} + \frac{\alpha G_b h^2}{6}\right) \tag{11}$$

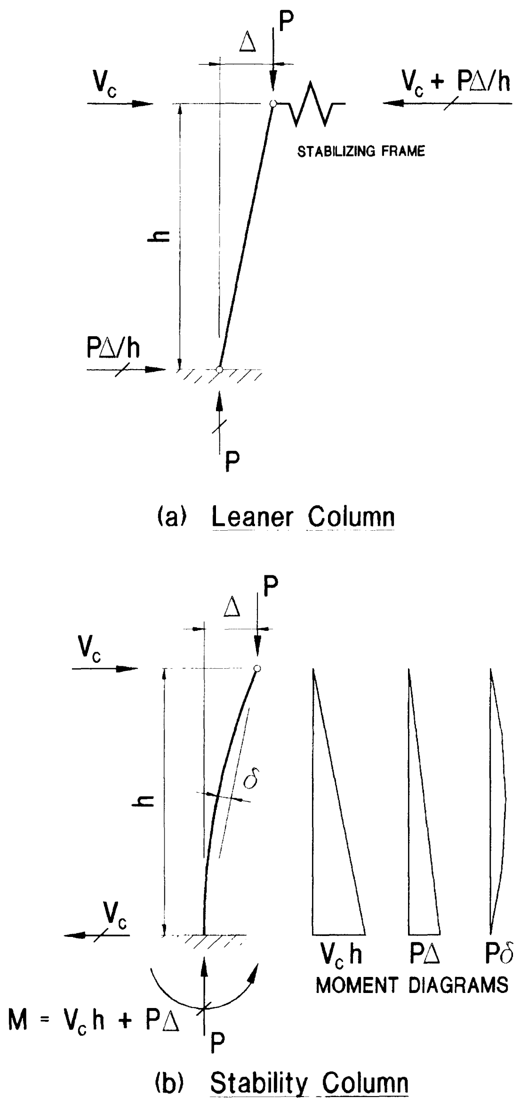


Fig. 1. Second order behavior of columns.

$$\Delta_{V_c}(x) = \int\theta(x)dx = \left(\frac{V_c}{EI}\right)\left(\frac{\alpha h x^2}{2} - \frac{x^3}{6} + \frac{\alpha G_b h^2 x}{6}\right) \tag{12}$$

$$\Delta_{V_c}(h) = \frac{V_c h^3}{\beta EI} \tag{13}$$

Where Δ_V is the lateral deflection induced by the load V_c and:

$$\beta = \frac{6(G_a + G_b) + 36}{2(G_a + G_b) + G_a G_b + 3} = \frac{1}{\left(\frac{\alpha}{2} + \frac{\alpha G_b}{6} - \frac{1}{6}\right)} \tag{14}$$

The lateral deflection caused by the application of a vertical load to the deformed column, as shown in Figure 3a, is calculated as follows: the *P*-delta effects are considered as having two separate components: the first, $P\Delta$, being the application of a shear load $P\Delta / h$, or $V_{P\Delta}$, as shown in Figure 3b; and the second, $P\delta$, being the application of an axial force *P* to the deformed shape as shown in Figure 3c.

Deflection due to shear load $V_{P\Delta}$ is proportional to the lateral load deflection, from Equation (13):

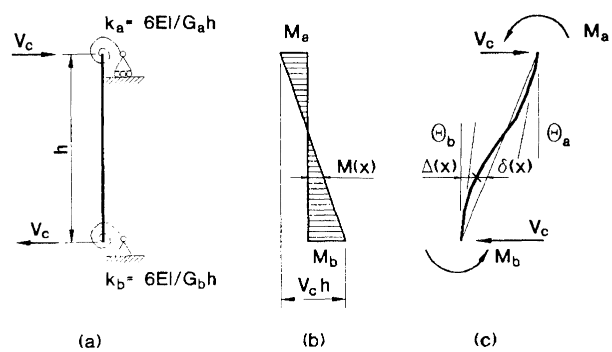


Fig 2. First order deflection of a stability column.

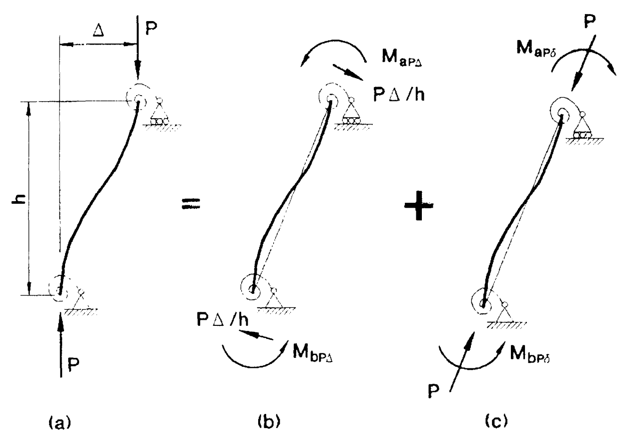


Fig. 3. P-delta effects on a stability column.

$$V_{P\Delta} = \frac{P\Delta_{V_c}}{h} = \left(\frac{P}{h}\right)\left(\frac{V_c h^3}{\beta EI}\right) = \frac{PV_c h^2}{\beta EI} \quad (15)$$

$$\Delta_{P\Delta}(h) = \frac{V_{P\Delta}h^3}{\beta EI} = \frac{PV_c h^5}{(\beta EI)^2} \quad (16)$$

Where $\Delta_{P\Delta}$ is the lateral deflection induced by $V_{P\Delta}$.

Deflection due to $P\delta$ loading (reference Figures 2c and 3c) is a function of the column's end restraint. From Equations (12) and (13):

$$\delta(x) = \Delta_{V_c}(x) - \Delta_{V_c}(h)\left(\frac{x}{h}\right)$$

$$= \left(\frac{V_c}{EI}\right)\left(\frac{\alpha h x^2}{2} - \frac{x^3}{6} + \frac{\alpha G_b h^2 x}{6}\right) - \frac{V_c h^2 x}{\beta EI} \quad (17)$$

$$M_{P\delta}(x) = M_{bP\delta} - P\delta(x) \quad (18)$$

$\delta(x)$ is the deflection of the column from an imaginary chord linking its ends, $M_{P\delta}(x)$ is the internal moment, and $M_{bP\delta}$ is the reaction at end 'b', resulting from the application of an axial load to the deformed shape. Neglecting axial deformation, the total energy, U, stored in the axially loaded system is:

$$U = \frac{1}{2}\int_{x=0}^{h} M_{P\delta}(x)\frac{d\theta}{dx}dx + \frac{1}{2}M_{bP\delta}\theta_{bP\delta} + \frac{1}{2}M_{aP\delta}\theta_{aP\delta} \quad (19a)$$

Where $d\theta / dx = M_{P\delta}(x) / EI$ and $\theta_n = M_n G_n h / 6EI$. By statics, $M_{bp\delta}$ must equal $M_{ap\delta}$ therefore:

$$U = \left(\frac{1}{2EI}\right)\int_{x=0}^{h} (M_{bP\delta} - P\delta(x))^2 dx +$$

$$\left(\frac{M_{bP\delta}^2 h G_b}{12EI}\right) + \left(\frac{M_{bP\delta} h G_a}{12EI}\right) \quad (19b)$$

Reactions $M_{aP\delta}$ and $M_{bP\delta}$ must be such they minimize the total energy in the system, i.e., $\partial U / \partial M_{bP\delta} = 0$. Solving Equation (19b) for $x = h$ and differentiating:

$$M_{bP\delta} = \frac{\mu P V_c h^3}{EI} \quad (20)$$

where:

$$\mu = \frac{\frac{\alpha}{6} + \frac{\alpha G_b}{12} - \frac{1}{2\beta} - \frac{1}{24}}{1 + \frac{(G_a + G_b)}{6}} \quad (21)$$

The deflection due to $P\delta$ can be calculated from Equations (17), (18) and (20):

$$M_{P\delta}(x) = \left(\frac{\mu P V_c h^3}{EI}\right) - \left(\frac{PV_c}{EI}\right)\left(\frac{\alpha h x^2}{2} - \frac{x^3}{6} + \frac{\alpha G_b h^2 x}{6} - \frac{h^2 x}{\beta}\right) \quad (22a)$$

$$M_{P\delta}(x) = \left(\frac{PV_c}{EI}\right)\left(\mu h^3 - \frac{\alpha h x^2}{2} + \frac{x^3}{6} - \frac{\alpha G_b h^2 x}{6} + \frac{h^2 x}{\beta}\right) \quad (22b)$$

$$\theta_{P\delta}(x) = \left(\frac{PV_c}{(EI)^2}\right) \times$$

$$\left(\mu h^3 x - \frac{\alpha h x^3}{6} + \frac{x^4}{24} - \frac{\alpha G_b h^2 x^2}{12} + \frac{h^2 x^2}{2\beta} + \frac{\mu G_b h^4}{6}\right) \quad (23)$$

$$\Delta_{P\delta}(x) = \left(\frac{PV_c}{(EI)^2}\right) \times$$

$$\left(\frac{\mu h^3 x^2}{2} - \frac{\alpha h x^4}{24} + \frac{x^5}{120} - \frac{\alpha G_b h^2 x^3}{12} + \frac{h^2 x^3}{6\beta} + \frac{\mu G_b h^4 x}{6}\right) \quad (24)$$

$$\Delta_{P\delta}(h) = \frac{\gamma P V_c h^5}{(EI)^2} \quad (25)$$

Where $\Delta_{P\delta}(h)$ is the lateral deflection induced by the application of the axial force, P, to the deformed column, and:

$$\gamma = \frac{\mu}{2} - \frac{\alpha}{24} - \frac{\alpha G_b}{36} + \frac{1}{6\beta} + \frac{\mu G_b}{6} + \frac{1}{120} \quad (26)$$

From Equations (16) and (25) the total P-delta deflection would then be:

$$\Delta_{P\text{-}delta} = \Delta_{P\Delta}(h) + \Delta_{P\delta}(h) = \left(\frac{1}{\beta^2} + \gamma\right)\left(\frac{PV_c h^5}{(EI)^2}\right) \quad (27)$$

Express R_{C1} as the first-order lateral stiffness of the column, R_{C2} as the effective second-order lateral stiffness, and CP / h as the loss of stiffness due to vertical loading, where C is a constant for an individual column with given end restraint conditions:

$$R_{c1} = \frac{V_c}{\Delta_{V_c}} \quad (28)$$

$$R_{c2} + \frac{V_c}{\Delta_{V_c} + \Delta_{P\text{-}delta}} \quad (29)$$

$$R_{c2} = R_{c1} - \frac{CP}{h} \quad (30)$$

$$C = \left(\frac{h}{p}\right)(R_{c2} - R_{c1}) = \left(\frac{V_c h}{P}\right)\left(\frac{\Delta_{P\text{-}delta}}{(\Delta_{P\text{-}delta} + \Delta_{V_c})(\Delta_{V_c})}\right) \quad (31)$$

From Equations (13) and (27):

$$C = \left(\frac{V_c h}{P}\right)\left(\frac{\left(\frac{1}{\beta^2} + \gamma\right)\left(\frac{PV_c h^5}{(EI)^2}\right)}{\left[\left(\frac{1}{\beta^2} + \gamma\right)\left(\frac{PV_c h^5}{(EI)^2}\right) + \left(\frac{V_c h^3}{\beta EI}\right)\right]\left(\frac{V_c h^3}{\beta EI}\right)}\right) \quad (32)$$

Simplifying Equation (32):

$$C = \frac{\left(\frac{1}{\beta^2} + \gamma\right)}{\left(\frac{1}{\beta^3} + \frac{\gamma}{\beta}\right)\left(\frac{Ph^2}{EI}\right) + \frac{1}{\beta^2}} = \frac{1 + \beta^2\gamma}{\left(\frac{1}{\beta} + \beta\gamma\right)\left(\frac{Ph^2}{EI}\right) + 1} \quad (33)$$

As shown in Equation 33, the factor C is reduced by increasing axial load and/or story height. However, increasing axial loads also result in incremental increases in deflections, as calculated in P-delta iterations. The increasing deflections directly increase the C factor, as shown in Figure 4. The C factor consistently approaches a value which can be directly calculated using Equation 33 with no axial load, i.e. $P = 0$:

$$C = \frac{1 + \beta^2\gamma}{\left(\frac{1}{\beta} + \beta\gamma\right)\left(\frac{0 \times h^2}{EI}\right) + 1} = 1 + \beta^2\gamma \quad (34)$$

Where:

$$\alpha = \frac{G_a + 3}{G_a + G_b + 6} \quad (7)$$

$$\beta = \frac{1}{\left(\frac{\alpha}{2} + \frac{\alpha G_b}{6} - \frac{1}{6}\right)} \quad (14)$$

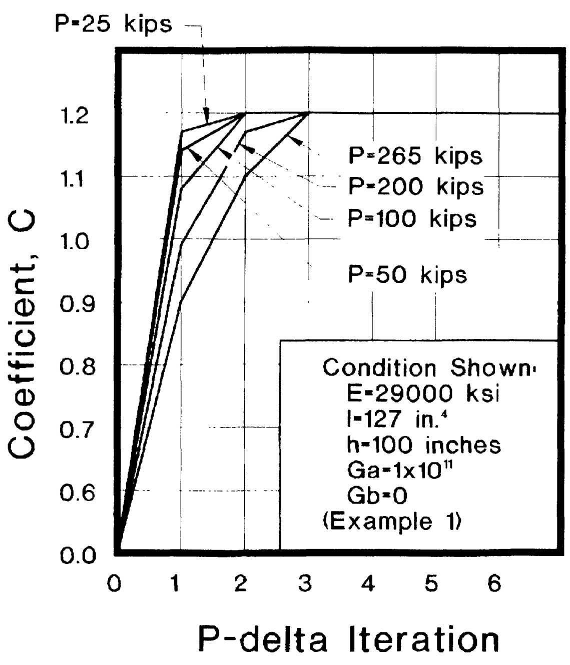


Fig. 4. Effect of P-delta iterations on second order defelctions.

$$\mu = \frac{\frac{\alpha}{6} + \frac{\alpha G_b}{12} - \frac{1}{2\beta} - \frac{1}{24}}{1 + \frac{(G_a + G_b)}{6}} \quad (21)$$

$$\gamma = \frac{\mu}{2} - \frac{\alpha}{24} - \frac{\alpha G_b}{36} + \frac{1}{6\beta} + \frac{\mu G_b}{6} + \frac{1}{120} \quad (26)$$

Le Messurier[6] has given a similar amplification factor, C_L, which he deemed a "clarification factor." C_L varies between 0 and 0.216 and is added to unity to obtain a factor analogous to the C factor proposed herein. Le Messurier has provided approximate and "exact" (based on Euler buckling equations) solutions for C_L. Use of $C = 1 + C_L$, using the "exact" solution, gives solutions that are within 1.5 percent of those calculated using $C = 1 + \beta^2\gamma$. Schilling also discusses the derivation alternatives, buckling equations versus P-delta deflections, for what he called the "correction factor."[7]

Equation 34 gives values for the coefficient C ranging from 1 to 1.2. The use of $C = 1.2$ as a constant has been suggested by Shilling and others.[4,6,7,11] A constant C is reasonable for preliminary designs, but is unnecessarily conservative for computerized structural analyses. Also, blanket use of $C = 1.2$ may significantly impact the design of all the columns in a frame as it affects the approximate second-order amplification factor, as shown in the discussion on individual members.

FRAME BEHAVIOR

Since all columns in a story must equally deflect, the destabilization of a frame must be the sum of the effects on all columns in a story. Expressing R_n as first-order lateral stiffness of a story under consideration, R_2 as effective second-order lateral stiffness, $\Sigma P_L / h$ as the sum of leaner column gravity loads divided by their story heights, and $\Sigma CP_S / h$ as the sum of stability column gravity loads times the individual C factors divided by their story heights; the general form of the second-order lateral stiffness equation for a frame in the elastic range is:

$$R_2 = R_n - \sum\frac{P_L}{h} - \sum\frac{CP_S}{h} \quad (35)$$

Bifurcation occurs when the frame laterally deflects without being subject to lateral loads; it happens when the gravity loads reduce the frame's effective second order lateral stiffness to zero:

$$R_2 = R_n - \left(\sum\frac{P_L}{h}\right)_{cr} - \left(\sum\frac{CP_S}{h}\right)_{cr} = 0 \quad (36)$$

The subscript "cr" denotes critical loading.

In frames subject to lateral loads, gross deflection will occur as the second-order lateral stiffness becomes small and failure results from increasing $P\Delta$ moments.[2]

Expressing R_u as the required stiffness to prevent sidesway buckling for an imposed set of gravity loads:

$$R_2 = R_u - \sum \frac{P_L}{h} - \sum \frac{CP_S}{h} = 0 \tag{37}$$

$$R_u = \sum \frac{P_L}{h} + \sum \frac{CP_S}{h} \tag{38}$$

The SSRC[9] shows many columns still buckle at less than pure elastic buckling loads when axial stress is 40 percent of yield. This reduction in elastic buckling strength dissipates as axial stress is decreased. The AISC[1] imposes an additional factor of safety of $(0.877)^{-1}$, or 1.14, for elastic buckling in order to account for this inelastic reduction in strength. Since the inelastic reduction of elastic buckling loads does not apply to pure elastic buckling, only the stability column term of the frame stiffness requirement need include the additional factor of safety:

$$R_u = \sum \frac{P_L}{h} + 1.14 \sum \frac{CP_S}{h} \tag{39}$$

The requirement for stability would be:

$$\phi R_n \geq R_u \tag{40}$$

Thus it is possible for the designer to show a frame has adequate stiffness to resist sidesway buckling for a given set of gravity loads. The frame stiffness approach allows the designer to take full advantage of live load reductions and the variability of moving loads (e.g., cranes) in the sidesway buckling analysis.

The need for lateral stiffness beyond this stability requirement will be demonstrated in the discussion of individual members.

INELASTIC BUCKLING OF FRAMES

As noted by Yura[10] and others, significant axial stress decreases the lateral stiffness of columns. Schilling[8] proposed the use of an effective moment of inertia, I_{eff}, in the calculation of frame lateral stiffness. The individual member properties of the columns in the frame must be modified in order to account for this inelastic softening:

$$EIS_r \approx E_t I \approx EI_{eff} \tag{41}$$

Where E_t is the tangent modulus and S_r is a stiffness reduction factor.

AISC[1] provides stiffness reduction factors, $F_{cr,\text{ inelastic}} / F_{cr,\text{ elastic}}$, that can be used to adjust the modulus of elasticity of columns for the frame analysis. The axial stress used for the determination of stiffness reduction is based on the ultimate axial load, P_u, divided by the cross sectional area times the reduction factor, ϕA.[5] The stiffness reduction factor is calculated as follows:

$$S_r = \frac{(F_{cr})_{\text{inelastic}}}{(F_{cr})_{\text{elastic}}} \tag{42}$$

For $\dfrac{P_u}{\phi A} \geq \dfrac{0.877}{2.25} F_y$:

$$\lambda_{S_r} = \left(\frac{\log\left(\dfrac{P_u}{\phi A F_y}\right)}{\log(0.658)} \right)^{0.5} \tag{43}$$

$$S_r = \frac{\lambda_{S_r}^2 (0.658)^{\lambda_{S_r}^2}}{0.877} \tag{44}$$

For $\dfrac{P_u}{\phi A} < \dfrac{0.877}{2.25} F_y$:

$$S_r = 1 \tag{45}$$

The stiffness reduction curves using Equations 44 and 45 are shown in Figure 5. Structural analyses of frames with columns stressed in the inelastic range can become arduous as there will be different lateral stiffnesses for each load combination. Fortunately, however, most frames subject to lateral loads are not axially stressed into the inelastic range, unless lateral stiffness is provided by bracing, in which case the stiffness reduction of the columns is moot.

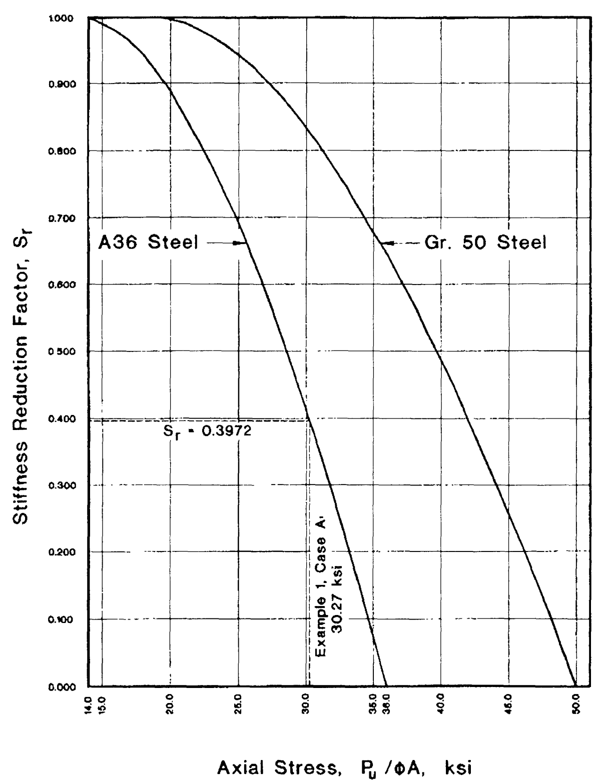


Fig. 5. Stiffness reduction curves.

ANALYSIS OF INDIVIDUAL MEMBERS

Members should be analyzed following the procedures given in the AISC *Specification*.[1] Following SSRC[9] recommendations, the effective length factor for individual columns need not exceed unity, once the stability of the story has been verified using Equation 40. The story has been demonstrated to be stable, therefore the columns need to be evaluated for buckling between story levels. The use of an effective length factor of less than unity in an unbraced frame is currently in violation of the AISC *Specification*.[1] The required flexural strength, M_u, should be in accordance with AISC Equation H1-2 restated here as Equation 46:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{46}$$

Where M_{nt} is the required member flexural strength for gravity loads assuming no lateral translation, and M_{lt} is the required member flexural strength as a result of lateral translation. The second-order moment amplification factor, B_2, is the ratio of the second-order deflection to the first-order deflection:[6]

$$B_2 = \frac{\Delta_2}{\Delta_1} \tag{47}$$

Where Δ_1 is the first order lateral deflection, and Δ_2 is the second order lateral deflection:

$$\Delta_2 = \frac{V}{R_2} = \frac{V}{R_n - R_u} \tag{48}$$

Combining Equations 47 and 48 gives:

$$B_2 = \frac{\Delta_2}{\Delta_1} = \frac{\dfrac{V}{R_n - R_u}}{\dfrac{V}{R_n}} = \frac{R_n}{R_n - R_u} = \frac{1}{1 - (R_u / R_n)} \tag{49}$$

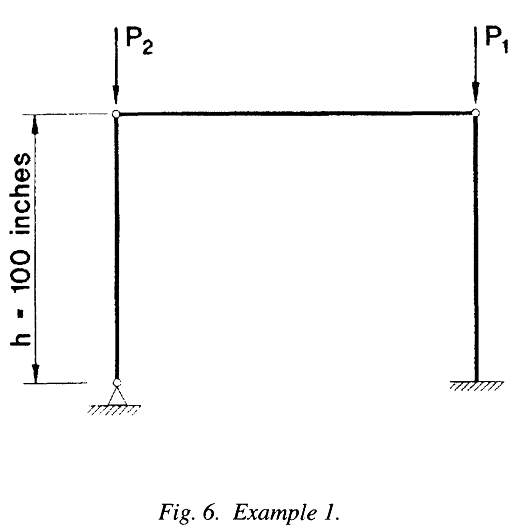


Fig. 6. Example 1.

Determination of B_2 in this manner is consistent with the proposed method of Kanchanalai and Lu.[11]

It is at this level of the analysis where overestimation of the required frame stiffness, R_u, can significantly impact the design of all the members in the frame. Such an overestimation can result from the use of approximate C factors and the neglect of moving load effects and live load reductions.

The moment amplification method addresses only second-order effects in rigid frame columns, whereas the stability equations contained herein are applicable to both braced and unbraced frames. As noted in the AISC *Commentary*,[1] the designer should consider the amplification effects of gravity loads on braced frame. If the required lateral stiffness, R_u, exceeds 10 percent of the provided lateral stiffness, R_n, then lateral load amplification effects cannot be considered negligible and should not be ignored.

The effect of Equation 49 is that the designer cannot provide a marginally stable ($R_u \rightarrow R_n$) laterally loaded frame. As the actual frame stiffness approaches the stiffness required, the deflections and bending moments are greatly amplified. The frame stability problem becomes a matter of drift control.

As shown in Example 1, the individual member analyses for stability columns may not govern. The size of these columns often will be driven by Equation 40, and the stress ratios for these columns may be low.

While the stress ratio of the typical stability column may not have much meaning, the benefit to analyzing members in this fashion is a more accurate analysis of the frame as a whole, and of columns that are either subject to far greater loads or are more slender than their peers.

EXAMPLE 1

Consider the statically determinate frame consisting of a cantilevered column, a leaner column and a simple beam; the frame is loaded with two point loads, P_1 and P_2 as shown in Figure 6, for a total factored load of 265 kips. Considering strong-axis buckling only (in the plane of the frame), verify the adequacy of the cantilevered column for the three frame cases:

Case A represents the loading condition: P_1 = 265 kips, P_2 = 0 kips. The cantilevered column is a W8×35 (A = 10.3 in.^{2}, I_x = 127 in.^{4}, ASTM A36 steel.) Checking stability criteria:

$$\frac{P_u}{\phi A F_y} = \frac{265 \text{ kips}}{0.85 \times 10.3 \text{ in.}^2 \times 36 \text{ ksi}} = 0.8408 \tag{50}$$

$$\lambda_{S_r} = \left(\frac{\log(0.8408)}{\log(0.658)} \right)^{0.5} = 0.6437 \tag{51}$$

$$S_r = \frac{\lambda_{S_r}^2 (0.658)^{\lambda_{S_r}^2}}{0.877} = \frac{0.6437^2 \times 0.658^{0.6437^2}}{0.877} = 0.3972 \tag{52}$$

$$R_n = \frac{3EIS_r}{h^3} = \frac{3 \times 29000 \text{ ksi} \times 127 \text{ in.}^2 \times 0.3972}{(100 \text{ in.})^3}$$

$$= 4.39 \text{ kips/in.} \qquad (53)$$

$$\alpha = \frac{\infty + 3}{\infty + 0 + 6} = 1 \qquad (54)$$

$$\beta = \frac{1}{\left(\frac{1}{2} + \frac{1 \times 0}{6} - \frac{1}{6}\right)} = 3 \qquad (55)$$

$$\mu = \frac{\frac{1}{6} + \frac{1 \times 0}{12} - \frac{1}{2 \times 3} - \frac{1}{24}}{1 + \frac{\infty + 0}{6}} = 0 \qquad (56)$$

$$\gamma = \frac{0}{2} - \frac{1}{24} - \frac{1 \times 0}{36} + \frac{1}{3 \times 6} + \frac{0 \times 0}{6} + \frac{1}{120} = 0.02222 \qquad (57)$$

$$C = 1 + 0.02222 \times 3^2 = 1.2 \qquad (58)$$

$$R_u = \sum \frac{P_L}{h} + \sum \frac{1.14CP_S}{h} = \frac{1.14 \times 1.2 \times 265 \text{ kips}}{100 \text{ in.}}$$

$$= 3.63 \text{ kips/in.} \qquad (59)$$

$$\phi R_n = 0.85 \times 4.39 \text{ kips / in.}$$

$$= 3.73 \text{ kips / in.} > 3.63 \text{ kips / in.} \quad \textbf{o.k.} \qquad (60)$$

Frame lateral stability has been verified, check the column using an effective length factor of one:

$$\lambda_c = \left(\frac{Kh}{r\pi}\right)\sqrt{\frac{F_y}{E}}$$

$$= \frac{1.0 \times 100 \text{ in.}}{3.51 \text{ in.} \times \pi}\sqrt{\frac{36 \text{ ksi}}{29000 \text{ ksi}}} = 0.3195 \qquad (61)$$

$$F_{cr} = (0.658^{\lambda_c^2})F_y = 34.49 \text{ ksi} \qquad (62)$$

$$P_n = 10.3 \text{ in.}^2 \times 34.49 \text{ ksi} = 355 \text{ kips} \qquad (63)$$

$$\phi P_n = 0.85 \times 355 \text{ kips} = 302 \text{ kips} > 265 \text{ kips} \quad \textbf{o.k.} \qquad (64)$$

The W8×35 is adequate for the applied load. Although there are no lateral loads on this frame, it is worth noting that the second order moment amplification factor using Equation 49 would be:

$$B_2 = \frac{1}{1 - \frac{R_u}{R_n}} = \frac{1}{1 - \frac{3.63}{4.39}} = 5.78 \qquad (65)$$

By comparison, the AISC[1] second order moment amplification factor would be:

$$B_2 = \frac{1}{1 - \frac{265 \text{ kips}}{11.06 \text{ kips / in.} \times 100 \text{ in.}}} = 1.32 \qquad \text{(H1-5) (66)}$$

Where 11.06 kips/in. is the elastic stiffness of the frame; or:

$$B_2 = \frac{1}{1 - \frac{265 \text{ kips}}{908 \text{ kips}}} = 1.41 \qquad \text{(H1-6) (67)}$$

Where 908 kips $= AF_y / \lambda_c^2 = 10.3 \text{ in.}^2 \times 36 \text{ ksi} \div (2 \times 0.3195)^2$.

The amplification factors given in Equations 65, 66, and 67 emphasize the difference in using the three approaches to determine second-order effects.

Case B represents the loading condition: P_1 = 132.5 kips and P_2 = 132.5 kips. The cantilevered column is a W8×21 (A = 6.16 in.^{2}, I_x = 75.3 in.^{4}, ASTM A36 steel.) Checking stability criteria:

$P_u / \phi AF_y$ = 0.703; S_r = 0.675; R_n = 4.42 kips/in.; as calculated for Case A, C = 1.2; R_u = 3.14 kips/in.; ϕR_n = 3.76 kips/in. > R_u **o.k.** Checking column adequacy: λ_c = 0.3213; F_{cr} = 34.48 ksi, ϕP_n = 180 kips > P_u **o.k.**

The column size required using Yura's method[10] is the same as for Case A, i.e., W8×35.

Case C represents the loading condition: P_1 = 0 kips, P_2 = 265 kips. The cantilevered column is a W8×15 (A = 4.44 in.^{2}, I_x = 48.0 in.^{4}, ASTM A36 steel.) Checking stability criteria: P_u / ϕA = 0 ksi; S_r = 1; R_n = 4.18 kips/in.; as calculated for Case A, C = 1.2; R_u = 2.65 kips/in.; ϕR_n = 3.55 kips/in. > R_u **o.k.**

Once again the column size required using Yura's method[10] is the same as for Case A, i.e., W8×35.

EXAMPLE 2

Consider the statically determinate frame shown in Figure 7, the frame consists of six columns supporting a roof and crane. The frame supports factored roof loads of 150 kips at interior columns and 75 kips at exterior columns. For Load Combination 1 the frame supports a factored crane load of 150 kips applied to an individual interior column, in addition to the roof loads. For Load Combination 2 the frame is subject to a 10 kip factored lateral load, a factored crane load of 100 kips applied to an individual interior column, and the previously described roof loads. Verify the adequacy of W8×48 A36 columns, assuming the columns are laterally supported in the weak axis (out of the plane of the frame) and limiting drift to h / 120.

For Load Combination 1, exterior columns without crane load:

$P_u / \phi AF_y$ = 75 kips/(0.85×14.1 in.^{2} × 36 ksi)
= 0.174 < 0.3898

$\therefore S_r = 1, G_b = \infty,$

$G_a = (\Sigma S_r I_c / L_c) / (\Sigma I_g / L_g) = (1 \times 184 / 15) / (2370 / 20) = 0.1035$, $\alpha = 0$, $\beta = (0/3 + (0.1035 + 3)/6 - 1/6)^{-1} = 2.852$, $\mu = 0$, $\gamma = 0.02222$, $C = 1 + (2.852)^2 \times 0.02222 = 1.18$.

For Load Combination 1, interior columns without crane load:

$$P_u / \phi A F_y = 0.348 < 0.3898$$

$\therefore S_r = 1$, $G_b = \infty$, $G_a = 0.0518$, $\alpha = 0$, $\beta = 2.924$, $\mu = 0$, $\gamma = 0.02222$, $C = 1.19$.

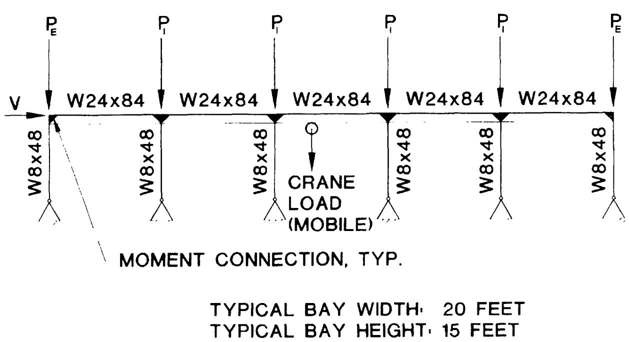
For Load Combination 1, interior column with crane load: $P_u / \phi A F_y = 0.695$, $\lambda_{S_r} = (\log(0.695) / \log(0.658))^{0.5} = 0.932$, $S_r = 0.689$, $G_b = \infty$, $G_a = (0.689 \times 184/15) / (2 \times 2370/20) = 0.0357$, $\alpha = 0$, $\beta = 2.947$, $\mu = 0$, $\gamma = 0.02222$, $C = 1 + (2.947)^2 \times 0.0222 = 1.19$.

For Load Combination 1, the lateral stiffness required for stability is:

$$(R_u)_{LC1} = 1.14 \times \frac{(1.18 \times 150\text{ kips}) + (1.19 \times 750\text{ kips})}{180\text{ in.}}$$

$$= 6.77\text{ kips / in.} \qquad (68)$$

For Load Combination 2, the only change in C factor is due to the lower crane load. For Load Combination 2, interior column with crane load: $P_u / \phi A F_y = 0.579$, $\lambda_{S_r} = 1.143$, $S_r = 0.862$, $G_b = \infty$, $G_a = 0.0446$, $\alpha = 0$, $\beta = 2.935$, $\mu = 0$, $\gamma = 0.02222$, $C = 1.19$.



LOADING CONDITIONS·	
LOAD COMBINATION 1	LOAD COMBINATION 2
P_E - 75 KIPS P_I -150 KIPS V - 0 KIPS CRANE - 150 KIPS	P_E - 75 KIPS P_I -150 KIPS V - 10 KIPS CRANE - 100 KIPS

Fig. 7. Example 2.

Determine the required frame stiffness based on the drift limitation criteria for Load Combination 2:

$$(R_u)_{LC2} = 1.14 \times \frac{(1.18 \times 150\text{ kips}) + (1.19 \times 700\text{ kips})}{180\text{ in.}}$$

$$= 6.40\text{ kips / in.} \qquad (69)$$

$$(\Delta_2)_{max} = \frac{h}{120} = \frac{180\text{ in.}}{120} = 1.5\text{ in.}$$

$$= \frac{B_2 V}{(R_n)_{min}} = \frac{V}{(R_n)_{min} - R_u} \qquad (70a)$$

$$(\Delta_2)_{max} = 1.5\text{ in.} = \frac{10\text{ kips}}{(R_n)_{min} - 6.40\text{ kips / in.}} \qquad (70b)$$

$$(R_n)_{min} = \frac{10\text{ kips}}{1.5\text{ in.}} + 6.40\text{ kips / in.}$$

$$= 13.07\text{ kips / in.} \qquad (71)$$

$$R_n \approx \sum \frac{S_r \beta E I}{h^3} \qquad (72a)$$

$$(R_n)_{LC2} = \frac{((2 \times 2.852) + (3 \times 2.924) + (0.862 \times 2.935)) \times 29000\text{ ksi} \times 184\text{ in.}^4}{(180\text{ in.})^3} \qquad (72b)$$

$$(R_n)_{LC2} = 15.56\text{ kips / in.} \qquad (72c)$$

$$(\Delta_2)_{LC2} = \frac{10\text{ kips}}{15.56\text{ kips/ in.} - 6.40\text{ kips/ in.}}$$

$$= 1.092\text{ in.} < 1.5\text{ in.} \quad \textbf{o.k.} \qquad (73)$$

Similarly, for Load Combination 1:

$$(R_n)_{LC1} = 15.10\text{ kips / in.} \qquad (74)$$

$$(\phi R_n)_{LC1} = 0.85 \times 15.1 = 12.84\text{ kips / in.} > R_u \quad \textbf{o.k.} \qquad (75)$$

Story drift is in an acceptable range for Load Combination 2, and stability is verified for Load Combination 1. Columns are then checked using an effective length of one. For Load Combination 1, evaluate the worst case column:

$$\lambda_c = \left(\frac{Kh}{r\pi}\right)\sqrt{\frac{F_y}{E}}$$

$$= \frac{1.0 \times 180\text{ in.}}{3.61\text{ inches} \times \pi}\sqrt{\frac{36\text{ ksi}}{29000\text{ ksi}}} = 0.5592 \qquad (76)$$

$$F_{cr} = (0.658^{\lambda_c^2})F_y = 31.58\text{ ksi} \qquad (77)$$

$$P_n = 14.1\text{ in.}^2 \times 31.58\text{ ksi} = 445\text{ kips} \qquad (78)$$

$$\phi P_n = 0.85 \times 445 \text{ kips} = 378 > 300 \text{ kips} \quad \textbf{o.k.} \qquad (79)$$

The column is adequate for Load Combination 1. Check columns for Load Combination 2:

$$M_{lt} = \frac{S_r\beta(1-\alpha)Vh}{\Sigma S_r\beta} = \frac{0.862 \times 2.935 \times 10 \text{ kips} \times 180 \text{ in.}}{2 \times 2.852 + 3 \times 2.924 + 0.862 \times 2.935}$$

$$= 268 \text{ in.-kips} \qquad (80)$$

$$B_2 = \frac{1}{1 - \frac{(R_u)_{LC2}}{(R_n)_{LC2}}} = \frac{1}{1 - \frac{6.40}{15.56}} = 1.699 \qquad (81)$$

$$M_u = 1.699 \times 268 \text{ in.-kips} = 455 \text{ in.-kips} \qquad (82)$$

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_u}{\phi M_n}\right)$$

$$= \frac{250 \text{ kips}}{378 \text{ kips}} + \frac{8}{9}\left(\frac{455 \text{ in.–kips}}{1433 \text{ in.–kips}}\right) = 0.944 < 1.0 \quad \textbf{o.k.} \qquad (83)$$

The W8×48 columns are adequate for both load combinations. An approximate method of determining lateral stiffness has been used in this example, however, any method of determining frame stiffness which accounts for inelastic behavior would be acceptable.

CONCLUSION

The frame stiffness approach is an accurate method of verifying the stability of frames, braced or unbraced. The approach is simple and linear where columns are elastic or where bracing provides lateral stiffness. Conditions where inelasticity complicates the analysis can be handled with somewhat more effort. The analysis can be easily accomplished by hand calculations or by computer analysis.

The stability analysis accurately accounts for the variability of column stiffnesses and of load distribution. The method provides an improved estimation of second-order effects and allows for a more accurate assessment of column capacities without resorting to a second-order inelastic analysis.

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